Wesley Andersen
1922 U of U 1925
169 South 13 E.
Salt Lake City, Utah.
From
Brigham City, Utah.

Digitized by the Internet Archive
in 2025

https://archive.org/details/bwb_Y0-CIN-128

WORKS OF
THE LATE PROF. J. B. JOHNSON

PUBLISHED BY

JOHN WILEY & SONS, Inc.

BY JOHNSON, WITHEY AND ASTON

The Materials of Construction.

A treatise for Engineers on the strength and other properties of Engineering Materials. All the Data newly compiled from the latest home and foreign tests. Fifth Edition, Rewritten by M. O. Withey, Professor of Mechanics in the University of Wisconsin, and James Aston, Metallurgist with the A. M. Byers Co., of Pittsburg. Edited by F. E. Turneaure, Dean of the College of Engineering of the University of Wisconsin. xix + 840 pages, 6×9, Illustrated. Cloth, $6.00 *net.*

BY JOHNSON, BRYAN, AND TURNEAURE

Theory and Practice in the Designing of Modern Framed Structures.

PART I. SIMPLE STRUCTURES. xii+328 pages, 6×9, cloth, $3.50 *net.*

PART II. STATICALLY INDETERMINATE STRUCTURES AND SECONDARY STRESSES. xvi+538 pages, 6×9, cloth, $5.00 *net.*

PART III. DESIGN. xii+486 pages, 6×9, cloth, $5.00 *net.*

BY JOHNSON AND SMITH

The Theory and Practice of Surveying.

Designed for the use of Surveyors and Engineers generally, but especially for the use of Students in Engineering. Seventeenth Edition, Rewritten and Enlarged, by Leonard S. Smith, C.E. xxxii+921 pages, 5¼×8, 263 figures, 7 plates, cloth, $4.00 *net.*

Published by McGraw-Hill Book Co.

Engineering Contracts and Specifications.

Including a brief synopsis of the Law of Contracts, and illustrative examples of the General and Technical Clauses of various kinds of Engineering Specifications. Designed for the use of Students, Engineers and Contractors. Price $3.00, *net.*

THE THEORY AND PRACTICE
OF
SURVEYING

DESIGNED FOR THE USE OF

SURVEYORS AND ENGINEERS GENERALLY

BUT ESPECIALLY FOR THE USE OF

STUDENTS IN ENGINEERING

By J. B. JOHNSON, C.E.

LATE DEAN OF THE COLLEGE OF MECHANICS AND ENGINEERING OF THE UNIV. OF WISCONSIN

SEVENTEENTH EDITION, REWRITTEN

By LEONARD S. SMITH, C.E.

PROFESSOR OF CITY PLANNING AND HIGHWAY ENGINEERING, UNIVERSITY OF WISCONSIN.
M. AM. SOC. C. E., M. AM. INST. C. P.

TOTAL ISSUE, FIFTY-NINE THOUSAND

NEW YORK

JOHN WILEY & SONS, Inc.

London: CHAPMAN & HALL: Limited

GEORGE E. MAYCOCK

Copyright, 1886, 1900,
by
J. B. JOHNSON

Copyright, 1910,
by
Mrs. J. B. JOHNSON and LEONARD S. SMITH

Copyright of 1886 Renewed, 1914,
by
PHOEBE E. JOHNSON

3/22

PRESS OF
BRAUNWORTH & CO.
BOOK MANUFACTURERS
BROOKLYN, N. Y.

3531-1

PREFACE TO THE SEVENTEENTH EDITION

THE reviser has long shared the opinion that the engineering profession needed better, rather than more, surveying texts and manuals. Because of the many and rapid improvements in apparatus as well as in field methods, frequent revisions are a prime necessity. Dean Johnson recognized this fact in his preface to the 15th edition of Johnson's Surveying by the following statement: "The author's desire and purpose to keep this work fully abreast of the American practice is his only excuse for the numerous additions and changes which have been made in the various successive editions."

The reviser has taken advantage of a needed resetting of the book to thoroughly revise, and in large part rewrite the entire book. The changes may be grouped as follows: 1. The rearrangement of the subject matter. 2. The omission of certain portions, notably the chapter on Railroad Topography. 3. The use of over 100 new and additional illustrations of apparatus and methods of its use. 4. The addition of over 100 pages to the book as a result of the rewriting of the chapters on the Engineer's Level and Transit, Land, Topographic, Hydrographic, City, and Geodetic Surveying. 5. The supplying at the end of each chapter of many leading references to the best literature on the subject.

The following is a brief statement of the extent of these changes:

The determination of the meridian by the solar compass has been omitted as obsolete, while the discussion of the solar attachment, refraction, etc., is transferred from Chapter II to Chapter

IV, where the subjects are discussed in connection with the determination of the meridian by the "direct solar observation." The addition of the many figures explaining the adjustments of the various instruments should be a welcome addition to those teachers who believe in teaching such matters as problems of Descriptive Geometry, rather than as mechanical rules to be learned. In Chapter III, illustrations and descriptions of the "Dumpy" level have been added. This instrument is becoming quite popular, but not more so than its merits justify.

In connection with the discussion of surveying instruments, the new Appendix F, on the specifications for engineers' transits and levels prepared by the writer should be of value.

The new discussions of "adverse possession of land" and its important relation to the land surveying Arts. 150–151, as well as the lawful methods of rerunning old traverses, Art. 202, are not found in any other text with which the writer is familiar. It would be difficult to find two more important subjects connected with the resurvey of land. The forty principles and court decisions bearing on the resurvey of private lands have been carefully revised and extended to conform with the later decisions of the courts.

The descriptions of the government methods of subdividing the public lands have been entirely rewritten and illustrated with new figures.

The changes in the chapter on Topography include a simpler proof of the stadia formula, improved form of keeping field notes, new data on the accuracy of stadia measurements, method of constructing stadia reduction charts, methods of contouring, illustrated by several examples.

The chapter on Railroad Topography has been omitted, because the subject is usually taught from some of the many excellent field manuals on this subject. The space thus gained has been used to better advantage by a new chapter on Stream Measurement. This chapter of hydrographic methods describes present practice as completely as the needs of the civil engineer

seem to justify. The thirty new illustrations used in this chapter add much to its value.

A new article on the surveying of bore-holes has been added to the Mining chapter, together with eight problems on mining surveying and a few new illustrations.

The chapter on City Surveying has been rewritten and expanded to about double the size of the old chapter. Special mention may be made of the practical treatment of city, land, and topographic surveys, city planning, street design, and the laying out of circular streets.

The old chapter on Geodetic Surveying has been subdivided and rewritten in five chapters, viz., Triangulation and Base Line, Field Astronomy, Trigonometric Levelling, Precise Spirit Levelling, and Map Projection.

Geodetic field and office methods have undergone extensive and important changes in the past ten years, due largely to the adoption by the Coast Survey of standard specifications.[1]

A few of the most important new features are the following: A discussion of the figure of the earth, strength of triangulation figures, base-line apparatus, the use of nickel steel tapes, directions for adjusting and using precise levels, both wye and dumpy, with sample field notes, limits of error, speed and cost of precise levelling.

Most noteworthy is the new chapter on Field Astronomy, prepared by Professor George C. Comstock, University of Wisconsin. This chapter is largely an abridgment of his well-known text on Astronomy, and presents not only his ingenious approximate methods, designed for plane surveying, but also the precise methods for determining latitude, azimuth, and longitude.

The chapter on Map Projection and Map Lettering has been rewritten and greatly improved by the use of new figures and plates.

The reviser wishes to acknowledge his indebtedness to members

[1] These specifications were drawn by Mr. John F. Hayford.

of the teaching and engineering professions who have assisted in preparing the revision, but especially to Professor George C. Comstock of the University of Wisconsin, for his chapter on Field Astronomy, and to Professor John F. Hayford of Northwestern University, for valuable assistance in connection with the chapters on Triangulation and Precise Spirit Levelling. His thanks are also due to Mr. George L. Teeple of Whitewater, Wis., Mr. Horace Andrews of Albany, N. Y., Professor David A. Molitor of Cornell University, Professor Charles A. Holden of Thayer School of Civil Engineering for many valuable suggestions, and to Professor J. C. Tracy of Yale University for permission to use Chapter II on Errors from his "Plane Surveying" as Appendix D. Other acknowledgments are made in the text.

In a revision of this character there is great danger of errors, especially in page and article references. The reviser will consider it a favor if his attention is called to any errors in this edition.

L. S. S.

Madison, Wis., September, 1910.

PREFACE TO THE FIRST EDITION

No apology is necessary for the appearance of a new book on Surveying. The needs of surveyors have long been far beyond the accessible literature on this subject, to say nothing of that which has heretofore been formulated in text-books. The author's object has been to supply this want so far as he was able to do it.

The subject of surveying, both in the books and in the schools, has been too largely confined to Land Surveying. The engineering graduates of our technical schools are probably called upon to do more in any one of the departments of Railroad, City, Topographical, Hydrographical, Mining, or Geodetic Surveying than in that of Land Surveying. Some of these subjects, as for example, City, Geodetic, and Hydrographical Surveying, have not been formulated hitherto, in any adequate sense, in either English or any other language, to the author's knowledge. In the case of Geodetic Surveying there has been a wide hiatus between the matter given in text-books and the treatment of the subject in works on Geodesy and in special reports of geodetic operations. The latter was too technical, prolix, and difficult to give to students, while the former was entirely inadequate to any reasonable preparation for this kind of work on even a small scale. The subjects of City and Hydrographical Surveying as here presented are absolutely new.

Part I treats of the adjustment, use, and care of all kinds of instruments sued in surveying, either in field or office.[1] In describing the adjustments of instruments the object has been to present to the mind of the reader the geometrical relations from which a rule or method of adjustment would naturally follow. The author has no sympathy with descriptions of adjustments as mechanical processes simply to be committed to memory, any more than he has with that method of teaching geometry wherein the student is allowed to memorize the demonstration.

Many surveying instruments not usually described in books on surveying are fully treated, such as planimeters, pantographs, barometers, protractors, etc. The several sets of problems given to be worked out by the aid of the corresponding instruments are designed to teach the capacity and limitations of such instruments, as well as the more important sources of error in their use. This work is such as can be performed about the college campus, or in the near vicinity, and is supposed to be assigned for afternoon or Saturday practice while the subject is under consideration by the class. More extended surveys require a special field-season for their successful prosecution.[2]

The methods of the differential and integral calculus have been sparingly used, as in the derivation of the barometric formula for elevations, and of the *L M Z* formulæ in Appendix D. Such demonstrations may have to be postponed to a later period of the course.

Part II includes descriptions of the theory and practice of Surveying Methods in the several departments of Land, Topo-

[1] Certain special appliances, as for example, helitropes, filar micrometers, current metres, etc., are treated in the subsequent chapters.

[2] At Washington University all the engineering Sophomores go into the field for four weeks at the end of the college year, and make a general land and topographical survey, such as shown in Plate II. At the end of the Junior year the civil-engineering students go again for four weeks, making then a geodetic and railroad survey. Some distant region is selected where the ground, boarding facilities, etc., are suitable.

graphical, Railroad, Hydrographical, Mining, City, and Geodetic Surveying; Surveys for the Measurement of Volumes; and the Projection of Maps, Map Lettering, and Topographical Signs. The author has tried to treat these subjects in a concise, scientific, and practical way, giving only the latest and most approved methods, and omitting all problems whose only claim for attention is that of geometrical interest.

In treating the trite subject of Land Surveying many problems which are more curious than useful have been omitted, and several new features introduced. The subjects of computing areas from the rectangular co-ordinates, and the supplying of missing data, are made problems in analytical geometry, as they should be. A logarithmic Traverse Table for every minute arc of from zero to 90°, arranged for all azimuths from zero to 360°, to be used in connection with a four-place logarithmic table, serves to compute the co-ordinates of lines when the transit is the instrument used. A traverse table computed for every 15 minutes of arc is no longer of much value. The isogonic declination-curves shown in Plate I will be found to embody all the accessible data up to 1885, and are reduced from the U. S. Coast Survey chart. Appendix A will be found of great value as outlining the Judicial Functions of the Surveyor by the best possible authority.

The Chapter on Mining Surveying was written by Mr. C. A. Russell, C.E., U. S. Deputy Mineral Surveyor of Boulder, Colorado.[1] He has had an extended experience in Hydrographic Surveying, in addition to many years' practice in surveying mines and mining claims.

The chapter on City Surveying was written by Mr. Wm. Bouton, C.E., City Surveyor of St. Louis, Mo. Mr. Bouton has done a large proportion of the city surveying in St. Louis

[1] This chapter has now been replaced by another by Professor R. S. Stockton, of the Colorado School of Mines.

for the last twenty years, and has gained an enviable reputation as a reliable, scientific, and expert surveyor.

It is believed that the ripe experience of these gentlemen which has been embodied in their respective chapters will materially enhance the value of the book.

The author also desires to acknowledge his indebtedness to his friend H. S. Pritchett, Professor of Astronomy in Washington University,[1] for valuable assistance in the preparation of the matter on Time in Chapter XIV.

Although the theorems and the notation of the method of least squares are not used in this work, yet two problems are solved by what is called the *method of the arithmetic mean* (which, when properly defined, is the same as the method of least squares), which will serve as a good introduction to the study of the method of least squares. These problems are the Rating of a Current-metre, in Chapter X, and the Adjustment of a Quadrilateral, in Chapter XIV. The author has found that such solutions as these serve to make clear to the mind of the student exactly what is accomplished by the least-square methods of adjusting observations.

The chapter of Measurement of Volumes is not intended to be an exhaustive treatment of the subject of earthwork, but certain fundamental theorems and relations are established which will enable the student to treat rationally all ordinary problems. The particular relation between the Henck prismoid and the warped-surface prismoid is an important one, but one which the author had nowhere found.[2]

An earthwork table (Table XI) has also been prepared which gives volumes directly, without correction, for the warped-surface prismoid. The author has no knowledge that such a table has ever been prepared before.

[1] Since made Superintendent of the U. S. Coast and Geodetic Survey and now (1900) President of the Massachusetts Institute of Technology.

[2] The author's attention has since been called to the fact that this relation was published in Henck's "Field Book" in 1881.

A former work by the author on Topographical Surveying by the Transit and Stadia is substantially included in this book.

The methods recommended for measuring base-lines with steel-tapes are new; but they have been thoroughly tested, and are likely to work a material change in geodetic methods.

The author wishes to acknowledge his obligations to many instrument-manufacturers for the privilege they have very kindly accorded to him of having electrotype copies made from the original plates, for many of the cuts of instruments given throughout the book; persons familiar with the valuable catalogues published by these firms will recognize the makers among the following: W. & L. E. Gurley, Troy, N. Y.; Buff & Berger, Boston, Mass.; Fauth & Co., Washington, D. C.; Queen & Co. and Young & Sons, Philadelphia, Pa.; Keuffel & Esser, New York; and F. E. Brandis Sons & Co., Brooklyn, N. Y. Also to Mr. W. H. Searles for the privilege of using copies of plates from his Field-book for Tables I, VI, and VII.

Hoping this work will assist in lifting the business of surveying to a higher professional plane, as well as to enlarge its boundaries, the author submits it to surveyors and engineers generally, but especially to the instructors and students in our engineering schools, for such crucial tests as the field and the class-room only can give.

J. B. J.

St. Louis, Sept. 23, 1886.

TABLE OF CONTENTS

PAGE

INTRODUCTION 1

BOOK I

SURVEYING INSTRUMENTS

CHAPTER I

INSTRUMENTS FOR MEASURING DISTANCES

THE CHAIN:

1. The Engineer's Chain 7
2. Gunter's Chain 7
3. Testing the Chain 8
4. The Use of the Chain 10

THE STEEL TAPE:

5. Varieties 12
6. The Use of Steel Tapes 13
7. Other Instruments 14
8. Sources of Error 14
9. Common Mistakes 14

EXERCISES WITH THE CHAIN AND TAPE:

Practical Problems 15, 16

CHAPTER II

INSTRUMENTS FOR DETERMINING DIRECTIONS

THE COMPASS:

10. The Surveyor's Compass Described 17
11. Methods of Reading a Magnetic Bearing 18
12. The General Principle of Reversion 20

PAGE

13. To Make the Plate Perpendicular to the Axis of the Socket 20
14. To Make the Plane of the Bubbles Perpendicular to the Axis of the Socket 20
15. To Detect Errors in the Adjustment of the Needle 21
16. To Straighten the Needle 21
17. To Centre the Pivot Point 22
18. To Remagnetize the Needle 22
19. To Make the Plane of the Sights Normal to the Plane of the Bubbles. 22
20. The Earth's Magnetism 23

THE DECLINATION OF THE NEEDLE:

21. The Declination Defined 23
22. The Daily Variation 23
23. The Secular Variation 23
24. Isogonic Lines 24
25. Other Variations of the Declination 26
26. Observation for Declination 27

USE OF THE NEEDLE COMPASS:

27. The Use of the Compass 27
28. To Set Off the Declination 28
29. Local Attractions 29
30. To Establish a Line of a Given Bearing 30
31. To Find the True Bearing of a Line 30
32. To Retrace an Old Line 30

THE PRISMATIC COMPASS:

33. The Prismatic Compass Described 31

EXERCISES:

Exercises for the Compass or Compass and Chain 31, 32

CHAPTER III

INSTRUMENTS FOR DETERMINING HORIZONTAL LINES

PLUMB LINE AND BUBBLE:

34. Their Universal Use in Surveying and Astronomical Work 33
35. Level Surface Defined 33
36. Proposition I 34
37. Proposition II 34
38. The Accurate Measurement of Small Vertical Angles 35
39. The Angular Value of One Division of the Bubble 36
40. General Considerations 37

THE ENGINEER'S WYE LEVEL:

41. The Level Described 38

PAGE

ADJUSTMENT OF THE WYE LEVEL:
42. Adjustment of the Collimation Cross-Wires ... 39
43. Lateral Adjustment of Bubble ... 42
44. Adjustment of Level Axis Parallel to Line of Sight ... 42
45. The Direct or Peg Adjustment ... 43
46. The Wye Adjustment ... 44
47. Relative Importance of Adjustments ... 44
THE DUMPY LEVEL:
48. General Statements ... 45
ADJUSTMENTS OF DUMPY LEVEL:
49. The Cross-Wire Adjustment ... 45
50. The Level-Vial Adjustment ... 45
51. The Line of Sight Adjustment ... 47
52. Focussing and Parallax ... 48
53. The Levelling-Rod ... 49
THE USE OF THE LEVEL:
54. The Level is Used ... 52
DIFFERENTIAL LEVELLING:
55. Differential Levelling Defined ... 53
56. Length of Sights ... 54
57. Bench-Marks ... 55
58. The Record ... 56
59. Precautions in Levelling Work ... 57
PROFILE LEVELLING:
60. Profile Levelling Defined ... 58
61. The Record ... 59
LEVELLING FOR FIXING A GRADE:
62. The Work Described ... 61
THE HAND LEVEL:
63. Locke's Hand Level ... 62
EXERCISES WITH THE LEVEL:
Practical Problems ... 62

CHAPTER IV

INSTRUMENTS FOR MEASURING ANGLES

THE TRANSIT:
64. The Engineer's Transit Described ... 64
THE TELESCOPE:
65. Description ... 67
66. Objective ... 67

PAGE

67. The Eye Piece ... 68
68. Magnifying Power ... 69
69. The Vernier ... 69
70. Transit Verniers ... 71

ADJUSTMENTS OF TRANSIT:

71. General Description ... 74
72. Plate-Bubble Adjustment ... 75
73. Cross-Wire Adjustment ... 75
74. Collimation Adjustment ... 76
75. The Standard Adjustment ... 77
76. Bubble Adjustment ... 78
77. Vertical Circle Adjustment ... 79
78. Object Slide Adjustment ... 79
79. Relative Importance of Adjusts ... 79

INSTRUMENTAL CONDITIONS AFFECTING ACCURATE MEASUREMENTS:

80. Eccentricity of Centres and Verniers ... 80
81. Inclination of Vertical Axis ... 81
82. Inclination of Horizontal Axis ... 82
83. Error in Collimation Adjustment ... 83

THE USE OF THE TRANSIT:

84. Setting up the Transit ... 83
85. To Measure a Horizontal Angle ... 84
86. To Measure an Angle by Repetition ... 84
87. To Lay off a Given Angle by Repetition ... 85
88. To Run a Straight Line ... 85

TERMINALS NOT VISIBLE:

89. To Prolong a Straight Line ... 86
90. To Measure a Vertical Angle ... 87
91. To Run a Traverse ... 87
92. Targets Used in Lining ... 89
93. Field Signals ... 89

THE GRADIENTER ATTACHMENT:

94. The Gradienter Described ... 90

CARE OF THE TRANSIT:

95. How Cared for ... 90

THE SOLAR ATTACHMENT:

95. Description of Saegmuller Attachment ... 91
97. The Celestial Sphere ... 92
98. Observations on the Sun with Solar Attachment ... 95
99. To Lay off the Co-Latitude ... 96
100. Adjustments of the Saegmuller Attachment ... 97

PAGE
101. Determination of the Meridian by Direct Solar Observation 98
102. To Find the Declination of the Sun 100
103. To Correct Declination for Refraction 101
104. Example of Computation of Refraction 102
105. Field Method of Computing Refraction 106
Exercises with the Transit 109
THE SEXTANT:
106. The Sextant Described 111
107. The Theory of the Sextant 113
108. The Ajustment of the Index Glass 114
109. The Adjustment of the Horizon Glass 114
110. The Adjustment of the Telescope to the Plane of the Sextant 115
111. The Use of the Sextant 115
EXERCISES WITH THE SEXTANT:
Practical Problems 116
112. The Cross-Section Polar Protractor 116

CHAPTER V

THE PLANE TABLE

113. The Plane Table Described 119
114. Adjustment of the Plate Bubbles 121
115. Adjustment of Horizontal Axis 121
116. Adjustment of Vernier and Bubble to Telescopic Line of Sight 121
THE USE OF THE PLANE TABLE:
117. General Description of its Use 122
118. Location by Resection 125
119. Resection on Three Known Points 125
120. Resection on Two Known Points 126
121. The Measurement of the Distances by Stadia 128
EXERCISES WITH THE PLANE TABLE:
Practical Problems 128

CHAPTER VI

ADDITIONAL INSTRUMENTS USED IN SURVEYING AND PLOTTING

THE ANEROID BAROMETER:
122. The Aneroid Described 129
123. Derivation and Use of Barometric Formulæ 130
124. Use of the Aneroid 138

PAGE

THE PEDOMETER:
125. The Pedometer Described 140
The Length of Men's Steps 141
THE ODOMETER:
126. Description and Use 143
THE CLINOMETER:
127. Description and Use 144
THE OPTICAL SQUARE:
128. Description and Use 145
THE PLANIMETER:
129. Description 145
130. Theory of the Polar Planimeter 147
131. To Find the Length of Arm to Use 152
132. The Suspended Planimeter 155
133. The Rolling Planimeter 155
134. Theory of the Rolling Planimeter 156
135. To Test the Accuracy of a Planimeter 160
136. The Use of the Planimeter 160
137. Accuracy of Planimeter Measurements 163
THE PANTOGRAPH:
138. Description and Theory 164
Various Styles of Pantographs 166
139. Use of the Pantograph 167
PROTRACTORS:
140. Various Styles Described 168
PARALLEL RULERS:
141. Description and Use 171
SCALES:
142. Various Kinds Described 172
143. The Pocket Slide Rule 173

BOOK II

SURVEYING METHODS

CHAPTER VII

Land Surveying

PAGE

144. Land Surveying Defined 180
145. Laying out Land 180
146. Land Monuments 181
147. Significance and Authority of Monuments 182
148. Lost Monuments 183
149. Rerunning Old Survey from a Deed 184
150. Significance of Adverse Possession 185
151. Kinds of Adverse Possession 186

Principles and Laws Bearing on the Resurvey of Private Lands:

152. The Problem Stated 187
153. The Interpretation of Descriptions in Deeds and the Identification of Boundaries 189
154. Water Boundaries and Meandered Lines 192
155. Treatment of Surplus and Deficiency 193

The U. S. System of Laying Out the Public Lands:

156. General Description 194
157. The Details of the System 195
158. Running Out Standard Parallels 198
159. The Secant Method 198
160. The Tangent Method 201
161. Initial Points 204
162. Base Line 204
163. The Principal Meridian 204
164. Standard Parallels 205
165. Guide Meridians 205
166. Township Exteriors 205
167. Subdivision of Townships 207
168. The Convergence of the Meridians 210
169. Requirements of the Work 211
170. Fractional Sections 212
171. Other Systems Used 212
172. Marking Corners 214

PAGE

173. Witnessing Corners 214
174. Witness Corners 215
175. Meander Corners 215
176. Meandering 215
177. Objects and Data to be Noted 215
178. Original Notes, where Kept 217
179. Subdivision of Sections 218
180. Relocating Lost Corners 220

FINDING THE AREA OR SUPERFICIAL CONTENTS OF LAND WHEN THE LIMITING BOUNDARIES ARE GIVEN:

181. The Area Defined 223

By Triangular Subdivision:

182. By the Use of the Chain Alone 223
183. By the Use of the Compass or Transit and Chain 224
184. By the Use of the Transit and Stadia 225

From Bearing and Length of the Boundary Lines:

185. The Common Method 225
186. The Field Notes 226
187. Method of Computation Stated 229
188. Latitudes, Departures, and Meridian Distances Defined 229
189. Computation of Latitudes and Departures 231
190. Example of Such a Computation 232
191. Balancing the Survey 233
192. The Compass Rule for Balancing 235
193. The Transit Rule for Balancing 236
194. Comparison of Compass and Transit Rule 236
195. The Error of Closure 237
196. The Form of Reduction 238
197. Correction from Erroneous Length of Chain 242

AREA FROM THE RECTANGULAR CO-ORDINATES OF THE CORNERS:

198. Conditions of Application of the Method 244
199. Theory of the Method 245
200. The Form of Reduction 247

SUPPLYING MISSING OR ERRONEOUS DATA:

201. Equations for Supplying Missing Data—Four Cases 247
202. To Locate an Old Traverse 251

PLOTTING:

203. Plotting the Survey 256

IRREGULAR AREAS:

204. The Method by Offsets at Irregular Intervals 256
205. The Method by Offsets at Regular Intervals 258

PAGE

THE SUBDIVISION OF LAND:

206. The Problems of Infinite Variety 260
207. To Cut from a Given Tract of Land a Given Area by a Right Line Starting from a Given Point in the Boundary 261
208. To Cut a from Given Tract of Land a Given Area by a Right Line Running in a Given Direction 263
Examples in Land Surveying 267
References on Land Surveying 269

CHAPTER VIII

TOPOGRAPHICAL SURVEYING BY THE TRANSIT AND STADIA

209. Topographical Survey Defined 271
210. Available Methods 271
211. Method by Transit and Stadia Stated 272

THEORY OF STADIA MEASUREMENTS:

212. Fundamental Relations 273
213. The Determination of Stadia Interval 274
214. To Set Adjustable Wires 274
215. The Use of the Interval Factor 274
216. A Simple Determination of the Wire Interval 278
217. The Prevention of Systematic Errors of Stadia Measurements 279
218. Adaptation of Formulæ to Inclined Sights 282
219. Description of Table V 283

THE INSTRUMENTS:

220. The Transit 284
221. Setting the Cross Wires 285
222. Graduating the Stadia Rod 286
223. Disadvantages of Painting Rods to Fit the Interval 288

GENERAL TOPOGRAPHICAL SURVEYING:

224. Topography 289
225. Methods of Field Work 289
226. Check Readings 294
227. Azimuth Checks 296
228. Reducing the Field Notes 296
229. Stadia Reduction Diagrams 297
230. Plotting the Stadia Traverse 301
231. Plotting the Side Readings 303
232. The Contour Lines 307
233. The Final Map 311

PAGE

234. Topographic Symbols 313
235. Accuracy of Stadia Measurements 313
236. Accuracy of Levels Run by Stadia and Transit 316
237. The Cost of Topographic Maps 320
References on Topographic Surveys 321

CHAPTER IX

Hydrographic Surveying

238. Hydrographic Surveying Defined 322
The Location of Soundings:
239. Enumeration of Methods 323
240. By Two Angles Read on Shore 324
241. By Two Angles Read in the Boat—The Three-point Problem 324
242. By One Range and One Angle 327
243. Buoys, Buoy Flags, and Range Poles 328
244. By One Range and Time Intervals 328
245. By Means of Intersecting Ranges 329
246. By Means of Cords or Wires 329
Making the Soundings:
247. The Lead 329
248. The Line 330
249. Sounding Poles 331
250. Making Soundings in Running Water 331
251. The Water-surface Plane of Reference 332
252. Lines of Equal Depth 333
253. Soundings on Fixed Cross-Sections in Rivers 333
254. Area of Cross-Section 334
Bench-marks, Gauges, Water Levels, and Water Slope:
255. Bench-marks 334
256. Water Gauges 335
257. Chain Gauges 336
258. Self-registering Gauge 337
259. Water Levels 339
260. River Slope 341
261. Transit and Stadia Locations 341
262. Sounding by "Sweeping" 342

CHAPTER X

Stream Measurement

PAGE

The Discharge of Streams

263. General Statement of the Problem 344
264. Factors of Accuracy of Measurement 345
265. Methods of Measuring Mean Velocity 346

Current Meter

266. Current Meter Methods 346
267. Rating the Current Meter 347
268. The Graphical Solution 352
269. The Analytical Solution 352
270. Variations of Velocity in a Cross-Section 354
271. Velocity in the Vertical Plane—Open Conditions 355
272. Velocity in the Vertical Plane—Frozen Conditions 357
273. Velocity from Bank to Bank 357

Rod Floats:

274. The Use of Floats to Determine Velocity 360
275. The Subsurface Float 360
276. Rod or Tube Floats 362
277. Compilation of Discharge—Rod Float Method 363
278. Advantages of Rod Floats 364
279. Discharge Measurements by Current Meter 364
280. Computations of Current Meter Measurements 368
281. Current Meter Measurements of Large Rivers 369

Slope Method of Measuring Stream Velocity:

282. Method Described 369
283. General Conclusions 372
284. Application of Stream Measurement 373

Weir Method of Measuring Stream Flow:

285. General Statement 376
286. Sharp-crested Weirs 377
287. Submerged Weirs 380
288. Weirs of Varying Section 381
289. Depth of Water on the Weir 384
290. The Use of Dams as Weirs 386
291. The Miners' Inch 388

I. References on General Hydraulics 398
II. References on Stream Gauging 389
III. References on Weir Measurements 389

CHAPTER XI

Mining Surveying

PAGE

Surveying Mining Claims:

292. Title to Mining Claims 391
293. Conditions Governing the Location of Claims 392
294. Location Surveys 393
295. Surveying Lode Claims 393
296. Patent Surveying 397
297. Instrumental Work 398
298. Stations and Methods of Measuring 399
299. Field Work and Adjustment of Claim 400
300. Conditions Controlling the Patent Survey 401
301. The Field Notes 402
302. Preparing the Plat 406
303. Placer Claims 408
304. Mill Sites 404
305. Amended Surveys 409
306. Adverse Surveys 409

Underground Surveys:

307. Underground Surveying 410
308. Instruments 411
309. Use of Top Telescope 411
310. Use of Side Telescope 416
311. Stations 418
312. Carrying the Meridian into the Mine by a Transit 420
313. Use of Plumb Lines 421
314. Where There are Two Shafts 422
315. Where There is One Shaft 423
316. The Wire Method 426
317. Alignment of Wire from the Surface 427
318. Underground Traversing 427
319. Underground Levelling 430
320. Mapping the Survey 431
321. Problems of Underground Surveying 433
322. Making an Underground Connection 434
323. Establishing Property Lines Underground 436
324. To Find the Bearing and Length of a Tunnel to Intersect a Known Vein 437
325. Surface Surveys 438
326. Court Maps 438

PAGE
327. Surveying Bore-holes 439
328. To Determine the Dip and Strike of a Vein 441
Definitions of Mining Terms 441
Exercises in Mining Surveying 442
References on Mining Surveying 444

CHAPTER XII

City Surveying

329. Land Surveying Methods Inadequate 445
330. What a City Survey May Include 445
331. City Engineer's Transit and Level 447
332. The Steel Tape 449
City Land Surveys. Original:
333. The Requirements of Original City Surveys 449
334. Subdivisional Work, Field and Office 451
335. Monuments 452
336. Defective Plats 454
City Land Surveys. Resurveys:
337. Resurveys in Monumented Cities 456
338. Resurveys in Non-monumented Cities 457
339. The Value of an Existing Monument 459
340. The Preservation of Lines 461
341. The Cause of Disagreement between Surveyors 461
342. City Standards 462
City Cadastral and Topographic Surveys:
343. General Description 463
344. Triangulation System 463
345. Steel Tape Traverses 464
346. Precise Level Bench-marks 464
347. The Datum Plane 464
348. City Topographic Methods 465
The City Plan:
349. The Location of the Site 467
350. The Street Plan 467
351. Location of Streets 468
352. Size of City Blocks and Lots 471
353. Width of City Streets 474
354. Street Grades 474
355. Minimum and Maximum Grades 476
356. Vertical Curve at Grade Intersections 476

PAGE

CURVED STREETS:

357. Curved Streets. General Description ... 477
358. Principles of a Circular Curve ... 478
359. Laying Out Circular Curves ... 479
360. Computation of Deflection Angles and Chords ... 482
361. To Set Lot Stakes at Fixed Arc Distances ... 483
362. Form of Field Notes ... 484
363. When the Whole Curve is not Visible from One End ... 475
364. Street Cross-Section ... 486
365. Surveys for Sewers ... 489
366. The Sewer Field Notes ... 489
367. Map and Profiles ... 490
368. Indexing City Plans and Records ... 492

REFERENCES TO LITERATURE IN CITY SURVEYING:

I. The City Layout ... 492
II. City Resurvey ... 493
III. City Planning ... 493
IV. Street Grades and Cross-Sections ... 494
V. Cadastral and City Topographic Survey ... 494

CHAPTER XIII

THE MEASUREMENT OF VOLUMES

369. Proposition Explained ... 495
370. Grading over Extended Surfaces ... 497
371. Approximate Estimates by Means of Contours ... 500
372. The Prismoid ... 504
373. The Prismoidal Formula ... 505
374. Areas of Cross-Section ... 507
375. The Centre and Side Heights ... 507
376. The Area of a Three-level Section ... 508
377. Cross-sectioning ... 508
378. Three-level Sections, the Upper Surface Consisting of Two Warped Surfaces ... 510
379. Construction of Tables for Prismoidal Computation ... 512
380. Three-level Sections, the Surface Divided into Four Planes by Diagonals ... 517
381. Comparison of Volumes by Diagonals and by Warped Surfaces ... 519
382. Preliminary Estimates from the Profiles ... 521
383. Borrow-pits ... 523
384. Shrinkage of Earthwork ... 524
385. Excavations under Water ... 524

BOOK III

GEODETIC SURVEYING

CHAPTER XIV

Triangulation and Base Line

PAGE

386. The Objects of a Geodetic Survey 528
387. The Size and Figure of the Earth 529
388. Minor Irregularities in the Shape 530

Triangulation:

389. Triangulation Figures 531
390. Classification of Triangulation 533
391. Primary Triangulation 534
392. Tertiary Triangulation 534
393. Secondary Traingulation 534
394. Strength of Triangulation Figures 535
395. Base Line Sites 538
396. Connection of Base with Triangulation 538
397. The Reconnoisance 541
398. Instrumental Outfit 542
399. The Direction of Invisible Stations 543
400. The Heights of Stations 543
401. Intervisibility of Stations 545
402. Construction of Stations 548
403. Targets 551
404. Heliotropes 553
405. Geodetic Night Signals 556
406. Station Marks 557

Measurement of Base Lines:

407. Preparation of the Line 559
408. Base Apparatus 559
409. Contact Apparatus 561
410. Optical Apparatus 561
411. Compensating Apparatus 563
412. The Iced-bar Apparatus 563
413. The "Duplex" Bars 563
414. Field Methods with Duplex Bars 564
415. Secondary Base Apparatus 566
416. Steel Tape Apparatus 566

PAGE
417. Methods of Mounting and Stretching Tapes 568
418. Thermometers 568
419. Balances 568
420. M. Jäderin's Method 571
421. Nickel Steel Tapes 573
422. Comparative Errors of Invar Tapes 573
423. Comparison of Costs 574
424. The True Length of the Tape 574
425. Coefficient of Expansion of Steel Tapes 575
426. Modulus of Elasticity 576
427. Effect of Sag 577
428. Temperature Correction 579
429. Temperature Correction when a Metallic Thermometer is Used 579
430. Correction for Alignment 581
431. Correction for Sag 583
432. Correction for Pull 585
433. To Reduce a Broken Base to a Straight Line 585
434. To Reduce Base to Sea Level 585
435. Summary of Corrections 586
436. To Compute any Portion of a Base not Directly Measured 588
437. Accuracy of Steel Tapes 590

MEASUREMENT OF THE ANGLES:

438. Instruments for Measuring Angles 592
439. The Repeating Instrument 592
440. The Direction Instrument 594
441. The Altazimuth Instrument 594
442. Adjustments of the Theodolite 598
443. The Filar Micrometer 599
444. Connection for Run 601
445. Reading the Circle 601
446. Programme with Repeaters 602
447. Programme with Non-repeaters 604
448. Time for Observing 606
449. Reduction to Centre 606

ADJUSTMENT OF THE MEASURED ANGLES:

450. Condition Equations 610
451. Local or Station Adjustment 611
452. Spherical Excess 612
453. Adjustment of a Triangle 613
454. Adjustment of a Quadrilateral General Statement 614
455. The Geometric Conditions 615

PAGE
456. The Angle-equation Adjustment 616
457. The Side-equation Adjustment 617
458. Rigorous Method of Adjustment 621
459. Adjustment on Primary Triangulation 624
460. Computing the Sides of the Triangles 626
461. Computing the L. M. Z 630
462. The Inverse Problem 634
LITERATURE:
I. Figure and Size of the Earth 635
II. Triangulation 636
III. Base Lines 637

CHAPTER XV

FIELD ASTRONOMY

FIELD ASTRONOMY:
463. Fundamental Concepts 638
464. Relation of Siderial to Mean Solar Time 639
465. Longitude and Time 642
466. Conversion of Time 643
467. Finding the Stars 643
468. Tables. Time, Stars 645
469. Tables, Polaris Orientation 648
470. Azimuth, Time, Latitude, Polaris Orientation Method 651
471. Latitude by Circum-meridian Altitudes of the Sun 655
472. Azimuth by Elongation of a Circum-polar Star 656
473. The Two-star Method—Time and Azimuth 660
474. Latitude from Differences of Meridian Altitudes 664
475. Longitude 669
References in Astronomy 672

CHAPTER XVI

TRIGONOMETRICAL LEVELLING

TRIGONOMETRICAL LEVELLING:
476. General Description 673
477. Instruments and Methods 673
478. Refraction 675
479. Refraction. How Measured 676
480. Coefficient of Refraction 677
481. To Find Difference of Elevation 679
482. Levelling by the Sea Horizon 682
References on Trigonometric Levelling 683

CHAPTER XVII

Precise Spirit Levelling

PAGE

Instruments:

483. General Description 684
484. Levelling Instruments 685
485. The Coast Survey Level 688
486. Level Rods 691
487. Rod Supports 691

Adjustments of Precise Levels.

488. Adjustment of the Kern Type 694
489. Adjustment of the Coast Survey Instrument 695
490. Instrumental Constants 696

Field Methods:

491. Field Methods 701
492. General Instructions on the Coast Survey 703
493. Bench-marks 705
494. Limits of Error 705
495. Speed and Cost 706
496. Adjustment of Level Net 707
497. Elevation of Mean Sea Level 708
References on Precise Levelling 708

CHAPTER XVIII

Projection of Maps. Map Lettering, Etc.

Projection of Maps:

498. General Statement 710
499. Cylinder Projection 710
500. Trapezoidal Projection 712
501. Conic Projection 713
502. Mercator's Conic Projection 715
503. Bowne's Projection 716
504. The Polyconic Projection 716
505. Formulæ Used in Map Projection 718
506. Explanation of Table VIII 720
507. Summary 722
508. Angle of Convergence of Meridians 723

Map Lettering:

509. Map Lettering 724
510. Topographic Symbols 725

APPENDIX A

PAGE
THE JUDICIAL FUNCTIONS OF SURVEYORS..... 729

APPENDIX B

INSTRUCTIONS TO U. S. DEPUTY MINERAL SURVEYORS..... 739

APPENDIX C

FINITE DIFFERENCES..... 782

APPENDIX D

ERRORS..... 788

APPENDIX E

GEOGRAPHICAL POSITIONS OF BASE-LINES AND PRINCIPAL MERIDIANS GOVERNING THE PUBLIC SURVEYS..... 797

APPENDIX F

SPECIFICATIONS OF AN ENGINEER'S TRANSIT AND LEVEL..... 800

APPENDIX G

THE OWNERSHIP OF SURVEYS AND WHAT CONSTITUTES A SURVEY..... 807

APPENDIX H

MICHIGAN LAWS FOR MAKING TOWN, CITY AND VILLAGE PLATS. 813

APPENDIX I

RESTORATION OF LOST OR OBLITERATED CORNERS..... 817

TABLES.

PAGE

I. Trigonometrical Formulæ 853
II. For Converting Metres, Feet, and Chains 839
III. Logarithms of Numbers to Four Places 840
IV. Logarithms of Trigonometrical Functions to Four Places 842
V. Stadia Reductions for Horizontal Distance and for Elevation 846
VI. Natural Sines and Cosines 854
VII. Natural Tangents and Cotangents 863
VIII. Co-ordinates for Polyconic Projection 875
IX. Values of Coefficient in Kutter's Formula 876
X. Diameters of Circular Conduits, by Kutter's Formula 877
XI. Earthwork Table—Volumes by the Prisomoidal Formula 878
XII. Length of Circular Arcs Radius 888
XIII. Azimuth of Polaris at Elongation 889

SURVEYING

INTRODUCTION

Surveying is the art of making such field observations and measurements as are necessary to determine positions, areas, volumes, or movements on the earth's surface. The field operations employed to accomplish any of these ends constitute a survey. Accompanying such survey there is usually the field record, the computation, and the final maps, plats, profiles, areas, or volumes. The art of making all these belongs, therefore, to the subject of surveying. When the survey is of such limited areas that the effects of the earth's curvature need not be considered, it is called *Plane Surveying*. Where the survey covers so large an area that the effect of the earth's curvature must be considered, it is called *Geodetic Surveying*.

Inasmuch as all fixed engineering structures or works involve a knowledge of that portion of the earth's surface on which they are placed, together with the necessary or resulting changes in the same, so the execution of such works is usually accompanied by the surveys necessary to obtain the required information. Thus surveying is seen to be intimately related to engineering, but it should not be confounded with it. All engineers should have a thorough knowledge of surveying, but a surveyor may or may not have much knowledge of engineering.

The subject of Surveying naturally divides itself into—

I. The Adjustment, Use, and Care of Instruments.

II. Methods of Field Work.

III. The Records, Computations, and Final Products.

All the ordinary instruments that a surveyor may be called upon to use in any of the departments of the work will be discussed in the following pages. The most approved methods only will be given for obtaining the desired information, and many problems that are more curious than useful will not be mentioned. The student is assumed to possess a knowledge of geometry, and of plane and spherical trigonometry. He is also supposed to be guided by an instructor, and have access to most of the instruments here mentioned, with the privilege of using them in the field.

Accuracy of Measurements. The field work of surveying consists wholly of *measuring* distances, angles, and time, and it is well to remember that no measurement can ever be made exactly. The first thing the young surveyor needs to learn, therefore, is the *proportionate error* allowable in the special work assigned him to perform. It is of the utmost importance to his success that he shall thoroughly study this subject. He should know what all the sources of error are, and their relative importance; also the relative cost of diminishing the size of such errors. Then, with a given standard of accuracy, he will know how to make the survey of the required standard with the least expenditure of time and labor. He must not do all parts of the work as accurately as possible, or even with the same care. For, if the expense is proportioned to the accuracy of results, then he is the most successful surveyor who does his work just good enough for the purpose. The relative size of the various sources of error is of the utmost importance. One should not expend considerable time and labor to reduce the error of *measurement* of a line to 1 in 10,000 when the unknown error in the length of the measuring unit may be as high as 1 in 1000.

Compensating vs. Cumulative Errors. The surveyor must carefully discriminate, also, between compensating errors and cumulative errors. A compensating error is one which is as likely to be plus as minus, and it is therefore largely compensated in, or eliminated from, the result. A cumulative error is one which always enters with the same sign, and therefore it accumulates in the result. Thus, in chaining, the error in setting the pin is a compensating error, while the error from erroneous length of chain is a cumulative error. If a mile is chained with a 66-foot chain, there are 80 measurements taken. Suppose the error of setting the pin be 0.5 inch, and the error in the length of the chain be 0.1 inch. Now the theory of probabilities shows us that in the case of compensating errors the square root of the number of errors *probably*[1] remains uncompensated. The probable error from setting the pins is therefore 9×0.5 inch $= 4.5$ inches. The error from erroneous length of chain is 80×0.1 inch $= 8$ inches. Thus we see that although the error from setting the pins was five times as great as that from erroneous length of chain, yet in running one mile the resulting error from the latter cause was nearly twice that from the former. A careful study of the various sources of error affecting a given kind of work will usually enable the surveyor either to add greatly to its accuracy without increasing its cost, or to greatly diminish its cost without diminishing its accuracy.[2]

The surveyor should have no desire except to arrive at the truth. This is the true scientific spirit. He should be most severely honest with himself. He should not allow himself to change or "fudge" his notes without sufficient warrant, and then a full explanation should be made in his note-book. Neither should he make his results appear more accurate than

[1] The meaning of this statement is that *on the average* this will occur oftener than any other combination, and that any *single* result will, *on the average*, be nearer to this result than to any other.

[2] For a discussion of probable error see Appendix D.

they really are. He should always know what was about the relative accuracy with which his field work was done, and carry his results only so far as the accuracy of the work would warrant. He is either foolish or dishonest who, having made a survey of an area, for instance, with an error of closure of 1 in 300, should carry his results to six significant figures, thus giving the area to 1 in 500,000. It is usual to carry the computations one place farther than the results are known, in order that no additional error may come in from the computation. It is not unusual, however, to see results given in published documents to two, three, or even four places farther than the observations would warrant.

The student should make himself familiar with the structure and use of every part of every instrument put into his hands. The best way of doing this is to take the instrument all apart and put it together again. If this is not practicable for each student in college, when he is given an instrument in real practice, he should then make himself thoroughly familiar with it before attempting to use it.

The adjustments of instruments should be studied as problems in descriptive geometry and not as mechanical manipulations, learned in a mechanical way; and when adjusting an instrument the geometry of the problem should be in the mind rather than the rule in the memory. Rarely, if ever, should the instructor do the actual work of adjusting an instrument for the student. Instead the beginner should be encouraged to make the adjustments frequently. The most important adjustment should be made daily.

Students of engineering in technical schools are urged to make themselves familiar with every kind of instrument in the outfit of the institution, and to do in the field every kind of work herein described if possible. Otherwise he may be called upon to do, or to direct others to do, what he has never done himself, and he will then find that his studies prove of little avail without the real knowledge that comes only from experience.

An experience of fifteen years in the teaching of surveying to engineering students has convinced the writer that the hardest task which beginners in this work find is the proper recording of their field notes.

The criterion to be applied is that these notes should show so clearly the exact work done in the field that a stranger to the work would give to them only one interpretation, and that the true one. Students seem inclined to regard field work and its recorded notes an end in themselves, when in fact their main purpose is for preparing maps or reports.

In no other way can the field engineer increase the reliability of his work and so win the confidence and esteem of his employers as by excelling in the manner of keeping his field records. What may seem trivial points to the beginner are often of vital importance. For example, in the case of so simple a matter as that of dating each day's work, a case once arose in the writer's experience where the date in his field book, by showing that certain grades had been given at a certain date, was worth several thousands of dollars to his employers.

The right-hand page should be used freely in explanatory notes and sketches neatly executed. Especially is this necessary in describing bench marks or transit stations, and yet it is here that the beginner most often fails.

Next in importance to the completeness and accuracy of the field notes is the adoption of some system for the logical arrangement of the data and the computations based upon them. The beginner is unlikely to realize the importance of this matter in securing accurate results, and too frequently covers with his computation the largest sheet of paper he can find. In practical work it has been found necessary to make the computations in a blank book kept only for this purpose. This encourages compactness and neatness in the arrangement of the computations and when properly labelled is always ready for future inspection.

Smith's Field Manual[1]

Professor Leonard S. Smith, of the University of Wisconsin, has prepared a field manual of notes and problems for the use of students in surveying, especially designed to accompany this work. Teachers and students using this work as a text-book will find the Manual of Professor Smith very helpful. It contains sixty problems, and while a number of them have reference to particular marks on the campus of the University of Wisconsin, these could be readily changed to suit any set of local conditions. This manual is neatly bound in red morocco and can be obtained from the University Coöperative Company, Madison, Wis. Price, $1.25.

[1] This note added in the sixteenth and subsequent editions.

BOOK I

ADJUSTMENT, USE, AND CARE OF INSTRUMENTS

CHAPTER I

INSTRUMENTS FOR MEASURING DISTANCES

THE CHAIN[1]

1. The Engineer's Chain is 50 or 100 feet long, and should be made of No. 12 steel wire. The links are one foot long, including the connecting rings. All joints in rings and links should be brazed to prevent giving. The connections are designed so as to admit of as little stretch as possible. Every tenth foot is marked by a special form of brass tag. If the chain is adjustable in length, it should be made of standard length by measuring from the inside of the handle at one end to the outside of the handle at the other. If it is not adjustable, measure from the outside of the handle at the rear end to the standard mark at the forward end.

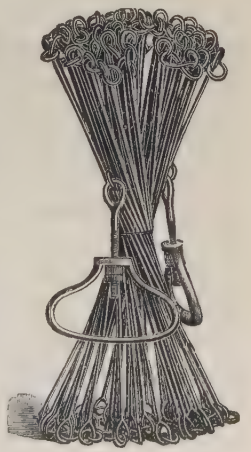


FIG. 1.

2. Gunter's Chain is 66 feet long, and is divided into 100 links, each link being 7.92 inches in length. This chain is

[1] The *metric chain* is usually 20 meters long. It is divided into 100 links, each of 2 decimeters in length.

mostly used in land-surveying, where the acre is the unit of measure. It was invented by Edmund Gunter, an English astronomer, about 1620, and is very convenient for obtaining areas in acres or distances in miles. Thus,

One mile = 80 chains; also,
One acre = 160 square rods,
= 10 square chains,
= 100,000 square links.

If, therefore, the unit of measure be chains and hundredths (links), the area is obtained in square chains and decimals, and by pointing off one more place the result is obtained in acres. This is the length of chain used on all the U. S. land surveys. In all deeds of conveyance and other documents, when the word *chain* is used it is Gunter's chain that is meant.

3. Testing the Chain. No chain, of whatever material or manufacture, will remain of constant length. The length changes from temperature, wear, and various kinds of distortion. A change of temperature of 70° F. in a 100-foot chain will change its length by 0.05 foot, or a change of 1 in 2000.

If the links of a chain are joined by three rings, then there are eight wearing surfaces for each link, or eight hundred wearing surfaces for a 66- or 100-foot chain. If each surface should wear 0.01 inch, the chain is lengthened by eight inches. It is not uncommon for a railroad survey of, say, 300 miles to be run with a single chain. If such a chain were of exactly the right length at the beginning of the survey, it might be six inches too long at the end of it.

The change of length from distortion may come from a flattening out of the connecting rings, from bending the links, or from stretching the chain beyond its elastic limit, thus giving it a permanent set. Both the wear and the distortion are likely to be less for a steel chain than for an iron one. When a bent link is straightened it is permanently lengthened.

When we remember that all unknown changes in the length of the chain produce cumulative errors in the meas

ured lines, we see how important it is that the true length of the chain should be always known, or better, that the standard length (50, 66, or 100 feet) should be properly measured from one end of the chain and marked at the other. This chain test is most readily accomplished by the aid of a standard steel tape which is at least as long as the chain.[1] By the aid of such a tape a standard length may be laid off on the floor of a large room, or two stones may be firmly set in the ground at the proper distance apart and marks cut upon their upper surfaces. If stones are used they should reach below the frost-line. Or a short tape, or other standard measuring unit, may be used for laying off such a base-line. Some ready means should at all times be available for testing the chain. Since a chain always grows longer with use, the forward end of the chain will move farther and farther from the standard mark. A small file-mark may be made on the handle or elsewhere, and then removed when a new test gives a new position. Care must be exercised to see that there are no kinks in the chain either in testing or in use.

In laying out the standard base the temperature at which the unit of measure is standard should be known (this temperature is stamped on the better class of steel tapes), and if the base is not laid out at this temperature, a correction should be made before the marks are set. The coefficient of expansion of iron and steel is very nearly 0.0000065 for 1° F. If T_0 be the temperature at which the tape is standard, T the temperature at which the base is measured, and L the length of the base, then $0.0000065\ (T-T_0)L$ is the correction to be applied to the measured length to give the true length.

When the chain is tested by this standard base the temperature should be again noted, and if this is about the mean temperature for the field measurements no correction need be

[1] The National Bureau of Standards at Washington, D. C., will test any tape for a very small fee.

made to the field work. If it is known, at the time the chain is tested, that the temperature is very different from the probable mean of the field work, then the standard mark can be so placed on the chain as to make it standard when in use.

4. The Use of the Chain. The chain is folded by taking it by the middle joint and folding the two ends simultaneously. It is opened by taking the two handles in one hand and throwing the chain out with the other.

Since *horizontal* distances are always desired in surveying, the chain should be held horizontally in measuring. Points vertically below the ends of the chain are marked by iron pins, the head chainman placing them and the rear chainman removing them after the next pin is set. The chain is lined in either by the head or rear chainman, or by the observer at the instrument, according as the range-pole is in the rear, or in front, or not visible by either chainman. When chaining on level ground, the rear chainman brings the outside of the handle against the pin, and the head chainman sets the forward side of his pin even with the standard mark on the chain. By this means the centers of the pins are the true distance apart. On uneven ground both chainmen cannot hold to the pin; one end being elevated in order to bring the chain to a horizontal position. In this case there are three difficulties to be overcome. The chain should be drawn so taut that the stretch from the pull would balance the shortening from the sag; the chain should be made horizontal; the elevated end-mark must be transferred vertically to the ground. It is practically impossible to do any of these exactly. The first could be determined by trial. Stretch the chain between two points at the same elevation, having it supported its entire length. Then remove the supports, and see how strong a pull is required to bring it to the marks again. This should be done by the chainmen themselves, thus enabling them to judge how hard to pull it when it is off the ground. To hold the chain horizontal on sloping ground is very difficult, on account of the judgment

being usually very much in error as to the position of a horizontal line. In all such cases the apparently horizontal line is much too nearly parallel with the ground. Sometimes a level has been attached to one end of the chain, in which case it should be adjusted to indicate horizontal end-positions for a certain pull, this being the pull necessary to overcome the shortening from sag. To hold a plumb-line at the proper mark, with the chain at the right elevation, and stretched the proper amount, requires a steady hand in order that the plumb-bob may hang stationary. This should be near the ground, and when all is ready, it is dropped by the chainman letting go the string. The pin is then stuck and the work proceeds. It is common in this country for the rear chainman to call "stick" when he is ready, and for the head chainman to answer "stuck" when he has set the pin. The rear chainman then pulls his pin and walks on.

Chaining downhill has been found more accurate than chaining uphill, because in the first case the rear end is held firmly at a point on the ground. This allows the head chainman to pull steadily on the chain and to transfer the chain length to the ground by use of the plumb-bob.

On fairly even slopes it has been found more accurate to measure the angle of the slope and then do the chaining on the surface of the ground. The true horizontal distances would be found by multiplying the slope measurements by the cosine of the slope angle.

There should be eleven pins, marked with strips of colored flannel tied in the rings to assist in finding them in grass or brush. In starting, the rear chainman takes a pin for the initial point, leaving the head chainman with ten pins. When the last pin is stuck, the head chainman calls "out," and waits by this station until the rear chainman comes up and delivers over the ten pins now in his posession. The eleventh pin is in the ground, and serves as the initial point for the second score. Thus only every ten chains need be scored.

Good chaining, therefore, consists in knowing the length of the chain, in true alignment,[1] horizontal and vertical, and in proper stretching, marking, and scoring.

The Steel Tape

5. Varieties. Steel tapes are now made from one yard to 1000 feet in length, graduated metrically, or in feet and tenths. A pocket steel tape from three to ten feet long should always be carried by the surveyor. A 50-foot tape is best fitted to city surveying where there are appreciable grades. For cities without grades a 100-foot tape might be found more useful. For measuring base-lines, or for some kinds of mining surveying, a 300- or 500-foot tape is best. These are of small cross-section, being about 0.1 inch wide and 0.02 inch thick. A tape about 0.33 inch wide and 0.02 inch thick (Figs. 2 and 3) is perhaps best suited to general surveying. During the past few years a special form of steel tape, called a band chain, has come much into use. It is composed of narrow and thicker steel and is less likely to become broken. Its cost is less than half that of the old-style steel tape, and it is much more difficult to break.

---

[1] The error in length due to poor alignment may be computed from the approximate formula, x, the error $= h - b = \frac{v^2}{2b}$.

Thus in the right angle

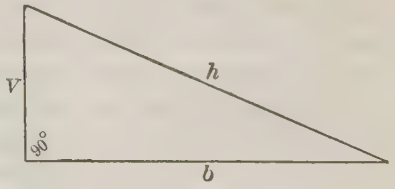
$$h^2 - b^2 = v^2,$$
$$h = b + x.$$

Substituting we have

$$b^2 + 2bx + x^2 - b^2 = v^2;$$
$$2bx + x^2 = v^2,$$

or neglecting second power of x,

$$2bx = v^2,$$
$$x = \frac{v^2}{2b}.$$



This formula is correct to the nearest $\frac{1}{100}$ foot even when $v = 14$ feet and $b = 109$ feet

Besides the steel tape, cloth and metallic tapes are sometimes used. The cloth tape stretches so easily as to be of little

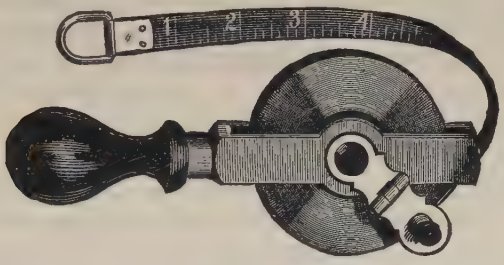


FIG. 2.

use in surveying. The so-called metallic tapes are made of cloth with fine brass wire woven in to prevent the stretching. They should not be used except for short measurements.

FIG. 3.

6. The Use of Steel Tapes. Steel tape-measures are used just as chains are. They are provided with handles, but the

end graduation-marks are usually on the tape itself and not on the handle. They are graduated to order, the graduations being either etched or made on brass sleeves which are fastened on the tape. Their advantages are many. They do not kink, stretch, or wear so as to change their length, so that, with careful handling, they remain of constant length except for temperature. They are used almost exclusively in city and bridge work, and in the measurement of base-lines. The same precautions must be taken in regard to alignment, pull, and marking with the tape, as was described for the chain.[1]

The *Odometer* is an instrument which when attached to the wheel of a carriage, records the number of revolutions of the wheel. See Art. 126.

7. Other Instruments. Because of the ease and rapidity with which they can be taken, stadia measurements are becoming very popular where great accuracy is not required. The work is done by a transit provided with two extra horizontal cross-wires so spaced that they intercept a foot on a speaking rod for every 100 feet distant. See Chapter VIII for a full discussion.

8. Common Sources of Error in Measurement of Distances.

1. Insufficient pull on tape or chain.
2. Careless plumbing.
3. Incorrect alignment.
4. Effect of wind.
5. Variation of temperature.
6. Erroneous length of chain or tape.

9. Common Mistakes in Reading and Recording Measurements.

1. Failure to observe the position of the zero point of the tape.
2. Omitting a whole chain or tape length.

[1] For methods of using the steel tape in accurate measurements, see Chapter XIV, Base-Line Measurements.

3. Reading from the wrong end of chain, as 30 feet for 70 feet, or in the wrong direction, as 35 feet for 53 feet.
4. Transposing figures, thus 38.62 for 38.26, or reading tape upside down, thus 6 for 9 or 86 for 98.
5. Reading wrong foot mark, thus 39.62 for 38.62.

EXERCISES

To be worked out on the ground by the use of the chain or tape alone.

1. To chain a line over a hill between two given points, not visible from each other.

Range-poles are set at the given points. Then the two chainmen, each with a range-pole, range themselves in between the two fixed points, near the top of the hill, by successive approximations. The line can then be chained.

2. To chain a line across a valley between two fixed points.

Establish other range-poles by means of a plumb-line held on range between the points.

3. To chain a line between two fixed points when woods intervene, and the true line is not to be cleared out.

Range out a trial line by poles, leaving fixed points. Find the resulting error at the terminus, and move all the points over their proportionate amount. The true line may then be chained.

4. To set a stake in a line perpendicular to a given line at a given point.

All multiples of 3, 4, and 5 are the sides of a right-angled triangle; also any angle in a semicircumference is a right angle.

5. To find where a perpendicular from a given point without a line will meet that line.

Run an inclined line from the given point to the given line. Erect a perpendicular from the given line near the required point, extend it till it interescts the inclined line, and solve by similar triangles.

6. To establish a second point that shall make with a given point a line parallel to a given line.

Diagonals of a parallelogram bisect each other.

7. To determine the horizontal distance from a given point to a visible but inaccessible object.

Use two similar right-angled triangles.

8. To prolong a line beyond an obstacle, in direction and distance.

First Solution: By an equilateral triangle.

Second Solution: By two rectangular offsets on each side of the obstacle.

Third Solution: By similar traingles, as in Fig. 4.

From any point as A run the line AB, fixing the half and three-quarter points at x and y. From any other point as C, run CxD, making $xD = Cx$. From D run DyE making $DE = 4Dy$, fixing the mdidle point z. From B run BzH, making

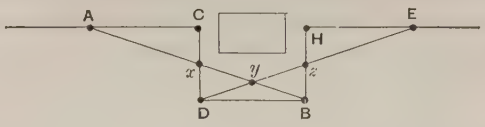


FIG. 4.

$zH = Bz$. Then is HE parallel and equal to DB, AC, and CH. Stakes should be set at all the points lettered in the figure. Check: Measure HE and AC. If they are equal the work is correct.

9. To measure a given angle.

Lay off equal distances, b, from the vertex on the two lines, and measure the third side a of the triangle. Then $\sin \frac{1}{2}A = \frac{a}{2b}$.

10. To lay out a given angle on the ground.

Reverse the above operation. A is known; assume b and compute a. Then from A measure off $AB = b$. From B and A strike arcs with radii equal to a and b respectively, giving an intersection at C. Then CAB is the required angle. If b is assumed not greater than 0.6 the length of the chain, angles may be laid out up to 90°.

CHAPTER II

INSTRUMENTS FOR DETERMINING DIRECTIONS

The Compass

10. The Surveyor's Compass consists essentially of a line of sight attached to a horizontal graduated circle, at the centre of which is suspended a magnetic needle free to move, the whole conveniently supported with devices for levelling. Fig.

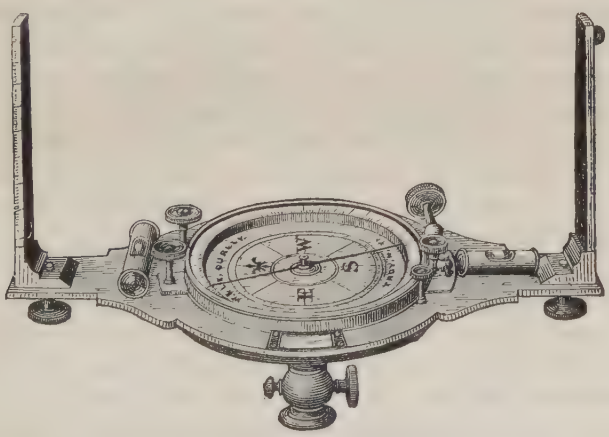


Fig. 6.

6 shows a very good form of such an instrument. In addition to the above essential features, the instrument here shown has a tangent-screw and vernier-scale for setting off the declination of the needle; a tangent-scale on the edge of the vertical sight for reading vertical angles, the eye being placed at the sight-disk shown on the opposite standard; and an

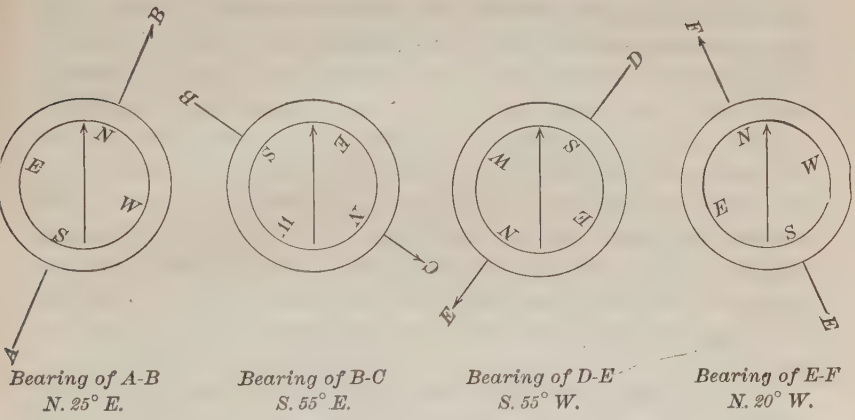
auxiliary graduated circle, with vernier, shown on the front part of the plate, for reading angles closer than could be done with the needle. The compass is mounted either on a tripod or on a single support called a Jacob's-staff. It is connected to its support by a ball-and-socket joint, which furnishes a convenient means of levelling.

Although the needle-compass does not give very accurate results, it is one of the most useful of surveying instruments. Its great utility lies in the fact that the needle always points in a known direction, the magnetic meridian, and therefore the direction of any line of sight may be determined by referring it to the needle-bearing. The needle points true north in only a few localities; but its *declination from* the north point is readily determined for any region, and then the true azimuth, or bearing of a line, may be found. It has grown to be the universal custom, in finding the direction of a line by the compass, to refer it to *either* the north *or* the south point, according to which one gives an acute angle. Thus, if the bearing is 100° from the south point it is but 80° from the north point, and the direction would be defined as north, 80° east or west, as the case might be: thus no line can have a numerical bearing of more than 90°. In accordance with this custom, all needle-compasses are graduated from both north and south points each way to the east and west points, the north and south points being marked zero, and the east and west points 90°. When the direction of a line is given by this system it is called the *bearing* of the line. When it is simply referred to the position of the needle it is called the *magnetic* bearing. When it is corrected for the declination of the needle, either by setting off the declination on the declination-arc or by correcting the observed reading, it is called the *true* bearing, being then referred to the true meridian.

11. Method of Reading a Magnetic Bearing. Because the graduated circle is attached to the line of sight and moves with it, while the needle remains stationary, *E* and *W* are placed

on the compass-circle in *reversed* position. Otherwise the needle would not give the proper quadrant. See Fig. 7. Thus when the line of sight is northeast, the north end of the needle points to the left of the north point on the circle, and hence *E* must be put on this side of the meridian line. In a ship's compass the graduated circle moves with the needle and hence *E* and *W* are not reversed.

In reading the compass, always keep the north end of the circle pointing forward along the line, and read the north end of the needle.



Bearing of A-B *N. 25° E.* | *Bearing of B-C* *S. 55° E.* | *Bearing of D-E* *S. 55° W.* | *Bearing of E-F* *N. 20° W.*

FIG. 7.—Showing How to Make Readings of Bearings.

The north end of the needle is usually shaped to a special design, or, if not, it may be distinguished by knowing that the south end is weighted by having a small adjustable brass wire slipped upon it to overcome the tendency the north end has to dip. See Art. 20.

Care should be taken not to leave the axe, chain, or marking pins near the compass, because of their effect in changing the needle reading. Even a bunch of keys on a person, or the iron wire in a stiff hat, has been known to cause an error in the needle reading.

Adjustments of the Compass

12. The General Principle of almost all instrumental adjustments is the *Principle of Reversion, whereby the error is doubled and at the same time made apparent.* A thorough mastery of this principle will nearly always enable one to determine the proper method of adjusting all parts of any surveying instrument. It should be a recognized principle in surveying that no one is competent to handle any instrument who is not able to determine when it is in exact adjustment, to locate the source of the error if not in adjustment, to discuss the effect of any error of adjustment on the work in hand, and to properly adjust all the movable parts. The methods of adjustment should not be committed to memory—any more than should the demonstration of a proposition in geometry. The student in reading the methods of adjustment should *see* that they are correct, just as he sees the correctness of a geometrical demonstration. Having thus had the method and the reason therefor clearly in the mind, he should trust his ability to evolve it again whenever called upon. He thus relies upon the accuracy of his reasoning, rather than on the distinctness of his recollection.

13. To Make the Plate Perpendicular to the Axis of the Socket. This must be done by the maker. It is here mentioned because the axis is so likely to get accidentally bent. Instruments made of soft brass must be handled very carefully to prevent such an accident. If this adjustment is found to be very much out, it should be sent to the maker. If much out, it will be shown by the needle, and also by the plate-bubbles.

14. To Make the Plane of the Bubbles Perpendicular to the Axis of the Socket. Level it in one position, turn 180°, and correct one-half the movement of each bubble by the adjusting-screw at the end of the bubble-case. Now level up again, and revolve 180°, and the bubbles should remain at

the centre. If not, adjust for one-half the movement again, and so continue until the bubbles remain in the centre for all positions of the plate. This adjustment makes the plane of the bubbles parallel to plate.

For a diagram of this adjustment see Fig. 41.

15. To Detect Errors in the Adjustment of the Needle Circle. When the two ends of the needle do not read 180° apart it may be due to the pivot being bent out of its central position; or to the needle being bent, or to both causes.

If the difference between the two end readings is constant for all positions of the needle, then the pivot is in the centre of

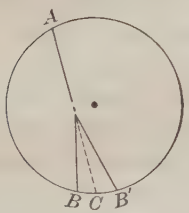


FIG. 8.—Compass Needle Bent.

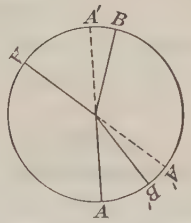


FIG. 9.—To Straighten the Needle.

the circle, but the needle is bent. For manner of testing this condition see Fig. 8. The error in one reading $A'B$ is found equal to the error in second reading $A'B'$.

16. To Straighten the Needle, set the north end A exactly at some graduation-mark, and read the south end thus, B in Fig. 9. Remove the glass cover and with compass axis still clamped, turn the needle on its pivot until the south end, B, now points at A, the same reading as the north end read before. Now read the north end B', then will BB' be twice the error. Remove the needle and bend it by one-half BB' in the direction from B' toward B. Since the amount of bending is only estimated the operation may need to be repeated several times before an accurate adjustment is reached.

17. To Centre the Pivot Point. If the difference between the two end readings is variable for different parts of the circle, then the pivot is bent, and the needle may or may not be straight.

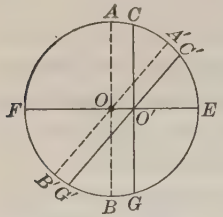


FIG. 10.—Pivot Point out of Centre.

To adjust the pivot, therefore, find the position of the needle which gives the *maximum* difference of end readings, remove the needle, and with small flat pliers *bend the pivot at right angles to this position by one-half the difference in the extreme variation of end readings.* See Fig. 10. Since the glass cover is removed from the compass-box in making this adjustment, the adjustment should be made indoors, to prevent any disturbance from wind.

When the adjustment is completed the differences in end-readings will be zero. Several trials will usually be required before this condition is satisfied.

18. To Remagnetize the Needle. Needles sometimes lose their magnetic properties. They must then be remagnetized. To do this take a simple bar-magnet and rub each end of the needle, from centre towards the ends, with the end of the magnet which attracts in each case. In returning the magnet for the next stroke lift it up a few inches to remove it from the immediate magnetic field, otherwise it would tend to nullify its own action. The needle should be removed from the pivot in this operation, and the work continued until it shows due activity when suspended. An apparently sluggish needle may be due to a blunt pivot. If so, this should be removed, and ground down on an oil-stone.

19. To Make the Plane of the Sights Normal to the Plane of the Bubbles. Carefully level the instrument and bring the plane of the sights upon a suspended plumb-line. If this seems to traverse the farther slit, then that sight is in adjustment. Reverse the compass, and test the other sight in like manner. If either be in error, its base must be reshaped

to make it vertical. Otherwise tin foil may be used to plumb the sights.

20. The Earth's Magnetism. The earth is known to be a great magnetic field. In the northern hemisphere the lines of magnetic force point downward and northward, while in the southern hemisphere upward and northward. A magnet, free to turn, like a magnetic needle, will therefore have its north end dip downward in the northern hemisphere and upward in the southern hemisphere. To maintain a horizontal position a coil of fine wire is placed on the south end of the needle.

The Declination of the Needle

21. The Declination[1] **of the Needle** is the horizontal angle it makes with the true meridian. At no place on the earth is this angle a constant. The *change* in this angle is called the *variation* of the declination.

22. The Daily Variation in the Declination consists in a swinging of the needle through an arc of about eight minutes daily, the north end having its extreme easterly variation about 8 A.M. and its extreme westerly position about 1.30 P.M. It has its *mean* or *true* declination about 10.30 A.M. and 8 P.M. It varies with the latitude and with the season, but the following table gives a fair average for the United States. A more extended table may be found in the Report of the U. S. Coast and Geodetic Survey for 1881, Appendix 8.[2]

23. The Secular Variation[3] of the magnetic declination is probably of a periodic character, requiring two or three centuries to complete a cycle. The most extensive set of obser-

[1] Formerly called *variation* of the needle, and still so called by navigators and by many surveyors.

[2] See also "Principal Facts of the Earth's Magnetism," U. S. C. and G. Survey, 1909, and Magnetic Tables and Charts for 1905 by L. A. Bauer, U. S. C. and G. Survey, 1908, Washington, D. C.

[3] Discovered in 1634.

vations bearing on this subject have been made at Paris, where records of the magnetic declination have been kept for about three and a half centuries. The secular variation for Paris is shown in Fig. 11, and that for Baltimore, Md., in Fig. 12.[1]

TABLE OF CORRECTIONS TO REDUCE OBSERVED BEARINGS TO THE DAILY MEAN

Month.	Add to N.E. and S.W. bearings. Subtract from N.W. and S.E. bearings.					Add to N.W. and S.E. bearings. Subtract from N.E. and S.W. bearings.							
	6 A.M.	7 A.M.	8 A.M.	9 A.M.	10 A.M.	11 A.M.	12 M.	1 P.M.	2 P.M.	3 P.M.	4 P.M.	5 P.M.	6 P.M.
January	1′	1′	2′	2′	1′	0′	2′	3′	3′	2′	1′	1′	0′
April	3	4	4	3	1	1	4	5	5	4	3	2	1
July	4	5	5	4	1	1	4	5	5	4	3	2	1
October	1	2	2	2	1	1	3	3	3	2	1	0	0

This table is correct to the nearest minute for Philadelphia, where the observations were made.

Whether or not either of these curves will return in time to the same extreme limits here given is unknown, as is also the cause of these remarkable changes. The extraordinary variation in the declination at Paris of some 32°, and that at Baltimore of some 5°, shows the necessity of paying careful attention to this matter. No reliance should be placed on old determinations of the declination unless the *rate of change* be known, and even then this rate is not likely to be constant a great many years. They also show necessity of recording the date and the declination of the needle on all plats and records of surveys, with a note stating whether the bearings given were the true or magnetic bearings at the time they were taken.

24. Isogonic Lines are imaginary lines on the earth's surface joining points whose declinations are equal at any given

[1] These taken from the Coast Survey Report of 1882.

time. The isogonic line joining points having no declination is called the *agonic line.* There is such a line crossing the

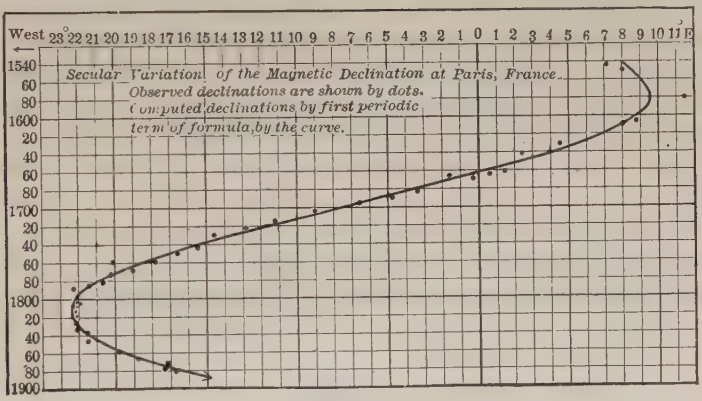

Secular Variation of the Magnetic Declination at Paris, France.
Observed declinations are shown by dots.
Computed declinations by first periodic term of formula, by the curve.
West
E

FIG. 11.

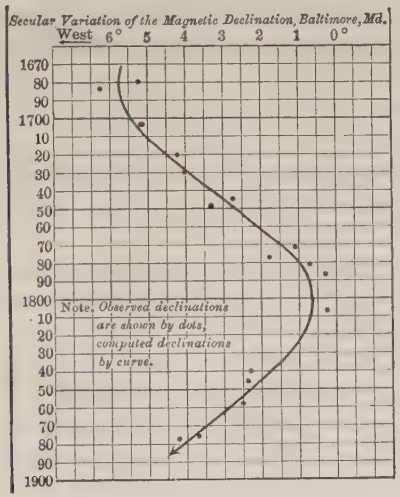

Secular Variation of the Magnetic Declination, Baltimore, Md.
West
Note. Observed declinations are shown by dots; computed declinations by curve.

FIG. 12.

United States passing just east of Charleston, S. C., and just west of Detroit, Mich. All points east of this line have a

western declination, and all points west of it have an eastern declination. The isogonic lines for 1915 for the whole of the United States are shown on Plate I.[1] It will be noted that where the observations are most thickly distributed, as in Missouri for instance, there the isogonic lines are most crooked, showing that if the declinations were accurately known for all points of this map the isogonic lines would be much more irregular, and would be changed very much in position in many places.

The isogonic lines given on this chart are all moving westward, so that all western declinations are increasing and all eastern declinations are decreasing. They are not all moving at the same rate, however, those in New Brunswick and also those near the eastern boundaries of California and Oregon being about stationary. For many points in the United States and Canada the rate of change in the declination has been observed, and formulæ determined for computing the declination for each point, which formulæ will probably remain good for the next twenty years. Tables giving this information can be obtained from the Superintendent of the U. S. Coast and Geodetic Survey, Washington, D. C.

25. Other Variations of the Declination. In addition to the daily and secular changes in the declination, there are others worthy of mention.

The annual variation is very small, being only about a half-minute of arc from the mean position for the year. It may therefore be neglected.

The lunar inequalities are still smaller, being only about fifteen seconds of arc from the mean position.

Magnetic disturbances are due to what are called magnetic storms. They may occur at any time, and cannot be predicted. They may last a few hours, or even several days. "The following table of the observed disturbances, in a bi-hourly series,

[1] Reduced from the U. S. Coast and Geodetic Survey Charts.

at Philadelphia, in the years 1840 to 1845, will give an idea of their relative frequency and magnitude:

Deviations from Normal Direction.	Number of Disturbances.
3′.6 to 10′.8	2189
10′.8 to 18′.1	147
18′.1 to 25′.3	18
25′.3 to 32′.6	3
Beyond ...	0

"At Madison, Wis., where the horizontal magnetic intensity is considerably less, very much larger deflections have been noticed. Thus, on October 12, 1877, one of 48′, and on May 28, 1877, one of 1° 24′, were observed." [1]

The geometric axis of a needle may not coincide with its *magnetic axis*, and hence the readings of two instruments at the same station may differ slightly when both are in adjustment. In this case the declination should be found for each instrument independently.

26. Observation for Declination. This involves a determination of the true meridian. Methods of doing the field work are given in Arts. 98 and 101. When the line of sight is on the meridian the needle should read the declination.

Use of the Needle-Compass

27. The Use of the Needle-Compass is confined almost exclusively to land-surveying, where an error of one in three hundred could be allowed. As the land enhances in value, however, there is an increasing demand for more accurate means of determining areas than the compass and chain afford. The original U. S. land surveys were all made with the needle, or with the solar compass and Gunter's chain. Hence all land

[1] From Report of the U. S. Coast and Geodetic Survey for 1882.

boundaries in this country have their directions given in compass-bearings, and their lengths in chains of 66 feet each.

The compass is used, therefore—

1. To establish a line of a given bearing.
2. To determine the bearing of an established line.
3. To retrace old lines.

If the true bearing is to be used, the declination of the needle from the meridian must be determined and set off by the vernier.

If the magnetic bearing is used, the declination of the needle at the time the survey was made should be recorded on the plat.

If old lines are to be retraced, the declinations at the times of both surveys must be known.

The needle should be read to the nearest five minutes. This requires reading to sixths of the half-degree spaces. The use of the verniers will improve the work.

Always lift the needle from the pivot before moving the instrument.

If the needle is sluggish in its movements and settles quickly it has either lost its magnetic force or it has a blunt pivot. In either case it is likely to settle considerably out of its true position. The longer a needle is in settling the more accurate will be its final position. It can be quickly brought very near its true position by checking its motion by means of the lifting screw. In its final settlement, however, it must be left free.

Careful attention to the instrumental adjustments, to local disturbances, and close reading of the needle are all essential to good results with the compass.

28. To Set off the Declination we have only to remember that the declination arc is attached to the line of sight and that the vernier is attached to the graduated circle. If the declination is west, then when the line of sight is north the north end of the needle points to the left of the zero of the graduated circle. In order that it may read zero, or north, the cir-

cle must be moved towards the left, or opposite to the hands of a watch. On the other hand, if the declination is east, the circle to which the vernier is attached should be moved with the hands of a watch. This at once enables the observer to set the vernier so that the needle-readings will be the true bearings of the line of sight.

29. Local Attractions may disturb the needle by large or small amounts, and these often come from unknown causes. The observer should have them constantly in mind, and keep all iron bodies at a distance from the instrument when the needle is being read. The glass cover may become electrified from friction, and attract the needle. This can be discharged by touching it with a wet finger, or by breathing upon it. Reading-glasses should not have gutta-percha frames, as these become highly electrified by wiping the lens, and will attract the needle. Such glasses should have brass or German-silver frames. No nickel coverings or ornaments should be near, as this metal has magnetic properties. A steel band in a hat-brim, or buttons containing iron have been known to cause great disturbance. In cities and towns it is practically impossible to get away from the influence of some local attraction, such as iron or gas pipes in the ground, iron lamp-posts, fences, building-fronts, etc. For this reason the needle should never be used in such places.

In many regions, also, there are large magnetic iron-ore deposits in the ground, which give special values for the declination at each consecutive station occupied. It is practically impossible to use magnetic bearings in such localities.

The test for local attraction in the field-work is to read the bearing of every line from both ends of it. If these are not the same, and no error has been made, there is some local disturbance at one station not found at the other. If there are known to be mineral deposits in the region it may perhaps be laid to that. If not, the preceding station should be occupied again, and the cause of the discrepancy inquired into. If the forward and reverse bearings of all lines agree except the bear-

ings taken from a single station, then it may be assumed there is local attraction at that station.

30. To Establish a Line of a Given Bearing set the compass up at a point on the line, turn off the declination on the declination-arc, and bring the north end of the needle to the given bearing. The line of sight now coincides with the required line, and other points can be set.

31. To Find the True Bearing of a Line set the compass up on the line, turn off the declination by the vernier, bring the line of sight to coincide with the line with the south part of the graduated circle towards the observer, and read the north end of the needle. This gives the forward bearing of the line.

32. To Retrace an Old Line,[1] set the compass over one well-determined point in the line and turn the line of sight upon another such point. Read the north end of the needle. If this reading is not the bearing as given for the line, move the vernier until the north end of the needle comes to the given bearing, when the sights are on line. The reading of the declination-arc will now give the declination to be used in retracing all the other lines of the same survey. If a second well-determined point cannot be seen from the instrument-station, a trial line will have to be run on an assumed value for the declination, and then the value of the declination used on the first survey computed. Thus, if the trial line, of length l, comes out a distance x to the *right* of the known point on the line, the vernier is to be moved *in the direction* of the hands of a watch an angular amount whose tangent is $\frac{x}{l}$. If the trial line comes out to the *left* of the point, move the vernier in a direction *opposite* to the hands of a watch.

[1] This is treated more fully in Arts. 149 and 202.

Prismatic Compass

33. The Prismatic Compass is a hand instrument provided with a glass prism so adjusted that the needle can be read while taking the sight. A convenient form is shown in Fig. 13, which is carried in the pocket as a watch. The line of sight is established by means of the etched line on the glass cover *S*. It is

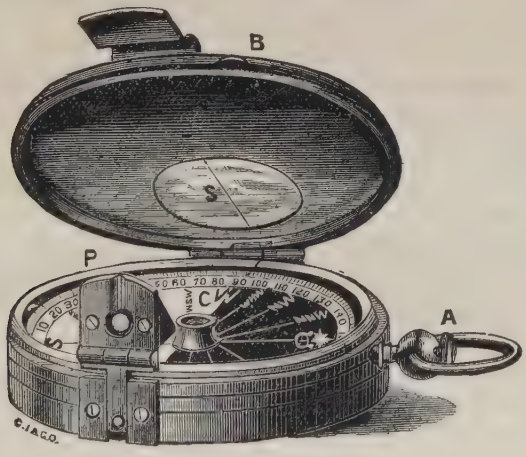


Fig. 13.

used in preliminary and reconnoissance work, in clearing out lines, etc.

EXERCISES FOR COMPASS ALONE OR FOR COMPASS AND CHAIN

1. Run out a line of about a mile in length, on somewhat uneven ground, establishing several stations upon it, using a constant compass-bearing. Then run back *by the reverse bearings* and note how nearly the points coincide with the former ones. The chain need not be used.

2. Select some half dozen points that inclose an area of about forty acres (one quarter mile square) on uneven ground. Let one party make a compass-and-chain survey of it, obtaining bearing, and length of each side. Then let other parties take these field notes and, all starting from a common point, *run*

out the lines as given by the field notes, setting other stakes at all the remaining corners, each party leaving special marks on their own stakes. Let each party plot their own survey and compare errors of closure.

3. Select five points, three of which are free from local attraction, while two consecutive ones are known to be subject to such disturbance. Make the survey, finding length and forward and reverse bearings of every side. Determine what the true bearing of each course is, and plot to obtain the error of closure.

4. A traverse was run over a closed field with the following bearings: N. 40° E., N. 34° E., S. 2° W., N. 52° W. Compute the interior angles of the field. What should their sum be?

5. The magnetic declination at a given place is 5° 30′ E. What will be the magnetic bearing of true north? The true south? The true east?

6. A line of an old survey is recorded as N. 12° E. magnetic bearing. It now reads N. 14° 30′ E. What has been the change in declination in direction and amount?

7. A line of an old survey is recorded as N. 26° 15′ E. magnetic bearing. It now reads N. 24° 45′ E. and the magnetic declination is now 5° 30′ E. What was the declination at the time of the original survey?

8. A line of an old survey is recorded as N. 28° E. magnetic bearing, declination 10° E. The present declination being 8° 15′ E., what magnetic bearing should be used to retrace the old line?

9. At a certain place the declination was 4° 25′ W. in 1700, 1° 39′ W. in 1750, 0° 30′ E. in 1800, 1° 0′ W. in 1850; 4° 0′ W. in 1900. If a line had a bearing of S. $65\frac{1}{4}$° W. in 1900 what was its bearing in 1700, 1750, 1800, 1850?

10. The following bearings were observed by a compass: *AB* N. 27° 30′ E.; *BA* S. 25° 15′ W.; *BC* S. 88° W.; *CB* N. 87° 45′ E.; *CD* N. 47° 15′ W.; *DC* S. 47° 15′ E. Find the true bearing of *AB*. Where is the local attraction? Which way is the needle deflected at each point, and how much?

CHAPTER III

INSTRUMENTS FOR DETERMINING HORIZONTAL LINES

Plumb-line and Bubble

34. The Plumb-Line and the Bubble-Tube are at once the most simple, universal, and essential of all appliances used in surveying and astronomical work. Without them neither the zenith nor the horizon could be effectually determined, and the determination of altitudes and of horizontal lines and planes would be out of the question. Even azimuths, bearings, and horizontal angles require that the circle by which they are obtained shall be brought into a horizontal position.

The direction of the plumb-line is by definition a vertical line, pointing to the zenith, and a plane at right angles to this line is *for that point* a horizontal plane. As no two plumb-lines can be parallel, so no two planes, respectively horizontal at two different positions on the earth's surface, can be parallel.

Parallel horizontal planes can only be planes at different elevations, all horizontal for a single position on the earth's surface.

35. A Level Surface is not a plane, but a curved surface which is at every point perpendicular to a plumb-line at that point. The surface of still water is a level surface.

A *Spirit Level* is a round glass tube bent or ground so that its inside upper surface is circular on a longitudinal section. This is nearly filled with ether, the remaining space being occupied with ether-vapor, which forms the bubble. This tube is usually graduated to assist in determining the exact

position of the bubble in the tube. If the tube has been ground to a perfect circular longitudinal section, then a *longitudinal line tangent to this inner surface at the centre of the air-bubble is a horizontal line*, in whatever part of the tube the bubble may lie. If this were not a horizontal line, the centre of gravity of the bubble would not occupy its highest possible position and would move until it did. Since the position of the centre of a bubble in a tube is determined by reading the position of its ends and taking the mean, it is necessary that the arc shall be of uniform curvature—that is, circular.

A line tangent to the inner surface of the bubble-tube at its centre, as defined by the graduations (or another line parallel to it) it is called *the axis of the bubble tube*. When the bubble is in the centre of the tube, therefore, its axis is horizontal.

36. Proposition I. If a bubble-tube be rigidly attached to a frame, and if this frame be reversed on two supports lying in the vertical plane through the bubble-axis, the supporting points are level when the bubble occupies *the same portion* of the tube in both positions of frame, whether this be the centre or not; providing, of course, that the points of support on the frame were identical in the two cases.

For, the tangent horizontal lines being identical in the two positions of the bubble, the vertical distances from this line to the points of support must be equal, otherwise the direct and reversed positions would not give identical tangent lines. The points of support are therefore in a horizontal line.

37. Proposition II. If a bubble-tube be revolved about an axis in such a way that *the bubble keeps a constant position in the tube*, the axis of revolution is vertical.

For, since the bubble-tube maintains a constant inclination to the horizon (this inclination being zero when the bubble is in the centre), the plane of motion can have no vertical component, and, therefore, the axis of revolution must be vertical.

Cor. 1. Similarly we may say that if a bubble-tube be revolved 180° about an axis, and if the bubble have the same

reading in the two positions, then the plane of revolution has no vertical component in the direction of the bubble-axis, and therefore the axis of revolution lies in a vertical plane at right angles to the bubble-axis. If the same test be made for any other two horizontal positions 180° apart (preferably 90° from first position) and the bubble have the same reading in the two cases, then the axis of revolution lies in a vertical plane at right angles to these new positions of bubble-axis, and therefore it lies at the intersection of these two vertical planes, or it is vertical. If two bubble-tubes (not parallel to each other and preferably at right angles) be rigidly attached to a frame that revolves about an axis, and if each bubble has the *same* reading in two positions of frame 180° apart, the axis of revolution is vertical, even though the two bubbles do not read alike nor either is at the middle of its tube.

Cor. 2. In all cases where a bubble-tube has been shifted 180° in the same supports, or axis, the angular difference between the two positions of the bubble is twice the angular deviation of the supports from a horizontal, or of the axis from a vertical.

38. The Accurate Measurement of Small Vertical Angles is accomplished by means of the bubble with greater readiness and precision than by any other device known. For this purpose the bubble-tube should be ground accurately to the arc of a circle with a long radius, and uniformly graduated. Then a given bubble-movement in any part of the tube corresponds to a known angular change, when the angular value of a movement of one division in the graduated scale has been determined. These graduations are usually made on the top of the glass tube. To measure a small angle by means of the bubble, read the two ends of the bubble to divisions and tenths, and take the one-half difference of end readings.[1] Shift the

[1] Bubbles are read from the middle outwards towards the ends. Then the half-difference of end readings is the distance of the centre of the bubble from the centre of the scale.

bubble a given amount and read both ends again, taking one-half the difference. The difference of these two results in divisions of the scale, multiplied by the angular value of one division on the scale, is the vertical angle through which the tube was shifted.

39. The Angular Value of One Division of the Bubble may be found in various ways.

(*a*) *By a Telescopic Line of Sight.* Attach the bubble-tube rigidly to a mounted telescope, putting the bubble axis in the plane of the telescope. Measure off a convenient base-line on level ground of from 200 to 500 feet. Set the telescope at one end of this base, and hold a rod vertically at the other. Bring the bubble near one end of its tube by moving the telescope vertically, and read the two ends. Read the height of the cross wires on the rod. Bring the bubble near the other end of tube and read both the bubble and rod. Repeat many times. Reduce the work by taking the half difference of the two end readings in each case, thus giving the distance of the centre of the bubble from the centre of tube for each position. Take the mean of these results for each set of end readings separately. If these mean results were for opposite ends of the tube, add them together and this gives the *average movement of bubble.* Similarly take the mean of the upper readings and the mean of the lower readings on the rod, and take the difference, and this is the *average movement of the line of sight.* Calling the bubble movement in divisions of scale D, the movement on the rod, in feet, R, and the length of the base, in feet, B, we would have, in seconds of arc,

$$\text{angular value of 1 div. of bubble} = \frac{R}{BD \sin 1''}.^{1}$$

(*b*) *By a Large Vertical Circle.* Mount the bubble rigidly upon the circle, having its axis parallel to the plane of the circle.

[1] Log sin $1'' = 4.6855749$.

Move the bubble from end to end of tube, as before, reading the corresponding angular changes directly upon the circle. Divide the mean angular movement by the mean movement of bubble.

This requires a large circle with micrometer attachments, such as is used on astronomical instruments.

(*c*) *By a Level Trier.* This consists of a beam hinged at one end and moved vertically by means of a micrometer screw at the other. The bubble-tube is placed upon the beam, and the bubble moved back and forth by means of the screw, each revolution of which gives a known angular movement to the beam.

40. General Considerations. A bubble is sensitive directly as the length of the radius of curvature, or indirectly as its rate of curvature. It is also sensitive in proportion to its length, a long bubble [1] settling much more quickly and accurately than a short one. Some bubble-tubes have a chamber at one end connected with the main space by a small hole through the bottom of the dividing partition. This enables the length of the bubble to be under control. As ether expands and contracts very largely with temperature, the bubble is apt to be too long in winter and too short in summer if the chamber is not used. The bubble-tube should not be rigidly confined by metallic fastenings about its centre, if the value of one division is significant, as the changes of temperature will change its curvature. Bubble-tubes, or level-vials, as they are often called, may be sealed by glass stoppers set in a glue made by dissolving isinglass in hot water, and covering with gold-beater's skin set with the same glue, the whole varnished over when dry.

[1] This refers to the length of the air-bubble itself, and not to the glass tube.

The Engineer's Level

41. The Engineer's Wye Level consists of a telescopic horizontal line of sight joined to a spirit-level, the whole properly

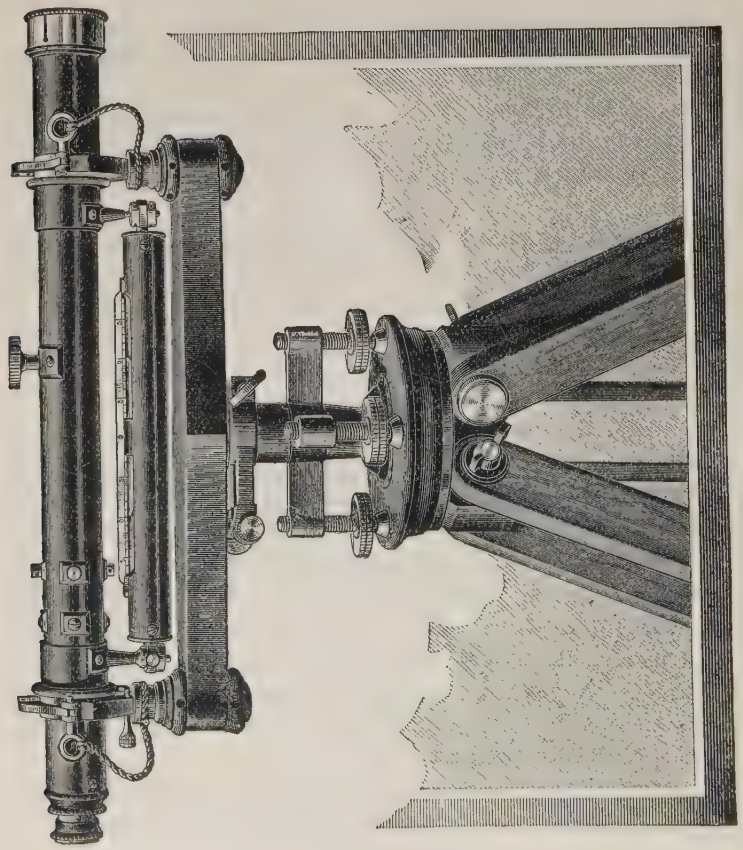


Fig. 14.

supported and revolving on a vertical axis. Such an instrument is shown in Fig. 14. The vertical parts of the frame which support the telescope are called wyes, and the cylindrical bear-

ings on the telescope-tube are called the pivot-rings. The telescope can be lifted out of the wyes by loosening the clips over the rings, these being held by the small pins attached to strings and shown in the cut. A clamp and tangent-screw are connected with the axis for holding it to a given pointing or for moving it horizontally while clamped. The attached bubble enables the line of sight in the telescope to be brought into a horizontal position.

The construction of the instrument is best shown by the sectional view given in Fig. 15.

The objective is a compound lens, the two parts having different refractive powers in order that the image may be flat, A simple lens gives a spherical image. The image is formed at the plane of the cross-wires, which are attached to the reticule held in place by the capstan-screws shown in the cut. The line of sight is the line joining the two corresponding points in object and image with which the intersection of the cross-wires coincides. Evidently this line of sight may lie anywhere in the field of view within the limits of movement of the reticule. The line of collimation is simply the true position of the line of sight. The eye-piece serves only to magnify the image, and sometimes to invert it, as is the case in the sectional view of Fig. 15. The image itself is always inverted; and if this be examined by an eye-piece of two lenses, which simply magnifies but does not invert, the object is seen in an inverted position. If four lenses are used in the eye-piece, it re-inverts the image so that the object is seen erect. This results in a loss of light and of distinctness.

Adjustments of the Wye Level

42. First. The Collimation or Cross-Wire Adjustment. This consists of two parts:

(*a*) To make the horizontal wire truly horizontal when the instrument is levelled. This adjustment can be tested on a plumb-

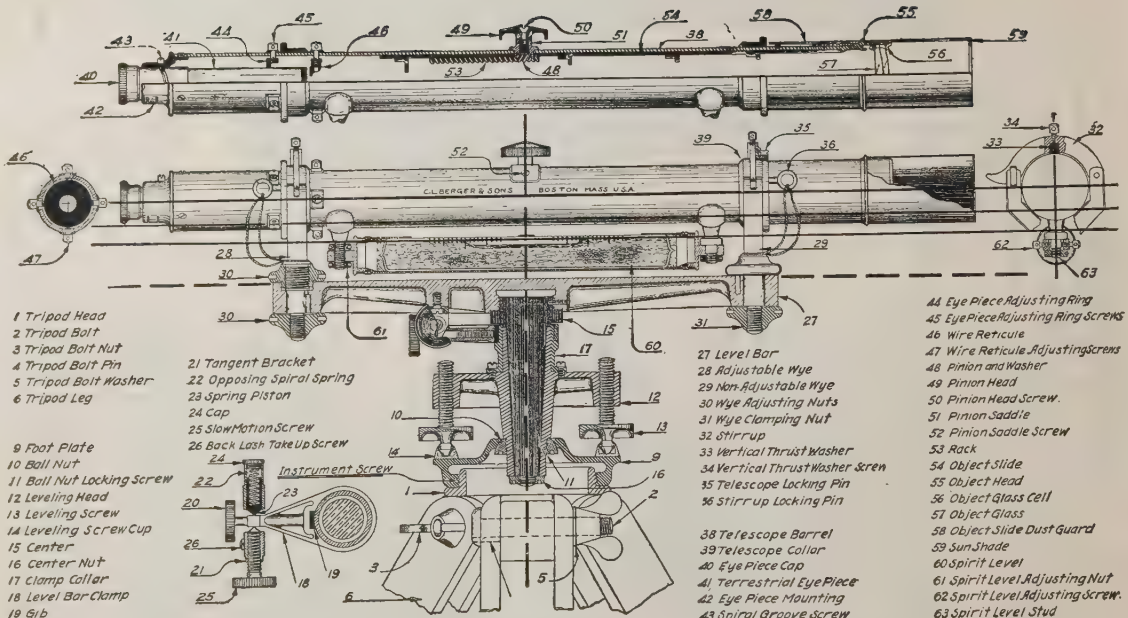

C.L.BERGER & SONS BOSTON MASS. U.S.A.
1 Tripod Head
2 Tripod Bolt
3 Tripod Bolt Nut
4 Tripod Bolt Pin
5 Tripod Bolt Washer
6 Tripod Leg
9 Foot Plate
10 Ball Nut
11 Ball Nut Locking Screw
12 Leveling Head
13 Leveling Screw
14 Leveling Screw Cup
15 Center
16 Center Nut
17 Clamp Collar
18 Level Bar Clamp
19 Gib
20 Clamp Screw
21 Tangent Bracket
22 Opposing Spiral Spring
23 Spring Piston
24 Cap
25 Slow Motion Screw
26 Back Lash Take Up Screw
Instrument Screw
27 Level Bar
28 Adjustable Wye
29 Non-Adjustable Wye
30 Wye Adjusting Nuts
31 Wye Clamping Nut
32 Stirrup
33 Vertical Thrust Washer
34 Vertical Thrust Washer Screw
35 Telescope Locking Pin
36 Stirrup Locking Pin
38 Telescope Barrel
39 Telescope Collar
40 Eye Piece Cap
41 Terrestrial Eye Piece
42 Eye Piece Mounting
43 Spiral Groove Screw
44 Eye Piece Adjusting Ring
45 Eye Piece Adjusting Ring Screws
46 Wire Reticule
47 Wire Reticule Adjusting Screws
48 Pinion and Washer
49 Pinion Head
50 Pinion Head Screw.
51 Pinion Saddle
52 Pinion Saddle Screw
53 Rack
54 Object Slide
55 Object Head
56 Object Glass Cell
57 Object Glass
58 Object Slide Dust Guard
59 Sun Shade
60 Spirit Level
61 Spirit Level Adjusting Nut
62 Spirit Level Adjusting Screw.
63 Spirit Level Stud

FIG. 15.

line or on the edge of a building and noting whether the vertical wire is coincident with the plumb-line.

If not it can be made to coincide by loosening the reticule screws and turning the reticule. As the horizontal and vertical cross-wires are placed at right angles by the maker, when the vertical wire is vertical the horizontal wire must needs be truly horizontal.

(*b*) The line of sight should be made to coincide with the axis of the telescope rings or pivots. See Fig. 16.

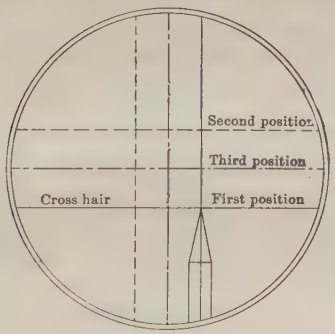


FIG. 16.—Adjustment of Cross-wires. 1st Part.

This relation can be established as follows: Loosen the clips which hold the telescope in the rings so that the telescope is free to move. Sight the intersection of the cross-wires on some well-defined point, using the level screws for the vertical motion and the clamp and tangent screw for the horizontal motion. Rotate the telescope 180° in the wyes so that the level vial is now directly above the telescope.

If in adjustment, the intersection of the cross-wire should still be upon the point. If not move the horizontal wire half way back to its first position by means of the lower and upper adjusting screws to the reticule. By the use of the other pair of screws the vertical cross-wire should be brought half way back

to its first position. The adjustment should be repeated until it is perfect and care taken to move the adjusting screws to a firm seat.

43. Second. The Lateral Adjustment of the Level-Vial. To put the axis of the level-vial in the same vertical plane with the line of sight, bring the bubble to the centre of its tube and rotate the telescope in the wyes a few degrees. If in adjustment, the bubble will continue in the centre. Otherwise the bubble will run to the higher end. Move the screws controlling the lateral movement of the level-vial until the bubble remains in the centre. With modern instruments this adjustment is not important, and seldom needs attention.

44. Third. To Make the Axis of the Level-Vial Parallel to the Line of Sight. For this purpose either of two methods is

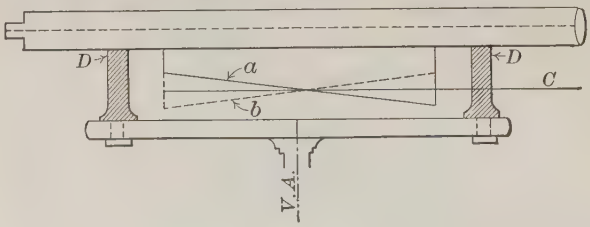


FIG. 17.

available, the *indirect* method or the *direct* or so-called *peg* method. Either method can be used with the wye level, but only the *peg* method is applicable to the dumpy level.

The Indirect or Wye Method. With the instrument clamped over one set of levelling screws, bring the bubble to the centre of its tube. Then lift the telescope out of the wyes and turning it end for end carefully replace it so that the eye end will come where the object end was before. See Fig. 17. If this operation be done with great care the bubble will remain in the centre if the instrument is in adjustment. Otherwise the run of the bubble is twice the actual error of adjustment. Bring the bubble half way back by the screws con-

trolling the vertical adjustment at one end of the bubble-tube. This test should be repeated to prove that the operation was skilfully performed.

45. The Direct or Peg Method. To make the axis of the bubble parallel to the line of sight. Fig. 18. This adjustment is commonly made in several different ways. The following is perhaps the simplest.

The instrument is set up midway between two hubs *N* and *S*. With the bubble in the centre the rod is held and read on these points. Even if out of adjustment the difference between the rod readings gives the true difference of level between *N* and *S*.

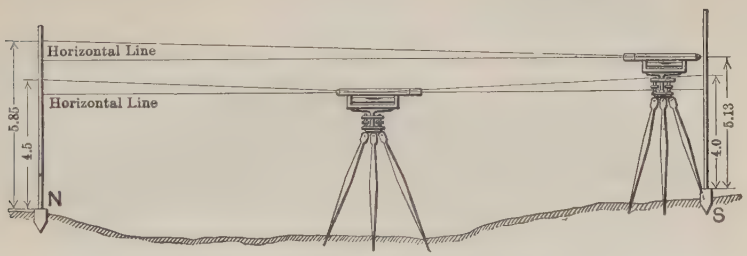


FIG. 18.—Adjustment by Peg Method.

The instrument is next set up close to the higher hub so that looking through the telescope with the eye at the *object* end a pencil point or target can be set in the exact centre of the small field of view and the reading taken. The rod is next held on the distant hub and read in the usual way. If the true difference of level between the hubs be added [1] to the near hub reading, it will give what the distant rod reading should be when the instrument is in adjustment. The difference between this amount and the actual distant rod reading represents the error in adjustment. To correct the error, keeping the line of sight at the

[1] For another method of making the peg adjustment preferred by many see Art. 489.

correct reading on the distant hub, move the adjusting screws on the bubble tube until the bubble returns to the centre. Both line of sight and axis of bubble are now horizontal and hence parallel. See Fig. 19. Example:

Instrument midway between N and S:		
Rod reading on S	=	4.00
Rod reading on N	=	4.50
True diff. of level	=	0.50 between N and S.
Instrument at S:		
Rod reading on S	=	5.13
Difference of level	=	0.50
		5.63
Curvature and Refraction Correction		.01
Correct reading on S	=	5.64
Actual reading on S	=	5.85
Error	=	0.21

46. Fourth. The Wye Adjustment. To make the axis of the wyes perpendicular to the vertical axis of the instrument (Fig. 20). Secure the telescope by fastening the two clips over it. Level the instrument and bring the bubble exactly to the centre of its tube over one set of levelling-screws, and then turn the instrument 180° about its vertical axis. If the bubble moves from the centre it should be brought half way back by means of the adjusting nuts at the foot of one wye support, and the other half corrected by the levelling-screws.

47. Relative Importance of the Adjustments. It should be noted that the wye adjustment in no way affects the accuracy of levelling, but by insuring that the bubble will always remain

very near the centre, saves some time in relevelling. The most important adjustment is that one which insures the parallelism of the line of sight with the axis of the level-vial. With this

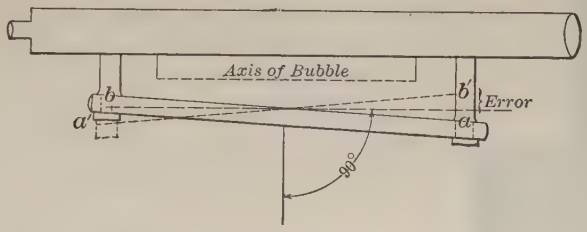


FIG. 19.—Adjustment of the Wyes.

relation established the line of sight must needs be horizontal when ever the bubble is in the centre.

It is inevitable that a small residual error will always be left in the adjustments. This can be overcome by a proper balancing in the lengths of sight.

THE DUMPY LEVEL

48. General Statement. The dumpy form of level is illustrated by Fig. 20, has always been most popular among European engineers. Because of its simpler construction and its cheapness this type of level is deserving of much greater popularity in this country. The following adjustments should be made:

ADJUSTMENTS OF THE DUMPY LEVEL

49. First. The Cross-Wire Adjustment. This is made as described in the first part (*a*) of Art. 42.

50. Second. The Level-Vial Adjustment. This consists in making the axis of the level-vial perpendicular to the vertical axis of

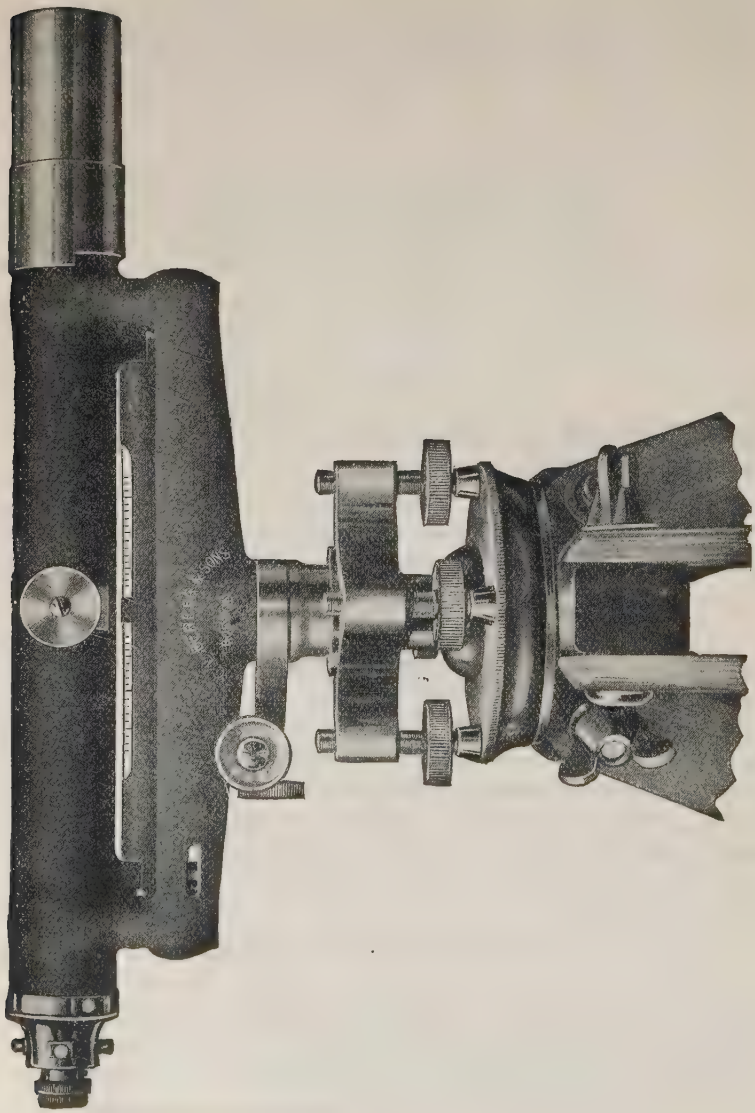


FIG. 20.—The Dumpy Level.

the instrument. Because of the manner in which the dumpy level is constructed, this adjustment must be made *before* making the line of sight parallel to the bubble axis. It is tested by the same programme as that used in the wye adjustment of the wye level, viz., after levelling the instrument over both sets of levelling-screws, the level is turned about its vertical axis 180° (see Fig. 21). If the bubble remains in the centre the adjustment is perfect. If the bubble does not remain in the centre, bring it half way back by means of the adjusting screws at one end of the bubble-tube. Bring the bubble back the remaining

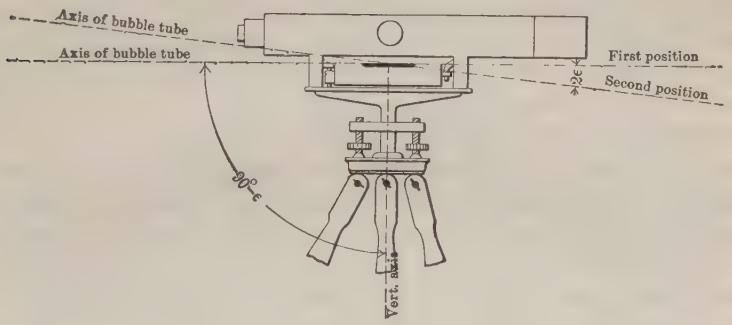


FIG. 21.

half by the levelling-screws and repeat the operation as a check.

51. The Line of Sight Adjustment. This consists in making the line of sight parallel to the axis of the bubble. It is tested by the so-called peg adjustment (see Art. 45), but the actual adjustment is made quite differently from the wye level. Thus after determining the true horizontal reading (5.70) to the distant point, *the line of sight* is adjusted to it by the use of the *reticule* screws, taking care that the bubble is at the same time in the middle of its tube. This should insure that both the line of sight and the axis of the bubble are now horizontal, and hence

parallel. This is the most important adjustment of the level, and corresponding care should be taken to insure its accuracy.

52. Focussing and Parallax. The function of the object glass is to gather and bring to a focus at the cross wires the many divergent rays of light from the object viewed. The eye-piece, however, serves primarily to magnify this image. In instruments constructed so that the object is seen in an erect position, the compound eye-piece also reinverts this image. The large magnifying powers used in ordinary field instruments (viz., 20 to 35 diameters) result in the focal length of the eye-piece being very small, about one-sixteenth of an inch. Both the image formed by the objective and the cross-wires should lie in the focus of the eye-piece in order that both may be seen at once. Now the image may be moved back and forth by moving the objective in or out, but the plane of the cross-wires is fixed. If the two are brought into the same plane, therefore, the image must be brought upon the wires. To accomplish this, first focus the eye-piece on the wires, so that they appear black and distinct. In doing this there should be no image visible, so that either the objective is thrown out of focus or the telescope is turned to the sky. The eye-piece is most accurately focussed by finding its inner and outer limits for distinct vision of the wires, and then setting it at the mean position. The objective may now be moved until the image also comes into focus. This will have to be done for each pointing if the distances are different. If the image is not brought into exact coincidence with the cross-hairs, these will seem to move slightly on the image as the eye is moved behind the eye-piece. This angular displacement of the wires on the image is called *parallax*, and can only occur when they are not in the same plane. It is removed by focussing the objective, thus moving the image, until there is no perceptible relative movement of wires and image as the eye is shifted, when they are practically in coincidence. If there is a parallax, the reading may be in error by its maximum angular amount. If the eye were always held at the centre of the eye-piece there would be

no parallax, and it is to accomplish this that the eye-piece is covered by a shield with a small hole in its centre. Still, the slight movement of the eye thus allowed is sufficient to cause some parallactic error if the wires and image are not practically coincident. When the eye-piece is once adjusted to distinct vision on the cross-wires it requires no further attention so long as the instrument is used by the same person. Another person, having eyes of a different focal range, would have to readjust the eye-piece. The eye-piece adjustment, therefore, is personal, and is made once for all for a given individual; while the objective adjustment depends on the distance of the object from the instrument, is made for each pointing, and is considered perfect when the parallax is removed.

The Levelling-Rod

53. The Levelling-Rod is used to measure the vertical distance from the line of sight down to the turning-point or bench-mark. There are two general classes, Self-reading, or Speaking, and Target Rods.

A Self-reading, or Speaking, Rod is one so graduated as to enable the observer to note at once the reading of the point which lies in the line of sight, this reading being in all cases the distance to the bottom of the rod. The rod-man here has nothing to do but to hold the rod vertical. The observer notes and records the reading.

A Target-rod is furnished with a sliding target moved by the rod-man is response to signals from the observer until it accurately coincides with the line of sight. Its position is then read with great accuracy by means of a vernier scale.

The most commonly used self-reading rod, called the Philadelphia, is shown in Fig. 23. Neither the New York nor Boston rods (Figs. 24 and 25) are self-reading. The latter one is rarely used.

It is necessary that the rod be held vertical when in use, and on sloping ground, or when the wind is blowing, it is difficult to do this. To insure a vertical rod, therefore, especially in the plane of the line of sight, two level-bubbles are sometimes attached, such as are shown in the accompanying cuts. When not in use they can be removed and folded up as shown.

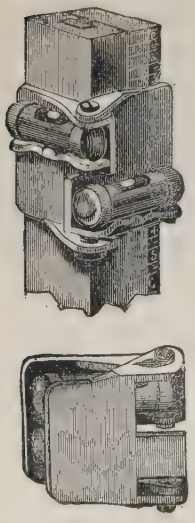


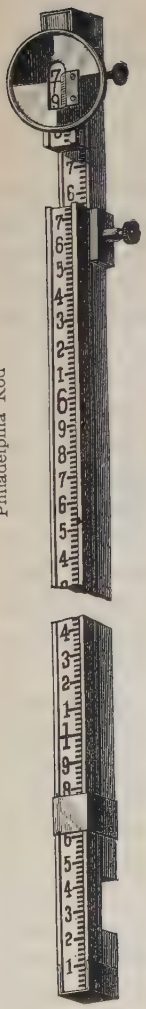
Fig. 22.

Another method of attaining the same end is by means of Thompson's Levelling Target, shown in Fig. 26. This target is bent at right angles, and so lies against two faces of the rod. If held so that both faces show, the middle dividing line will appear as a broken line when the rod is not vertical.

Most targets slide on the rod, and have a clamp screw and springs. When the rod is wet, the target is apt to stick and move with a jerking motion. The target shown in Fig. 27 is mounted on rollers in order to obviate this difficulty.

Various patterns of self-reading rods are used. For rough work a 12- or 14-foot rod, 2 inches wide and $1\frac{1}{4}$ inches thick, painted and fitted with an iron or brass shoe at bottom, graduated to hundredths of a foot, will be found very efficient. The graduations should be so distinct that they can be read through the telescope at a distance of 500 or 600 feet. The stadia rods shown in Fig. 110 are well adapted for rapid levelling.

Note. —In view of the now well-established fact that not only cheaper, but more accurate levels can be run with the speaking-rod, this method deserves to have a much wider use. Even on the most precise levels lines of the U. S. Coast and Geodetic Survey, speaking-rods are exclusively read. The greater accuracy appears to come partly from a more perfect compensation in the errors made in reading the rod.

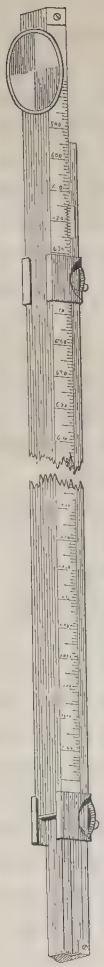


Philadelphia Rod

FIG. 23.

New York Rod

FIG. 24.



Boston Rod

FIG. 25.

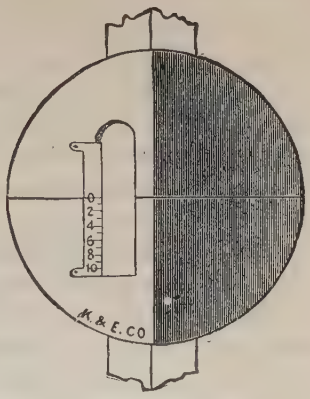


FIG. 26.

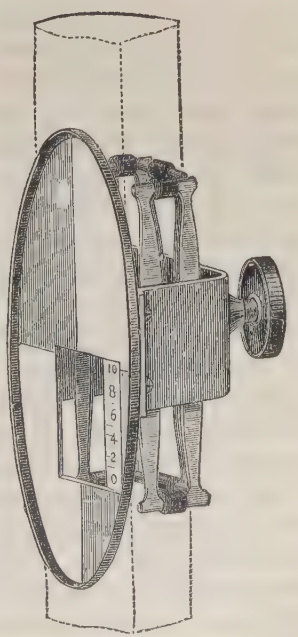


FIG. 27.

The Use of the Level

54. The Level is Used—

(*a*) To find the difference in elevation of points a considerable distance apart.

(*b*) To obtain the profile of a line.

(*c*) To establish a grade.

These objects may be more or less intermingled in any given piece of work. Whatever may be the ultimate object of the work, however, the immediate object for any given setting of the instrument is to find how much higher or lower a certain forward, or unknown, point is than a certain other back, or known, point. Thus, the rod being held on the known point, the line of sight is turned upon it and the rod-reading gives at once the height of instrument above that point. If the rod be now held on the forward, or unknown, point, and the line of sight turned upon it, this rod-reading gives the distance of that point below the line of sight. The reading on the known point is called the *back sight*, and that on the unknown point is called the *fore sight*. If the elevation of the known point be given, we find the elevation of the line of sight by adding the rod-reading at that point. By subtracting from this elevation the reading on the unknown point, the elevation of that point is obtained. Thus we have found the relative elevations of the two points by referring them both to the horizontal plane through the instrument. Since the back-sight reading gives the elevation of the instrument, and since this is always greater than the elevation of that point, it follows that the back-sight reading is essentially positive. For a similar reason the fore-sight reading is essentially negative, since any point on which the rod is held is lower than the line of sight. It will also be seen that there can be but one back sight (unless the height of the instrument is to be found from readings

on several known points, and the mean taken), while there can be any number of fore sights from one instrument position. Thus, the height of the instrument having been determined, the elevations of any number of points, in any direction, may be determined by referring them all to the horizontal plane through the instrument, whose elevation has been obtained by the single back-sight reading. It is also important to remember that the terms "back sight" and "fore sight" have no reference to directions or points of the compass, but they do have a rational significance when we think of the work proceeding from the known point to the unknown point or points. Thus, we refer *back* to the known point for height of instrument, and then transfer this knowledge *forward* to the points whose elevations we wish to find.

Differential Levelling

55. Differential Levelling consists in finding the difference of elevation of points a considerable distance apart. The elevation of the first point being known or assumed, the difference of elevation between this and any other point is found and added algebraically, thus giving the elevation of the second point. The "plane of reference" is the surface of zero-elevation and is generally called the "datum plane." This is not really a *plane*, but a *level* surface, according to the definition given in Art. 35. It is, however, universally denominated the "plane of reference," "datum plane," or simply "datum." The problem, then, is to find the difference of elevation between two distant points. If the points were near together and had not too great a difference of elevation, a single setting of the instrument would be sufficient. If they are too far apart for this, either in distance or in elevation, then more than one setting of the instrument must be made. In this case the intervening points occupied by the rod are called *turning-points*, the terminal

points being called *bench-marks.* The successive differences of elevation of these turning-points is determined by setting the level equally distant from them, and so they serve to divide up the total distance between terminal points into a series of short spaces, each of which can be covered by a single setting of the instrument. The successive differences of elevation of turning-points being found, their algebraic sum would be the difference of elevation of the terminal points, or bench-marks. But since all the back sights are essentially positive and all the fore sights are essentially negative, we may at once add all the back sights together and all the fore sights together, and take the difference of the sums. This is the difference of elevation between terminal points, and has the sign of the larger sum, the back sights being positive and the fore sights negative. This difference of elevation added algebraically to the elevation of the initial point gives the elevation of the final point. Evidently the route travelled in passing from one bench-mark to another is of no consequence so long as the true difference of elevation is obtained.

56. Length of Sights. Where the ground is nearly level it is desirable to make the length of sights (distance from instrument to rod) as long as practicable, in order to increase the rate of progress. For the best work this distance may be from 100 to 300 feet, according to the state of the atmosphere. When the air and ground differ greatly in temperature there result innumerable little upward and downward currents of air, the upward being warmer than the downward currents. The warmer air is more rarefied than the colder, and thus a ray of light passing from the rod to the instrument passes alternately through denser and rarer media, each change producing a slight refraction of the ray. This causes a peculiar tremulous condition of the image in the telescope, so that it is difficult to determine just what part of it is covered by the cross-hairs. At such times the air is said to be "boiling," or "dancing" or "unsteady." It always occurs more or less in clear weather, owing to

the earth then being hotter than the air, and it varies with the quality of the soil, cinders or gravel being very bad. When the air is in this condition the length of sights should be shortened.

The back and fore sights for any setting of the instrument should always be equal in length. Levelling is the only kind of field surveying wherein the instrumental errors may be thoroughly eliminated without duplicating the observations. This may be done in levelling by making the back and fore sights of equal length. For, since the *difference* between back and fore sights is always the quantity used, it follows that if both are too large or too small by the same amount, the *difference* will be unchanged. If, when the bubble is in the middle of its tube, the line of sight is inclined upwards by a given small angle, then it has this relation to the horizontal on both fore and back sights, and *if the lengths of sights were equal* the fore and back rod-readings were equally in error. It is therefore very desirable that these sights should be made of equal length. Moreover, the effect of the earth's curvature is eliminated by so doing, however long the sights may be.[1] There are other kinds of errors that are not eliminated by this means, but those that are eliminated are of sufficient importance to warrant great care to secure equal sights for each setting. If it is impossible to do this at any time, the inequality should be balanced off at the next one or two settings, by making them unequal in the opposite direction by the same amount. The equality of sights can be determined by pacing with sufficient accuracy.

57. Bench-Marks are fixed points of more or less permanent character whose elevations are determined and recorded for future reference. The general and particular location of a bench-mark should be so distinctly described that anyone could find it from its description. Whenever the work is temporarily interrupted a temporary bench-mark is set, such as a substantial

[1] The student should prepare a drawing showing the proof of these statements.

stake driven into the ground, or a spike in the root of a tree. The prime requisite of a good bench-mark is that it shall not change its elevation during the period in which it is to be used. If this period is not more than two or three years, a spike driven in the spreading root of a tree near the trunk and well above ground will serve. The wood should be trimmed away from it so as to leave a projecting spur that will not be overgrown. The tree itself should then be marked by notching or otherwise, and carefully located in the description.

If the mark is to serve for from five to fifty years, stone or brick structures or natural rock should be selected. The water-tables, or corners of stone steps, of buildings, copings of foundation and retaining walls, piers and abutments of bridges, or copper bolts leaded in natural rock may serve. If artificial structures are chosen, those should be selected which have probably settled to a fixed position, and for this reason old structures are preferable to new ones.

When stakes are used for temporary benches it is often advisable to set two or even three for a check. In this case the *mean elevation* is the elevation used. In starting from such a series of benches there would be as many back sights for the first setting of the instrument as there were benches, the mean of which, added to the mean elevation of the benches, would give the height of instrument. In running a continuous line of levels it is advisable to set a bench-mark at least as often as one to the mile.

58. The Record in differential levelling is very simple. The bubble always being put in the middle of the tube, and the rod positions chosen equally distant from the instrument, the bubble-reading and the length of sights may be omitted from the record, unless some knowledge of the distance run is desired, when the length of sights may be inserted. If it is inconvenient to make corresponding back sights and fore sights equal their totals between bench-marks should be made equal as in the following example:

FORM OF RECORD FOR DIFFERENTIAL LEVELLING [1]

Sta.	B. S. +	H. I.	F. S. −	Elevation	Distance, B. S.	Distance, F. S.	Remarks.
B.M. 1				886.35			[1] Cross on foundation of Science Hall, U. W. Campus 7 ft. north of E. steps.
	9.42				40		
		895.77	0.74	895.03		50	
	12.16				60		*Party.*
		907.19	2.34	904.85		70	Levelman, D. Show.
	10.47				90		Rodman, H. Beach.
		915.32	1.74	913.58		70	Oct. 1, 1909.
	8.97				40		
		922.55	3.72	918.83		60	
	10.72				50		
B.M. 2		929.55	7.50	922.05		50	Top of hydrant N.E. cor. Main and North Sts.

[1] Some engineers prefer to put *both* the F. S. and B. S. on any turning-point on the *same* line.

If preferred the H. I. and Elevation columns may be omitted. The difference between the sum of the B. S. and F. S. columns gives directly the difference of level. In the above example if this difference, +35.70, be added to the elevation of B. M. 1, 886.35, we obtain the elevation of B. M. 2, 922.05.

59. Precautions in Level Work should always be taken if the best results are to be obtained. The adjustments should be tested every day, especially the parallelism of bubble-axis and line of sight. The instrument and rod should both be set in firm ground. An iron pin, with a spherical top, 6 to 8 inches long, and tapering to a point, should be used for the turning-point. A rope or leather noose should be passed through an eye at top to serve as a handle. To hold the rod upright the rodman should stand squarely behind it and keep it balanced on the pin. When the target is set and clamped the rodman reads it and records it on a paper he carries for the purpose. He then carries it to the observer, if it was a back-sight reading, or he awaits the coming of the observer if it was a fore-sight reading, when the observer also reads it

and records it in his note book. The rodman then calls off his reading, and the observer notes its agreement with his recorded reading. In this way two wholly independent readings are obtained and any erroneous reading corrected. Errors of one foot or one tenth are not very uncommon in reading target rods. The rodman should be especially careful to protect the turning-point from all disturbances between the forward and back readings upon it. The observer must not only obtain an accurate bisection on the target, but he must know that the bubble is accurately in the centre of the tube when this bisection is obtained. When the observer walks forward to set his instrument he counts his paces, and takes as long a sight as the nature of the ground or the condition of the atmosphere will allow. When the rodman comes up he counts his paces to the instrument and then goes *the same distance* in advance. Thus the observer controls the length of sights, making them whatever he likes; and it is the business of the rodman to see that the back and fore sight for every instrument-station are equal.

Profile Levelling

60. In Profile Levelling the object is to obtain a profile of the surface of the ground on certain established lines. Here both the distances from, and the elevation above, some fixed initial point must be obtained. When the line is laid out stakes are usually driven every hundred feet, these positions being obtained by a chain or tape. It is now the business of the leveller to obtain the elevation of the ground at each of these stakes, and at as many other intermediate points as may be necessary to enable him to draw a fairly accurate profile of the ground. The 100-foot stakes are usually numbered, and these numbers are entered on the level record. The intermediate points are called pluses. Thus, a point 40 feet beyond the twenty-fifth 100-foot stake is called 25+40, being really 2,540 feet from the initial point. It is evident that no plus-

distance can be more than 100 feet, and these are usually paced by the rodman. The intermediate points are selected with reference to their value in determining the profile. These are points where the slope changes, being mostly maximum and minimum points, or the tops of ridges and bottoms of hollows. Turning-points are selected at proper distances, depending on the accuracy required, and these may or may not be points in the line whose profile is desired. The levelling instrument also is not set on line, if it is found more convenient to set it off the line.

In profile levelling, since absolute elevations with reference to the datum-plane are to be obtained from every instrument-position, it is necessary to find the height of instrument above datum for every setting, and from this height of instrument, obtained by a single back-sight reading on the last turning-point, the elevations of any number of points are found by subtracting the readings upon them.

61. The Record in profile levelling is much more elaborate than in differential levelling. The form on following page is considered very convenient for profile work where the line has been laid out and 100-foot stakes set.

It will be noted that there is but one back sight and one height of instrument for each setting. The back-sight and fore-sight readings from the same instrument-station are not found here on the same line, but the fore- and back-readings on the same turning-point are on the same line. Thus, the rod was first read on the bench-mark whose elevation was known to be 886.347 feet above the datum. The reading on this bench was 1.00, thus giving a height of instrument of 887.347. This is marked B. M. in the station column, and evidently has but one reading upon it in starting the work from it. A series of intermediate sights are then taken at various 100-foot stakes and pluses, the readings on which, when subtracted from the H. I., give the surface elevations at those points. When the work has progressed as far in front of the instrument as the B. M. was back of it, a turning-point is set, and the reading upon it

recorded in the column of fore sights. This reading was 11.794, which, subtracted from the H. I. 887.347, gives 875.553 as the elevation of the turning-point. The instrument is now moved forward and a back-sight reading taken upon this T. P. of 1.249.

FIELD NOTES, PROFILE LEVELLING

Station.	B. S. +	H. I.	I. S. –	F. S. –	Elevation	Remarks.
S.H.B.M. *	1.00	.....	.....	.....	886.347	* B.M. on foundation
0+00	.....	887.347	1.2	.....	886.1	of Science Hall U.
+11	.....	.....	2.1	.....	885.2	W. Campus 9 ft.
+21	.....	.....	3.5	.....	873.8	north of E. steps.
1+3.3	.....	.....	6.7	.....	880.6	*Party.*
+27	.....	.....	9.1	.....	878.2	Levelman, D. Jones.
T.P.	1.249	.....	.....	11.794	875.553	Rodman, H. Price.
1+75	.....	876.802	5.9	.....	870.9	.....
+95	.....	.....	7.9	.....	868.9	June 1, 1910.
2+14	.....	.....	4.2	.....	872.6	.....
+50	.....	.....	7.2	.....	869.6	.....
T.P.	0.941	.....	.....	11.420	865.382	.....
2+95	.....	866.323	3.7	.....	862.6	.....
3+00	.....	.....	1.7	.....	884.6	.....
+75	.....	.....	5.7	.....	880.6	.....
4+0	.....	.....	5.7	.....	880.6	.....
T.P.	11.032	.....	.....	4.973	861.351	
4+18 B.M.	6.728	872.383	.....	6.728	865.655	* B.M. Top of large rock
.....	9.727	872.383	.....	2.282	870.101	shore of lake 50 feet
.....	.....	879.828	.....	3.736	876.092	east of Park St.
.....	12.021	888.113	.....	1.772	886.341	

In the above headings, B. S. denotes back sight; F. S., fore sight; I. S., intermediate sight; H. I., height of instrument; T. P., turning-point; B. M., bench-mark; Sta., station.

which added to 875.553 gives 876.802 as the new H. I. The readings on bench-marks and turning-points are made to thousandths, while the intermediate sights for surface elevation are read only to tenths of a foot. The last elevation obtained is checked by adding the back sights and fore sights, taking the difference and applying it to the elevation of the initial point with its proper

sign, remembering that back sights are positive and fore sights negative. The profile is now constructed by the data found in Elevation and Station columns. In constructing the profile, some kind of profile or cross-section paper is used, and the horizontal scale made much smaller than the vertical. Thus, if the horizontal scale were 400 feet to the inch, the vertical scale might be 10 or 20 feet to the inch.

Levelling for Fixing a Grade

62. In Fixing a Grade the profile may be obtained and the grade marked upon it. The vertical distance between the surface line and the grade line at any point is the depth of cut or fill at that point, and this may be marked on the line stakes at once, without the aid of the level or rod, if only the centre depths are desired, as in the case of a ditch or trench. If the sides are to have a required slope, however, the level and rod are necessary to fix the horizontal distance of the limiting or "slope" stakes from the centre stakes whenever the ground is not strictly a level surface. This operation is called "cross-sectioning," and is described in Chapter XIII, on Determination of Volumes.

If the grade be known before the profile is determined, together with the absolute elevation of the initial point, as is sometimes the case with ditches and trenches for pipe lines or sewers, then the depth of cut (or fill) may be at once determined and marked on the line stakes when the profile is taken. The form of record might be the same as given above for profile levelling, with the addition of two columns after the "Station" column, one being Elevation of Grade, and the other Cut or Fill. The elevation of grade would be found for each profile point by adding if an up, and subtracting if a down, grade, the differences of elevation corresponding to the successive distances in the profile. The difference between the corresponding "surface elevation" and "elevation of grade" would be the cut or fill at each point,

which could be at once taken out and marked on the line stake. This point is explained more fully in Art. 366.

The Hand-Level

63. Locke's Hand-Level is a very convenient little instrument for rough work, such as is done on reconnaissance expeditions. It consists of a telescope with a bubble attached in such a way that the position of the bubble is seen by looking through the telescope. A horizontal line of sight is thus readily determined. If the instrument be out of adjustment the cross-wire can be adjusted by the use of the peg adjustment, using screws provided for this purpose. See Fig. 28.

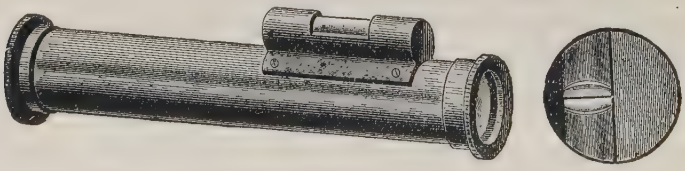


Fig. 28.

EXERCISES WITH THE LEVEL

1. Adjust the bubble to the line of sight by the first, or indirect, method, and then test it by the second, or direct, method. If this second method does not show it to be in adjustment, where does the error lie?

2. Cause the line of sight and bubble-axis to make a considerable angle with each other (that is, put it badly out of adjustment in this particular), and level around a block or two, closing on the starting-point, being careful to make back and fore sights as nearly equal as possible. Of course the final elevation of the point should agree with the assumed initial elevation. The difference of these elevations is the error of closure of the level polygon. If the back and fore sights were exactly equal this should be zero, notwithstanding the erroneous adjustment.

3. Put the instrument in accurate adjustment, and level over the same polygon as before, making the back and fore sights quite unequal, and note the error of closure. If the instrument were in exact adjustment and there were no errors of observation, should the error of closure be zero?

4. Range out a line on uneven ground about a half-mile in length, and set stakes every hundred feet. Let each student determine the profile independently. When all have finished, let them copy their profiles on the same piece of tracing-cloth, starting at a common point. The vertical scale should be large, so as to scatter the several profile lines sufficiently on the tracing. Each profile should be in a different color or character of line.

5. Select a line on nearly level ground, about a half-mile in length. Establish a substantial bench-mark at each end. Let each student determine the difference of elevation of these benches twice, running forward and back. See if the results are affected by the direction in which the line is run.

If each student could do this several times some evidence would be obtained as to there being such a thing as "personal equation" in levelling; that is, each person tending to always obtain results too high or too low. Why is it improbable that there could be any personal equation in levelling?

CHAPTER IV

INSTRUMENTS FOR MEASURING ANGLES

The Transit

64. The Engineer's Transit is the most useful and universal of all surveying instruments. Besides measuring horizontal and vertical angles it will read distances by means of stadia wires, determine bearings by means of the magnetic needle, do the work of a solar compass by means of a special attachment, and do levelling by means of a bubble attached to the telescope. It is therefore competent to perform all the kinds of service rendered by any of the instruments heretofore described, and is sometimes called the "universal instrument." A cut of this instrument is shown in Fig. 29. Fig. 30 is a sectional view through the axis of a transit of different manufacture.

The telescope, needle-circle, and vernier plates are rigidly attached to the inner or solid spindle which turns in the hollow outer spindle, Fig. 30. This portion of the instrument is called the *alidade,* as it is the part to which the line of sight is attached. The outer centre or spindle carries the horizontal limb and may itself revolve in the outer socket attached to the levelling-head. The inner and outer centres are held to the tripod by a ball-and-socket joint marked 7 in the cut. Either or both of these spindles or centres may be made rigid by means of proper clamping devices. If the spindle carrying the horizontal limb be clamped rigidly to the levelling-head and the alidade spindle be allowed to revolve, then horizontal angles may be read by noting the vernier readings on the fixed horizontal limb for the different pointings of telescope. If the horizontal limb itself be set and clamped so that

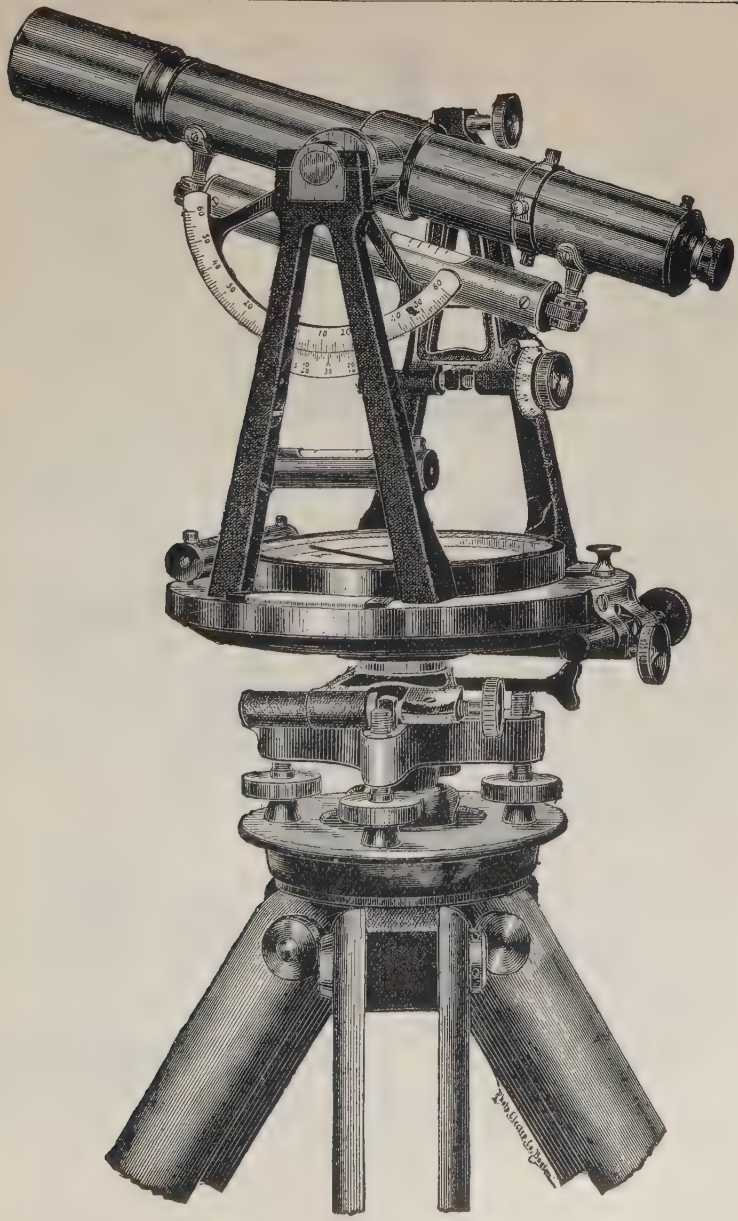


FIG. 29.

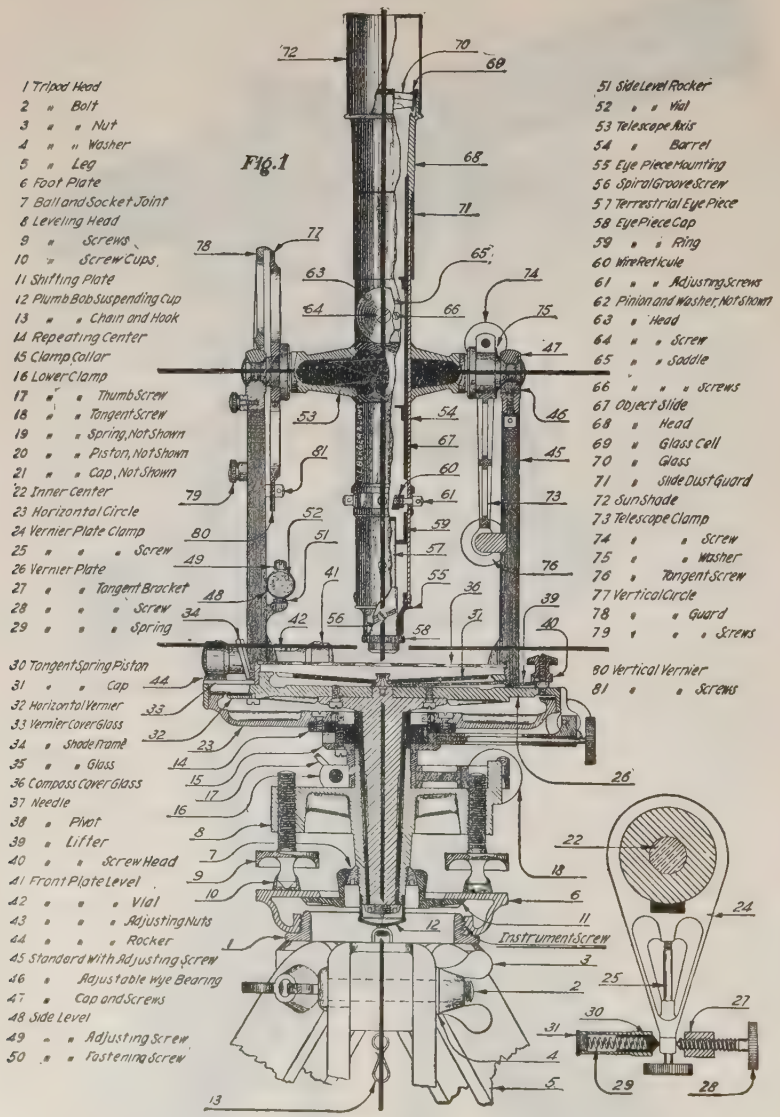


Fig. 1
1 Tripod Head
2 " Bolt
3 " " Nut
4 " " Washer
5 " Leg
6 Foot Plate
7 Ball and Socket Joint
8 Leveling Head
9 " Screws
10 " Screw Cups
11 Shifting Plate
12 Plumb Bob Suspending Cup
13 " " Chain and Hook
14 Repeating Center
15 Clamp Collar
16 Lower Clamp
17 " " Thumb Screw
18 " " Tangent Screw
19 " " Spring, Not Shown
20 " " Piston, Not Shown
21 " " Cap, Not Shown
22 Inner Center
23 Horizontal Circle
24 Vernier Plate Clamp
25 " " " Screw
26 Vernier Plate
27 " " Tangent Bracket
28 " " " Screw
29 " " " Spring
30 Tangent Spring Piston
31 " " Cap
32 Horizontal Vernier
33 Vernier Cover Glass
34 " Shade Frame
35 " " Glass
36 Compass Cover Glass
37 Needle
38 " Pivot
39 " Lifter
40 " " Screw Head
41 Front Plate Level
42 " " " Vial
43 " " " Adjusting Nuts
44 " " " Rocker
45 Standard with Adjusting Screw
46 " Adjustable Wye Bearing
47 " Cap and Screws
48 Side Level
49 " " Adjusting Screw
50 " " Fastening Screw
51 Side Level Rocker
52 " " Vial
53 Telescope Axis
54 " Barrel
55 Eye Piece Mounting
56 Spiral Groove Screw
57 Terrestrial Eye Piece
58 Eye Piece Cap
59 " " Ring
60 Wire Reticule
61 " " Adjusting Screws
62 Pinion and Washer, Not Shown
63 " Head
64 " " Screw
65 " " Saddle
66 " " " Screws
67 Object Slide
68 " Head
69 " Glass Cell
70 " Glass
71 " Slide Dust Guard
72 Sun Shade
73 Telescope Clamp
74 " " Screw
75 " " Washer
76 " Tangent Screw
77 Vertical Circle
78 " " Guard
79 " " " Screws
80 Vertical Vernier
81 " " Screws
Instrument Screw

FIG. 30.

one of the verniers reads zero when the telescope is on the meridian, then for any other pointing of the telescope the reading of this same vernier gives the true azimuth of the line. See Art. 91. It is necessary, therefore, to have two independent movements of telescope and horizontal limb on the same vertical axis. The magnetic needle is shown in the compass-box. The plumb-line is attached, as shown; this should always be in the vertical line passing through the centre of the graduated horizontal circle. This will be the case when it is attached directly to the axis itself, for this must always be made vertical.

The limb is graduated from zero to 360°, and sometimes with a second set of figures to 90° or 180°. There are two verniers reading on the horizontal limb 180° apart. Both the instruments shown in Figs. 29 and 30 have shifting centres, enabling the final adjustment of the instrument over a point to be made by moving it on the tripod-head. The telescope is shorter than those used in levelling-instruments in order that it may be revolved on its horizontal axis without having the standards too high. The movement of this axis is controlled by a clamp and slow-motion screw. Two level-vials are provided on the alidade, one parallel and one at right angles to the horizontal axis, for levelling the instrument, making the limbs truly horizontal.

The Telescope

65. The Telescope. The construction of a transit telescope is very similar to the level telescope described in Art. 41. Its essential parts the objective, cross-wires and eye-piece, are shown in Fig. 30.

66. The Objective. As already explained, the function of the objective is to gather the rays of light from the object sighted. The advantage of a large objective is shown by the fact that an objective 1.25 inches in diameter will gather 50 per cent more light than an objective one inch in diameter, while on

objective 1.5 inches in diameter has 2.25 times the capacity of the 1-inch objective.

The cross-wires are placed in the principal focus of the objective, near the eye-piece. The objective is screwed into the end of a tube inside the telescope tube proper. A rack-and-pinion screw provides for bringing the image of the object to a focus at the cross-wires. The construction of the instrument insures that the motion of the tube shall be parallel to the line of collimation.

67. The Eye-Piece has a single purpose, the magnification of the image. A suitable arrangement is provided for moving the focal plane of the eye-piece into coincidence with the plane of the cross-wires, and if at the same time the plane of the image has been brought to coincide with that of the cross-wires, the instrument is said to be focussed. The transit eye-piece may be either of the erecting or the inverting type. The former is much more common in the United States in spite of its disadvantages. It is worthy of consideration that with the same length of telescope a much greater magnifying power and clearer images may be obtained by using the inverting eye-piece. The necessity of transiting the transit telescope limits its allowable length. As the advantages of the inverted eye-piece become better known and appreciated it seems certain to replace the erecting eye-piece.

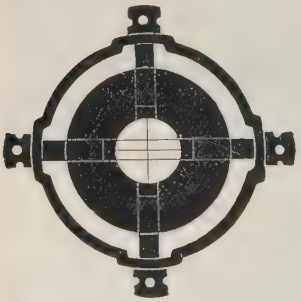


FIG. 31.—Cross-section of Telescope showing the Wire Diaphragm in Natural Size.

The cross-wires are commonly made of fine opaque spider threads, though sometimes platinum wires are used. These are stretched across a metallic ring at right angles to each other and fastened by shellac. This ring, called the reticule, is held in place by four capstan-headed screws. See Fig. 31.

Care should be taken not to tighten one of these screws without first loosening the opposing screw, otherwise the brass screw could be twisted off. In addition to the horizontal and vertical cross-wires, the engineer's transit is usually provided with two additional horizontal wires on opposite side of and equally distant from the central horizontal cross-wire. These extra wires are essential in stadia measurements. See Art. 212.

68. Magnifying Power. The magnifying power of a telescope should depend upon the relative focal lengths of the objective and eye-piece. The greater the power of the eye-piece, the smaller the field of view and the dimmer the image, a result which should be counterbalanced by increasing the diameter of the object glass. The powers used on good engineers' transits varies between 20 and 30 diameters.

Of the several ways of measuring the magnifying power of a telescope the following is the most simple: A level-rod or stadia rod is held vertically at a distance of 50 to 100 feet away from the transit. While one eye is reading the number of symbols included between the stadia wires in the telescope field of view, the other eye can observe outside the telescope the number of symbols corresponding. Thus at a distance of 50 feet the stadia wires are seen to subtend five tenths of a foot while in the same visual angle the unaided eye sees 100 tenths. The power would then be $\frac{100}{5} = 20$.

The magnifying power of the telescope and the least count of the vernier should be designed so that the least noticeable motion of the cross-wires on an object should cause a barely perceptible change of the vernier and vice versa.

69. The Vernier. This is a device, named after its inventor, for measuring smaller subdivisions of the scale or circle than can be done by estimation. The device depends upon the fact that the eye can judge when two lines coincide much more accurately than it can estimate the fractional part of a

space.[1] To understand the use of a vernier suppose that two bars of equal length, the one divided into N parts and the other into $N+1$ parts, are placed side by side as shown in Fig. 32(I). Now suppose that one of these bars, say the left one, remains stationary and the right one slides along it. Just after it starts to move no line on one bar will coincide with a line on the other, but after the lower bar has moved upward the distance *a-b*, the first line marked *a* will come into coincidence with the line marked *b* of the left bar. This gives a vernier reading of 3.01, the distance *ab* being $\frac{1}{10}$ of the smallest division on the scale. It is obvious that to make *c* coincide with *d*, the lower bar must move upward to twice the distance *ab*, hence such a reading (see Fig. 32 (II)) would indicate 3.02. And in general the number of the line on the right-hand bar (the vernier) which coincides with a line of the scale will be the number of tenths of the smallest division of the scale that the zero has passed the next lower division on the scale. This type of vernier is commonly used on level rods. It will be noted that *the zero of the vernier scale always marks the point* on the limb [2] whose reading is desired. Such a reading consists of two parts: First, noting the last division on the scale or limb passed by the zero of the vernier, and second, counting the number of lines *on the vernier* between its zero and the point of coincidence with the scale. (The particular division on the scale which happens to be in coincidence is of no consequence.) The scale reading is then

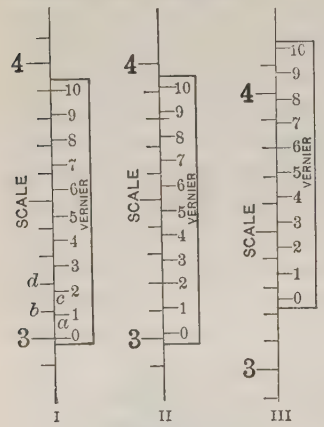


Fig. 32.

[1] The introduction of this invention in 1631 revolutionized the construction of angle-measuring instruments as well as the methods of using them. The device is named after its inventor, Pierre Vernier.

[2] The graduated scale on a transit or compass is called the limb.

obtained by adding the latter number of units to the first. Verniers are distinguished as direct or retrograde. In the former a division of the vernier is less than the smallest division on the scale, while in the latter the vernier division is greater. The direct vernier reads forward from its zero, while the retrograde vernier reads backward from its zero point.

70. Transit Verniers. Since angles may be measured to the right or to the left, the verniers used on transits are usually double, i.e., there is a single vernier on each side of the zero or index point. This will be better understood by reference to Fig. 33, which shows a form of vernier most commonly used

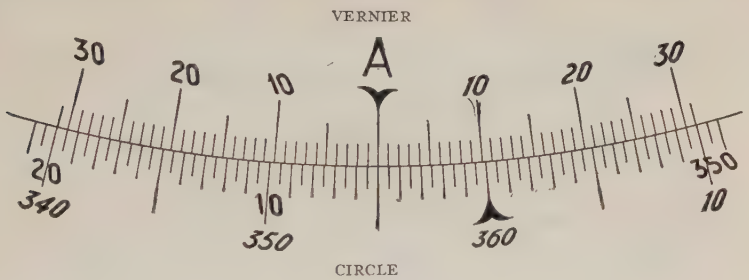


FIG. 33.—Vernier reads to minutes set at 5° or 355°.

when reading only to single minutes. In this case the circle is divided into degrees and half degrees while the vernier is constructed by dividing the length of 29 half degrees into 30 equal parts. Each space on the vernier is then equal to $\frac{29}{30} \times 30' = 29'$. The difference then between the length of the 30′ division of the circle and the 29′ division of the vernier is one minute of arc. In Fig. 33 the zero of the vernier coincides with the 5° and 355° marks on the circle. The first graduation on the vernier to the right of the zero fails to coincide with the 30′ line by exactly 1′ of arc. The second line on the vernier falls 2′ short of the 1° mark and the third line 3′ short of the 1° 30′ mark, etc. Hence if the vernier were moved 1′ to the left its first line would coincide

and the reading would be 0° 01′. If the vernier were moved 1′ more the second line would coincide and the reading would be 0° 2′, etc. In general the number of the line on the vernier which is in coincidence with some line on the circle is the number of minutes to be added to the last adjacent graduation on the limb. After the vernier has passed the half-degree line on the circle care should be taken to add 30′ to the vernier reading.

The following figures show the more common types of vernier used on transits.

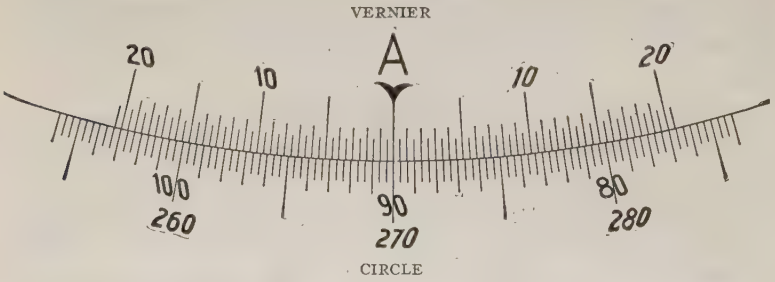


FIG. 34.—This is a double vernier. The circle is divided into 20′ spaces, and it will be noted that 40 spaces on the vernier correspond in length to 39 divisions on the circle. The vernier therefore reads to $\frac{1}{40}$ of the smallest division on the circle, 20′, or $\frac{1}{2}' = 30''$. Read that part of the double vernier on the side of zero toward which the angle in question is increasing. Thus the reading of the inner figures is 90° while of the outer figures is 270°.

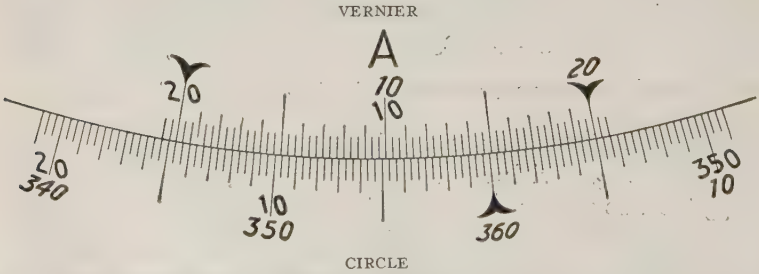
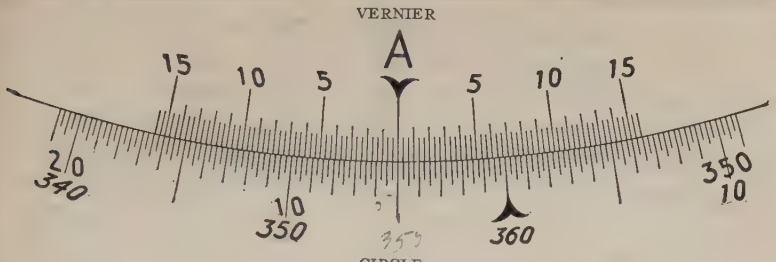
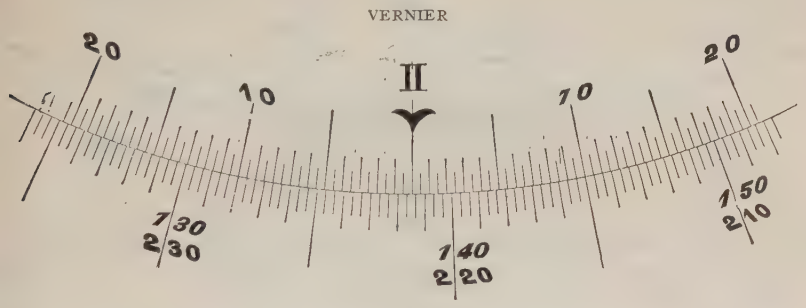


FIG. 35.—This vernier is also a double vernier. The arrow at the right is zero when reading angles to the left (clockwise), while the arrow at the left is zero for angles increasing to the right. 1 division on circle = 20′. 59 divisions on circle = 60 divisions on vernier. Vernier reads to $\frac{1}{60} \times 20' = \frac{1}{3} = 20''$. Reading of inner row of figures, 355°–0′. Reading of outer row of figures, 345°–20′.



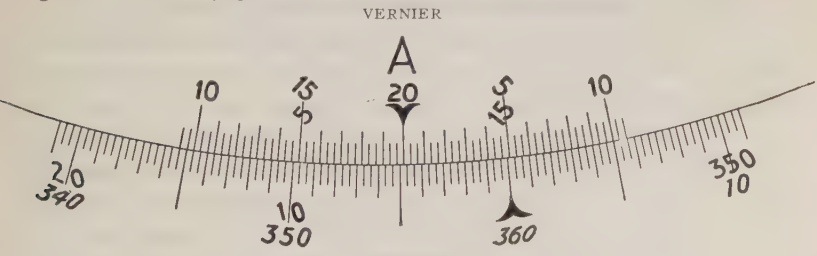
CIRCLE

IG. 36.—This is a double vernier also, each degree of the circle being divided into 4 parts or 15′. Note that 44 divisions of the circle is equal to 45 divisions of the vernier. The least count of this vernier is therefore $\frac{1}{45} \times 15' = \frac{1}{3} = 20''$. The reading on the inner figures is 5°, on the outer, 355°.



CIRCLE

IG. 37.—Double vernier reading to 30″. 39 divisions of circle divided to equal 40 divisiont of the vernier. Inner row of figures reads 138° 30′ 30″. Outer row of figures reads 221° 29′ 30″.



CIRCLE

IG. 38.—Double vernier reading to 20″. If the odd number of minutes is not greater than ten the reading is made by counting divisions from the index mark (zero). If the number of minutes is greater than ten the reading is secured by looking for coincidence between the 10 mark on the other half of the vernier and the index point. Inner row of figures reads 4° 50′. Outer row of figures reads 355° 10′.

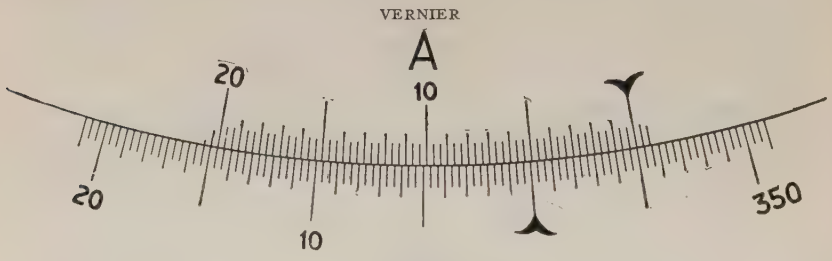


FIG. 39.—Single vernier reading to 20″. Reading of circle is 355°

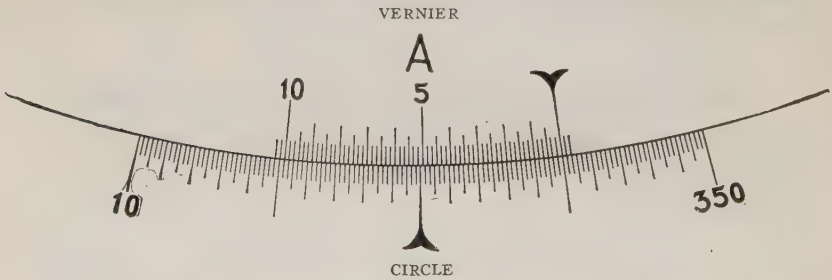


FIG. 40 Single vernier, circle divided into 10-minute spaces, 59 of which are divided to make 60 divisions of the vernier. Vernier reads to $\frac{1}{60} \times 10' = 10''$. Circle reads 355°.

ADJUSTMENTS OF THE TRANSIT

71. The Adjustments of the Engineer's Transit are such as to cause (1) the instrument to revolve in a horizontal plane about a vertical axis, (2) the line of collimation to generate a vertical plane through the instrument axis when the telescope is revolved on its horizontal axis, (3) the axis of the telescope bubble to be parallel to the line of collimation, thus enabling the instrument to do levelling, and (4) the vernier on the vertical circle so adjusted that its readings shall be the true altitude of the line of collimation. These four results are attained by making the following five adjustments, nearly all being based upon the principle of reversion:

72. First. The Plate-Bubble Adjustment. *To make the axis of each plate bubble lie in a plane perpendicular to the vertical axis.* This adjustment is the same as with the compass. (One of the plate bubbles is usually set on one pair of standards.) Bring both bubbles to the centre, revolve 180°, correct one-half the movement on the levelling-screws and the other half by raising or lowering the adjustable end of the bubble-tube. In Fig. 41, (1) shows plate levelled up; (2) shows plate turned 180° about the vertical axis; (3) shows the bubble brought back half way to the centre by the bubble-tube adjusting-screws; and (4) shows the bubble brought entirely back to the centre by the levelling-

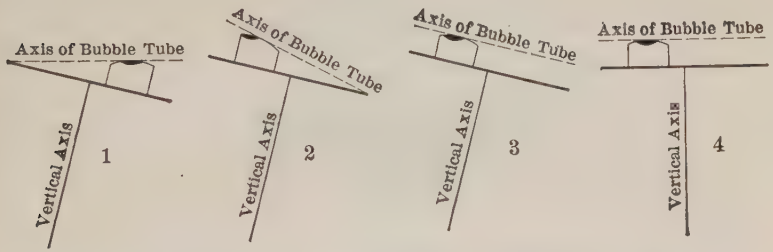


FIG. 41.—Plate Bubble Adjustment.

screws. Each bubble should be brought parallel to a set of opposite levelling-screws in making this adjustment, so that the correcting for one bubble does not throw the other out. When either bubble will maintain a fixed position in its tube as the instrument is revolved horizontally, the axis of revolution is vertical. One bubble is therefore sufficient for making this axis vertical, but two are somewhat more convenient, especially for indicating when the axis has become inclined from unequal settling or expansion while in use.

73. Second. The Cross-Wire Adjustment. This consists in two parts. (*a*) *To make the vertical cross wire lie in a vertical plane.* This adjustment can be made as in (*a*), Art. 42, or as follows: Sight the upper part of the vertical wire on some well-

defined point, and with both plates clamped rotate the telescope upon its horizontal axis, as shown in Fig. 42. If in adjustment, the point should travel along the entire length of the vertical wire. If not, the reticule screws should be loosened and the reticule ring made to rotate slightly, by lightly tapping one screw, until the above condition is satisfied. When completed the reticule screws should all be brought to a firm footing, but without straining the screw (*b*). The second part is called the collimation adjustment.

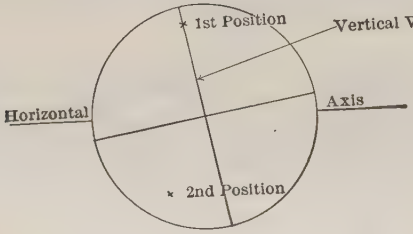


FIG. 42.

74. The Collimation Adjustment. *To make the line of sight perpendicular to the horizontal axis of the telescope.*[1] When this is done, the line of sight will generate *a plane* when the telescope is revolved on its horizontal axis. If the line of sight is not perpendicular to the horizontal axis, it generates the surface of a cone when the telescope is revolved, the axis of the cone being the horizontal axis and the apex being at the intersection of the line of sight with this axis.

Set up the instrument on nearly level[2] ground, where a view can be had in opposite directions. (See Fig. 43.) Set the line

[1] This is called the *Adjustment for Collimation*, since it consists in bringing the line of sight into coincidence with the line of collimation, which is simply the true position for the line of sight.

[2] If not level the small error due to the horizontal axis will be introduced into this test.

of sight on a definite point A, a few hundred feet away. Reverse the telescope and set another point B in the opposite direction. Revolve the alidade upon its vertical axis until the line of sight comes upon the first point A. Reverse the telescope again and fix a third point C on the line of sight beside the second point set. Measure off one-fourth the distance between these two points *from the last point set*, to a point D, and bring the line of sight to this position by moving the reticule laterally. This movement of the reticule is direct in an erecting instrument and reversed in an inverting instrument. The adjustment should be repeated as a check.

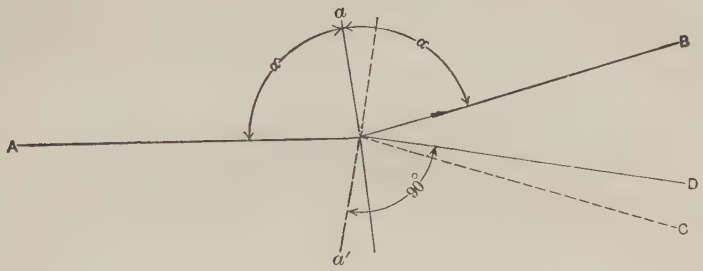


Fig. 43.—Collimation Adjustment.

75. Third. The Standard Adjustment. *To make the horizontal axis of the telescope perpendicular to the vertical axis of the instrument.* (See Fig. 44.) With the transit levelled sight on some high point, as a steeple S, lower the telescope and set a point R below S on about the level of the transit. Reverse the telescope and turn the transit upon its vertical axis and again sight at S. Again lower the telescope and set a point L opposite R. A point S' midway between L and R must be in the same vertical plane with S. Now loosen the screws in the pivot cap and raise or lower the adjustable end of the horizontal axis, by means of the capstan-headed screws, under the one end of the axis. The high end of the axis is always on the same side as the last point set.

If this side of axis is not adjustable the other end can be raised instead. The test should be repeated until the line of sight coincides with *SS'*. Care should be taken to leave the cap screws tight enough to insure that the axis rests on its bearing, but not tight enough to cause friction in turning the axis.

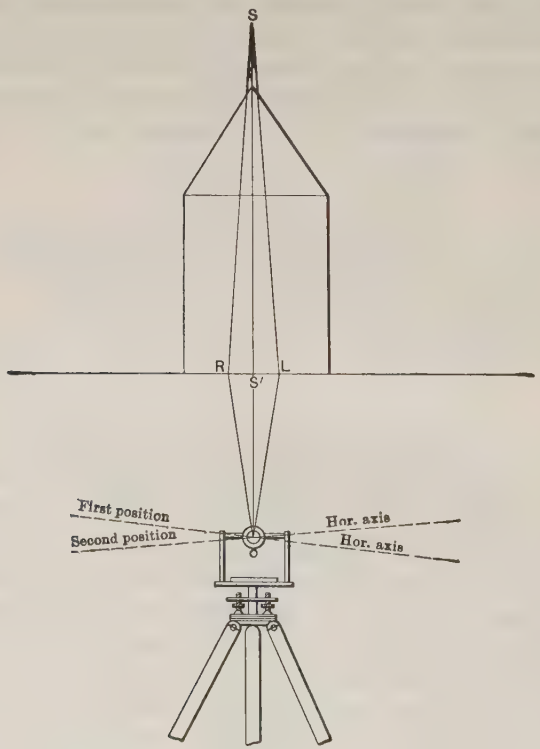


FIG. 44.

76. Fourth. Telescope Bubble Adjustment. *To make the axis of the telescope bubble parallel to the line of sight.* This adjustment is performed by means of the "peg adjustment," as described in Art. 45. This consists in first determining a level line and then correcting the level tube so that the bubble is in the centre

at the same time as the telescope is horizontal, as determined by this level line. With a sensitive bubble[1] very good levels can be run with a transit.

77. Fifth. Vertical Circle Adjustment. *To make the vernier read zero when the line of sight is horizontal.* The fourth adjustment having been first made, bring the telescope bubble to the centre. If the vertical circle does not now read zero loosen the screws holding the vernier and shift same until the zeros do coincide. Tighten the screws, taking care that the vernier neither rubs on the vertical circle, nor is at a perceptible distance away from it. The most improved transits provide a slow-motion screw for making this adjustment.

78. Object Slide Adjustment. In addition to the above adjustments, provision of some kind should be made for making the object glass move, when focussed, in a right line perpendicular to the horizontal axis. This is sometimes accomplished by making the telescope work through a ring which can be adjusted like the ring carrying the cross-wires. One way of adjusting the slide is as follows: After adjusting the line of sight as in Art. 74, using long sights, repeat the operation, using very short sights, the first requiring the telescope drawn well in, the second requiring the telescope run out. If the slide is out of adjustment the line of sight in the second test will not be in adjustment. In this case move the adjusting screws of the objective slide so as to apparently increase the error by one-quarter the error found.

Most makers make this adjustment permanently. In the higher grades of instrument the whole slide except the middle third is made to fit the inside of the telescope tube, a condition requiring considerable skill.

79. Relative Importance of the Adjustments. The first adjustment is important in all horizontal and vertical angular

[1] The writer suggests a sensibility of bubble which corresponds to $\frac{1}{10}$-inch = 12 to 15 seconds. This requires a finer tangent screw than usually furnished. See Appendix F for Specifications of a Transit.

measurements. In measuring vertical angles the error may be the full amount of the deviation of the vertical axis from the vertical, and in measuring horizontal angles something very much less than this.

The second adjustment is more important in the running of a straight line by revolving the telescope than in any other kind of work, for here the error in the continuation of the line is twice the error of adjustment. It is also important in measuring horizontal angles between points not in the same horizontal plane.

The third adjustment is most important in the measurement of horizontal angles between points not in the same horizontal plane, as in the determination of the azimuth of a line by an observation on a circumpolar star.

The fourth and fifth adjustments are important only in levelling operations, either by reading the vertical angle or by the use of the bubble. A proper programme of observation will eliminate all the errors of adjustment.

Instrumental Conditions Affecting the Accurate Measurement of the Horizontal Angles [1]

80. Eccentricity. This is of two kinds: (1) *eccentricity of centres*, and (2) *eccentricity of verniers*. If the axis of the conical outer socket, Fig. 30, is not exactly in the centre of the graduated limb 23, then when the telescope with the vernier plates 26 are revolved in this socket, the verniers will have an eccentric motion with reference to the graduated limb. If the line joining the zeros of the verniers passes through the axis of the socket, it is evident that there is but one position of

[1] For extended discussions of this subject, see Bauernfeind's "Vermessungskunde," § 144, vol. i., and Jordan's "Handbuch der Vermessungskunde," § 88. vol. i. Also translations from these, by Prof. Eisenmann, in Jour. Asso. Eng. Soc., Vol. IV, p. 196.

these verniers which will give readings on the limb 180° apart, and that is when both centres lie in this diametral line. For all other positions of the verniers, one of them will read as much too large as the other does too small; so that if the mean of the two vernier readings be taken, this error from eccentricity would be eliminated.

Eccentricity of verniers is due to their zeros not falling on a diametral line through the axis of the spindle; in other words, they are not 180° apart. This involves no error in measuring horizontal angles. It is convenient, however, to have the verniers read exactly 180° apart. In any case, reading of both verniers and taking the mean eliminates all errors from eccentricity. An eccentricity of centres of *one one-thousandth of an inch* would cause a maximum error of 1′ 08″ on a six-inch circle if but one vernier were read. It is not unusual for an instrument to have an eccentricity of centres of several times this amount, either from wear or from faulty construction, or both. The necessity for reading both verniers in all good work is therefore apparent.

81. Inclination of Vertical Axis. The horizontal angle between points at different elevations is obtained by measuring the horizontal angle subtended by two vertical planes passing through these points and the point of observation. These vertical planes are the planes described by the line of sight as the telescope is revolved. By this means the points may be said to be projected vertically on the horizontal plane and then the angle measured. If the vertical axis of the instrument is somewhat inclined, these projecting planes are not vertical, neither do they have the same inclination to the horizon on different parts of the limb. The projecting planes through two points will therefore neither be vertical nor equally inclined to the horizon. The measured horizontal angle thus obtained will therefore be in error. The vertical axis is always inclined when the plate bubbles are not in adjustment or when they do not show a level position.

If the axis be inclined 5′ from the vertical, and readings be taken on points 60° apart, one being 10° above and the other 10° below the horizon, the maximum error from this source would be about 1′. If the inclination in this case were 1°, the maximum error would be 18′. This shows the importance of keeping the plate levels in adjustment and of watching them during the progress of the work to see that they remain in the centre.

82. Inclination of Horizontal Axis of Telescope. This causes the plane generated by the line of sight to be inclined from the vertical as much as the axis of revolution is from the horizontal. The projecting planes are therefore all equally inclined, and the resulting error in horizontal angle is a function of the difference of elevation of the two points. If one point is 10° above and the other 10° below the horizon, and if the inclination of the axis is 5′, the resulting error in the measurement of the horizontal angle is 1′ 45″. This error is not a function of the size of the horizontal angle, and would be the same for two points in the same vertical plane, the instrument indicating a horizontal angle of 1′ 45″ between them for the case here chosen. In making the adjustment of the horizontal axis by means of the plumb-line, if the line be 15 feet distant and suspended 15 feet above the instrument, then the pointing to the top will have an altitude of 45°. In this case the angular error made in bisecting the plumb-line will be the angular divergence of the axis of rotation from the horizontal. If the combined error of the two bisections be 0.05 in., the angular error in the adjustment will be 1′. The adjustment may readily be made closer than this.

Errors from this source are eliminated by revolving the telescope and reading the same angle in the reversed position. The mean of the two values will be independent of this error. If many measurements are made of one angle, there should be an equal number with telescope direct and reversed.[1]

[1] The student should show by a figure how this elimination is effected by the reversal of the telescope.

83. The Line of Sight not being Perpendicular to the Horizontal Axis. This causes the projecting planes to be conical surfaces, which become vertical on the horizon. Since the error of collimation is necessarily a small angle, thus causing the conical surface to be very nearly a plane, and since this surface is vertical on the horizon, the resulting error in measuring horizontal angles is very small unless the difference in the elevations of the points is very great. If the points are distant, as they always are in the accurate measurement of horizontal angles, then their angular elevation is necessarily small, so that this source of error is insignificant in this kind of work. When straight lines are prolonged by reversing the telescope, however, this adjustment becomes very important, for the error then enters the work with twice its angular amount. It is eliminated by revolving the alidade until the line of sight, with telescope reversed, falls again on the rear point, and again revolving the telescope. The point now falls as far on one side of the true position as it before did on the other. The middle point lies therefore in the line prolonged.

Let the student illustrate by diagram.

The Use of the Transit

84. Setting Up the Transit. Time can be saved if the following suggestions are followed in setting up a transit or level. Place one leg of the tripod in nearly the right position on the ground and holding the other two move the instrument in a way to bring the blumb-bob over the point and at the same time keeping the levelling head approximately level. The latter condition can be estimated easily by sighting along the top of the base at the line of the distant horizon. Steadiness and rigidity can be obtained only by spreading the tripod sufficiently, by pressing the tripod shoes well into the earth, and by tightening the wing nuts at the top of the tripod legs until they can barely sustain their own weight when raised from the ground. When

too tight as unstable a condition results as when they are too loose. If it is found that the circles and levelling base are not approximately level, when the beginner has succeeded in getting the plumb-bob over the point the trouble can generally be overcome by moving one of the tripod legs in the arc of a circle.

If the plumb-bob is now within $\frac{1}{2}$ inch of the exact point, the remaining error can be overcome by loosening any two adjacent screws and moving the instrument on the shifting plate. In the final levelling up of the plates each plate bubble should be brought parallel to one pair of *opposite* screws. To level the bubbles grasp a pair of opposite screws between the fore-fingers and thumbs and turn the screws so that the thumbs move either *toward* each other or *away from* each other. This insures that one screw will tighten as fast as the *opposing* one loosens.

As the levelling may have disturbed the plumb-bob it should be re-examined for centering. The beginner should practice setting up the instrument both on the level and on side hill, until the operation is thoroughly mastered.

85. To Measure a Horizontal Angle. Having centred the transit over the vertex of the angle set the zero of one of the verniers opposite the zero of the circle, using the upper clamp and tangent screw to bring the zeros into exact coincidence. It has been found to give better results if the tangent screw is moved so as to *compress* the spring against which it works. Then turn the plates about the lower motion until the vertical wire nearly bisects one of the points, say the left-hand one, when the lower motion is clamped and an exact bisection is made by using the *lower* tangent screw. The upper clamp is now loosened and the alidade turned, *by the upper motion*, upon the right-hand point. Then by use of the upper clamp and slow-motion screw, the vertical wire is made to bisect this point. The reading of the vernier first set on zero will now give the angle required.

86. To Measure an Angle by Repetition. In the work described in article 85 errors will appear in the result, because of the accidental errors in pointing the telescope and in reading the vernier. A more accurate result, due to the balancing of errors, can be obtained by repeating the angle by the following programme: After the first measurement of the angle has been obtained, leaving the two plates clamped together, loosen the *lower* clamp and turn by the lower motion on the first or left-hand point. By means of the lower clamp and slow-motion screw make the pointing exact. Then loosen the *upper* clamp and set on the second or right-hand point, thus adding another angle to the first onto the circle. If the angle be added on the circle three times with the telescope direct and then three times more with the telescope reversed, a very reliable value for the angle will be obtained by dividing the total angle by the number of repetitions. The reversing of the telescope eliminates all error due to lack of adjustment. It has been found that increasing the speed of the work decreases the errors of the work. This method is discussed more at length in Geodetic Surveying Art. 439.

87. To Lay off a Given Angle by Repetition. It is sometimes necessary to lay off an angle very exactly, e.g., the locating of bridge piers, when the following method may be used. With the verniers reading zero set the telescope by the lower motion on the known leg of the angle. Then *loosen* the upper motion and turn off the required angle as near as the verniers will determine, placing a target on the line so determined. The angle between the target and the given line can then be accurately measured as in Art. 86. The difference between this result and the angle that was to be turned is the angular error of the target. If the *distance* from the instrument to the target be now approximately determined, its product with the tangent of the angle of error gives the corresponding linear error to be measured on the target at right angles to the line of sight.

88. To Run a Straight Line: Terminals not Intervisible. In such a case a random line can be run between the points, the error of closing noted and a corrected line run back as explained in Art. 89. Otherwise if both points can be seen from some intermediate point, the following method could be used. Set up the transit at the intermediate point as near the line as can be estimated and a back sight on one of the points made and the transit clamped. The telescope is next reversed on its horizontal axis. If in so doing the vertical cross-wire strikes the second point the transit is in line (provided it is in perfect adjustment). If not the error in the position of the instrument is estimated and a second trial made and the process repeated until the correct point is found. The final tests must be made with the telescope in both direct and reversed position, in order to eliminate the errors of adjustment.

FIG. 45.
To Prolong a Straight Line.

89. To Prolong a Straight Line. Wherever it is possible, a more accurate alignment will be secured by setting points on a constant fore sight instead of running from a back sight. Thus to stake out the line AX, set instrument at A and with X, the other terminus, as a fore sight, set B in line (Fig. 45). This can be repeated as long as the point X can be seen. But if X cannot be seen beyond B the line may be prolonged by the back-sight method as follows: With transit centred and levelled over B set the line of sight on A; transit the telescope and set C' beyond B. Next revolve the alidade about the *vertical axis*, relevel and set the line of sight again on A; transit the telescope again and set C'' opposite C'. If the transit be in perfect adjustment, C' and C'' will coincide. In any case C, a point

midway between C' and C'' will be on the prolongation of the required line. The line can be continued by this method for any distance. It will be noted, however, that with this method the errors in alignment accumulate.

90. To Measure a Vertical Angle. By a vertical angle is meant the angle of depression below, or of elevation above, the horizon. To measure a vertical angle clamp the horizontal axis with the object near the horizontal cross-wire, bringing it into exact coincidence with the slow-motion screw controlling the vertical motion. Then read the proper vernier on the vertical circle. If the telescope be now levelled and the vertical circle again read, it will give the index error of the circle. The correct vertical angle may be obtained by subtracting (algebraically) such index error from the vertical angle. Otherwise the index error can be corrected by adjustment. See Art. 77. In some transits the vernier is placed on a separate standard carrying also a level-vial. By bringing this bubble to the centre of its tube, by means of its tangent screw, each time a reading is made, the index error is reduced to zero. If the transit is provided with a complete vertical circle the vertical angle may also be read with the telescope reversed. The *average* of both vertical angles will then give the true angle.

91. To Run a Traverse. A traverse in surveying consists in a series of consecutive courses whose lengths and bearings or azimuths have been determined. A line has a *forward* and a *back* azimuth, corresponding to its two directions. Thus the forward azimuth of the line AB is the horizontal angle at A between the meridian [1] passing through A and the line AB, always measuring such angle clockwise. The back azimuth of the same line is the angle at B between the meridian and the line BA, measuring as before in a clockwise direction.

[1] In this treatise azimuth is always reckoned from the south point in the direction S. W. N. E. to 360°. The direction of the line is thus given by its numerical value, without the aid of letters.

The field work proceeds as follows: The transit is set up at A and the *known* azimuth laid off on vernier A. With this reading the telescope is directed by the lower motion at the object whose azimuth is given and the instrument clamped. This orients the transit, i.e., places the zero circle reading in the meridian. The transit alidade is now unclamped, and the telescope turned by the upper motion until the vertical wire coincides with the forward point B. The reading on vernier A (260°) now gives the forward azimuth of the line AB. To

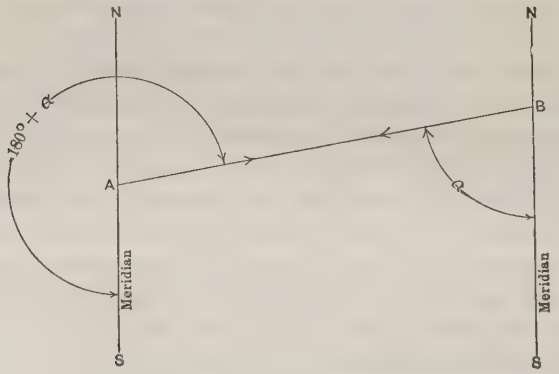


FIG. 46.—To run an Azimuth Traverse.

find the back azimuth move the transit to B and set the A vernier to read what the B vernier read at station A, viz., 80°. By the lower motion set the telescope on station A and clamp the lower motion. This orients the transit again. In this way a traverse may be run with the transit, the field notes showing both forward and back azimuth of every course.[1]

The telescope should never be reversed in such work, as errors

[1] For a method of computing the co-ordinates of the courses, and the use of the traverse table, see chapter on Land Surveying (Arts. 225–228).

in adjustment are likely to creep in. The distances may be gotten by chaining or by stadia. See Arts. 211 and 212.

92. Targets Used in Lining. The shorter the distance the smaller should be the target. Thus a pencil may be used with distances of less than 200 feet. A properly painted pole $1\frac{1}{2}$ inches in diameter may be used up to a quarter of mile or more. An effort should always be made to point as near the bottom of the rod as possible. For long distances a kite-shaped target of white cloth may be fastened with one diagonal coinciding with the transit pole.

93. Field Signals. Signals are generally less likely to be misunderstood than spoken orders. The more common ones are the following:

"Right" or "Left." The hand is extended in the direction required, moving slowly for long distances and fast for short distances.

"All right." Both hands are extended horizontally and moved vertically.

"Plumb the pole." The arm is extended above the head and moved slowly in a direction to plumb the pole.

"Give a fore sight." The transitman desiring a fore sight motions to the rodman by holding one arm vertically above his head.

"Turning point." If the transitman desires that the fore sight shall also be a turning point he whirls his handkerchief in a circle.

"Give line." When the rodman desires to be placed "on line" he holds his transit pole horizontally with both hands over his head and then places the rod vertically. If this is to be also a turning point he waves the top of the rod in a circle before bringing same to a vertical position. For long distances flags are attached to the transit poles.

The Gradienter Attachment

94. The Gradienter is a tangent screw with a micrometer head attached to the horizontal axis of the telescope for the purpose of turning off vertical angles that are expressed in terms of its tangent as so many feet to the hundred. Such a device is shown in Fig. 29. In railroad work the grade or slope is expressed in this manner, as 26.4 feet per mile, or as 0.5 foot per 100 feet. The micrometer head is graduated so that one revolution raises or lowers the telescope by 1 foot or 0.5 foot in 100 feet. It is divided into 100 or 50 parts, so that each division on the head is equivalent to 0.01 foot in 100 feet. This attachment is found very convenient in railroad work. It is also of general utility in obtaining approximate distances. On level ground the distance is read directly, but on sloping ground the rod is still held vertical, and the distance read is too great. The true horizontal distance may be found by multiplying the distance read by the factors for horizontal distance given in table V.[1] Thus, if one revolution of the screw raises the line of sight 1 foot at a distance of 100 feet, and if at a certain unknown distance one revolution of screw caused the line to pass over 5.5 feet on the rod, then the distance was 550 feet if the ground was horizontal. If the rod readings had a mean vertical angle of 15°, the horizontal distance was $550 \times \frac{93.3}{100} = 513$ feet.

Care of the Transit

95. The Transit should be protected from rain and dust as much as possible. A silk gossamer waterproof bag should be carried by the observer to be used for this purpose. If water gets inside the telescope, remove the eye-piece and let it dry out. If moisture collects between the two parts of the objec-

[1] This table is for reduction of stadia measurements, and is explained in the chapter on Topographical Surveying, Art. 219.

tive, unscrew it, and dry it with a gentle heat over a stove or lamp, but *do not separate the glasses.* If dust settles on the wires it may be blown off by removing both objective and eye-piece and blowing gently through the tube. Dust should be removed from the glasses by a camelhair brush, which should always be carried for the purpose. A clean handkerchief may be used with a gentle pressure to prevent scratching in case the dust is gritty. Use alcohol for cleansing greasy or badly soiled glasses. No part exposed to dust should be oiled, as this serves to retain all the dust that may fall on it. The centres should be cleaned occasionally with chamois skin, and oiled with a very little pure watch-oil. In the absence of watch-oil plumbago will be found to serve. A soft leadpencil may be scraped and a little rubbed on the spindles with the finger. The tripod legs should have no lost motion either at the head or in their iron shoes. If the legs are split, as in Fig. 30, and fastened by thumb-nuts, these should be loosened when the instrument is carried and tightened again after setting. For a method of putting in new cross-wires see chapter on Topographical Surveying, Art. 221.

The Solar Attachment

96. Description of the Solar Attachment. This is a device which can be fastened to the telescope of an engineer's transit. By means of observations on the sun with it, the true meridian can be determined. One of the most common forms known as the Saegmuller attachment is shown in Figs. 47 and 48. This device has two essential parts, first a polar axis, which is so attached to the transit telescope as to be perpendicular to its horizontal axis and line of sight, and second a small telescope which can revolve about this polar axis as well as on its own horizontal axis. A level vial is also attached to the auxiliary telescope.

In another form of the attachment shown in Fig. 49, a lens and screen on which the sun's image can be thrown, replace

the telescopic line of sight. This form also has an auxiliary vertical arc on which the declination can be laid off. All the various solar attachments work on the similar general principles.

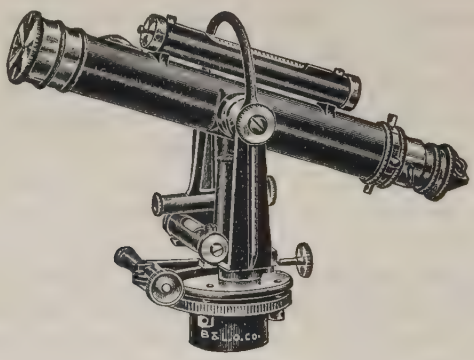


FIG. 47.—Saegmuller Solar Attachment.

97. The Celestial Sphere. A few astronomical terms must first be explained before the theory of the attachment can be understood. The celestial sphere is the one upon which the sun and other stars are seen projected. Fig. 50 represents that half of the celestial sphere which is visible at any one time. In the present problem it is sufficient to regard the centre of the earth as the centre also of the celestial sphere (whose radius is the distance of the sun from the earth). The circle NHS' is the observer's celestial horizon. The point Z is the observer's zenith, the point where the plumb-line produced upward pierces the celestial sphere. The circle $S'ZPN$ is the observer's meridian, being a vertical plane through the north pole. EQW is the plane of the earth's equator produced until it intersects the celestial sphere, i.e., the celestial equator. The circle LSR is the circle cut from the celestial sphere by a plane parallel to the celestial equator. If this circle be at the same angular distance from the equator as the sun the angle SQ measures the declination of the sun and the circle LSR represents

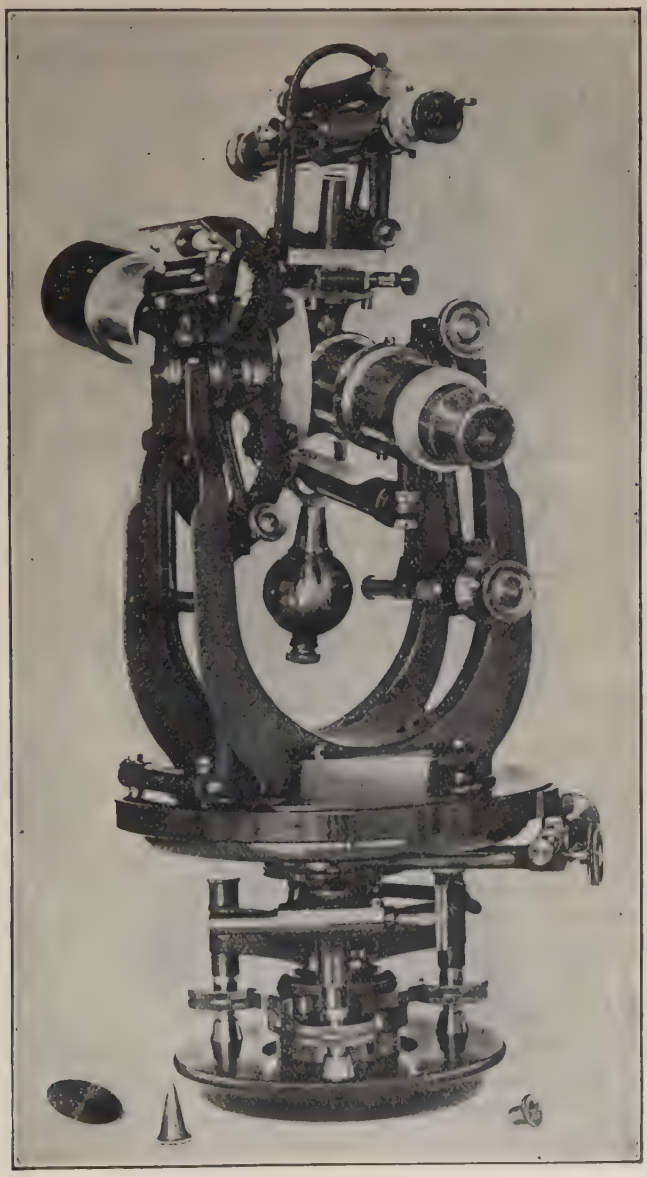


FIG. 48.—Manner of Mounting Solar Attachment.

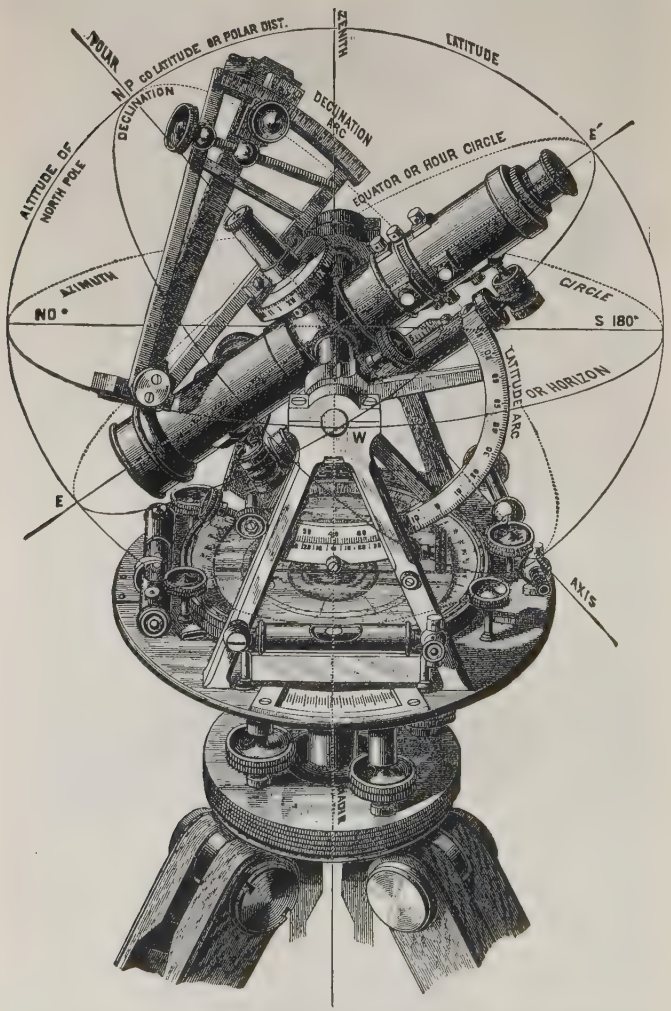


FIG. 49.—Graphic Illustration of the Solar Apparatus (Gurley).

the sun's path. The sun's declination then is defined as its angular distance from the equator, being considered *positive* when north and negative when south. The *polar distance* of the sun is obviously the complement of the declination, shown in the figure *SP*.

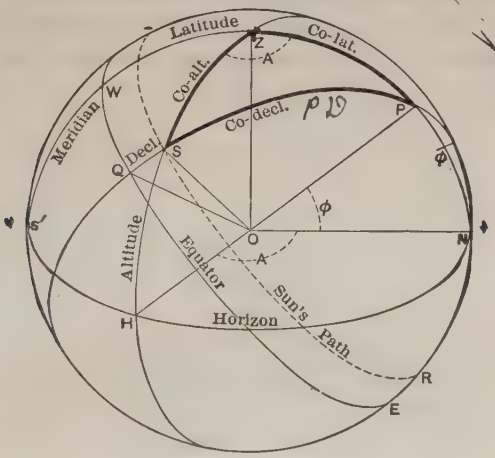


FIG. 50.

98. Observations on the Sun for the Meridian with Solar Attachment. It will be seen from Fig. 51 that if the polar axis is made parallel to the earth's axis the large telescope will be parallel to the celestial equator. If the solar telescope be now inclined to the large telescope by an angle equal in amount and sign to the declination of the sun for the day, then the small telescope can be made to follow the sun in its daily path, by simply turning it upon the polar axis.

To lay off the declination of the sun, first put both telescopes in the same vertical plane by sighting both on the same distant object and clamp the solar telescope. Next turn the large telescope down or up, according as the declination of the sun is north or south until the vertical circle reads the amount of the

declination. Then turn the solar telescope on its *horizontal* axis until the bubble of its attached level comes to the centre of its tube. The angle now formed by the two telescopic lines of sight is the declination angle.

99. To Lay off the Co-latitude, incline the large telescope upward until the vertical circle reads the co-latitude. For reason see Fig. 51. The instrument is now ready for a determination of the meridian. The circle being set at 0, the instrument can

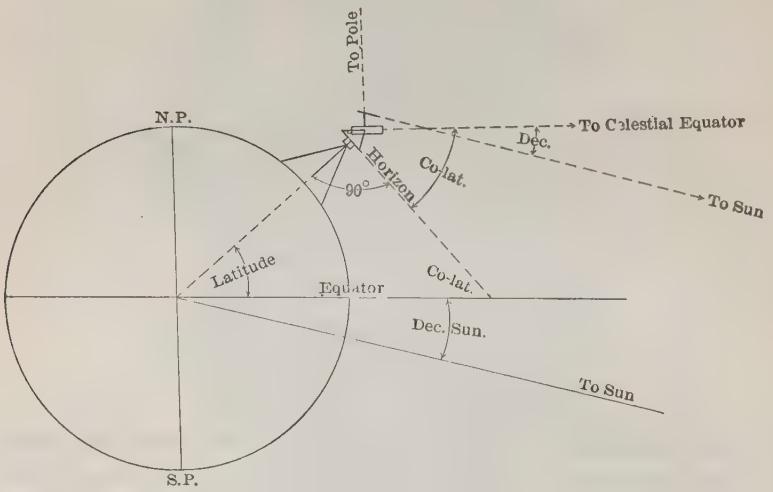


FIG. 51.—Solar Attachment Oriented for Determining the Meridian.

be put approximately in position [1] by using the magnetic needle to turn the object end of the large telescope to the south point. Keeping the solar telescope carefully clamped on its horizontal axis it is turned on its polar axis toward the sun. If the first trial was nearly correct the sun's image will be seen in the field of the solar telescope. If so, with the lower motion of the large

[1] This requires only that the magnetic declination be known approximately, say as near as it could be taken from an isogonic chart

telescope clamped, the whole instrument is turned on its vertical axis by the use of the lower tangent screw, until the image of the sun in the solar telescope is brought to the centre of the square formed by the four cross-wires. The large telescope is now on the meridian, *because it is only in this position that the sun can be seen at the cross-wires.* In this position the vertical axis points to the zenith, the polar axis to the pole, the large telescope parallel to the equator and the solar telescope to the sun. The instrument in fact has solved mechanically the spherical triangle whose vertices are *ZPS*, Fig. 50. The meridian can now be marked by unclamping the horizontal axis and driving a stake in line, or the true azimuth to any object to which the large telescope is now directed may be read directly on the vernier.

100. Adjustments of the Saegmuller Attachment. *First.* All the adjustments of the transit must be as perfect as possible, but especially the plate and telescope bubbles, the vernier of the vertical circle, and the transverse axis of the telescope.

Second. Polar Axis Adjustment. To make the Polar Axis Perpendicular to the Plane of the Line of Collimation and Horizontal Axis of the Main Telescope. Carefully level the instrument and bring the telescope bubble to the middle of its tube. The line of sight and horizontal axis of this telescope should now be horizontal, so that the polar axis is to be made vertical. To test this, revolve the auxiliary telescope about the polar axis, and see if the bubble on the small telescope maintains a constant position. If not, correct half the movement by means of the adjusting screws at the base of the small disk, and the other half by revolving the auxiliary telescope. Repeat over the other set of screws. These adjusting-screws are exactly analogous to the levelling-screws of the main instrument.

Third. Cross-wire Adjustment. To make the Line of Sight of the Small Telescope Parallel to the Axis of its Attached Bubble. Make the large telescope horizontal by bringing its attached bubble to the middle of its tube. Bring the small telescope in the same plane and make it also horizontal by means of its bub-

ble, clamping its vertical motion. Measure the vertical distance between the axes of the two telescopes, and lay off this distance on a piece of paper by two plain horizontal lines. Set this paper up at a convenient distance from the instrument, and on about the same level. Bring the line of sight of the large telescope on the lower mark, and see if that of the small telescope falls on the upper mark. If not, adjust its reticule until its line of sight come on the upper mark. Revolve back to the horizontal to see if both bubbles again come to the middle simultaneously.

When this adjustment is completed, there should be five lines in the instrument parallel to each other when instrument and telescopes are level, viz., the axes of the two telescope bubbles and of the plate bubble on the standards, and the two lines of sight, and, in addition, the vernier on the vertical circle, should read zero.

The seven adjustments (five of the transit and two of the attachment) must all be carefully made and frequently tested if the best results are desired. When this is done, this attachment will give the meridian to the nearest minute of arc, if observations be taken when the sun is more than one hour from the horizon and two hours from the meridian. The advantages of the Saegmuller attachment consist mainly in having a telescopic line of sight, and in the use of the vertical limb of the transit for setting off the declination and co-latitude. The effect of small errors in the latitude and declination angles, such as may be due to errors in the adjustments, is shown by the table, Art. 105.

101. Determination of the Meridian by Direct Solar Observation. Knowing the latitude of the place of observation and the declination of the sun, the angle which a vertical plane through the sun makes with the meridian plane, counting from the north point, may be found by simply observing the sun's altitude. Thus, in Fig. 50, let Z be the zenith point, P the pole, and S the sun. Then knowing l, the latitude of the place, d, the declination, and h, the altitude of the sun, the three

sides of the spherical triangle *ZPS* become known, since these sides are respectively the co-latitude, the co-declination, and the co-altitude. Knowing these three sides of the spherical triangle, the solid angle *A*, or *PZS*, the angle of the sun from the North, may be computed from the formula,

$$\text{Cos } A = \frac{\sin d}{\cos h \cdot \cos l} - \tan h \cdot \tan l^1 \quad . \quad . \quad . \quad . \quad (1)$$

The sign of the first term will depend upon the sin of *d* being + when the sun is north of the equator and − when the sun is south of the equator.

This method of determining an azimuth does not require a solar attachment. It is much used in the mining regions. It does require an auxiliary disk, mounted or held just back of the eye end of the telescope. This disk may even be a plain sheet of white paper held in the hand some four to six inches back of the eye-piece. By having the cross-wires in good focus, and the objective focussed on the sun, so as to give a clearly defined image, this image may be brought centrally upon the shadow of the cross-wires by the vertical and horizontal slow-motion screws, within the limits of accuracy of reading the graduated circles. The reading of the vertical circle gives the altitude of the sun, from which the co-altitude is found.

The declination of the sun at the time of the observation is obtained from the nautical almanac, the same as if it were to be used with the solar attachment. The correction for refraction is also applied.[2] The time of the observation is recorded and the declination of the sun at this time can be afterwards determined. The horizontal circle is also read, and a pointing

[1] From Spherical trigonometry we have, Cos (90-*d*) = cos (90-*h*) cos (90-*l*) + sin (90-*h*) sin (90-*l*) cos *A*. Solving this for cos *A* gives equation (1).

[2] In this case the correction is subtracted from the observed altitude, but not from the declination of the sun.

made to the azimuth mark, and the horizontal circle read again. These observations can be repeated as often as desired; preferably three sets of readings would usually be taken.

The time of day best suited to this observation, like that for an observation with the solar attachment, is near the middle of the sun's path from the horizon to the meridian, either before or after noon. That is to say, the altitude of the sun should be changing rapidly at the time of the observation. If taken too near the horizon the correction for refraction becomes large and uncertain, and if taken too near noon the altitude is changing too slowly to furnish a good argument. It needs scarcely to be said that the transit must be in as good adjustment for this kind of direct observation as is required when the solar attachment is used. As in that case also, the mean of two observations taken at symmetrical times before and after noon will be free from the errors of adjustment, and of the latitude and declination used. This method is now commonly employed in the mining regions of the West, where it is rapidly replacing the use of the solar attachment.

102. To Find the Declination of the Sun. On account of the inclination of the earth's axis to the plane of its orbit, the sun is seen north of the celestial equator in summer, and south of it in winter. This deviation, north or south of the equator, is called north or south declination, and is measured from any point on the earth's surface in degrees of arc.

On about the 21st of June the sun reaches its most northern declination, and begins slowly to return. Its most southern point is reached about December 21st. In June and December, therefore, the sun is changing its declination most slowly, while at the intervening quadrant-points of the earth's orbit, March and September, it is changing its declination most rapidly, being as much as one minute in arc for one hour in time. It is evident, therefore, that we must regard the declination of the sun as a constantly changing quantity, and,

for any given day's work, a table of declinations must be made out for each hour of the day. The American Ephemeris and Nautical Almanac gives the declination of the sun for noon of each day of the year for both Greenwich and Washington. Since the time universally used in this country is so many hours from Greenwich, it is best to use the Greenwich declinations. Since, also, we are five, six, seven, or eight hours west of Greenwich, the declination given in the almanac for Greenwich noon of any day will correspond to the declination here at 7, 6, 5, or 4 o'clock A.M. of the same date, according as Eastern, Central, Mountain, or Western time is used. As this standard time is seldom more than 30 minutes different from local time, and as this could never affect the declination by more than 30 seconds of arc, it will here be considered sufficient to correct the Greenwich declination by the change, as found for the standard time used. Thus, if Central (90th meridian) time is used, the declination given in the almanac is the declination at 6 o'clock A.M. at the place of observation. To this must be added (algebraically) the hourly change in declination, which is also given in the almanac. A table may thus be prepared, giving the declination for each hour of the day.

103. To Correct the Declination for Refraction. All rays of light coming to the earth from exterior bodies are refracted downward, thus causing such bodies to appear higher than they really are. This refraction is zero for vertical lines, and increases towards the horizon. It varies with the temperature and pressure of the atmosphere. Tables of refraction give only average values, and these may differ considerably from the values found for any given time, especially for lines near the horizon. It is for this reason that all astronomical observations made near the horizon are very uncertain. There is but one setting on the solar transit that has reference to the position of the sun in the heavens, and that is the declination. Now, the refraction changes the apparent *altitude* of the body; and by so much as a change in the altitude changes the declination, by so much

does the *apparent* declination differ from the *true* declination. Evidently it is the *apparent* declination that should be set off. When the sun is on the meridian, the change in altitude has its full effect in changing the declination, but at other times the change in declination is less than the change in altitude.

The correction to the declination due to refraction is found from the following equation:[1]

$$d\delta = +dh \cos q,$$

where dh represents the amount of the refraction in altitude and q is the spherical angle formed at the star between the zenith and the celestial north pole. The value of q may be found in any given case by the methods of spherical trigonometry.

In this manner the following table has been computed for the latitude of 40°. For any other latitude the correction is found by multiplying the correction given in the table by the corresponding coefficient, as given in the table of Latitude Coefficients. These coefficients were obtained by plotting the ratios of the actual refraction at the different latitudes to that at latitude 40°, for each hour from 7 A.M. to 5 P.M. and for the various declinations. It was found that this ratio was almost a constant, except for very low altitudes, where the inherent uncertainties of an observation are very large, from the actual refraction varying so largely from the mean, as given in the tables. A mean value of this ratio was chosen, therefore, which enables the corrections at other latitudes to be found in terms of those in latitude 40° without material error. These ratios are given in the table of Latitude Coefficients.

EXAMPLE

104. Let it be required to prepare a table of declination settings for a point whose latitude is 38° 30′, which lies in the "Central Time Belt," and for April 5, 1910.

[1] See Chauvenet's "Spherical Astronomy," vol. i., p. 171, and Doolittle's "Practical Astronomy," p. 159.

TABLE OF REFRACTION CORRECTIONS TO BE APPLIED TO THE DECLINATIONS

Date.	Refraction Correction. Latitude 40°.	Date.	Refraction Correction. Latitude 40°.	Date.	Refraction Correction. Latitude 40°.	Date.	Refraction Correction. Latitude 40°.
Jan.		Feb.		Mar.		May.	
	[1] 1 h. 1′ 58″	13	1 h. 1′ 16″	30	1 h. 42″	14	1 h. 23″
1	2 2 16	14	2 1 25	31	2 47	15	2 27
2	3 3 04	15	3 1 48	April 1	3 57	16	3 34
3	4 6 23	16	4 2 47	2	4 1′ 19	17	4 49
		17	5 8 39	3	5 2 18	18	5 1′ 18
4	1 1 54	18	1 1 12	4	1 39	19	1 22
5	2 2 11	19	2 1 20	5	2 44	20	2 26
6	3 2 59	20	3 1 40	6	3 54	21	3 33
7	4 6 01	21	4 2 31	7	4 1 14	22	4 47
8		22	5 6 49	8	5 2 08	23	5 1 15
9	1 1 51	23	1 1 07	9	1 36	24	1 21
10	2 2 07	24	2 1 15	10	2 41	25	2 25
11	3 2 51	25	3 1 33	11	3 51	26	3 32
12	4 5 40	26	4 2 18	12	4 1 10	27	4 46
13		27	5 5 29	13	5 1 58	28	5 1 13
14	1 1 46	28	1 1 03	14	1 34	29	1 20
15	2 2 01	Mar. 1	2 1 10	15	2 38	30	2 24
16	3 2 40	2	3 1 27	16	3 48	31	3 31
17	4 5 00	3	4 2 06	17	4 1 06	June 1	4 44
18		4	5 4 39	18	5 1 49	2	5 1 11
19	1 1 42	5	1 0 59	19	1 32	3	1 19
20	2 1 56	6	2 1 06	20	2 36	4	2 23
21	3 2 31	7	3 1 21	21	3 45	5	3 30
22	4 4 35	8	4 1 56	22	4 1 02	6	4 43
23		9	5 4 04	23	5 1 42	7	5 1 10
24	1 1 37	10	1 55	24	1 30	8	1 18
25	2 1 50	11	2 1 02	25	2 34	9	2 22
26	3 2 22	12	3 1 15	26	3 42	10	3 29
27	4 4 07	13	4 1 47	27	4 58	11	4 43
28		14	5 3 34	28	5 1 36	12	5 1 09
29	1 1 32	15	1 52	29	1 28	13	1 18
30	2 1 44	16	2 58	30	2 32	14	2 22
31	3 2 13	17	3 1 10	May 1	3 39	15	3 29
Feb. 1	4 3 41	18	4 1 39	2	4 55	16	4 42
2		19	5 3 08	3	5 1 30	17	5 1 08
3	1 1 26	20	1 48	4	1 26	18	1 18
4	2 1 37	21	2 54	5	2 30	19	2 22
5	3 2 04	22	3 1 05	6	3 37	20	3 29
6	4 3 21	23	4 1 32	7	4 53	21	4 42
7		24	5 2 51	8	5 1 26	22	5 1 08
8	1 1 21	25	1 45	9	1 25	23	1 18
9	2 1 31	26	2 50	10	2 29	24	2 22
10	3 1 56	27	3 1 01	11	3 36	25	3 29
11	4 3 04	28	4 1 25	12	4 51	26	4 42
12		29	5 2 34	13	5 1 22	27	5 1 08

[1] The hours are counted each way from noon. Thus 9 A.M. and 3 P.M. would correspond to the 3d hour in the table.

TABLE OF REFRACTION CORRECTIONS TO BE APPLIED TO THE DECLINATIONS [1]—*Continued.*

Date.	Refraction Correction. Latitude 40°.	Date.	Refraction Correction. Latitude 40°.	Date.	Refraction Correction. Latitude 40°.	Date.	Refraction Correction. Latitude 40°.
June		Aug.		Oct.		Nov.	
28	1 h. 18″	17	1 h. 32″	6	1 h. 1′ 03″	20	1 h. 1′ 46″
29	2 22	18	2 36	7	2 1 10	21	2 2 01
30	3 29	19	3 45	8	3 1 27	22	3 2 40
July 1	4 43	20	2 1′ 02	9	4 2 06	23	4 4 59
2	5 1′ 09	21	5 1 42	10	5 4 39	24	
3	1 19	22	1 34	11	1 1 07	25	1 1 50
4	2 23	23	2 38	12	2 1 15	26	2 2 06
5	3 30	24	3 48	13	3 1 33	27	3 2 49
6	4 43	25	4 1 06	14	4 2 18	28	4 5 33
7	5 1 10	26	5 1 49	15	5 5 29	29	
8	1 20	27	1 36	16	1 1 12	30	1 1 54
9	2 24	28	2 41	17	2 1 20	Dec. 1	2 2 11
10	3 31	29	3 51	18	3 1 40	2	3 2 59
11	4 44	30	4 1 10	19	4 2 31	3	4 6 01
12	5 1 11	31	5 1 58	20	5 6 49	4	
13	1 21	Sept. 1	1 39	21	1 1 16	5	1 1 58
14	2 25	2	2 44	22	2 1 25	6	2 2 16
15	3 32	3	3 54	23	3 1 48	7	3 3 04
16	4 46	4	4 1 14	24	4 2 47	8	4 6 23
17	5 1 13	5	5 2 08	25	5 8 39	9	
18	1 22	6	1 42	26	1 1 21	10	1 2 00
19	2 26	7	2 47	27	2 1 31	11	2 2 19
20	3 33	8	3 57	28	3 1 56	12	3 3 09
21	4 47	9	4 1 19	29	4 3 04	13	4 6 38
22	5 1 15	10	5 2 18	30	5 11 01	14	
23	1 23	11	1 48	31	1 1 26	15	1 2 01
24	2 27	12	2 50	Nov. 1	2 1 37	16	2 2 20
25	3 34	13	3 1 01	2	3 2 04	17	3 3 11
26	4 49	14	4 1 25	3	4 3 21	18	4 6 47
27	5 1 18	15	5 2 34	4	5 13 57	19	
28	1 25	16	1 48	5	1 1 32	20	1 2 01
29	2 29	17	2 54	6	2 1 44	21	2 2 20
30	3 36	18	3 1 05	7	3 2 13	22	3 3 11
31	4 51	19	4 1 32	8	4 3 41	23	4 6 49
Aug. 1	5 1 22	20	5 2 51	9		24	
2	1 26	21	1 52	10	1 1 37	25	1 2 00
3	2 30	22	2 58	11	2 1 50	26	2 2 19
4	3 37	23	3 1 10	12	3 2 22	27	3 3 09
5	4 53	24	4 1 39	13	4 4 07	28	4 6 13
6	5 1 26	25	5 3 08	14		29	
7	1 28	26	1 55	15	1 1 42	30	
8	2 32	27	2 1 02	16	2 1 56	31	
9	3 39	28	3 1 15	17	3 2 31		
10	4 55	29	4 1 47	18	4 4 35		
11	5 1 30	30	5 3 34	19			
12	1 30	Oct. 1	1 59				
13	2 34	2	2 1 06				
14	3 42	3	3 1 21				
15	4 58	4	4 1 56				
16	5 1 36	5	5 4 04				

[1] Refractions at noon (hr. ang = 0) sensibly same as at 11 or 1 o'clock.

Since the time is 6 h. earlier than that at Greenwich, the declination given in the Ephemeris for Greenwich mean noon (5° 50′ 36″) is the declination for the given place at 6 A.M. If the point were in the "Eastern Time Belt" it would be the declination at 7 A.M., etc. Suppose it is desired to prepare declination settings from 7 A.M. to 5 P.M. From the table of

TABLE OF LATITUDE COEFFICIENTS.

Latitude.	Coefficient.	Latitude.	Coefficient.	Latitude.	Coefficient.
15°	.30	30°	.65	45°	1.20
16	.32	31	.68	46	1.24
17	.34	32	.71	47	1.29
18	.36	33	.75	48	1.33
19	.38	34	.78	49	1.38
20	.40	35	.82	50	1.42
21	.42	36	.85	51	1.47
22	.44	37	.89	52	1.53
23	.46	38	.92	53	1.58
24	.48	39	.96	54	1.64
25	.50	40	1.00	55	1.70
26	.53	41	1.04	56	1.76
27	.56	42	1.08	57	1.82
28	.59	43	1.12	58	1.88
29	.62	44	1.16	59	1.94

NOTE.—For any other latitude than 40° the refraction corrections given in the preceding table are to be multiplied by the coefficients given in this table to obtain the true refraction corrections for that latitude.

latitude coefficients we find that the refraction corrections will be .94 of those given in the table. The following table of declination settings may now be made out:

Hour.	Declination.	Refr.Cor.	Setting.	Hour.	Declination.	Refr.Cor.	Setting.
7	+5° 51′ 33″	+2′ 00″	+5° 53′ 33″	1	+5° 55′ 21″	+ 37″	+5° 55′ 58″
8	5 52 30	+1 10	5 53 40	2	5 56 18	+ 41	5 56 59
9	5 53 27	+0 51	5 54 18	3	5 57 15	+ 51	5 58 6
10	5 54 24	+0 41	5 55 5	4	5 58 12	+1′ 10	5 59 22
11	5 54 21	+0 37	5 54 58	5	5 59 9	+2 00	6 1 9

From March 20th to September 20th the declination is positive, while from September 20th to March 20th it is negative. From December 20th to June 20th the hourly correction is positive, while from June 20th to December 20th it is negative. The refraction correction is always positive. Particular attention must be given to all these signs in making out the table of declination settings.

105. A Simple Way of Correcting the Sun's Declination for the Effect of Refraction.[1] In case the observer should not have with him the refraction tables found on the preceding three pages, the refraction can be determined by the aid of his watch, as follows:

Having focussed the eye-piece and object glass of the transit so that a clear image of both the sun's disk and the cross-wires can be seen on the screen held behind the eye-piece (a piece of white paper held by the hand will do for a screen), set the horizontal circle of the transit to read some integral ten minutes and point on the sun by the lower motion.

The earth's diurnal motion will carry the sun across the vertical thread of the instrument. Note the time on a watch to the nearest second when the sun is tangent to the vertical wire. Keeping the lower motion clamped, unclamp the upper and turn the alidade in the direction of the sun's movement, i.e., toward the west, and set the vernier to read the next ten minutes. Note again the time when the sun is tangent to the vertical thread. Also read the vertical angle to the sun. Then if we call n the interval of time elapsed in seconds while the sun (really the earth) was passing through ten minutes of arc, and call h the vertical angle in degrees, the refraction d, in minutes, is given by the equation,

$$d = \frac{2000}{h \cdot n}.$$

[1] For a very complete explanation of this method see Comstock's "Astronomy," by Professor George C. Comstock, University of Wisconsin.

Experience in using this formula has shown that its maximum errors will not exceed 15″ when the sun is above 10° altitude, while its average error is less than half this amount. As the refraction correction, as ordinarily computed, is based upon average conditions of temperature and barometric pressure, seldom exactly realized in any given case, the writer has not been surprised to find that results obtained by the use of the above formula are quite as good as those obtained from the more complicated and pretentious formulas and tables.

A still more accurate determination of the refraction can be made by the use of the following equation:

$$d \text{ in minutes} = \frac{N}{n},$$

where d is the refraction in seconds, and N is obtained from the following table by entering it with the measured altitude of the sun as an argument:

h	N	Dif. for 1°.	h	N	Dif. for 1°.
10°	219		30°	60	
		15			2.3
15°	144		40°	37	
		8			1.5
20°	103		50°	22	
		5			1.0
25°	79		60°	12	
		3.6			0.7
30°	59		70°	5	

The altitude of the sun need only be observed to the nearest half degree.

The tabulated values of N correspond to a temperature of 50° F. and a barometric pressure of thirty inches. They may be adapted to any other temperature by diminishing d by one per cent for each five degrees by which the temperature

exceeds 50°, or by increasing one per cent for each five degrees below 50°.

This correction and the correction for variations of the barometer can usually be neglected. At great elevations, however, the barometric pressure becomes so much reduced that its variation must be taken into account, and this is done by diminishing d by one per cent for each 300 feet of elevation above the sea.

TABLE OF ERRORS IN AZIMUTH (BY SOLAR TRANSIT) FOR 1′ ERROR IN DECLINATION OR LATITUDE

Hour.	For 1′ Error in Declination.			For 1′ Error in Latitude.		
	Lat. 30°.	Lat. 40°.	Lat. 50°.	Lat. 30°.	Lat. 40°.	Lat. 50°.
11.30 A.M. } ... 12.30 P.M. }	8′.85	10′.0	12′.9	8′.77	9′.92	11′.8
11 A.M. } ..... 1 P.M. }	4 .46	5 .05	6 .01	4 .33	4 .87	5 .80
10 A.M. } ..... 2 P.M. }	2 .31	2 .61	3 .11	2 .00	2 .26	2 .70
9 A.M. } ..... 3 P.M. }	1 .63	1 .85	2 .20	1 .15	1 .30	1 .56
8 A.M. } ..... 4 P.M. }	1 .34	1 .51	1 .80	0 .67	0 .75	0 .90
7 A.M. } ..... 5 P.M. }	1 .20	1 .35	1 .61	0 .31	0 .35	0 .37
6 A.M. } ..... 6 P.M. }	1 .15	1 .30	1 .56	0 .00	0 .00	0 .00

Note.—Azimuths observed with erroneous declination or co-latitude may be corrected by this table by observing that for the line of collimation set *too high*, the azimuth of any line *from the south point* in the direction S.W.N.E. is found *too small* in the *forenoon* and *too large* in the *afternoon* by the tabular amounts for each minute of error in the altitude of the solar line of sight. The reverse is true for this line set too low.

Several important conclusions may be drawn from this table:

First—That the solar transit should never be used between 11 A.M. and 1 P.M., and preferably not between 10 A.M. and 2 P.M., if the best results are desired.

Second—That at 6 o'clock A.M. and P.M., when the line of collimation lies in a plane at right angles to the plane of the meridian, no small change in the latitude-arc will affect the accuracy of the result. At these times, however, occurs the maximum error in the refraction.

EXERCISES WITH THE TRANSIT

1. Establish three stations forming a triangle. Measure the three horizontal angles and see if their sum is 180°.

2. Prolong a line in azimuth and distance by carrying both around an imaginary obstruction, and then check the azimuth by a back sight and the distance by measurement. Thus, let A and B be two points establishing a line. The problem is to establish two other points, C and D, in the continuation of the line AB, with an imaginary obstruction to both sight and measurement between B and C. The distance BC is also to be obtained.

The equilateral triangle will be found most efficient.

3. Find both the distance to and the height of an inaccessible steeple, chimney, smokestack, or tree.

Measure a base-line such that its two extremities make with the given object approximately an isosceles triangle (it is desirable that no angle of the triangle should be less than 30° nor more than 120°). The top of the object only need be visible from the two ends of the base. Measure both the horizontal and vertical angles at the extremities of the base-line subtended by the other two points of the triangle. Let A and B be the extremities of the base and P the point whose distance and elevation are required. We then have for horizontal angles

$$\text{Sin } P : \sin A :: AB : BP;$$

also

$$\sin P : \sin B :: AB : AP.$$

In reading the vertical angles to the base-stations the reading should be taken on a point as high above the ground (or peg) as the telescope is above the peg over which it is set. The difference in the elevations of the two pegs is then obtained. The vertical angle to the point P is taken to the summit, and height of instrument added in each case to find its elevation above peg. If A be the lower of the two base-stations and if I_A and I_B be the heights of instru-

ment (line of sight) above the peg in the two cases, and if V_A, V_B, V_P and $V_{P'}$ be the vertical angles read to the corresponding points, we may write:

$$\text{Elevation of } B \text{ above } A = AB \tan V_B;$$
$$\text{“ “ } P \text{ “ } A = AP \tan V_P.$$

Also, from the vertical angles taken at B, we have:

$$\text{Elevation of } A \text{ below } B = AB \tan V_A;$$
$$\text{“ “ } P \text{ above } B = BP \tan V_{P'}.$$

We now have a check on both the relative elevations and on the distances AP and BP. Assuming the elevation of A to be zero, we have:

$$\text{Elevation of } P \text{ above } A = AP \tan V_P = AB \tan V_B + BP \tan V_{P'}.$$

This equality will not result unless the observations were well taken, the computations accurately made, and the instrument carefully adjusted. The adjustments mainly involved here are the plate-bubbles and the vernier on the vertical circle. If the points are a considerable distance apart, as over a half-mile, the elevations obtained by reading the vertical angles are appreciably too great, on account of the earth's curvature. This may be taken as eight inches for one mile and proportional to the square of the distance. Or, we may write:

Elevation correction on long sights, in inches,[1] $= 8$ (distance in miles)^2.

If the distances are all less than about half a mile, no attention need be paid to this correction in this problem.

4. Find the height of a tree or house above the ground, on a distant hill, without going to the immediate locality.

5. Find the horizontal length and bearing of a line joining two visible but inaccessible objects. Use the magnetic bearing if the true bearing of the base-line is not known.

6. Find the horizontal length and bearing of a line joining two inaccessible points both of which cannot be seen from any one position.

Let A and B be the inaccessible points. Measure a base CD such that A is seen from C, and B from D. Auxiliary bases and triangles may be used to find the lengths of AC and BD. Knowing AC and CD and the included angle, compute AD in bearing and distance. The angle ADB may now be found which, with the adjacent sides AD and BD known, enables the side AB to be found in bearing and distance.

[1] For a full discussion of this subject see Chapter XVI.

7. With the transit badly out of level, or with horizontal axis of the telescope thrown considerably out of the horizontal, measure the horizontal angle between two objects having very different angular elevations. Do this with both telescope normal and telescope reversed, and note the difference in the values of the angle obtained in the two cases.

8. Select a series of points on uneven ground, enclosing an area, and occupy them successively with the transit, obtaining the traverse angles. That is, knowing or assuming the azimuth of the first line, obtain the azimuths of the other connecting lines, or courses, with reference to this one, returning to the first point and obtaining the azimuth of the first course as carried around by the traversed line. This should agree with the original azimuth of this course. The distances need not be measured for this check.

9. Lay out a straight line on uneven ground by the method given in Art. 89, occupying from six to ten stations. Return over the same line and establish a second series of points, paying no attention to the first series, and then note the discrepancies on the several stakes. In returning, the two final points of the first line become the initial points of the second, this return line being a prolongation of the line joining these two points. If these deviate ever so little, therefore, from the true line, the discrepancy will increase towards the initial point.

Similar exercises to those given for the solar compass may be assigned for the solar attachment.

The Sextant

106. The Sextant is the most convenient and accurate hand-instrument yet devised for measuring angles, whether horizontal, vertical, or inclined. It is called a sextant because its limb includes but a 60° arc of the circle. It will measure angles, however, to 120°. It is held in the hand, measures an angle by a single observation, and will give very accurate results even when the observer has a very unstable support, as on board ship. It is exclusively used in observations at sea, and is always used in surveying where angles are to be measured from a boat, as in locating soundings, buoys, etc., as well as in reconnoissance work, explorations, and preliminary surveys. It has been in use since about 1730.

The accompanying cut shows a common form of this

instrument. The limb has a $7\frac{1}{2}$-inch radius, and the most accurate kind reads to 10 seconds of arc.

There is a mirror *M* (Fig. 53), called the *Index Glass*, rigidly attached to the movable arm *MA*, which carries a vernier reading on the graduated limb *CD*. There is another mirror, *I*, called the *Horizon Glass*, rigidly attached to the frame of the instrument, and a telescope pointing into this mirror, also rigidly attached. This mirror is silvered on its lower half, but

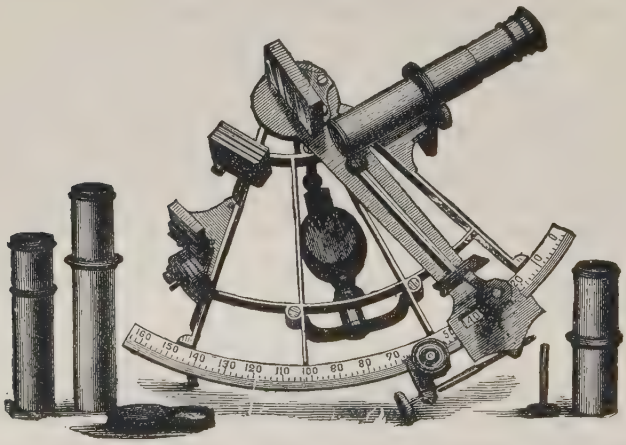


FIG. 52.

clear on its upper half. A ray of light coming from *H* passes through the clear portion of the mirror *I* on through the telescope to the eye at *E*. Also, a ray from an object at *O* strikes the mirror *M*, is reflected to *m*, and then through the telescope to *E*. Through one half of the objective come the rays from *H*, and through the other half the rays from *O*, each of which sets of rays forms a perfect image. By moving the arm *MA* it is evident these images will appear to move over each other, and for one position only will they appear to coincide. The bringing of the two images into exact coincidence is what the observation consists in, and however unsteady the motion of

the observer may be, he can occasionally see both images at once, and so by a series of approximations he may finally put the arm in its true position for exact superposed images. The angle subtended by the two objects is then read off on the limb.

107. The Theory of the Sextant rests on the optical principle that "if a ray of light suffers two successive reflec-

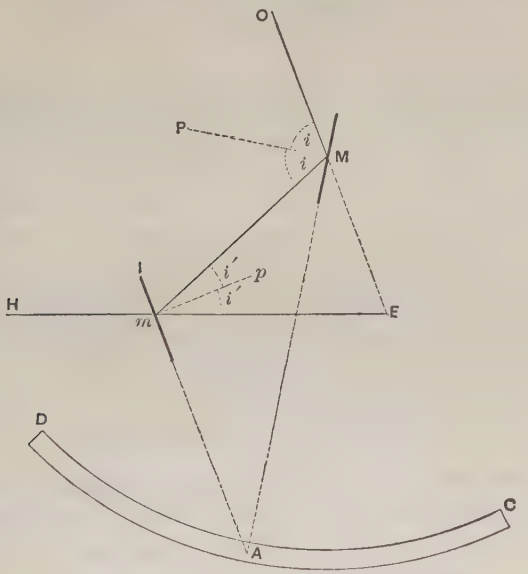


FIG. 53.

tions in the same plane by two plane mirrors, the angle between the first and last directions of the ray is twice the angle of the mirrors."

To prove this, let *OM* and *mE* (Fig. 53), be the first and last positions of the ray, the latter making with the former produced the angle *E*. The angle of the mirrors is the angle *A*. The angles of incidence and reflection at the two mirrors are the angles *i* and *i'*, *PM* and *pm* being the normals.

We may now write

$$\text{Angle } E = OMm - MmE.$$
$$= 2(i - i');$$
$$\text{angle } A = ImM - mMA$$
$$= (90° - i') - (90° - i)$$
$$= i - i';$$

Therefore $E = 2A$. Q.E.D.

When the mirrors are brought into parallel planes, the angle A becomes zero, whence E also is zero, or the rays OM and Hm are parallel. This gives the position of the arm for the zero-reading of the vernier. The limb is graduated from this point towards the left in such a way that a 60° arc of the circle will read to 120°. That is, a movement of 1° on the arc really measures an angle of 2° in the incident rays, so it must be graduated as two degrees instead of one. The very large radius enables this to be done without difficulty.

Adjustments of the Sextant

108. To Make the Index Glass Perpendicular to the Plane of the Sextant. Bring the vernier to read about 30° and examine the arc and its image in the index glass to see if they form a continuous curve. If the glass is not perpendicular to the plane of the arc, the image will appear above or below the arc, according as the mirror leans forward or backward. It is adjusted by slips of thin paper under the projecting points and corners of the frame, or by a screw provided for this purpose.

109. To Make the Horizon Glass Parallel to the Index Glass for a Zero-Reading of the Vernier. Set the vernier to read zero and see if the direct and reflected images of a well-defined distant object, as a star, come into exact coincidence.

If not, adjust the horizon glass until they do. If this adjustment cannot be made, bring the objects into coincidence, or even with each other so far as the motion of the arm is concerned, and read the vernier. This is the index error of the instrument and is to be applied to all angles read. The better class of instruments all allow the horizon glass to be adjusted. This adjustment is generally given as two, but it is best considered as one. If made parallel to the index glass after that has been adjusted, it must be perpendicular to the plane of the instrument.

110. **To Make the Line of Sight of the Telescope Parallel to the Plane of the Sextant.** The reticule in the sextant carries four wires, forming a square in the centre of the field. The centre of this square is in the line of collimation of the instrument.

Rest the sextant on a plane surface, pointing the telescope upon a well-defined point some 20 feet distant. Place two objects of equal height upon the extremities of the limb that will serve to establish a line of sight parallel to the limb. Two lead-pencils of same diameter will serve, but they had best be of such height as to make this line of sight even with that of the telescope. If both lines of sight come upon the same point to within a half-inch or so at a distance of 20 feet, the resulting maximum error in the measurement of an angle will be only about 1 second.

The Use of the Sextant

111. **To Measure an Angle** with the sextant, bring its plane into the plane of the two objects. Turn the direct line of sight upon the fainter object, which may require the instrument to be held face downwards, and bring the two images into coincidence. The reading of the limb is the angle required. It must be remembered that the angles measured by the sextant are *the true angles subtended by the two objects at the point of*

observation, and not the vertical or horizontal projection of these angles, as is the case with the transit. The true vertex of the measured angle is at *E*; Fig. 53. It is evident the position of *E* is dependent on the size of the angle, being at a great distance back of the instrument for a very small angle. The instrument should therefore not be used for measuring very small angles except as between objects a very great distance off. The sextant is seldom or never used for measuring angles where the position of the instrument (or the vertex of the angle) needs to be known with great accuracy.

EXERCISES FOR THE SEXTANT

1. Measure the altitude of the sun or a star at its culmination by bringing the direct image, reflected from the surface of mercury held in a flat dish on the ground, into coincidence with the image reflected from the index glass. Half the observed angle is the altitude of the body. The altitude of a terrestrial object may be obtained in the same manner, in which case the vessel of mercury should rest on an elevated stand; the sextant could then be brought near to it and the angular divergence of the two incident rays to the mercury surface and index glass reduced to an inappreciable quantity.

If the observation of a heavenly body be made on the meridian and the declination of the body at the time of observation be known, the latitude of the place is readily found.

2. Measure the angle subtended by two moving bodies, as of two men walking the street in the same direction, or of two boats on the water. (This is to illustrate the capacity of the sextant, for none but a reflecting instrument bringing two converging lines of sight into coincidence is competent to do this.)

Exercises 1, 3 and 4 for the transit may also serve for the sextant. Further applications of the sextant in locating soundings are given in Chapter X.

112. The Cross-Section Polar Protractor. The accompanying cuts (Figs. 54 and 55) illustrate an instrument recently invented and used on the New York Aqueduct for taking polar coördinates of the cross-sections of the tunnel. It consists of a plain circular disk, graduated to single degrees, and mounted on a tripod in such a way that it may be levelled up and also

have a vertical motion and a motion about the vertical axis.[1] The construction is shown clearly in the figures.

In use it is mounted with its centre in the axis of the tunnel. A light wooden measuring rod, not shown in the figures, tapering to a point and shod with brass of sufficient length, and graduated to feet and hundredths, lies upon the wooden

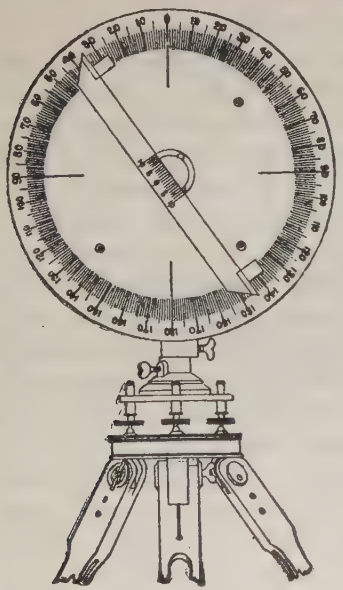


FIG. 54.

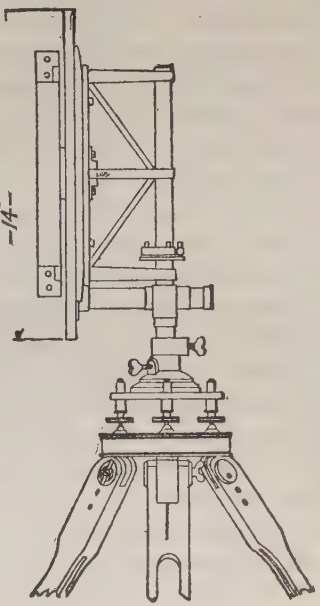


FIG. 55.

arm or rest, which revolves upon the face of the disk, and slides out to a contact with the surface at such points as are to be taken. If the only information desired is whether or not

[1] Those used on the New York Aqueduct were designed by F. W. Watkins and Alfred Craven, and were manufactured by Heller & Brightly, Philadelphia. See description in "Trans. Am. Soc. C. E.," 1890, and Engineering News, July 26, 1890.

the excavation is sufficient, or beyond the established lines, then the rod is set to the proper radius, and if it swings clear, the fact is determined. If a true copy of the actual cross-section is desired, then the rod is brought into contact with the significant points in the cross-section (mostly the points of greatest projection and depression of the surface), and the angle and distance read and recorded. In the instrument here shown, the graduation increases in both directions from the top to 180° at the bottom. Perhaps a better arrangement would be to have the angles increase continuously to 360°. The work could be plotted by means of such a protractor as shown in Figs. 113 or 114, Chapter VIII. The points being plotted, they should be joined by a free-hand line and the area determined by the planimeter.

If the cross-section contains one or more marks from which the axis of the tunnel may be found, as an alignment mark and a bench mark (which may be one and the same), then the instrument may be set up at random on this section, and these fixed marks pointed in and plotted, along with the cross-section points. The axis of the tunnel can then be laid off from the plotted marks, and by drawing in the established lines from this axial point, the question of clearance may be determined nearly as well as by setting the instrument in the axis of the tunnel itself. The actual cross-section and area are quite as well determined as if the instrument were carefully centred on the axial line.

CHAPTER V

THE PLANE TABLE

113. The Plane Table consists of a drawing-board properly mounted on which rests an alidade carrying a line of sight rigidly attached to a plain ruler with a fiducial edge. The line of sight is usually determined by a telescope, as in Fig. 56. This telescope has no lateral motion with respect to the ruler, but both may be moved at pleasure on the table. The telescope has a vertical motion on a transverse axis, as in the transit. It is also provided with a level tube, either detachable or permanently fixed. The table is levelled by means of one round or two cross bubbles on the ruler of the alidade. The line of sight of the telescope is usually parallel to the fiducial edge of the ruler, though this is not essential. It is only necessary that they should make a fixed horizontal angle with each other. The table itself must have a free horizontal angular movement and the ordinary clamp and slow-motion screw. The table corresponds to the graduated limb in the transit, the alidades in the two instruments performing similar duties. Instead, however, of reading off certain horizontal angles, as is done with the transit, and afterwards plotting them on paper, the directions of the various pointings are at once drawn on the paper which is mounted on the top of the table, no angles being read. The true relative positions of certain points in the landscape are thus transferred directly to the drawing-paper to any desired scale. The magnetic bearing of any line may be determined by means of the *declinator*, which is a small box carrying a needle which can swing some ten degrees either side of the zero-line. The zero-line being parallel to one edge of the box, the magnetic meridian

may be at once marked down on any portion of the map, and the bearing of any intersecting line determined by means of a

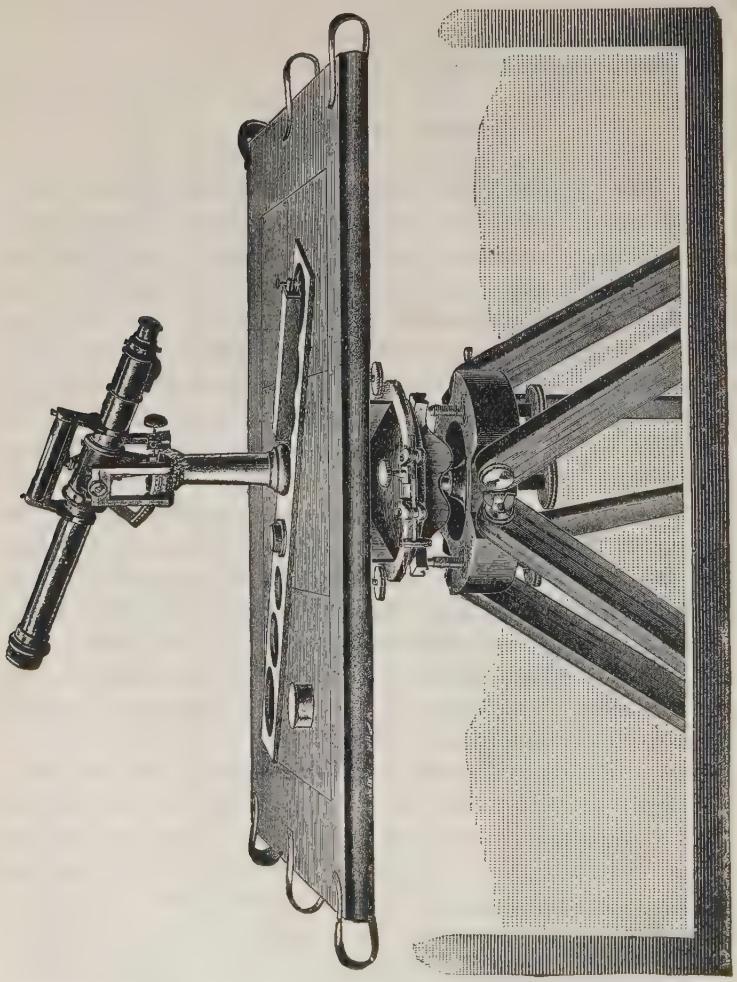


Fig. 56.

protractor. The instrument has been long and extensively used for mapping purposes, and is still the only instrument used

for the "filling in" of the topographical charts of the U. S. Coast and Geodetic Survey. An extended account of the instrument and the field methods in use on that service may be found in Appendix 7 of the Report of the U. S. Coast and Geodetic Survey for 1905. The following discussion is partly from that source.

Adjustments of the Alidade

114. To Make the Axes of the Plate Bubbles Parallel to the Plane of the Table. Level the table with the alidade in any position, noting the readings of the bubbles. Mark the exact position of the alidade on the table, take it up carefully, and, reversing it end for end, replace it by the same marks. If the bubbles now have the same readings as before, with reference to the table they are parallel to the plane of the table. If not, adjust the bubbles for one half the movement and try again.

115. To Cause the Line of Sight to Revolve in a Vertical Plane. This adjustment is the same as in the transit. It need not be made with such extreme accuracy, however, and the plumb-line test is sufficient. With the instrument carefully levelled, cause the line of sight to follow a plumb-line through as great an arc as convenient. If the line of sight deviates from the plumb-line raise or lower one end of the transverse axis of the telescope, until it will follow it with sufficient exactness.

116. To Cause the Telescope Bubble and the Vernier on the Vertical Arc to Read Zero when the Line of Sight is Horizontal. This adjustment is also the same as in the transit. The methods given for the transit may be used with the plane table, or a sea horizon may be used as establishing a horizontal line, or a levelling instrument may be set up beside the plane table having the telescopes at the same elevation, and both lines of sight turned upon the same point in the horizontal plane as determined by the level. The bubble and vernier are then both adjusted to this position of telescope.

This adjustment is important if elevations are to be determined either by vertical angles or by horizontal lines of sight. If only geographical position is sought this adjustment may be neglected.

The Use of the Plane Table [1]

117. In Using the Plane Table at least two points on the ground must be plotted on the paper to the scale of the map before the work of locating other points can begin. This requires that the distance between these points shall be known, which distance becomes the base-line for all locations on that sheet. Any error in the measurement or plotting of this line produces a like proportional error in all other lines on the map.

The plane table is set over one of these plotted points, the fiducial edge of the ruler brought into coincidence with the two points, and the table revolved until the line of sight comes on the distant point. The table is now clamped and carefully set by the slow-motion screw in this position, when it is said to be *oriented*, or *in position.*

In Figs. 57 and 58 let T, $T,'$ $T,''$ $T,'''$ represent the plane-table sheet and the points a and p the original plotted points. The corresponding points on the ground are A and P, the latter being covered by p in Fig. 57, and the former by a in Fig. 58. In Fig. 57, the plotted point p is centred over the point P, the ruler made to coincide with ap, and the telescope made to read on A by shifting the table. For plotting the directions of other objects on the ground, the alidade is made *to revolve about p* just as the transit revolves about its centre. A needle is sometimes stuck at this point, and the ruler caused to press against it in all pointings, but this defaces the sheet. Other pointings are now made to B, C, and D, which may be used as stations,

[1] For the more accurate methods of doing plane table problems see Appendix 8, U. S. C. and G. Survey Report for 1897–8, by D. B. Wainwright.

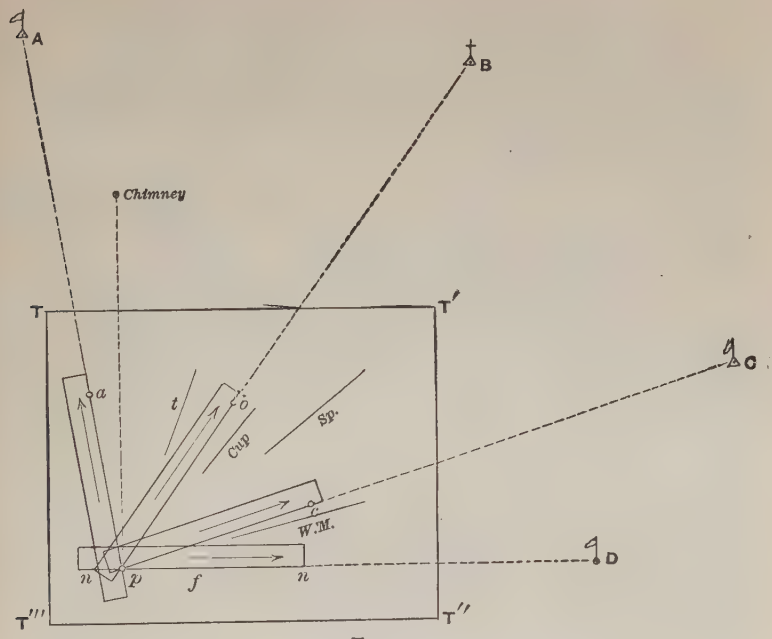


FIG. 57.

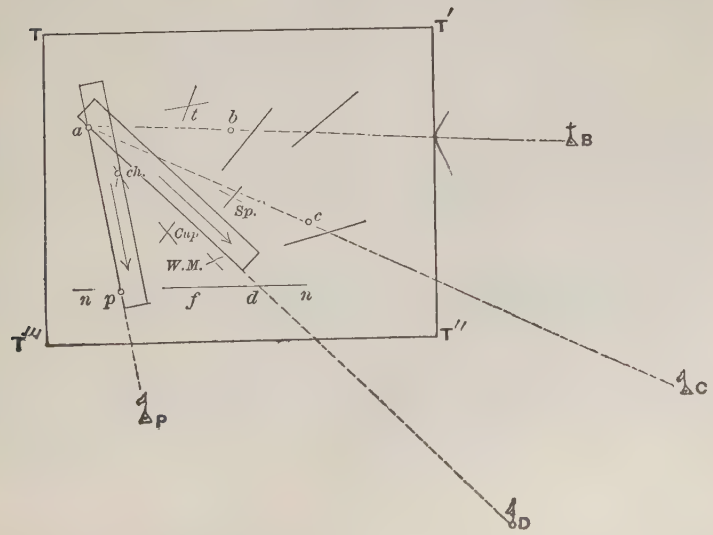


FIG. 58.—Resection with Plane Table.

and also to a chimney (*ch.*), a tree (*t.*), a cupola (*cup.*), a spire (*sp.*), and a windmill (*w.m.*). Short lines are drawn at the estimated distance from *p*, and these marked with letters, as in the figure, or by numbers, and a key to the numbers kept in the sketch- or note-book.

The table is now removed to *A*, the other known point, and set with the point *a* on the plot over the point *A* on the ground, when the table is approximately oriented. The ruler is now set as shown in Fig. 57, coinciding with *a* and *p*, but pointing towards *p*. The table is then swung in azimuth until the line of sight falls on *P*, when it is clamped. It is now oriented [1] for this station, and pointings are taken on all the objects sighted from *P*, and on such others as may be sighted from subsequent stations, the alidade now revolving about the point *a* on the paper. The intersections on the plot of the two pointings taken to the same object from *A* and *P* will evidently be the true position on the plot for those points with reference to, and to the scale of, the line *ap*. These intersections are shown in Fig. 58.

It is evident that if other points, as *D* or *C*, be now occupied, the table oriented on either *A* or *P*, and pointings taken on any of the objects sighted from both *A* and *P*, the third or fourth line drawn to the several objects should intersect the first two in a common point. This furnishes a check on the work, and should be taken for all important points. It is preferable also to have more than two points on the sheet previously determined. Thus, if *B* were also known and plotted at *b*, when the table had been oriented on any other station, and a pointing taken to *B*, the fiducial edge of the ruler should have passed through *b*.

As fast as intersections are obtained and points located the accompanying details should be drawn in on the map to the proper

[1] It will be noted that this process of orienting the plane table is practically identical with that by which the limb of the transit is oriented in traversing (Art. 91).

scale. If distances are read by means of stadia wires on a rod held at the various points (see Chapter VIII), then a single pointing may locate an object, the distance being taken off from a scale of equal parts, and the point at once plotted on the proper direction-line. It is now common to do this in all detail plane-table surveying.

118. Location by Resection. This consists in *locating the points occupied* by pointings to known and plotted points. The simple case is where a single pointing has been taken to this point from some known point, and a line drawn through it on the sheet. It is not known what point on this line represents the plotted position of this station. The setting of the instrument can therefore be but approximate, but near enough for all purposes. The table can be oriented as before, there being one pointing and corresponding line from a known point. A station is then selected, a pointing to which is as nearly 90 degrees from the orienting line as possible, and the alidade so placed that while the telescope sights the object the fiducial edge of the ruler passes through the plot of the same on the sheet. The intersection of this edge with the former line to this station gives the station's true position on the sheet. This letter operation is called *resection*. Another resection from any other determined point may be made for a check.

119. To Find the Position of an Unknown Point by Resection on Three Known Points. This is known as the *Three-point Problem*, and occurs also in the use of the sextant in locating soundings. It is fully discussed in that connection (see Art. 241), so that only a mechanical solution suitable for the problem in hand will be given here. It is understood there are three known points, A, B, and C, plotted in a, b, and c on the map. The table is set up over any given point (not in the circumference of a circle through A, B, and C). Fasten a piece of tracing-paper, or linen, on the board, and mark on it a point p for the station P occupied. Level the table, but of course it cannot be oriented. Take pointings to A, B, and C,

and draw lines on the tracing-paper from p towards a, b, and c, long enough to cover these distances when drawn to scale. Remove the alidade and shift the tracing-paper until the three lines drawn may be made to coincide exactly with the three plotted points a, b, and c. The point p is then the true position of this point on the sheet. This being pricked through, the table may now be oriented and the work proceed as usual.

120. To Find the Position of an Unknown Point by Resection on Two Known Points. This is called the Two-

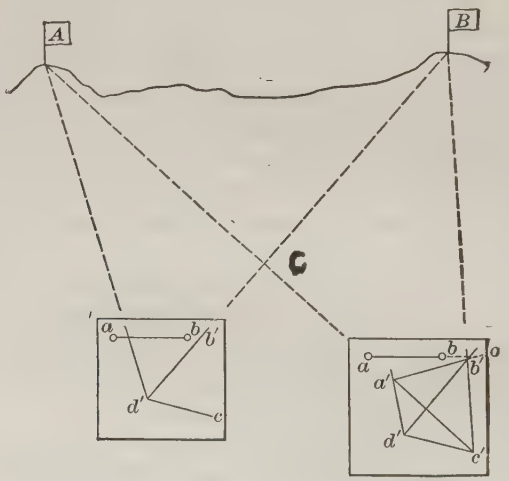


FIG. 59.—Two Point Method.

point Problem, and but one of several solutions will be given. It is evident that if the table could be properly oriented over the required point, its position on the sheet could be at once found by resection on the two known points. The table may be oriented in the following manner: Let A and B be the known points plotted in a and b on the sheet, Fig. 59. Let C be the unknown point whose position c on the sheet is desired. Select a fourth point D, which may be occupied, and so placed that intersections from C and D on A and B will give angles between 30 and

120 degrees. Fasten a piece of tracing linen or paper on the board, marking a point d' at random. Set up over D, orienting the table as nearly as may be by the needle or otherwise. Draw lines from d' towards A, B, and C. Mark off on the latter the estimated distance to C, to scale, calling this point C'. Set up over C, with c' over the station, orienting on D by the line $c'd'$. This brings the table parallel to its former position at D. From c' draw lines to A and B, intersecting the corresponding lines drawn from d' in a' and b'. We now have a quadrilateral $a'b'c'd'$ similar to the quadrilateral formed by the true positions of the plotted points *abcd*, but it differs in size, since the distance $c'd'$ was assumed, and also in position (azimuth), since the table was not properly oriented at either station. Remove the alidade, and shift the tracing until the line $a'b'$ coincides with a and b on the sheet. Replace the alidade on the tracing, bringing it into coincidence with $c'a'$, $c'b'$, or $c'd'$, and *revolve the table on its axis* until the line of sight comes upon A, B, or D, as the case may be. The table is now oriented, when the true position of c may be readily found by resecting from a and b, which, when pricked through, gives its position on the sheet.

The student may show how the same result could have been obtained without the aid of tracing-paper.

If the fourth point D may be taken *in range* with A and B, the table may be properly oriented on this range, and a line drawn towards C from any point on this range line on the plot. Then C is occupied, and the table again properly oriented by this line just drawn, when the true position of c may be found by resecting from a and b, as before.

In general, if the table can be properly oriented over any unknown point from which sights may be taken to two or more known (plotted) points, the position of this unknown point is at once found by resection from the known points.

The student would do well to look upon the table and the attached plot as analogous to the graduated horizontal limb in

the transit. The principles and methods of orienting are precisely similar, the pointings differing only in this, that with the transit the horizontal angle, referred to the meridian, is read off, recorded, and afterwards plotted, while with the plane table this bearing is immediately drawn upon the sheet.

121. The Measurement of Distances by Stadia. This method of determining short distances is now generally used in connection with the plane table. It is fully discussed in Chapter VIII, where the principles of its action and its use with the transit are given at length. The same principles, field methods, and tables apply to its use with the plane table, with such modifications as one accustomed to the use of the plane table would readily introduce. When used in this way it enables a point to be plotted from a single pointing, it being located by polar coördinates (azimuth and distance), instead of by intersections.

EXERCISES WITH THE PLANE TABLE

1. Make a plane-table survey of the college campus, measuring the length of one side for a base.

2. Having located several points on the sheet by *intersections*, occupy them and check their location by *resection*.

3. Locate a point (not plotted) by resection on three known points (Art. 119).

4. Locate a point (not plotted) by resection on two known points, first taking the auxiliary point D not in line with AB, and then by taking it in line with AB. This gives a check on the position of the point, and shows the advantages of the second method when it is feasible.

CHAPTER VI

ADDITIONAL INSTRUMENTS USED IN SURVEYING AND PLOTTING

The Aneroid Barometer

122. The Aneroid Barometer consists of a circular metallic box, hermetically sealed, one side being covered by a corrugated plate. The air is mostly removed, enough only being left in to compensate the diminished stiffness of the cor-

Fig. 58.—Aneroid Barometer.

rugated cover at higher temperatures. This cover rises or falls as the outer pressure is less or greater, and this slight motion is greatly multiplied and transmitted to an index pointer moving over a scale on the outer face. The motion of the index is com-

pared with a standard mercurial barometer and the scale graduated accordingly. Inasmuch as all barometric tables are prepared for mercurial barometers, wherein the atmospheric pressure is recorded in inches of mercury, the aneroid barometer is graduated so that its readings are identical with those of the mercurial column.

Fig. 60 shows a form of the aneroid designed for elevations to 4000 feet above or to 2000 feet below sea level. It has a vernier attachment and is read with a magnifying glass to single feet of elevation. It must not be supposed, however, that elevations can be determined with anything like this degree of accuracy by any kind of barometer. The barometer simply indicates the pressure at the given time and place, but for the same place the pressure varies greatly from various causes. All barometric changes, therefore, cannot be attributed to a change in elevation, when the barometer is carried about from place to place.

If two barometers are used simultaneously, which have been duly compared with each other, one at a fixed point of known elevation and the other carried about from point to point in the same locality, as on a reconnoissance, then the two sets of readings will give very close approximations to the differences of elevation. If the difference of elevation between distant points is desired, then long series of readings should be taken to eliminate local changes of pressure. The aneroid barometer is better adapted to surveys than the mercurial, since it may be transported and handled with greater ease and less danger. It is not so absolute a test of pressure, however, and is only used by exploring or reconnoissance parties. For fixed stations the mercurial barometer is to be preferred. It has been found from experience that the small aneroids of $1\frac{3}{4}$ to $2\frac{1}{2}$ inches diameter give as accurate results as the larger ones.

123. Barometric Formulas. In the following derivation of the fundamental barometric formula the calculus is used, so that the student will have to take portions of it on trust until

he has studied that branch of mathematics. All that follows Eq. (4) he can read.

Let H = height of the "homogeneous atmosphere" [1] in lat. 45°.

h = corresponding height of the mercurial column.

δ = the relative density of the "homogeneous atmosphere" with reference to mercury.

z = difference of elevation between two points, with barometric readings of h' and h_1, at the higher and at the lower point respectively.

Then from the equilibrium between the pressures of the mercurial column and atmosphere we have:

$$h = \delta H \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Also, for a small change in elevation, dz, the corresponding change in the height of the mercurial column would be

$$dh = \delta dz \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Substituting in (2) the value of δ as given by (1), we have:

$$dh = \frac{h}{H} dz;$$

or,

$$dz = H \frac{dh}{h} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Integrating (3) between the limits h' and h_1 we have:

$$z = H \int_{h'}^{h_1} \frac{dh}{h} = H \log_e \frac{h_1}{h'} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

[1] "Homogeneous atmosphere" signifies a purely imaginary condition wherein the atmosphere is supposed to be of uniform density from sea level to such upper limit as may be necessary to give the observed pressure at the observed temperature.

where the logarithm is in the Napierian system. Dividing by the modulus of the common system to adapt it to computation by the ordinary tables, we have:

$$z = 2.30258\, H \log_a \frac{h_1}{h'} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If H_o be the height of the homogeneous atmosphere at a temperature of 32° F., and if h_o be the height of the mercurial column at sea-level at same temperature, and if g_m and g_a be the specific gravities of mercury and air respectively, then, evidently,

$$H_o g_a = h_o g_m,$$

or,

$$H_o = \frac{h_o g_m}{g_a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

From experiment we have:

$$h_o = 29.92 \text{ inches},$$

$$g_m = 13.596$$

$$g_a = 0.001293$$

whence $H_o = 26{,}220$ feet.

This is on the assumption that gravity is constant to this height above sea level. When this is corrected for variable gravity we have:

$$H_o = 26{,}284 \text{ feet.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Equation (7) gives the height of the homogeneous atmosphere at a temperature of 32° F. But since the volume of a gas under constant pressure varies directly as the temperature, and

since the coefficient of expansion of air is 0.002034 for 1° F., we have for the height of the homogeneous atmosphere at any temperature:

$$H = H_o [1 + 0.002034 (t - 32°)] \quad . \quad . \quad . \quad (8)$$

If the temperature chosen be the mean of the temperatures at the two points of observation, as t' and t_1 for the upper and lower points respectively, then we should have:

$$H = H_o \left[1 + 0.002034 \left(\frac{t' + t_1}{2} - 32\right)\right]$$
$$= 26{,}284 [1 + 0.001017 (t' + t_1 - 64)] \quad . \quad . \quad . \quad (9)$$

Substituting this value of H in Eq. (5) we obtain:

$$z = 60{,}520 [1 + 0.001017 (t' + t_1 - 64)] \log \frac{h_1}{h'}. \quad . \quad . \quad (10)$$

If we wish to refer this equation to approximate sea level (height of mercurial column of 30 inches) and to a mean temperature of the two stations of 50° F., we may write:

$$\log \frac{h_1}{h'} = \log \frac{\frac{30}{h'}}{\frac{30}{h_1}} = \log \frac{30}{h'} - \log \frac{30}{h_1}.$$

Also, when $t' + t_1 = 100°$, we have

$$t' + t_1 - 64 = 36°.$$

Substituting these equivalents in Eq. (10), we obtain:

$$z = 60520\ (1 + 0.001017 \times 36)\left(\log \frac{30}{h'} - \log \frac{30}{h_1}\right),$$

$$z = 62737 \log \frac{30}{h'} - 62737 \log \frac{30}{h_1}. \quad \ldots\ldots\ (11)$$

In this equation, the two terms of the second member represent the elevations of the upper and lower points respectively, above a plane corresponding to a barometric pressure of 30 inches and for a mean temperature of the two positions of 50° F.

Table I. is computed from this equation, the arguments being the readings of the barometer, h' and h_1 at the upper and lower stations respectively, the tabular results being elevations above an approximate sea level. The difference between the two tabular results gives the difference of elevation of the two points, for a mean temperature of 50° and no allowance made for the amount of aqueous vapor in the air. For other temperatures, and for the effect of the humidity (which is not observed, but the average conditions assumed to exist), a certain correction needs to be applied, which correction is not an absolute amount, but is always a *certain proportion* of the difference of elevation as obtained from Eq. (11) or table I. If the two elevations taken from the table be called A' and A_1, and the correction for temperature and humidity be C, we would have:

$$z = (A' - A_1)\ (1 + C). \quad \ldots\ldots\ (12)$$

It is seen, therefore, that C is a coefficient which, when multiplied into the result obtained from table I., gives the correction to be applied to that result. The values of C are given in table II. for various values of $t' + t_1$.

The following example will illustrate the use of the tables:

TABLE I. BAROMETRIC ELEVATIONS[1]

Containing $A = 62737 \log \frac{30}{h}$. Argument, h.

h.	A.	Dif. for .01.	h.	A.	Dif. for .01.	h.	A.	Dif. for .01.
Inches.	Feet.	Feet.	Inches.	Feet.	Feet.	Inches.	Feet.	Feet.
11.0	27,336	—24.6	14.0	20,765	—19.5	17.0	15,476	—16.0
11.1	27,090	24.4	14.1	20,570	19.3	17.1	15,316	15.9
11.2	26,846	24.2	14.2	20,377	19.1	17.2	15,157	15.8
11.3	26,604	24.0	14.3	20,186	18.9	17.3	14,999	15.7
11.4	26,364	23.8	14.4	19,997	18.8	17.4	14,842	15.6
11.5	26,126	23.6	14.5	19,809	18.6	17.5	14,686	15.5
11.6	25,890	23.4	14.6	19,623	18.6	17.6	14,531	15.4
11.7	25,656	23.2	14.7	19,437	18.5	17.7	14,377	15.4
11.8	25,424	23.0	14.8	19,252	18.4	17.8	14,223	15.3
11.9	25,194	22.8	14.9	19,068	18.2	17.9	14,070	15.2
12.0	24,966	22.6	15.0	18,886	18.1	18.0	13,918	15.1
12.1	24,740	22.4	15.1	18,705	18.0	18.1	13,767	15.0
12.2	24,516	22.2	15.2	18,525	17.9	18.2	13,617	14.9
12.3	24,294	22.1	15.3	18,346	17.8	18.3	13,468	14.9
12.4	24,073	21.9	15.4	18,168	17.6	18.4	13,319	14.7
12.5	23,854	21.7	15.5	17,992	17.5	18.5	13,172	14.7
12.6	23,637	21.6	15.6	17,817	17.4	18.6	13,025	14.6
12.7	23,421	21.4	15.7	17,643	17.3	18.7	12,879	14.6
12.8	23,207	21.2	15.8	17,470	17.2	18.8	12,733	14.4
12.9	22,995	21.0	15.9	17,298	17.1	18.9	12,589	14.4
13.0	22,785	20.9	16.0	17,127	16.9	19.0	12,445	14.3
13.1	22,576	20.8	16.1	16,958	16.9	19.1	12,302	14.2
13.2	22,368	20.6	16.2	16,789	16.8	19.2	12,160	14.2
13.3	22,162	20.4	16.3	16,621	16.7	19.3	12,018	14.1
13.4	21,958	20.1	16.4	16,454	16.6	19.4	11,877	14.0
13.5	21,757	20.0	16.5	16,288	16.4	19.5	11,737	13.9
13.6	21,557	19.9	16.6	16,124	16.3	19.6	11,598	13.9
13.7	21,358	19.8	16.7	15,961	16.3	19.7	11,459	13.8
13.8	21,160	19.8	16.8	15,798	16.2	19.8	11,321	13.7
13.9	20,962	—19.7	16.9	15,636	—16.0	19.9	11,184	—13.7
14.0	20,765		17.0	15,476		20.0	11,047	

[1] This table was taken from Appendix 10, Report U. S. Coast and Geodetic Survey, 1881.

TABLE I. BAROMETRIC ELEVATIONS—*Continued*

Containing $A = 62737 \log \frac{30}{h}$. Argument, h.

h.	A.	Dif. for .01.	h.	A.	Dif. for .01.	h.	A.	Dif. for .01.
Inches.	Feet.	Feet.	Inches.	Feet.	Feet.	Inches.	Feet.	Feet.
20.0	11,047	—13.6	23.0	7,239	—11.8	26.0	3,899	—10.5
20.1	10,911	13.5	23.1	7,121	11.7	26.1	3,794	10.4
20.2	10,776	13.4	23.2	7,004	11.7	26.2	3,690	10.4
20.3	10,642	13.4	23.3	6,887	11.7	26.3	3,586	10.3
20.4	10,508	13.3	23.4	6,770	11.6	26.4	3,483	10.3
20.5	10,375	13.3	23.5	6,654	11.6	26.5	3,380	10.3
20.6	10,242	13.2	23.6	6,538	11.5	26.6	3,277	10.2
20.7	10,110	13.1	23.7	6,423	11.5	26.7	3,175	10.2
20.8	9,979	13.1	23.8	6,308	11.4	26.8	3,073	10.1
20.9	9,848	13.0	23.9	6,194	11.4	26.9	2,972	10.1
21.0	9,718	12.9	24.0	6,080	11.3	27.0	2,871	10.1
21.1	9,589	12.9	24.1	5,967	11.3	27.1	2,770	10.0
21.2	9,460	12.8	24.2	5,854	11.3	27.2	2,670	10.0
21.3	9,332	12.8	24.3	5,741	11.2	27.3	2,570	10.0
21.4	9,204	12.7	24.4	5,629	11.1	27.4	2,470	9.9
21.5	9,077	12.6	24.5	5,518	11.1	27.5	2,371	9.9
21.6	8,951	12.6	24.6	5,407	11.1	27.6	2,272	9.9
21.7	8,825	12.5	24.7	5,296	11.0	27.7	2,173	9.8
21.8	8,700	12.5	24.8	5,186	10.9	27.8	2,075	9.8
21.9	8,575	12.4	24.9	5,077	10.9	27.9	1,977	9.7
22.0	8,451	12.4	25.0	4,968	10.9	28.0	1,880	9.7
22.1	8,327	12.3	25.1	4,859	10.8	28.1	1,783	9.7
22.2	8,204	12.2	25.2	4,751	10.8	28.2	1,686	9.7
22.3	8,082	12.2	25.3	4,643	10.8	28.3	1,589	9.6
22.4	7,960	12.2	25.4	4,535	10.7	28.4	1,493	9.6
22.5	7,838	12.1	25.5	4,428	10.7	28.5	1,397	9.5
22.6	7,717	12.0	25.6	4,321	10.6	28.6	1,302	9.5
22.7	7,597	12.0	25.7	4,215	10.6	28.7	1,207	9.5
22.8	7,477	11.9	25.8	4,109	10.5	28.8	1,112	9.4
22.9	7,358	—11.9	25.9	4,004	—10.5	28.9	1,018	—9.4
23.0	7,239		26.0	3,899		29.0	924	

TABLE I. BAROMETRIC ELEVATIONS—*Continued*

Containing $A = 62737 \log \frac{30}{h}$. Argument, h.

h.	A.	Dif. for .01.	h.	A.	Dif. for .01.	h.	A.	Dif. for .01.
Inches.	Feet.	Feet.	Inches.	Feet.	Feet.	Inches.	Feet.	Feet.
29.0	924	—9.4	29.7	274	—9.2	30.4	—361	—9.0
29.1	830	9.4	29.8	182	9.1	30.5	451	8.9
29.2	736	9.3	29.9	91	9.1	30.6	540	8.9
29.3	643	9.3	30.0	00	9.1	30.7	629	8.8
29.4	550	9.2	30.1	—91	9.0	30.8	717	8.8
29.5	458	9.2	30.2	181	9.0	30.9	805	—8.8
29.6	366	—9.2	30.3	271	—9.0	31.0	—893	
29.7	274		30.4	361				

Wesley Anderson

TABLE II

CORRECTION COEFFICIENTS TO BAROMETRIC ELEVATIONS FOR TEMPERATURE AND HUMIDITY [1]

t_1+t'.	C.	t_1+t'.	C.	t_1+t'.	C.
0°	−0.1025	60	−0.0380	120	+0.0262
5	− .0970	65	− .326	125	+ .0315
10	− .0915	70	− .0273	130	+ .0368
15	− .0860	75	− .0220	135	+ .0420
20	− .0806	80	− .0166	140	+ .0472
25	− .0752	85	− .0112	145	+ .0524
30	− .0698	90	− .0058	150	+ .0575
35	− .0645	95	− .0004	155	+ .0626
40	− .0592	100	+ .0049	160	+ .0677
45	− .0539	105	+ .0102	165	+ .0728
50	− .0486	110	+ .0156	170	+ .0779
55	− .0433	115	+ .0209	175	+ .0829
60	− .0380	120	+ .0262	180	+ .0879

[1] This table was compiled from tables I and IV of Appendix 10 of Report of U. S Coast and Geodetic Survey for 1881.

Example.

From observations made at Sacramento, Cal., and at Summit, on the top of the Sierra Nevada Mountains, the annual means were:

$$h' = 23.288 \text{ in.} \qquad t' = 42.1\ F.$$
$$h_1 = 30.014 \text{ in.} \qquad t_1 = 59.9.$$

From table I we have:

$$A' = 6901.0 \text{ feet.}$$
$$A_1 = -12.7 \quad \text{“}$$
$$A' - A_1 = 6913.7 \quad \text{“}$$

From table II we find for $t' + t_1 = 102.0°$, $C = +.0070$.

$$\therefore \qquad z = 6913.7\ (1 + .0070) = 6962 \text{ feet.}$$

124. Use of the Aneroid. The aneroid barometer should be carried in a leather case, and it should not be removed from it. It should be protected from sudden changes of temperature, and when observations are made it should have the temperature of the surrounding outer air. It should not be carried so as to be affected by the heat of the body, and should be read out of doors, or at least away from all artificially warmed rooms. Always read it in a horizontal position. The index should be adjusted by means of a screw at its back, to agree with a standard mercurial barometer, and then this adjustment left untouched.

When but a single instrument is used it is advisable to *pass* between stations as rapidly as possible, but to *stop* at a number of stations during the day for a half-hour or so, reading the barometer on arrival and on leaving. The difference of these two readings shows the *rate of change* of barometric readings

due to changing atmospheric conditions, and from these isolated *rates* of change a *continuous correction-curve* can be constructed on profile or cross-section paper from which the instrumental corrections can be taken for any hour of the day.[1] The observations should be repeated the same day in reverse order, the corrections applied as obtained from this correction curve, and the means taken. Observations should be made when the humidity of the air is as nearly constant as possible, and never in times of changeable or snowy weather.

Let the student measure the heights of buildings, hills, etc., and then test his results by level or transit.

To interpolate elevations between two points whose elevations are known, take a reading at the first known point and pass rapidly toward the other known point, taking intermediate readings at determinate distances, and so continue until the second point is reached, when a reading is also taken here. The error in the determination between the two known points may now be distributed along the line either proportionately to the differences of elevation or to the intervening time, or to both, in the following manner. To correct for the time changes in the barometric readings, return at once to the first known point, and take duplicate readings at all the intervening points at which readings were previously taken. Note the discrepancy between the first and final barometric readings at the initial station, and correct all intermediate readings for their corresponding time intervals. This can be readily done graphically by taking time as one coördinate and the final discrepancy at the initial station as the other, and joining the two extreme points of these lines by a straight line, as indicated in Fig. 61

Having thus made the time correction, the barometric readings may be corrected for elevation by another graphical diagram, such as is shown in Fig. 62, wherein the co-ordinates are

[1] Mr. Chas. A. Ashburner, Geologist of the Penn. Geol. Survey, has used this method with good results.

the average corrected barometric readings at the several stations and the known difference of elevation of the terminal station. This known difference of elevation thus gives a true interpretation of all barometric differences independent of the use of any particular formula. Presumably the time corrections have included also temperature changes.

This method of levelling between known points is commonly employed in Switzerland, and usually by going over the route but once. In this case the time change could be corrected by reading a duplicate barometer continuously at some stationary point in the immediate vicinity.

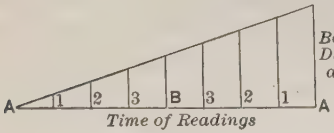


FIG. 61.

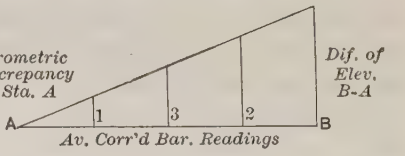


FIG. 62.

THE PEDOMETER

125. The Pedometer is a pocket-instrument for registering the number of paces taken when walking. It is generally made in the form of a watch, the front and back views being shown in Figs. 63 and 64.

When the instrument is attached to the belt in an upright position, as here shown, the jar given it at each step causes the weighted lever shown in Fig. 64 to drop upon the adjustable screw *S*. The lever recovers its position by the aid of a spring, and in so doing turns a ratchet-wheel by an amount proportional to the amplitude of the lever's motion. This may be adjusted to any length of pace by means of the screw *S*, which is turned by a key. The face is graduated like that of a watch, and gives the distance travelled in miles. This instrument may also be used on a horse, and when adjusted to the length

of a horse's step will give equally good results. The accuracy of the result is in proportion to the *uniformity* of the steps, after having been adjusted properly for a given individual. The instrument is only used on explorations, preliminary surveys, and reconnoissance-work.

The Length of Men's Steps has been investigated by Prof. Jordan,[1] of the Hanover Polytechnic School. From 256 step-measurements by as many different individuals, of lines from

FIG. 63.—Front View.

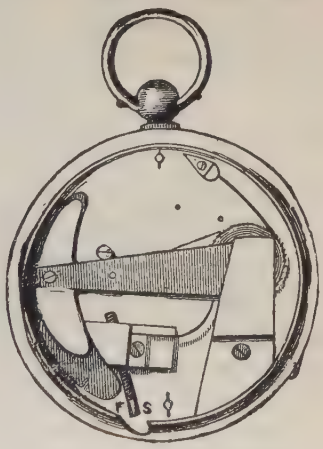


FIG. 64.—Back View.

650 to 1000 feet in length, carefully measured by rods and steel tapes, he concludes that the average length of step is 2.648 feet, ranging from 2.066 to 3.182 feet. The mean deviation from this amount for a single measurement was ± 0.147 feet, or $5\frac{1}{2}$ per cent. The average age of the persons making these step-measurements was 20 years. The length of step decreases with the age of the individual after the age of 25 to 30 years. It is

[1] See translation in Engineering News for July 25, 1885.

also proportional to the height of the person. The results for 18 different-sized persons gave the following averages:

Height of person .	5′.08	5′.25	5′.41	5′.58	5′.74	5′.90	6′.07	6′.23	6′.40	6′.56
Length of step ...	2 .46	2 .53	2 .56	2 .59	2 .62	2 .69	2 .72	2 .76	2 .79	2 .85

On slopes the step is always shorter than on level ground, whether one goes up or down. The following averages from the step-measurement of 136 lines on mountain-slopes along trails were found:

Slope	0°	5°	10°	15°	20°	25°	30°
Length of step in ascending	2′.53	2′.30	2′.03	1′.84	1′.64	1′.48	1′.25
Length of step in descending	2′.53	2′.43	2′.36	2′.30	2′.20	1′.97	1′.64

The length of the step is also found to increase with the length of the foot. One steps farther when fresh than when tired. The increase in the length of the step is also in nearly direct proportion to the increase of speed in walking.

When the proper personal constants are determined, and when walking at a constant rate, distances can be determined by pedometer, or by counting the paces, to within about two per cent of the truth. *One should always take his natural step*, and not an artificial one which is supposed to have a known value, as three feet, for instance. Let a base be measured off and each student determine the length of his natural step when walking at his usual rate, or, what is the same thing, find how many paces he makes in 100 feet. He then has always a ready means of determining distances to an approximation, which in many kinds of work is abundantly sufficient.

The Odometer

126. The Odometer is an instrument to be attached to the wheel of a vehicle to record the number of revolutions made by it. One form of such an instrument is shown in Fig. 65, attached to the spokes of a wheel.

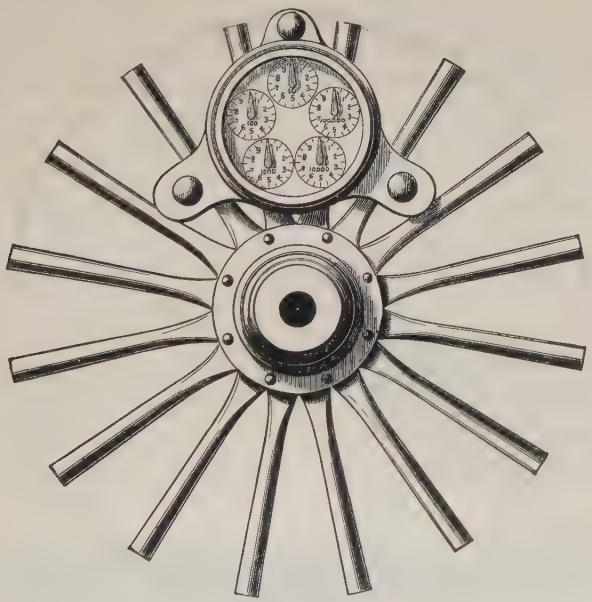


Fig. 65.

Each revolution is recorded by means of the revolution of an axis with reference to the instrument, this axis really being held stationary by means of an attached pendulum which does not revolve. The instrument really revolves about this fixed axis at each revolution of the wheel, and the number of times it does this is properly recorded and indicated by appropriate gearing and dials.

This method of measuring distances is more accurate than by pacing, as the length of the circumference of the wheel is a constant. This length multiplied by the number of revolutions is the distance travelled. It is mostly used by exploring parties and in military movements in new countries which have not been surveyed and mapped.

The Clinometer

127. The Clinometer is a hand-instrument for determining the slope of ground or the angle it makes with the horizon. It consists essentially of a level bubble, a graduated arc, and a line of sight, so joined that when the line of sight is at any angle to the horizon the bubble may be brought to a central position and the slope read off on the graduated arc. Such a combination is shown in Fig. 66. It is called the Abney level and

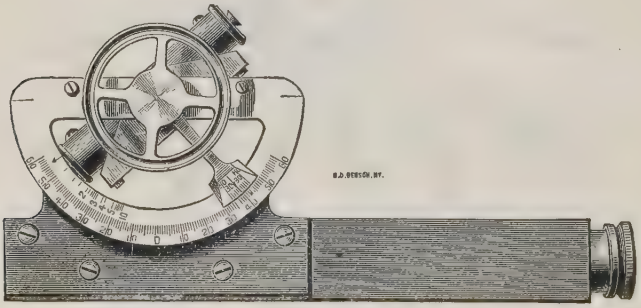


Fig. 66.—Clinometer.

clinometer, being really a hand level when the vernier is set to read zero. The position of the bubble is visible when looking through the telescope, the same as with the Locke hand level, shown in Fig. 28, p. 62. The body of the tube is made square, so that it may be used to find vertical angles of any surface by placing the tube upon it and bringing the bubble to the centre.

The graduations on the inner edge of the limb give the slope in terms of the relative horizontal and vertical components of any portion of the line; thus, a slope of 2 to 1 signifies that the horizontal component is twice the vertical. In reading this scale the edge of the vernier arm is brought into coincidence with the graduation.

This instrument is very useful in giving approximate slopes in preliminary surveys, the instrument being pointed to a position as high above the ground as its own elevation when held to the eye.

The Optical Square

128. The Optical Square is a small hand instrument used to set off a right angle. It is shown in Fig. 67, the method of its use being evident from the figure. Thus, while the rod at *o* is seen directly through the opening, the rod at *p* is seen in the glass as the prolongation downward of that of *o*, it being reflected from the mirrors *f* and *c* in succession, they having an angle of 45° with each other. By this means a line may be located at right angles to a given line at a given point, or a point in a given line may be found in the perpendicular to this line from a given point.

The Planimeter

129. The Planimeter is an instrument used for measuring areas that have been drawn to scale. It is a marked example of high mathematical analysis embodied in a very simple and useful mechanical appliance. Three of the best forms of the instrument will be described.

Fig. 68 is Amsler's Polar Planimeter. It consists of a metal arm, *ei*, carrying a needle point at *e* and pivoted at *i* to a frame through which slides a second metal arm, *h*, and to which is attached an axis, *ab*, carrying a rolling wheel, *c*, and a worm gear

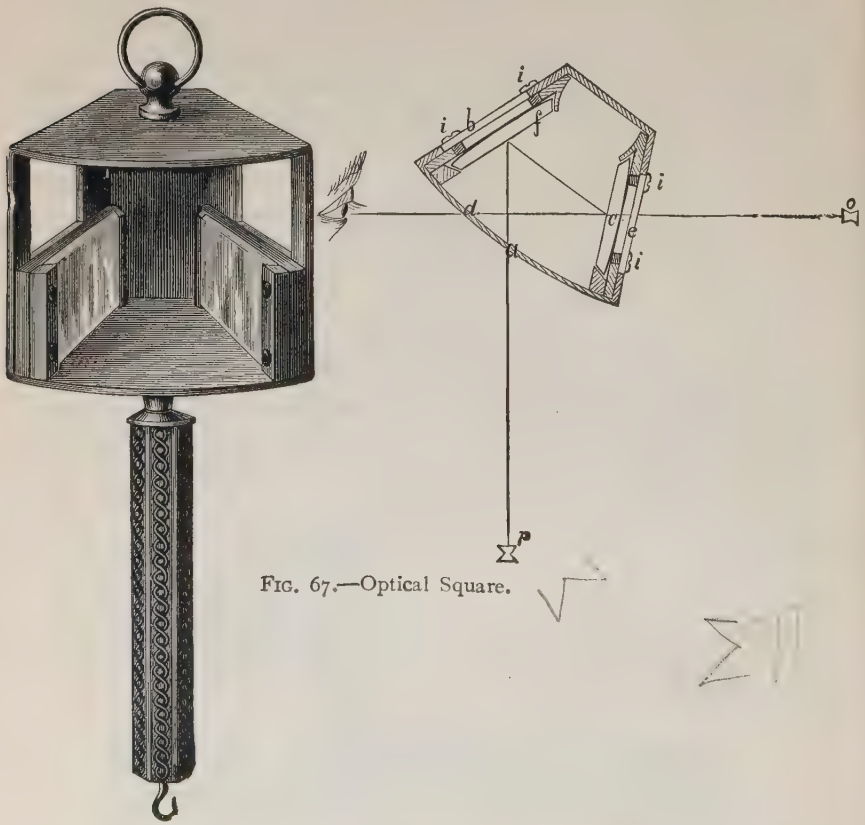


FIG. 67.—Optical Square.

FIG. 68.—Polar Planimeter.

which turns a record disk, *l*. The arm *h* may be adjusted so that any required length, within the limits of the instrument, from pivot *i* to tracing point *d* may be used.

When in use the instrument rests on three points, *e*, *d*, and the circumference of the wheel *c*. To measure an area, the needle point *e* is fastened in the drawing in a convenient position, and the point *d* made to circumscribe the required area. This causes the wheel *c* to rotate, and the number of revolutions made, as indicated by the record disk *l* and the vernier *m*, multiplied by a constant, is the required area. The determination of this constant, involving the theory of the instrument, will be given in such form as to be intelligible to students who have not studied the calculus.

130. Theory of the Polar Planimeter.[1] In Fig. 69 the essential parts of the instrument are lettered as in Fig. 68. The instrument is so constructed that the angle ϕ can never be less than 0° nor more than 180°; that is, neither *d* nor *c* can cross *ei*.

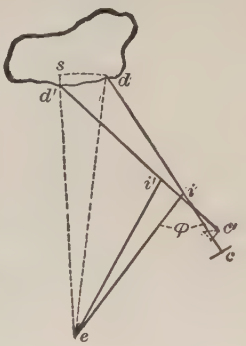


FIG. 69.

Any infinitesimal portion of *d*'s path, in circumscribing an area, as *dd'*, may be conceived to be the resultant of two infinitesimal component motions, *ds* and *sd'*; *ds* being described by a motion of *d* about *e* as a centre, the angle ϕ remaining fixed, while *sd'* is motion of *d* directly toward *e*, or normal to *ds*, the angle ϕ changing in value. Each of these motions has its due effect in moving the wheel *c* and causing it to turn. For a given motion of either class the amount the wheel *c* will turn depends on the value of the angle ϕ, as will presently be shown. It is evident that, in circumscribing a closed area, the tracing point will move as much *toward e* as it does *from e*, and that for each

[1] The following demonstration has been given the author by Professor Wm. G. Raymond, Dean of Engineering College, Iowa University.

element of motion toward e there will be a corresponding element from e with an equal value of ϕ. Hence the resulting turning of the wheel for *radial* motion of d is *nil* and need not be considered.

Corresponding right and left elements of circumferential motion of d are not made with equal values of ϕ, and hence the final record of the wheel is due to this class of motion. To show the effect of changes in ϕ (Fig. 70).

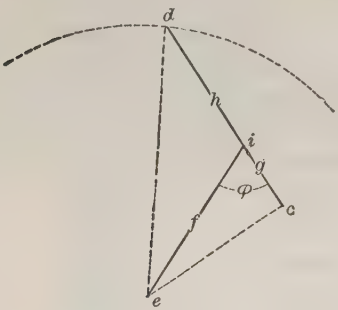


FIG. 70.

When d is so situated that *ecd* is a right angle, motion about e, ϕ remaining fixed, causes no rolling of the wheel c. For c rotates about e as a centre with radius ec, which is normal to the axis of the wheel, and hence the axis of the wheel lies in the direction of motion and the wheel slips. The circumference that would thus be traced by d, ϕ remaining fixed, in a complete revolution about e, is called the zero circumference. Its radius ed is easily shown to be:

$$R = \sqrt{f^2 + h^2 + 2gh},$$

and

$$\phi = \cos^{-1}\frac{g}{f}.$$

For ϕ less than this value, i.e., d farther from c, right-handed, or clockwise, motion of d about e will cause c to partly slip and partly roll, and a moment's consideration will show that, looking from c to d, its roll will be clockwise. Rolling in this direction is called *positive* and the wheel is accordingly graduated.

If, on the other hand, d be brought nearer to e than the zero circumference, clockwise motion of d about e causes c to both slip and roll, and the roll is counter clockwise, or *negative*.

Left-handed motion of d will cause c's roll to be *negative* for d *outside* the zero circumference and *positive* for d *inside* the zero circumference.

To determine the relation of the roll of the wheel to the area circumscribed (Fig. 71):

Let dd' be an infinitesimal circumferential component of d's motion due to the motion of the whole instrument about e through the infinitesimal angle Δ.

The wheel c moves through the arc cc', partly rolling and partly slipping. The roll is that component of its motion cc' normal to its axis. This component may be considered, in the infinitesimal triangle $cc's$, to be cs, normal to $c'i'$.

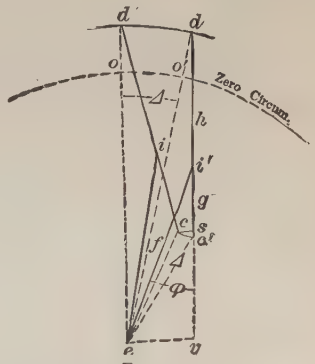


FIG. 71.

Let P be the arc corresponding to the small angle Δ when the radius is unity. Then

$$cc' = P \cdot c'e,$$

and the *roll* of the wheel is

$$cs = P \cdot c'e \cdot \cos c'cs.$$

Since the angle Δ is very small, cc' may be considered normal to $c'e$, hence

$$c'cs = ec'v,$$

ev being a perpendicular on $i'c'$ produced.

Then $$c'v = ec' \cdot \cos ec'v$$

$$= ec' \cos c'cs.$$

Whence, $cs = P \cdot c'v.$

Also, $c'v = f \cos \phi - g.$

Whence, $cs = P(f \cos \phi - g).$ . . . . . (1)

And this is the roli of the wheel due to motion of d through dd'.
To get an expression for the area $dd'o'o$:
By trigonometry,

$$ed = \sqrt{f^2 + h^2 + 2fh \cos \phi},$$

$$dd' = ed \cdot P.$$

The area of a sector is its arc multiplied by half its radius: whence,

$$\text{Area } edd' = \tfrac{1}{2}P(f^2 + h^2 + 2fh \cos \phi). \quad . \ . \ . \quad (2)$$

Similarly, using the value of eo previously found,

$$\text{Area } eoo' = \tfrac{1}{2}P(f^2 + h^2 + 2gh). \quad . \ . \ . \ . \quad (3)$$

Subtracting (3) from (2), it is found that

$$\text{Area } dd'o'o = Ph(f \cos \phi - g), \quad . \ . \ . \ . \quad (4)$$

which, it is observed, is equation (1) multiplied by h, whence the

Proposition.—*The distance rolled by the wheel c, for given motion of d, when multiplied by h, gives the area included between d's path, the zero circumference, and the radial lines to d's initial and final position.*

To show that in circumscribing a closed area with the tracing point d the roll of the wheel is correctly summed for motion of

d right handed and left handed, and inside or outside the zero circumference:

In Fig. 72, let the area $dd_1d_2d_3$ be traced, beginning at d, and moving clockwise. For motion from d to d_1 the wheel rolls positively an amount equal to the area $dd_1o'o$ divided by h; for motion from d_1 to d_2 the roll of the wheel will be neutralized by motion from d_3 to d; for motion from d_2 to d_3 the roll of the wheel is negative and equal to the area $d_2o'od_3$ divided by h. Hence the resulting roll of the wheel due to circumscribing the area $dd_1d_2d_3$ is that area divided by h.[1]

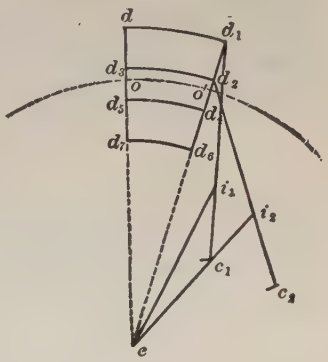


FIG. 72.

Since the area obtained by the wheel for a given motion of the tracer d is the area lying between d's path and the zero circumference, it follows that if e is placed within the area to be measured so that d makes a complete revolution about e there must be added to the result obtained, paying attention to its sign, the area of the zero circumference.[2] It is

$$A=\pi(f^2+h^2+2hg);$$

h being known, if f and g are not furnished by the maker, the area may nevertheless be found by circumscribing a known area with the point e within it and comparing it with the result obtained by the instrument.

Since the roll of the wheel for each area circumscribed must be multiplied by h to obtain the area, and since the roll is equal

[1] Let the student reason similarly of the areas $d_3d_2d_4d_5$ and $d_5d_4d_6d_7$.

[2] Let the student discuss with a diagram the three cases: 1. Perimeter wholly without the zero circumference; 2. Perimeter wholly within the zero circumference; and, 3. Perimeter partly within and partly without the zero circumference.

to the circumference of the wheel multiplied by the number of revolutions (the quantity read from the scale), it becomes necessary to determine values for h for the various kinds of areas that may be measured, such that the work of multiplying may be a minimum.

131. To Find the Length of Arm for Given Conditions. Let c be the circumference of the wheel and n the number of revolutions for a given area, A; then

$$A = hnc.$$

Since c is a fixed quantity, hc may be placed equal to a convenient constant and a value for h determined. The only caution to be observed is that h must be made a convenient length for use. If it is desired to measure square inches,

let $$hc = 10;$$

whence, $$h = \frac{10}{c}$$

Since c is usually between two inches and three inches, this will give a convenient value for h, which value being set, any area circumscribed by d is given by

$$A = 10n.$$

The reading of the instrument is usually to one one-thousandth of a revolution, the disk recording whole revolutions and the scale and vernier fractional revolutions.

The value of c is usually furnished by the maker, but for an additional charge. If not so furnished, it may be found thus: Circumscribe any known area, with e outside of it, with a known length of arm h, and note the record n, of the wheel.

Then, $$c = \frac{A}{hn}.$$

If the arm h is not graduated, it may be set by trial so that $A = 10n$ when the value of c is not required. If a diagram be drawn to a scale of f feet per inch, each square inch of paper represents f^2 square feet of actual area; and if it is desired to avoid multiplication, the arm h may be set so that a single revolution of the wheel shall correspond to a convenient number of units of actual area. Let A be the actual area represented by a on the diagram; then

$$f^2a = A,$$

and

$$a = hnc;$$

whence,

$$A = f^2hnc.$$

f^2hc may be assumed a convenient quantity and h determined.

Example.—The scale of a diagram is 40 feet per inch; then

$$A = 1600hnc.$$

Assume

$$1600hc = 20000;$$

then

$$h = \frac{20000}{1600c};$$

which will give a practical working value for h, and the figures representing the record of the wheel for any area circumscribed, without decimal point, multiplied by 20, will give the area in square feet. The instrument is supposed to read to thousandths of a revolution. If the horizontal scale is f feet per inch and the vertical scale is f' feet per inch, the equation for determining h would stand,

$$A = ff'hnc.$$

Similarly, if a number of areas are to be determined and multiplied by a constant length, d, as in railroad earth-work, the expression for a single volume v_1 of area a_1 would be, dividing by 27 to reduce to cubic yards,

$$v_1 = \frac{f^2 a_1 d}{27};$$

but, $$a_1 = hnc;$$

whence, $$v_1 = \frac{f^2 d}{27} hnc.$$

A convenient quantity may be chosen for $\frac{f^2 d}{27} hc$ and h determined.

The sum of a series of volumes, each of length d, is

$$V = v_1 + v_2 + v_3 + \text{etc.} = \Sigma v.$$

$$= \Sigma \frac{f^2 d}{27} hnc,$$

$$= \frac{f^2 d}{27} hc \Sigma n;$$

whence it is only necessary to circumscribe each area in succession, add the records of the wheel, and apply the coefficient. If all the areas are platted from the same base-line the tracer may be moved continuously around the areas, traversing each area as many times as it is used in the summing, and only the final single record read, which is of course the sum of all the records. A variety of other problems may be solved with the instrument.

132. The Suspended Planimeter. This is shown in Fig. 73. It is essentially a polar planimeter, the pole being at *C*. It has the advantage of allowing the wheel to move over the smooth surface of the plate *S*, instead of over the paper, thus giving an error about one-sixth as great as that of the ordinary polar instrument. The theory of its action is essentially the same as the other.

133. The Rolling Planimeter is the most accurate instrument of its kind yet devised. Its compass is also indefinitely increased, since it may be rolled bodily over the sheet for any

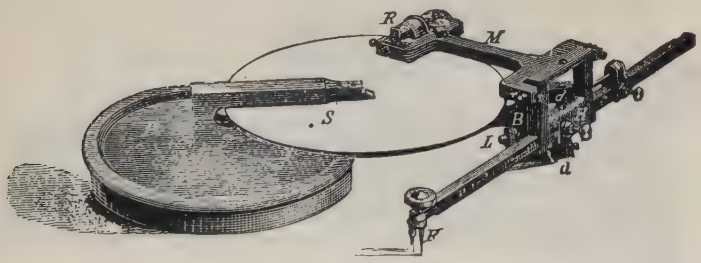


Fig. 73.

distance, on a right line, and an area determined within certain limits on either side. It is therefore especially adapted to the measuring of cross-sections, profiles, or any long and narrow surface. Fig. 74 shows one form of this instrument as designed by Herr Corradi of Zurich. It is a suspended planimeter, inasmuch as the wheel rolls on a flat disk which is a part of the instrument, but it could not be called a polar planimeter, the theory of its action being very different from that instrument. The frame *B* is supported by the shaft carrying the two rollers R_1. To this frame are fitted the disk *A* and the axis of the tracing arm *F*. The whole apparatus may thus move to and fro indefinitely in a straight line on the two rollers while the tracing point traverses the perimeter of the area to be measured. The shaft carries a bevel-gear wheel, R_2, which moves the pinion R_3.

This pinion is fixed to the axis of the disk, and turns with it, so that any motion of the rollers R_1 causes the disk to revolve a proportional amount, and the component of this motion at right angles to the axis of the wheel E is recorded on that wheel. If the instrument remains stationary on the paper (the rollers R not turning) and the tracing point moved laterally, it will cause no motion of the wheel, since its axis is parallel to the arm F, and turns about the same axis with F, but 90° from it; the wheel E, therefore, moves parallel with its axis and does not turn.

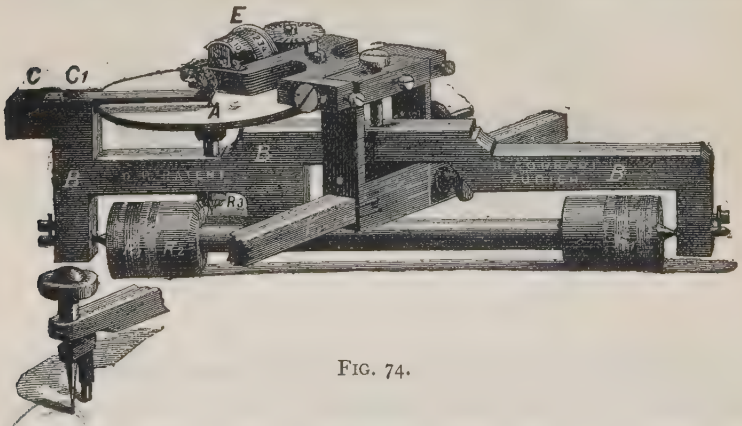


FIG. 74.

134. The Theory of the Rolling Planimeter.[1] This will be developed by a system of rectangular co-ordinates, the path of the fulcrum of the tracing arm being taken as the axis of abscissæ. The path of the tracing point will be considered as made up of two motions, one parallel to the axis of abscissæ and the other at right angles to it. The elementary area considered will be that included between the axis of abscissæ and two ordinates drawn to the extremities of an elementary portion of the path. But since this element of the perimeter is supposed to be made up of two right lines, one perpendicular to the axis of abscissæ

[1] A later and simpler explanation of this theory may be found in Jour. Asso. Engr'g. Societies, Feb., 1902, p. 67.

and the other parallel to it, our elementary area must also be divided in a similar manner. It will at once be seen that one part of this area is zero, since the two ordinates bounding it form one and the same line. This is the part generated by the motion at right angles to the axis of abscissæ. Now, we have just shown in the previous article that the wheel record for this part of the path is also zero.[1] We are brought therefore to this important

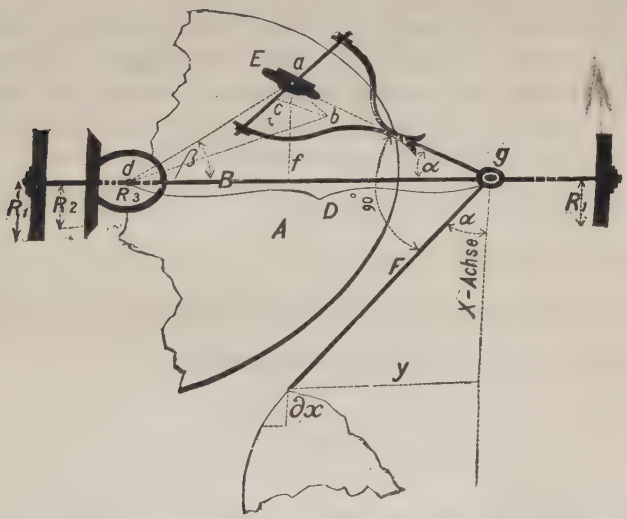


FIG. 75.

conclusion: *that all components of motion of the tracing-point at right angles to the axis of abscissæ have no influence upon the*

[1] This is not strictly correct, although it leads to correct results. The components of motion are really one parallel to the axis of x, and one about the pivot g as a centre. This latter movement makes no record on the wheel, but it does generate an elementary area included between this element of the path, the ordinates to its extremities, and the axis of x. These areas are either added to or cut from the areas made by the onward element of motion in such a way as to exactly balance when the tracing point starts and returns to the axis of x, or when it closes on the initial point, wherever it may be.

result. We will therefore only discuss a differential motion of the tracing-point in the direction of the axis of abscissæ.

In Fig. 75, which is a linear sketch of the instrument shown in Fig. 74, with the corresponding parts similarly lettered, it is to be shown that the motion of the wheel E caused by the movement of the tracing point over the path dx is equal to the corresponding area ydx multiplied by some constant which is a function of the dimensions of the instrument.

It is evident that a motion of the tracing point in the direction of the axis of abscissæ can only be obtained by moving the entire instrument on the rollers by the same amount, and therefore when the point moves over the path dx the circumferences of the rollers R_1 have moved the same amount. This causes a movement of the pitch circle of R_2 of $dx\frac{R_2}{R_1}$. This motion is conveyed to the disk through R_3, so that any point on this disk, as a, distant ad from the axis, moves through a distance equal to $dx\frac{R_2}{R_1}\cdot\frac{ad}{R_3}$. Let ab, Fig. 75, be this distance, then we have,

$$ab = dx\frac{R_2}{R_1}\cdot\frac{ad}{R_3}. \quad \ldots\ldots\ldots \quad (1)$$

The motion of that portion of the disk on which the roller rests, equal to ab, causes the circumference of the wheel E to revolve by an amount equal to the component of the distance ab perpendicular to the axis of the wheel. This component part of the disk's motion is bc, and this is the measure of the wheel's motion. It therefore remains to show that $bc = ydx$ multiplied by an instrumental constant.

Now, $$bc = ab \sin bac. \quad \ldots\ldots\ldots \quad (2)$$

But $bac = \alpha + \beta$, since gac and bad are both right angles. Also bac = supplement of $dag = a + \beta$.

Also, from the triangle *dag*, we have,

$$\sin dag : \sin agd :: D : ad,$$

or
$$\sin(\alpha+\beta)=\frac{D\sin\alpha}{ad}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Since *Fga* is also a right angle, we have the angle formed by *Fg* and the axis of abscissæ equal to α, whence $\sin\alpha=\frac{y}{F}$.

We may now write:

$$bc=ab\sin(\alpha+\beta)=ab\frac{D\cdot\sin\alpha}{ad}=ab\frac{Dy}{F\cdot ad}. \quad . \quad . \quad (4)$$

Now, substituting the value of *ab* from (1), we have,

$$cb=ydx\frac{D\cdot R_2}{F\cdot R_1\cdot R_3}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Since D, R_2, F, R_1, and R_3 are all constants for any one instrument, we see that the wheel record is a function of the area generated by the tracing point and the instrumental constants, which was to be shown. It now follows that the summation of all these elementary areas included between the path of the tracing point and the axis of abscissæ, is represented by the total *wheel movement*, or the difference between its initial and final readings, provided the tracing point starts from and closes on the axis of x, or closes on the starting point, wherever that may be.

The following comparison between the rolling and the polar planimeters may be made: The axis of x corresponds to the zero circle; the unrecorded motion about the pivot g, Fig. 75 corresponds to the balanced record of the motion about c, Fig.

69; and the significant forward motion of the former to the motion about P as a centre, in the latter.

As in the case of the polar instrument, the proper length of arm F, to be used with the rolling planimeter to give results in any desired unit, depends on the other instrumental constants. These being known, the value of F may be computed in the same manner as with the polar planimeter.

135. To Test the Accuracy of the Planimeter, there is usually provided a brass scale perforated with small holes. A needle point is inserted in one of these and made fast to the paper or board, while the tracing point rests in another. The latter may now be moved over a fixed path with accuracy. Make a certain number of even revolutions forward, or in the direction of the hands of a watch, noting the initial and final readings. Reverse the motion the same number of revolutions, and see if it comes back to the first reading. If not, the discrepancy is the combined instrumental error from two measurements due to slip, lost motion, unevenness of paper, etc.

If this test be repeated with the areas on opposite sides of the zero circle in the case of the polar planimeter, or on opposite sides of the axis of abscissæ in case of the rolling planimeter, with the same score in both cases, it proves that the pivot points a, b, k, and the tracing point d (Fig. 68), are in the same straight line, in case of the polar instrument, and that the corresponding points in the suspended and rolling planimeters form parallel lines; in other words, that the axis of the measuring wheel is parallel to the tracing arm. If the results differ when the areas lie on opposite sides of the axis or zero circle, these lines are not parallel and must be adjusted to a parallel position.

136. Use of the Planimeter. The paper upon which the diagram is drawn should be stretched smooth on a level surface. It should be large enough to allow the rolling wheel to remain on the sheet.

The instrument should be so adjusted and oiled that the parts move with the utmost freedom but without any lost

motion. This requires that all the pivot joints shall be adjustable to take up the wear. The rim of the measuring wheel must be kept bright and free from rust. The instrument must be handled with the greatest care. Having set the length of the tracing arm for the given scale and unit, it is well to test it upon an area of known dimensions before using. If it be found to give a result in error by $\frac{1}{n}$ of the total area, the length of the tracing arm must be changed by an amount equal to this same ratio of its former length. If the record made on the wheel was too small then the length of the tracing arm must be diminished, and *vice versa*. If the paper has shrunk or stretched, find the proportional change, and change the length of the tracing arm *from its true length as just found*, by this same ratio, making the arm longer for stretch and shorter for shrinkage. Or the true length of arm may be used, and the results corrected for change in paper.

To measure an area, first determine whether the fixed point, or pole, shall be inside or outside the figure. It is preferable to have it outside when practicable, since then the area is obtained without correction. If, however, the diagram is too large for this (in case of the polar planimeter) the pole may be set inside. In either case inspection, and perhaps trial, is necessary to fix upon the most favorable position of the pole, so that the tracing point may most readily reach all parts of the perimeter. If the area is too large for a single measurement, divide it by right lines and measure the parts separately. Having fixed the pole, set the tracing point on a well-defined portion of the perimeter, and read and record the score on the rolling wheel and disk. This is generally read to four places. Move the tracing point carefully and slowly over the outline of the figure, in the direction of the hands of a watch, around to the initial point. Read the score again. If the pole is outside the figure, this result is always positive when the motion has been in the direction here indicated. If the pole is inside the figure, the

result will be negative when the area is less than that of the zero circle, positive if greater. With the pole inside the figure, however, the area of the zero circle must always be added to the result as given by the score, and when this is done the sum is always positive, the motion being in the direction indicated. The area of this zero circle is found in Art. 130, to be $\pi(m^2+l^2+2nl)$. The value of l, which is the length of the tracing arm, is known. The values of m and n should be furnished by the maker. If these are unknown, the area of the zero circle can be found for any length of arm l, by measuring a given area with pole outside and inside, the difference in the two scores being the area of this circle. By doing this with two very different values of l we may obtain two equations with two unknown quantities, m and n, from which the absolute values of these quantities may be found. Thus we would have:

$$A = \pi(m^2+l^2+2nl);$$

$$A' = \pi(m^2+l'^2+2nl');$$

whence,

$$m^2+2nl = \frac{A}{\pi} - l^2;$$

$$m^2+2nl' = \frac{A'}{\pi} - l'^2;$$

wherein l, l', A, and A' are known. The values of m and n are then readily found.

In using the rolling planimeter, it is advisable to take the initial point in the perimeter on the basis of abscissæ, as in this position any small motion of the tracing point has no effect on the wheel, and so there is no error due to the initial and final positions not being exactly identical.

The planimeter may be used to great advantage in the solution of many problems not pertaining to surveying. In all cases

where the result can be represented as a function of the product of two variables and one or more constants, the corresponding values of the variables may be plotted on cross-section paper by rectangular co-ordinates, thus forming with the axis and end ordinates an area which can be evaluated for any scale and for any value of the constant functions by setting off the proper length of tracing arm. Thus, from a steam-indicator card the horse-power of the engine may be read off, and from a properly constructed profile the amount of earthwork in cubic yards in a railway cut or fill. Some of these special applications are further explained in Part II. of this work.

137. Accuracy of Planimeter Measurements. Professor Lorber, of Loeben, Austria, has thoroughly investigated the relative accuracy of different kinds of planimeters, and the results of his investigations are given in the following table. It will be seen that the relative error is less as the area measured is larger. The absolute error is nearly constant for all areas in the polar planimeter. The remarkable accuracy of the rolling planimeter is such as to cause it to be ranked as an instrument of precision.

TABLE OF RELATIVE ERRORS IN PLANIMETER MEASUREMENTS

Area in—		The error in one passage of the tracer amounts on an average to the following fraction of the area measured by—		
Square cm.	Square inches.	The ordinary polar planimeter, Unit of vernier: 10 sq. mm. = .015 sq.in.	Suspended planimeter, Unit of vernier: 1 sq. mm. = .001 sq.in.	Rolling planimeter, Unit of vernier: 1 sq. mm. = .001 sq.in.
10	1.55	$\frac{1}{75}$	$\frac{1}{625}$	$\frac{1}{1000}$
20	3.10	$\frac{1}{148}$	$\frac{1}{1111}$	$\frac{1}{2000}$
50	7.75	$\frac{1}{355}$	$\frac{1}{2500}$	$\frac{1}{3000}$
100	15.50	$\frac{1}{682}$	$\frac{1}{4167}$	$\frac{1}{5000}$
200	31.00	$\frac{1}{1274}$	$\frac{1}{7143}$	$\frac{1}{7693}$
300	46.50	....	$\frac{1}{9375}$	$\frac{1}{10000}$

The Pantograph

138. The Pantograph is a kind of parallel link-motion apparatus whereby, with one point fixed, two other points are made to move in a plane on parallel lines in any direction. The device is used for copying drawings, or other diagrams to the same, a larger, or a smaller scale. The theory of the instrument rests on the following:

Proposition: *If the sides of a parallelogram, jointed at the corners A, B, C, and D, and indefinitely extended, be cut by a*

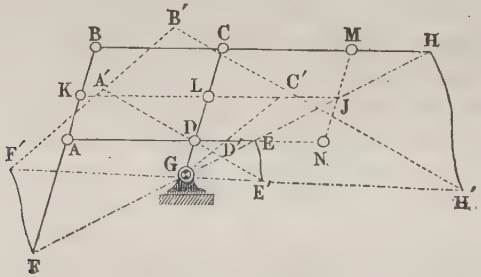


Fig. 76.

right line in four points, as E, F, G, and H, then these latter points will lie in a straight line for all values of each of the parallelogram angles from zero to 180°, *and the ratio of the distances EF, FG, and GH will remain unchanged.*

In Fig. 76, let *A*, *B*, *C*, *D* be the parallelogram, whose sides (extended) are cut by a right line in *F*, *G*, *E*, and *H*. It is evident that one point in the figure may remain fixed while the angles of the parallelogram change. Let this point be *G*.

Since *GC* and *GH*, radiating from *G*, cut the parallel lines *DE* and *CH*, we have,

$$GD : DE :: GC : CH.$$

Also, for similar reasons,

$$ED : DG :: EA : AF.$$

Now since the sides of the parallelogram, as well as all the intercepts, AF, GD, DE, and CH, remain constant as the angles of the figure change, when the figure has taken the position shown by the dotted lines, we still have,

also, $$GD' : D'E' :: GC' : C'H';$$

$$E'D' : D'G :: E'A' : A'F'.$$

From the first of these proportions we know that G, E', and H' are in the same straight line, and the same for G, E', and F'; therefore, they are all four in the same straight line.

To show that they are the same relative distance apart as before we have,

$$FG : GE : EH :: BC : DE : CH - DE;$$

also,

$$F'G : GE' : E'H' :: B'C' : D'E' : C'H' - D'E'.$$

But,

$$BC = B'C',\ DE = D'E', \text{ and } CH - DE = C'H' - D'E';$$

therefore we may write,

$$FG : GE : EH :: F'G : GE' : E'H'.$$

Q.E.D.

It is evident that two of the points E, F, G, and H may become one by the transversal passing through the point of intersection of two of the sides of the parallelogram. The above proposition would then hold for the three remaining points.

In the Pantograph only three of the four points E, F, G, and H (Fig. 76) are used. One of these *may* therefore be taken at

the intersection of two sides of the parallelogram, but it is not necessarily so taken. These three points are: the fixed point, the tracing point, and the copying point.

In Fig. 77 F is the fixed point, held by the weight P; B is the tracing point, and D is the copying point, or *vice versa* as

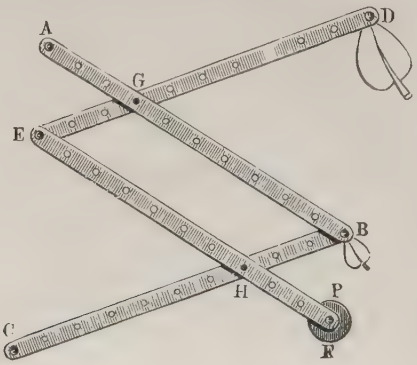


Fig. 77.

to B and D. The parallelogram is E, G, B, H. The points F, B, and D must lie in a straight line, B being at the intersection of two of the sides of the parallelogram. The points A, E, and

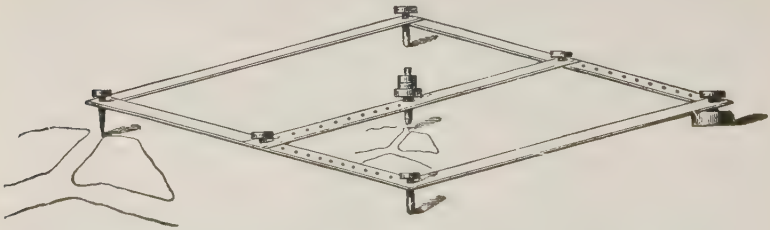


Fig. 78.

C are supported on rollers. In Fig. 78, the fixed point is the point of intersection of two of the sides of the parallelogram. The upper left-hand member of the frame is not essential to its

construction, serving simply to stiffen the copying arm, the fourth side of the parallelogram being the side holding the tracing point.

In Fig. 79, neither of the three points is at the intersection of two of the sides of the parallelogram, and hence there is a fourth point unused, having the same properties as the fixed, tracing, and copying points, it being at the intersection of the line joining these three points with the fourth side of the parallelogram.

From the theoretical discussion, and from the figures shown, it becomes evident that there may be an indefinite variety of

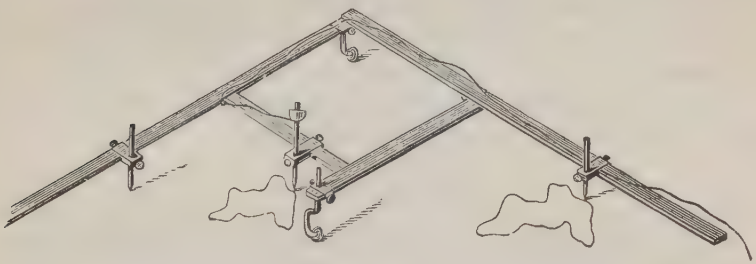


FIG. 79.

combinations which will do the work of a pantograph. *The only essential conditions are that the fixed, the tracing, and the copying points shall lie in a straight line on at least three sides of a jointed parallelogram, either point serving any one of the three purposes.*

139. Use of the Pantograph. The use of the instrument is easily acquired. Since both the tracing and copying points should touch the paper at all times, such a combination as that shown in Fig. 77 is preferable to those shown in Figs. 78 and 79, since in these latter the tracing point is surrounded by supported points, and so would not touch the paper at all times unless the paper rested on a true plane. In most instruments where the scale is adjustable, the two corresponding changes in position of tracing and copying points for different scales

are indicated. To test these marks, see that the adjustable points are in a straight line with the fixed point, and to test the scale see that the ratio $\frac{FD}{FB}$ (Fig. 77) is that of the reduction desired. Thus, if the diagram is to be enlarged to twice the original size, make $FD = 2FB$;

or make $$\frac{DE}{GE} = \frac{FE}{FH} = \text{scale of enlargement.}$$

If the drawing is to be reduced in size, make B the copying point and D the tracing point.

If the drawing is to be copied to the same scale, make $BF = BD$ and make B the fixed point. The figure is then copied to same scale, but in an inverted position.

In the best instruments the arms are made of brass, but very good work may be done with wooden arms.

Protractors

140. A Protractor is a graduated circle or arc, with its centre fixed, to be used in plotting angles. They are of various designs and materials.

Semicircular Protractors, such as shown in Fig. 80, are usually made of horn, brass, or German-silver. They are graduated to degrees or half-degrees, and the angle is laid off by holding the centre at the vertex of the angle, with the plain edge, or the 0 and 180 degree line on the given line from which the angle is to be laid off.[1]

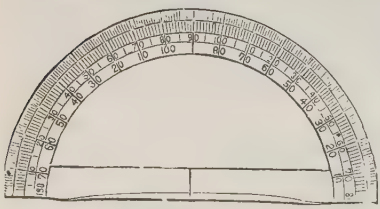


Fig. 80.

In the *full-circle protractor*, shown in Fig. 81, there is a movable arm with a vernier reading to from 1 to 3 minutes. The

[1] For a convenient style of protractor to be used in topographical work, see Fig. 113.

horn centre is set over the given point, the protractor oriented with the zero of the circle on the given line, and the arm set to the given reading when the other line may be drawn.

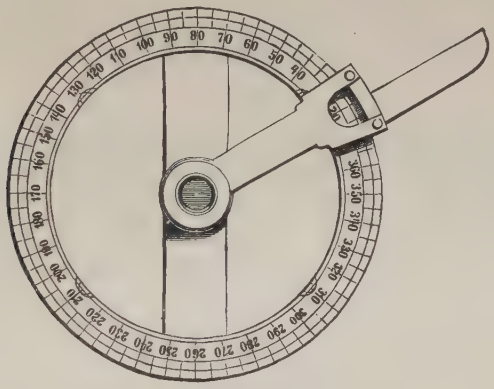


FIG. 81.

The *three-arm protractor*, Fig. 80, has one fixed and two movable arms by which two angles may be set off simultaneously.

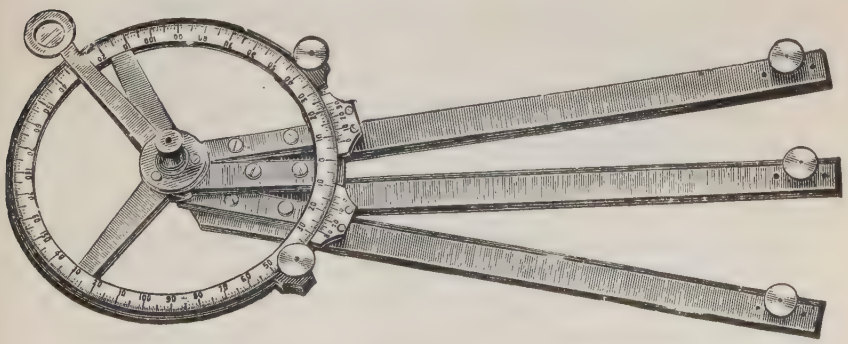


FIG. 82.

It is used in plotting observations by sextant of two angles to three known points for the location of the point of observation.

This is known as the three-point problem and is discussed in Chapter IX.

Paper protractors are usually full circled, from 8 to 14 inches in diameter, graduated to half or quarter degrees. They are printed from engraved plates on drawing or tracing paper or bristol board, and are very convenient for plotting topographical

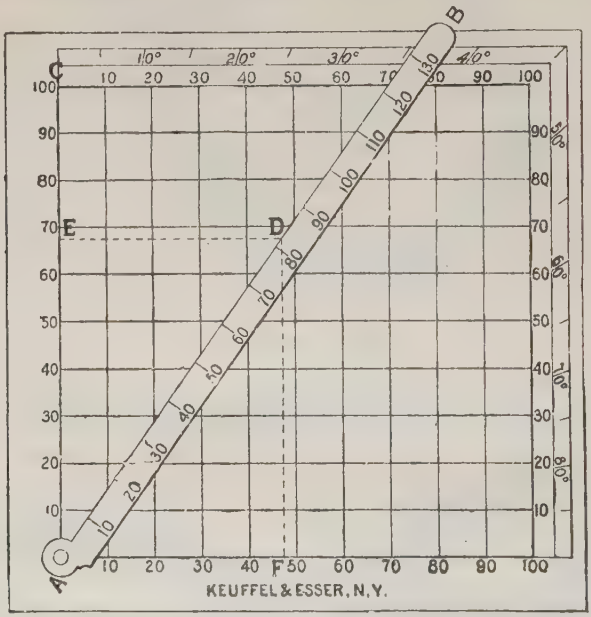


FIG. 83.—Co-ordinate Protractor.

surveys. The map is drawn directly on the protractor sheet, the bearing of any line being taken at once from the graduated circle printed on the paper. These "protractor sheets" can now be obtained of all large dealers.

The co-ordinate protractor[1] is a quadrant, or square, with angular graduations on its circumference, or sides, and divided

[1] Called a Trigonometer by the Keuffel, Esser Co., the makers.

over its face by horizontal and vertical lines, like cross-section paper. A movable arm can be set by means of a vernier to read minutes of arc, this arm being also graduated to read distances from the centre outward. Having set this arm to read the proper angle, the latitude is at once read off on the vertical scale and the departure on the horizontal scale for the given distance as taken on the graduated arm. A quadrant protractor giving latitudes and departures for all distances under 2500 feet to the nearest foot, or under 250 feet to the nearest tenth of a foot, has been used. The radius of the circle is $18\frac{3}{4}$ inches. Both the protractor and the arm are on heavy bristol board, so that any change due to moisture will affect both alike and so eliminate errors due to this cause. The instrument was designed to facilitate the plotting of the U. S. survey of the Missouri River. It has proved very efficient and satisfactory. A similar one on metal, shown in Fig. 83, is now manufactured, and serves the same purpose.

Parallel Rulers

141. The Parallel Ruler of greatest efficiency in plotting is that on rollers, as shown in Fig. 84. The rollers are made of exactly the same circumference, both being rigidly attached to the same axis. It should be made of metal so as to add to its weight and prevent slipping. It is of especial value in connection with the paper protractors, for the parallel ruler is set on any given bearing and then this transferred to any part of the sheet by simply running the ruler to place. Two triangles may be made to serve the same purpose, but they are not so rapid or convenient, and are more liable to slip. The parallel ruler is also very valuable in the solution of problems in graphical statics.

Fig. 84.

Scales

142. Scales are used for obtaining the distance on the drawing or plot which corresponds to given distances on the object or in the field. There is such a variety of units for both field and office work, and a corresponding variety of scales, that the choice of the particular kind of scale for any given kind of work needs to be carefully made. Architects usually make the scale of their drawings so many feet to the inch, giving rise to a duodecimal scale, or some multiple of $\frac{1}{1\frac{1}{2}}$. A surveyor who uses a Gunter's chain 66 feet in length plots his work to so many chains to the inch, making a scale of some multiple of $\frac{1}{79\frac{1}{20}}$. An engineer usually uses a 100-foot chain and a level rod divided to decimal

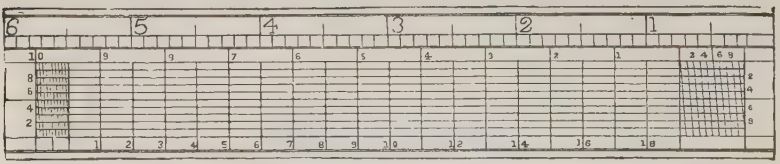


Fig. 85.

parts of a foot; so he finds it convenient to use a decimal scale for his maps and drawings, reduced to the inch unit, however. Here the field unit is feet and the office unit is inches, both divided decimally. This gives rise to a sort of decimal-duodecimal system, the scale being some multiple of $\frac{1}{1\frac{1}{20}}$. Various combinations of all these systems are found.

Figure 85 shows one form of an ivory scale of equal parts for the general draughtsman. The lower half of the scale is designed to give distances on the drawing for 4, 40, or 400 units to the inch when the left oblique lines and bottom figures are used, and for 2, 20, or 200 units to the inch when the right oblique lines and top figures are used. Thus, if we are plotting to a scale of 400 feet to the inch, and the distance is 564 feet, set one

point of the dividers on the vertical line marked 5, and on the fourth horizontal line from the bottom. Set the other leg at the intersection of the sixth inclined line with this same horizontal line, and the space subtended by the points of the dividers is 564 feet to a scale of $\frac{1}{4800}$.

Figure 86 is a cut of an engineer's triangular boxwood scale,

FIG. 86.

12 inches long, being divided into decimal inches. There are six scales on this rule, a tenth of an inch being subdivided into 1, 2, 3, 4, 5, and 6 parts, making the smallest graduations $\frac{1}{10}$, $\frac{1}{20}$, $\frac{1}{30}$, $\frac{1}{40}$, $\frac{1}{50}$, $\frac{1}{60}$ of an inch respectively. This is called an engineer's or decimal inch scale. The architect's triangular scale is divided to give $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{2}$, 2, 3, and 4 inches to the foot. Such a scale is of less service to the civil engineer.

THE POCKET SLIDE RULE[1]

143. The Slide Rule in some of its various forms is by far the most easy and rapid means of carrying on computations by multiplication and division that has ever been devised. One can work with it continuously without becoming mentally weary. The author, having had a large experience in the use of all kinds of mechanical computing machines and of logarithmic and multiplication tables, is fully convinced that the slide rule far exceeds them all in the ease, rapidity, and accuracy with which it can be used by anyone after an hour's instruction or practice. While this little implement is in almost universal use in Europe, not only by scientific, but by all classes of practical men, it has never come into general use in America. This is greatly to be deplored, and it is hoped

[1] While the use of the slide rule has no special relation to surveying, its use may properly be described here as one of the instruments of engineering computation.

our deficiency in this respect will now be remedied, by the placing of a cheap but accurate pocket slide rule upon the American market.

The rule in this pocket form (Fig. 87) gives results to three significant figures,[1] which is sufficiently accurate for most of the computations which have to be made in all kinds of physical investigations, and in the solution of engineering problems.

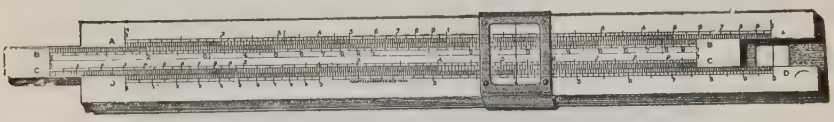


Fig. 87.

Where four or five significant figures are required, Thacher's slide rule (Fig. 88) will be used, but for a large part of the work for which this larger slide rule is employed a small pocket form is really sufficient, since this gives the results to a greater degree of accuracy than it is possible to obtain in the original data. After a little use of this instrument, it becomes almost a necessity. It is not too much

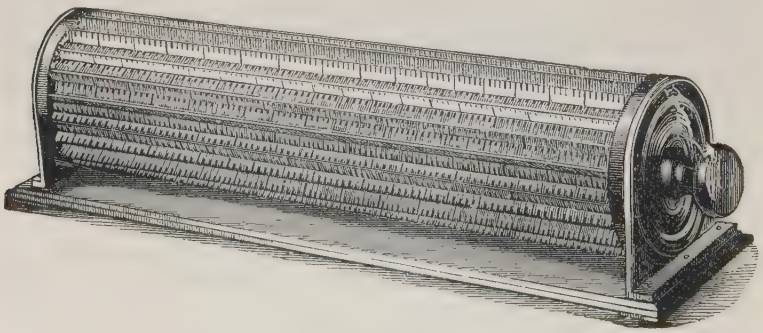


Fig. 88.

to say that it is indeed a great benefactor in preventing the mental fatigue which always accompanies continued numerical operations in multiplication and division.

The following simple rules and explanations are readily understood and followed with the slide rule in hand, and they fully exemplify most of its applications.

[1] This rule can now be purchased in America for $1.00, but a better one, printed on celluloid, and made adjustable so as to obviate all objections on the score of swelling, warping, or looseness from wear, is now manufactured by the Keuffel Esser Co., New York, and Eugene Dutzgen Co., Chicago, Ill.

A working knowledge of the use of logarithms is assumed. A small magnifying glass may sometimes be employed to increase the accuracy of the settings and readings.

THE SCALES A AND B

The scales A and B represent the logarithms of the numbers marked upon them, from 1 to 10, or from 10 to 100, or from 100 to 1000, according to the degree of accuracy required, to a uniform scale. By the movement of scale B, along scale A, to the right or left, these logarithms can be mechanically added or subtracted, or both simultaneously, and multiplication and division thus effected. Computations made on the slide rule, therefore, are mechanically performed by means of logarithms represented graphically. The two scales, A and B, are exactly alike, each containing two duplicate, complete, graphical logarithmic scales.

The slide rule is used to solve a problem in proportion, or in general to multiply two numbers together and to divide by a third, any one of which may, of course, be unity, the general problem being to perform the work indicated in the following: $\frac{ax}{b} = ?$

Since the scales increase to the right, a movement of the sliding scale B to the left subtracts a logarithm, and a movement to the right adds one. Thus, to divide by b we move the slide to the left until b comes under 1 of the A scale. Then to multiply by a we move it forward until b comes under a of that scale. But this is equivalent to setting b on the B scale opposite a on the A scale, at once. Then every number on the sliding (B) scale has been multiplied by a and divided by b relatively to the corresponding number on the A scale. Hence *any* number on the sliding sacle, as x, has opposite to it a number which is $\frac{a}{b}$ times x. Therefore we have the rule:

1. To solve $\frac{ax}{b}$ set b on B opposite a on A, then find x on B, and the opposite reading on A is the result.

EXAMPLE: Solve $\frac{3.04 \times 20.1}{59.6} = 1.025$.

Paying no attention to decimal points, treat the quantities as whole numbers, and setting 596 on B opposite 304 on A, find opposite 201 on B 1.025 on A. The decimal point is usually best located by a mental computation or inspection. Evidently the result is somewhat more than unity, hence the point comes after the 1.

When a series of results is to be obtained in which two of the numbers, as a and b, are constants, and the other varies, always set for the constant terms, after which the entire series of results can be read off for the varying values of x without further settings.

THE SCALES C AND D.

The log. scales C and D are so arranged that the numbers on them are the square roots of those in the same vertical line in scales A and B. Transfers are taken from scales C or D to scales A or B by means of the visible line on the transparent sliding indicator.

Thus if the problem be $\frac{ax}{b^2}$, move the transparent indicator to a on A, then move the sliding scale till b on C (which is the same as b^2 on B) comes to the line on the indicator, then look up x on B and find the result on A as before. This is the same as the former case, except b is now taken on the C, the square root scale, in place of b^2 on the B scale.

EXAMPLE: $\frac{4.69 \times 38.5}{(24.2)^2} = 0.308.$

Setting the line of the sliding indicator on 469 of the A scale, bring 242 on sliding scale C under same line. Then look up 385 on scale B and find 308 on A. The decimal point is located by a mental computation as before.

If the problem be $\frac{a^2x}{b}$, move the indicator to a on D, then bring b on the sliding scale B to the line, and look up x on B as before, finding the result again on A.

EXAMPLE: $\frac{(98.1)^2 \times 63.8}{452} = 1360.$

Here we set the sliding indicator on 981 on D (which is the position for $(981)^2$ on A), and bring 452 on sliding scale B to line, then looking up 638 on B find 136 on A. The location of the decimal point is made by a hasty approximate mental computation. Thus, $(98.1)^2$ is nearly 10,000, and 63.8 will go into 452 about 7 times. $\frac{10,000}{7}$ is about 1400. Hence the result is 1360.

To find the square or square root of a number simply set the sliding indicator on the number, finding this on A if the square root is sought, and on D if the square is to be found, the result being then under the same line on the other scale. In

finding square roots use only the left-hand half of scale A for an odd number of figures to the left of the decimal point, and the right-hand half for an even number, of figures before the decimal point.

EXAMPLES: Find the square of 34.8.

Setting the line of the transparent indicator on 348 of scale D, it is found to coincide with 121 of scale A. The answer is evidently 1210, to the nearest reading of the rule.

Find the square root of 7.56.

Set the line of the indicator on 756 of first half of scale A, and find under same line 275, the answer being evidently 2.75.

Find the square root of 13.9.

Set the line of the indicator on 139 on right half of scale A, and find below D 3.72 as the answer.

To find the cube of a number we have $a^3=\frac{a^2\times a}{1}$. Hence set 1 on B, opposite a on D, then for a on B we find a^3 on A.

EXAMPLE: Find the cube of 6.48.

Bring line of transparent indicator over 6.48 on D, and bring 1 on B to this mark. Then opposite 6.48 on B find 272 on A.

To find the cube root we reverse the process.

Thus $\sqrt[3]{a}=x$, wherein $\frac{x^2\times x}{1}=a$. Here we must find an unknown number such that when it falls on B under the given number on A, it is also found on D opposite unity (right or left) on C.

EXAMPLE: Find the cube root of 456.

Knowing approximately, by inspection, that it is in the vicinity of 8, we bring 8 on B under 456 on A (putting first the line on the transparent indicator at 456 on A for convenience); we then find that the right-hand index, or unity, on C falls a little to the left of 8 on D. By a little adjustment of the sliding scale we find that when 770 on B comes to the line (456 on A) that 770 is also opposite unity on C at the right end. The answer is therefore 7.7.

If the number had been 45.6, we should have had to use the right-hand scale of A and the left-hand index of C, thus finding the answer to be 3.57.

For carrying on continuous computations, as $\frac{aghx}{bfh}$, we may take $\frac{ag}{b}$ first, and by setting b on B to a on A, move the transparent indicator to g on B. Then set f on B to this line and move indicator to h on B. Then set h on B to this line again, and look up x on B, opposite to which is the answer on A.

Thus, $$\frac{2.42\times 16.9\times 64\times 0.132}{42.9\times 0.046\times 3.26}=537.$$

Here we set 429 on B opposite 242 on A, and move line on indicator to 169 on B. Then move 46 on B to line, and set indicator on 64 on B. Then move 326 on B to the line, and look up 132 on B. Opposite this find the answer 537 on A. To find the position of the decimal point we perform a mental computation like this: 2.42×16.9=about 40; 0.132=about $\frac{1}{7}$, and this into 64 is about 9; $9\times 40=360$; 0.046 is about $\frac{1}{20}$, and this into 42.9 is about 2; 2×3.26 is about 7; $\frac{1}{7}$ of 360 is about 50; hence the answer is 53.7, rather than 5.37 or than 537.

It goes without saying that any one or more of these numbers could have been a square, in which case scale C is used in place of B and scale D in place of A.

TRIGONOMETRICAL COMPUTATIONS.

On the *under* side of the slide three scales will be found, the upper one marked S, being a scale of natural sines, and the lower one marked T, a scale of natural tangents. Between these is a scale of equal parts, which gives the logarithms corresponding to the series of numbers on scale D.

In order to use these, place the slide in the groove with the under side uppermost, and the left and right indices coinciding. On A will then be found the sines of the angles given on S, those on the left of scale A having the characteristic -1, and those on the right of scale A, the characteristic 0, thus we find:

Sine $3^\circ=0.0523$, on left of scale A.
Sine $15^\circ\ 10'=0.262$, on right of scale A.

We have on D the tangents of the angles given on T, the characteristic being always 0; thus, tan $25^\circ=0.466$ on scale D. The scale gives the tangents from 6 to 45 degrees only; for angles less than 6° the tangent is practically the same as the sine; larger angles must be found by the formula,

$$tan\ a=\frac{1}{tan\ (90-a)}.$$

The sines and tangents of angles may be found *without reversing the slide*, by setting the given angle on scale S or T to the index line on the transparent disk on the under side of the rule, and reading off the sine on B or tangent on C opposite the right-hand index marks of the scales A and D.

The Logarithms of numbers are found in a similar manner by setting the left index of C to the given number on D, and reading off the logarithms on the scale of equal parts under the index on the reverse side of the rule.

With the scale of equal parts *the cube and other roots or powers may be extracted*, such as $x^{\frac{5}{2}}$, $x^{1.7}$ etc.

EXAMPLE: Find $4^{\frac{5}{2}}$ or $\sqrt{4^5}$.

By the above method log. $4 = .602$, and log. $4^{\frac{5}{2}} = .602 \times \frac{5}{2} = 1.505$. Now by placing 505 on the log. scale on the lower side of the slide at the index line on the under side of the rule, we find 32 on scale D under the left index of C, which is therefore equal to $4^{\frac{5}{2}}$, the logarithmic index being 1.

The position of the decimal point in all these cases will be easily ascertained by those accustomed to this class of calculations.

BOOK II

METHODS OF PLANE SURVEYING

CHAPTER VII

LAND SURVEYING [1]

144. Purposes. All surveys of land, properly so called, are made:

(*a*) For the purpose of establishing certain monuments, corners, lines, and boundaries, as in laying out and dividing land, or,

(*b*) For the purpose of identifying and locating such monuments, lines, and boundaries, after they have been established, as in all resurveys for location and area.

In all cases the boundary and dividing lines are the traces of vertical planes on the surface of the ground, and the area is the area of the horizontal plane included between the bounding vertical planes. In other words, the area sought is the area of the horizontal projection of the real surface.

145. In Laying Out Land the work consists in running the bounding and dividing lines over all the irregularities of the surface, leaving such temporary and permanent marks as the

---

[1] See Appendix G for the essential requirements of a survey and map.

work may demand. These lines to lie in vertical planes, and their bearings and horizontal distances to be found. The bearing of a line is the horizontal angle it makes with a meridian plane through one extremity, and its length is the length of its horizontal projection. This reduces the plot of the work to what it would be if the ground were perfectly level. If all the straight lines of a land-survey lie in vertical planes, and if their bearings and horizontal lengths are accurately determined, then as a land survey it is theoretically perfect, whatever the purpose of the survey may be.

Until recent years the needle compass and Gunter's chain have been almost exclusively used in land surveying. The transit and tape when available are now used. Most of the trouble which has resulted from the use of the compass has arisen from a failure to make frequent determinations of the declination. An observation on the sun by the method given in Arts. 98 or 101, or by the methods described in Chapter XV, should be made in each township, and a true meridian marked on the ground at every county-seat. The compass should be set up on this meridian as often as once a year, at about 10 o'clock A.M., and the declination noted. The annual change in declination found at the county-seat could then be attributed to each of the declinations found in the several townships of the county, and so a continuous corrected record of the true declination kept for all parts of the county.

146. Monuments. All marks of whatever description left on or near the surface of the ground, such as stones, stakes, mounds, holes, or trees specially marked and described, for the purpose of designating a particular point on the surface as a landmark, are called monuments or land bounds.

All land monuments set by surveyors should be stones, suitably cut and marked, and planted in the ground. Surveyors cannot insist too strongly on the necessity of setting permanent monuments to mark land boundaries at the time these boundaries are first established and before the survey passes beyond

their own control. It will not do to trust that someone interested will replace the temporary marks by those of a more permanent character afterwards, both because this is likely to be entirely neglected, and also because of the cloud thus thrown over the authority of the monument itself from the fact that it is not the original mark.

Monuments are used not only to mark corners of tracts of land, but also to mark points in straight lines, as in State boundaries, and points fixed by triangulation in geodetic, topographical, State, and municipal surveys.

147. Significance and Authority of Monuments. Whenever monuments are placed in any scheme of land subdivision, and these monuments are described in the conveyance of such lands when sold, they thereby acquire a perpetual and controlling significance. It matters not how frail and temporary a monument may have been—a mere peg stuck in the ground—if it did at the time designate a particular point in the boundary of the tract, and if such monument is recognized in the deed, its position controls the location absolutely. In any subsequent survey for the location of the boundary it becomes supremely important to identify with certainty the true position of such monument. The field notes of the original survey, or any description of the boundaries in the deed, or the area called for, have no weight in determining the position of the lines and corners as against the *certain* identification of the monuments also recognized in the conveyance. What the conveyer sold and the purchaser bought was a certain fixed tract of land which should have been marked at one time by visible monuments. In this case the field notes are material evidence of the original position of the monuments, but since errors in surveying are not uncommon, and since the supposed area of the tract is computed from these field notes, neither the area nor the description by course and distance, called for in the deed, are allowed to hold as against the proved location of the original monuments, also called for in the deed.

Surveys are always subject to revision and correction, but a monument once set and used in a conveyance cannot be changed, except by the free consent of all parties concerned, even though its position is not what it was intended to be, or not what it is said to be, in the written description. There is therefore an inviolableness and absoluteness of control in recognized monuments which does not pertain to any surveys or to any descriptions or areas dependent on surveys.[1]

148. Lost Monuments.[1] When monuments have once been established and used in conveyances and afterward disappear or are lost, they cannot be re-established as an absolute authoritative control by any survey or agreement of surveys. Nothing but consent or acquiescence of all the parties in interest, or a judgment of the court can replace a lost monument. Surveys and the judgment of surveyors are valuable evidence in determining where the original monument was placed, but the surveyor has no authority or right to replace or re-establish a lost monument, or to certify to its position, unless he can find such trace of the original monument itself, or of a witness point, which may serve to identify its position with certainty. He may then replace it by a more permanent mark, and by recording a full description of his work the new monument may be recognized as having all the authority of the original. But any location of a monument based on the field notes of the original survey, even in conjunction with other well-authenticated monuments a considerable distance off, cannot serve to "establish" such monument. It serves only as so much evidence, to be taken in connection with all other evidence, material and personal, such as fence lines, acknowledged boundaries, testimony of witnesses, etc., which evidence may, and often does, outweigh the evidence furnished by the survey. In such a case the surveyor is an expert witness, engaged to interpret the original field notes and to find where they would place the lost monument; but

[1] See chapter on City Surveying, also Appendix I.

inasmuch as the original field notes may not have agreed with the actual position of the monument, any number of resurveys or agreement of resurveys cannot of themselves be so conclusive evidence of its original position as to prevent an appeal to the courts.

149. Re-Running Old Survey from a Deed. Here the surveyor's efforts are mainly centred in locating on the ground the original position of the boundaries described in the deed. If the length and bearings do not exactly agree with the original monuments, or with the original location of the boundaries, then as stated before the former must yield to the monuments. Even where the old stakes are rotted away, a careful search will frequently discover the hole in which was located the old stake. Again in case of the obliteration of witness trees by natural or artificial causes, a skillful use of the pick and spade will often be sufficient to uncover their original location.

If the directions of the boundary lines were defined in the deed by magnetic bearings as was formerly the custom, the first step in the resurvey will be to find the declination of the needle at the time of the original survey. If this is not given in the original deed or plat its approximate value can sometimes be computed from data compiled by the U. S. Coast and Geodetic Survey,[1] provided the date of the original survey is known.

The change in declination since the original survey was made can now be determined and the bearings corrected to date. Or, better still, if one of the lines can be identified from the marks of the old survey, the present bearing of the same line can now be observed and the change in declination computed. As a transit would usually be used on the resurvey the interior angles could be computed from the bearing given in the deed and then turned off in proper order from the single re-established line. If more than one of the old boundaries can be identified, a com-

[1] See a special publication of the U. S. Coast and Geodetic Survey in 1902, entitled "Magnetic Declination Tables and Isogonic Chart for 1902;" also another with same title in 1908.

parison of the results obtained by using each as an initial line from which to turn the angles will give some idea of the accuracy of the original survey.

Where the records of the survey in question are found either insufficient or inconsistent, valuable aid may be secured by examining old deeds of adjoining lands. Such an examination should be made in the respective county Register of Deeds office or sometimes in the office of the city or town clerk. To save time in such searches, the register of deeds keeps an index of all transactions, both for the grantee and grantor. See also Art. 337.

150. Significance of Adverse Possession of Land.[1] Because of the sanction of the law, property lines are often fixed quite independent of descriptions in deeds, a fact which every surveyor should recognize. Adverse possession has been defined as "an actual and visible appropriation of land commenced and continued under claim of right, inconsistent with and hostile to the claim of the true owner."[2] Briefly stated, the law provides that if a person and his predecessor in title have continuously and exclusively claimed and occupied a piece of real estate for a certain term of years, and if this occupation has been open and notorious and hostile to the world, even if such person is not the rightful owner, the latter is estopped by law, after the end of the statutory period, from establishing his claim in a court of law. The continuous and exclusive use and possession of the land for the statutory period raises the presumption that the possession was adverse and puts the burden of proof on the plaintiff. While the operation of this law sometimes works an injustice, it has been found that the security of title and the general interests of society require it. Without such a law the doors of the courts would be open "to stale claims which experience has shown spring up at great distances of time when important witnesses are dead or material evidence is lost or destroyed."[3]

[1] See Engineering Contracting, 1910, article by George Teeple, Feb. 2, 1910.

[2] Texas Revised Statutes, Section 3198.

[3] Ballard on Real Property, Vol. I, p. 65.

151. Kinds of Adverse Possession. In general it may be stated that when claims of adverse possession are made under some color of title, as a deed or other written instrument, only ten years of adverse possession are required to establish the title. But when a claim is not founded on some color of title,[1] the statute provides that twenty years of adverse possession are necessary to insure title. Now it is evident that (1) when a fence is built to mark a boundary but not placed on the true line, the land between the fence and the true boundary is adversely held, and if so held for the legal time the title to the land will pass to the adverse holder. Any survey of such a line, after the elapse of such statutory period, is valueless.[2] Indeed this is always the case when the fence has been set by agreement of both parties that it shall be the boundary, right or wrong.

(2) But if the erroneous fence is built for convenience only, both parties recognizing that it is not the boundary, possession founded on such fence has been held *not* to be adverse.

(3) A third and most common case is the following: The fence is built without any distinct agreement, but both parties acquiescing in the fence, supposing it to be the true line. In far the greater part of the United States, the courts have held that in this case also, the possession of the controverted strip is adverse and will ripen in time into perfect legal title.[3]

(4) If the two descriptions in the deed of contiguous properties overlap, the adverse possession is evidently with color of title, but if both deeds call for the same boundary, the possession is "without color of title."

It is also a settled principle of law that while the public can get adverse possession against an individual, as for example by

[1] When the adverse user can show some written document which apparently should give him legal title, but which by reason of some defect fails to do so, he is said to hold adversely with "color of title." Without such written document his holding is said to be "without color of title."

[2] Unless the adverse possessor relinquished his rights.

[3] In Maine and Missouri the contrary has been held.

using private land as a road, the individual cannot get adverse possession against the public. Thus, for example, an individual might fence up a road for twenty or more years, but by doing so he secures no rights. The proper authority, town board or city council, could at any time order open the closed road.

The surveyor who has occasion to make relocations of old lines should become familiar with the provision of the law, and the supreme court decisions of his own State on this subject, both in his own interest and in the interests of his clients. By this it is not meant that he should usurp the place of judge and jury. His duty it is to report the facts to his client as near as he can determine them, but in many cases an understanding of the force of adverse possession would greatly assist in sifting out the facts. The total ignorance of or disregard for the application of adverse possession in determining boundaries is a fruitful course of expensive and useless lawsuits.

Principles and Laws Bearing on the Resurvey of Private Lands [1]

152. The Problem Stated. In all resurveys of private lands, whether for running boundaries, computing areas, or for parting off or dividing land, it is first necessary to examine the description of the tract as given in the deed of conveyance, and then to identify such marks, corners, boundaries, and monuments on the ground as have been used in the description. The identification of these monuments often taxes the expertness and skill of the surveyor to the utmost, and it is here that the greatest experience and judgment are required. The original monuments, if there were any placed, may have been entirely lost, and they may or may not have been replaced by others. If

[1] The rules laid down here are mostly derived from State Supreme Court decisions in cases which arose over boundaries established by the compass and chain, and hence do not apply so well to city and town surveys made with greater exactness. See also Appendices G and I.

they have been, it remains to be determined how reliable these secondary monuments are. In the absence of monuments specially set other natural or artificial features may have been used in the description or have by use acquired the force and authority of monuments. There may also be gross discrepancies between the position of the monuments and the description or area named in the deed. There may also be a controversy between the parties in interest as to the real boundaries which the surveyor may be wholly unable to decide. This much, however, is certain, that any location of a corner or line by course and distance is likely to be very uncertain and unsatisfactory, especially where the needle-compass was used in the original survey. Every effort should be made to find some trace of the original monuments, if any were set, or to decide from all the evidence available, both material and personal, what the real boundaries are.

The surveyor has no judicial authority to fix or establish anything.[1] He is simply an expert witness called in to assist by his knowledge and experience, and with the aid of his surveying instruments, to find the true lines and boundaries. His acts are at all times subject to review in the courts, and he should try and subject his decisions to the same rules and precedents which are likely to govern the court. He may thus save himself from much embarrassment and his client from unnecessary expense.

Where a surveyor makes gross blunders in his work, showing incompetency, he is often held for any damages which may result. In many cities licensed surveyors are under a heavy bond and are held liable for any erroneous locations they may make. The title to the map should state what and where the survey is, the scale used, date and name of the surveyor.

[1] See the valuable paper, by Chief-Justice Cooley, on "The Judicial Functions of Surveyors," read before the Michigan Association of Engineers and Surveyors, and printed in full in Appendix A. See also Appendices G and I, and Hodgman's Field Manual, p. 222.

Surveyors' records and plats should always be complete and definite, otherwise they cannot be admitted as evidence.

The following propositions are based on judicial decisions which are thought to have all the force and authority of law, in the absence of special statutes governing the case:

153. The Interpretation of Descriptions in Deeds and the Identification of Boundaries.—*General Rules.*—**1.** If the description is inconsistent, insufficient, doubtful, or capable of two or more constructions, the purchaser is to be given the reasonable benefit of such defects. That is to say, that interpretation of the description must be given which is most favorable to the purchaser.

But if the intention is evident on the face of the instrument, or if the parties by their acts have shown a mutual agreement or acquiescence in a certain interpretation of the description, this meaning will hold and bind the parties.

2. Where any inconsistency in the description arises from a false or impossible statement, and by rejecting such evident error the remaining description becomes consistent and possible, then such parts should be rejected and the deed allowed to stand.

Also, when parts of the description are certain and others uncertain or impossible, if the inconsistency can be removed by rejecting one or more of the uncertain portions, this may be done, but every call in the description must be answered if it can be done, and none rejected if all the parts can stand consistently together.

Again, an entire side which has evidently been omitted from the description may be supplied if the monuments and area may thereby be made to agree with the description. If possible, however, every call in the description should be satisfied by the surveyor in locating the property.[1]

3. The certain description must prevail over the uncertain.

4. A description must be interpreted in such a manner as will make it most consistent as a whole, disregarding conflicting details.

5. In case of overlapping descriptions, the prior description will take precedence over the later ones.

6. When a description calls for land owned and occupied, the actual line of occupation is a material call to be considered in locating the land lines.

7. When a description calls for unmarked lines of surrounding surveys, the position of which can be accurately located, and there is no evidence as to how the survey in question was made, then such unmarked lines in case of conflict, will prevail over courses and distances given in the description in question.

8. When reference is made in a description to a government or other survey,

[1] See also Art. 336, Chapter XII, on City Surveying.

such survey becomes a legal part of the description and must be used in its interpretation.

9. All descriptions in deeds must be construed in the light of what was known to be in the minds of the parties at the time the description was first written, and with reference to such plats, facts, and monuments as then existed.

10. The law presumes the deed was drawn with an honest intent to convey the property. The description must, therefore, be construed, if possible, in such a way as to make it effectual rather than void.

11. If the lines are fixed by monuments, and these can be clearly identified, and are used in the description, such lines are the true boundaries, and are to be determined by a resurvey, even though they differ from the plat, or from the description given in the deed. Any conclusive evidence of the original location of these lines and monuments will, therefore, overrule all surveys or other forms of evidence of where they *should* have been. If the boundaries were not marked at the time the plat was made, then the description is to govern, subject to the rules on excess and deficiency given below.

12. Where land is simultaneously subdivided into numerous tracts, as in the case of the United States land surveys and in the case of town plats, all the marks, lines, and monuments set in the original survey for subdivisions serve as marks, lines, and monuments for every tract or lot in the original survey and are some evidence of the location of each tract or lot. In the absence of monuments marking the location of a particular tract or lot other monuments of the same original survey may be used, but monuments placed in preceding or subsequent surveys, or in surveys of adjoining territory not a part of the given subdivision, cannot usually be so used.

13. In the absence of monuments which can be identified, *conclusive* evidence of the original position of such monuments, or of the lines themselves, may set aside the courses and distances called for in the deed. In short, boundaries may be proved on such testimony and evidence as may be adduced to establish any other fact. The surveyor should, however, give great weight to the courses and distances called for, as a part of such material evidence.

14. Where streets have been opened and used for a long period and the lines marked by fences or other material boundary, and these lines have been acquiesced in without protest, such marks obtain the force and authority of monuments and should not be disturbed because of any disagreement with the original plat and description.

15. All monuments established in United States land surveys are presumed to be equally well placed and have equal authority or weight in determining boundaries. Thus a quarter-section corner has the same weight as a township or section corner, even in fixing a township line. Also, section corners set on lines closing on the north and west sides of townships, though not lying on the original township lines, should still govern in locating the section lines.

16. In the United States land surveys the several sized tracts are section, quarter-section, half-quarter section, and quarter-quarter section. These are presumed to contain 640, 160, 80, and 40 acres respectively, and the Government sold them as containing these amounts.[1] The manner of subdividing a section is defined by law (see p. 218), and hence any actual subdivision of a section into quarters and a quarter-section into halves and quarters again is always subject to revision and correction until the law is satisfied, except that the quarter-section corners planted on the section lines by the United States Deputy Surveyors cannot be changed.

Particular Cases.—1. Where land is described as being "owned and occupied," the actual line of occupation is a material call of the deed.

2. Where a boundary line is defined by distance and terminus at a known point or line, this known terminus fixes the length of the line. If the position of the terminus is uncertain the distance governs.

3. Acquiescence in a given boundary erroneously placed does not alone fix the boundary if the issue has not arisen, and if the fact of such error has not come to the attention of the parties, but such acquiescence if continued for a sufficient period of time, usually 20 years, will in nearly all the States make such boundary the legal one.

4. A course described as running from point *A* to point *B* is presumably a straight line, but if not so stated it may be construed as a crooked or curved line if it is understood to follow some natural feature of the landscape.

5. The terms "southerly," "westerly," etc., are to be construed as meaning due south, due west, etc., if there is nothing to indicate the contrary. Also where terms of approximation are used, such as "about," "as near as may be," and the like, if the exact figures given fit the case and satisfy the description as well as any other, the interpretation is limited to the figures stated.

6. Where the described boundaries are complete and consistent, but inconsistent with the stated area, the boundaries hold as against the area. If the boundaries are doubtful the area may control.

7. Where the call is simply for a given area without dimensions it must be taken in the form of a square if such a rendering is not excluded by some other condition. If one side of the tract is given in line and distance it must be laid out as a rectangle upon such side.

8. Where lines are laid down on a map or plan and are referred to in a conveyance of land, the courses, distances, etc., appearing on such plan are to be as

[1] Whenever these legal subdivisions are mentioned *as such* in deeds of conveyance, as "the S. E. 40 acres of the N. E. quarter of section 10," etc., nothing more is intended than simply "the S. E. quarter of the N. E. quarter of section 10," etc., and the conveyor cannot be held for the full area named.

much considered the true description of the land conveyed as they would if expressly recited in the deed.

9. Land bordering on a public highway usually takes to the centre of the highway, unless expressly stated to the contrary. This usually applies to city and town lots where the streets have been reserved in the original plat.

10. All surveys and descriptions should close when platted, and the surveyor is usually at liberty to use his judgment in correcting either course, or distance, or both, where no monuments are identified with sufficient certainty to give them authority. Where monuments remain they control the boundaries so far as they go, in which case the description is not obliged to close, or if it is made to close the points marked by monuments must not be disturbed.

In closing a survey or description, there being no other guide, that method will be used which will convey the greater quantity of land in accordance with the principle that the description is to be construed in favor of the purchaser.

11. Where the bearings of all the courses are given and one course can be identified with certainty, the declination (or "variation") of the needle used in the original survey should be found by setting up on this course and the declination thus found used for all the courses.

12. When a course is defined as starting at a given point on a navigable stream or traveled highway and running a certain distance to another point on said stream or highway, the distance is to be measured along the line of the bank of the stream, or along the highway, and not in a straight line, unless it is specifically so stated. If the stream is not navigable and the presumption is against it being a boundary, the distance is to be measured in a straight line.

But where a *tract of land* is described as bordering on or fronting a certain distance on a stream, in the absence of other controlling facts such distance must be measured in a straight line between the extremities of the opposite boundaries.

154. Water Boundaries and Meandered Lines.[1]—1. Meandered lines on the United States land surveys were run for the purpose of outlining lakes and rivers, and are in no sense boundary lines. They serve for computing the areas of the fractional quarter-sections which were used in the first sales by the Government, but the real boundary is the centre of the stream if not navigable, and the line of ordinary high water, or line of vegetation, if navigable. In the case of lakes and ponds described as boundary lines the ownership is to the water's edge.

2. In extending side boundaries beyond the meandered lines to the river bank or lake shore, such extensions beyond the meandered lines should run *at right angles to the shore.* This rule also applies to city lots and to all lands fronting on bodies of water. An exception to the rule is the following:

3. When the waters of a lake recede from a drainage or any other cause, or when

[1] For numerous court decisions on water boundaries, see Hodgman's Surveying, p. 308–316.

a river or creek shifts its course, the accretion or "made land" belongs to the abutting property and should be divided in proportion to the original lengths of water frontage. If the land thus acquired is the valuable consideration then the original side boundaries are to be extended so as to divide the new *area* among the abutting tracts in proportion to their original frontages; but if the length of the new frontage is the desirable thing, then the new *line of frontage* is to be saved to the original tracts possessing such privileges, in due proportion. In either case the extension of the side boundaries will usually involve angles at the meandered or original water-line and these extensions will, in general, run nearly at right angles to the new shore line.

4. A "bank" of a stream is the continuous line where vegetation ceases. A "shore" is the exposed ground below the bank line.

5. The rights of ownership extend to the centre lines of non-navigable streams and lakes, but only riparian rights obtain in the beds of navigable streams or lakes.

6. Where land is specifically bounded by the "bank," or "along the bank" of a stream or lake, these words will exclude all ownership of the bed of such stream or lake. Whether they would exclude riparian rights also would depend on the circumstances of the case and on the understanding of the parties.

In computing the area of a survey the terms "from," "to," or "with" the shore of a stream mean to low-water mark in some States and to high-water mark in others.

7. In the case of meandered river banks or lake fronts on the United States land surveys, the computed areas included only up to the meandered line, all outside of that belong to the tract by a natural right. Hence in any subsequent sales of the tract the area should only be computed to the meandered line unless the conveyance specifically calls for an extension "to" or "along" the shore or bank, in which case the area would be computed to low-water or high-water mark, as above stated.

8. Similarly, when an area is to be laid off from a tract bounded in part by a meandered line, this area should be computed only up to the meandered line unless otherwise specifically stated.

9. Islands in streams unsurveyed by the United States and unappropriated belong to the abutting land on that side of the *filum aquæ* or the central thread of the low-water channel on which the island itself lies.

155. **Surplus and Deficiency.**[1]—1. Surplus or deficiency, either of distances or of areas, does not invalidate a conveyance.

2. In the case of contiguous tracts where no monuments were established the purchasers receive their full measure of ground, in the order of purchase from the original owner, the last purchaser receiving the surplus or losing the deficiency.

[1] See also Arts. 377, 378, Chapter XII, on City Surveying.

In the case of city lots, sold by number, any surplus or deficiency found on the ground should be divided proportionally among all the lots affected. This rule applies even when the suit concerns the last lot in the block, and the plat shows this to be fractional. But if the plat fails to show the dimensions of the *last* lot, its owner would have what was left of the block after giving the other owners in the block the full amounts shown in the plat.

3. A description which refers to a plat will generally be considered valid when the land conveyed can be located by means of the plat, even though the plat be imperfect and incomplete.

The United States System of Laying Out the Public Lands

156. General Description. The public lands of the United States have included all of that portion of our territory north of the Ohio River and west of the Mississippi River not owned by individuals previous to the dates of cession to the United States Government; also similar portions of the States of Florida, Alabama, Mississippi, and Tennessee. All of this territory, except the private claims, has been subdivided, or laid out, in rectangular tracts bounded by north and south and east and west lines, each tract having a particular designation, such that it is impossible for the patents or titles, as obtained from the Government, to conflict. This has saved millions of dollars to the land-owners in these regions by preventing the litigations that are common in the old colonial States, and is one of the greatest boons of our national Government. The system was probably devised by Gen. Rufus Putnam,[1] an American officer in the Revolutionary War. It was first used in laying out the eastern portion of the State of Ohio, in 1786–7, then called the Northwest Territory. This was the first land owned and sold

[1] See a paper read before the Engineers' Club, of St. Louis, by Col. H. C. Moore, and published in the "Journal of the Association of Engineering Societies," Vol. II, p. 282. See also a paper by W. A. Truesdell read before the Engineers' Club, of St. Paul, and published in the above journal, Vol. XLI, p. 207, Nov., 1908.

by the national Government. The details of the system have been modified from time to time, but its main features remain substantially unchanged. The following is a synopsis of the method now used, which is given in detail in the "Manual of Surveying Instructions," issued by the Commissioner of the General Land Office, at Washington, D. C.:[1]

157. The Details of the System. It should be understood that the following subdivision of a State into 24-mile square tracts, and later into township and sections, proceeded only as fast as a demand for the land developed, and as a rule covered many years before any State was entirely divided.

The system provided for the following work:

First, the establishment of

(*a*) An Initial Point by astronomical observations.

(*b*) A Principal Meridian passing through the initial point.

(*c*) A Base Line on true parallel of latitude passing through the initial point and extending both east and west therefrom.[2]

This initial operation and the ones following can be best understood by referring to Figs. 89 and 90.

Second, the division of the area to be surveyed into tracts approximately 24 miles square, by surveying,

(*a*) Standard parallels both east and west through the 24-mile points previously established on the principal meridian, and

(*b*) Guide meridians, being true meridians of longitude extending north and south through the 24-mile points previously established on the base line and standard parallels.

Since the guide meridians converge, these 24-mile tracts will be 24 miles wide on their southern and a little less on their northern boundaries. Though theoretically the eastern and the western boundaries should be exactly 24 miles in length, because of inevitable errors in measurements, this is rarely or never the case.

[1] The surveyor should obtain and follow the instructions in force at the time the original surveys were made in his locality. The manual may be purchased of the Supt. of Public Documents, Washington, D. C.

[2] Twenty-four such sets in all were established in the United States. For a description of each, see Appendix E.

Third, the division of each 24-mile tract into tracts approximately 6 miles square, called Townships, by the establishment of

(*a*) Range lines, being true meridians through the standard township corners already established on the base-line and standard parallels and extending north to their intersection with the next standard parallel, and

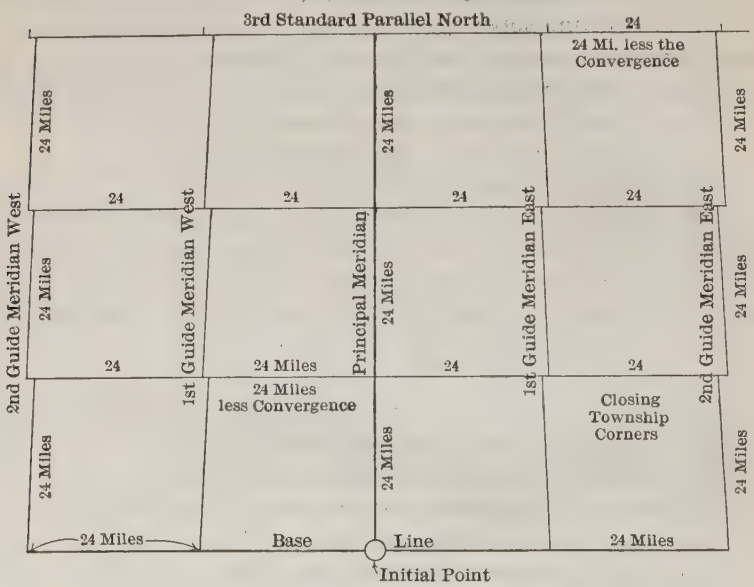


FIG. 89.—Division of Land into Tracts 24 miles square.

(*b*) Latitudinal Lines, called Township Lines, joining the township corners already established at intervals of six miles on the principal meridian, guide meridian and range lines. Fig. 90 shows the resulting division of the above third operation.

If we neglect the effect of discrepancies and error of measurement, both the east and west boundaries of all townships will be just six miles in length, while the north and south boundaries will be exactly 6 miles only on the parallel forming the southern

limit of the 24-mile tract. All the remaining north and south boundaries will be less than 6 miles, due to the convergence of the meridians.

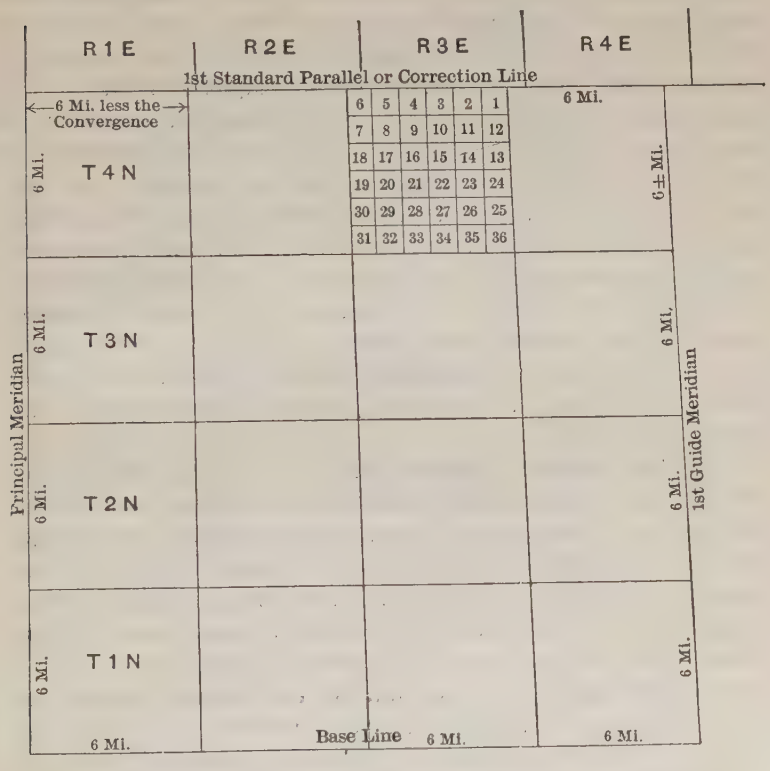


FIG. 90.—Showing the Division of Land into Townships and Sections.

Fourth, the subdivision of each township into Sections each approximately a mile square by the establishment of *section lines*, both meridional and latitudinal, at intervals of one mile apart. A complete explanation of this field work is given in Art. 166.

158. To Run Out a Standard Parallel of Latitude and Base Lines. A true east and west line is one which is at every point at right angles to a meridian passing through that point. It is therefore a constantly curving line, being always deflected toward the north in the northern hemisphere. Any line run on the earth's surface by prolongation by means of any surveying instrument will be a great circle. If the magnetic needle always pointed directly north, a line run at right angles to its direction would be a parallel of latitude. Since the solar compass, or attachment, always orients itself in the true meridian, any line run by it at right angles to the constantly observed meridian will be a true east and west line. This is the only instrument capable of running such a line directly. The transit, however, is usually employed in this work, using either the secant or tangent method. The following description of these methods is taken from the "Manual of Surveying Instructions for the Survey of the Public Lands," as prepared by the Commissioner of the general land office.

159. The Secant Method (Fig. 91). "This method consists in running a connected series of straight lines, each 6 miles long, on such courses that any one of the lines will intersect the curve of the parallel of latitude in two points separated by an interval of 4 miles; and, from the line thus established, measuring north or south, as the case may be, to attain other required points on the latitude curve." By reference to Fig. 91 it will be seen that the transit is set up south of the township corner from which the survey is to begin, and at a distance from the corner depending upon the latitude. In Table I, will be found the value of this offset for each degree of latitude, and in the column headed 0 miles. The offset for each half mile from the beginning up to 3 miles is also given. It will be noted that the offsets for the remaining 3 miles can be taken from the same table by entering the table at the bottom. Thus the offset at 4 miles is the same as at 2 miles.

The direction of the first secant at its initial point is determined

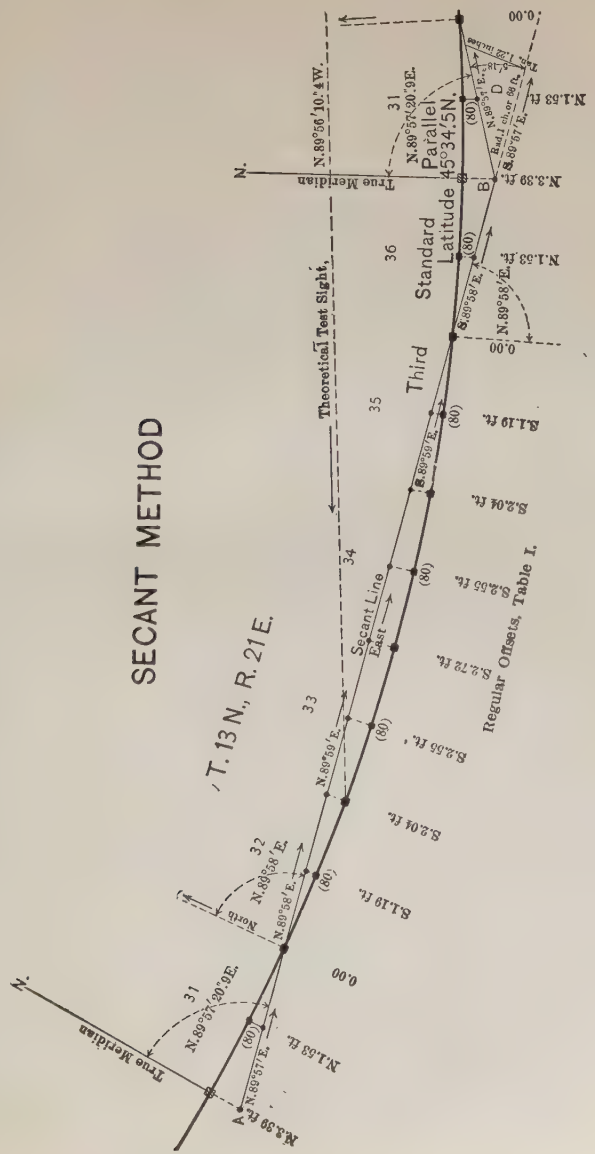

SECANT METHOD
T. 13 N., R. 21 E.
Third Standard Parallel
Latitude 45°34.5N.
True Meridian
Theoretical Test Sight,
Secant Line
East
North
Regular Offsets, Table I.
N.89°56'10".4W.
N.89°57'20".9E.
N.89°58'E.
N.89°57'E.
N.89°59'E.
S.89°59'E.
S.89°58'E.
N.89°58'E.
N.89°57'E.
N.1.53 ft.
N.3.39 ft.
S.1.19 ft.
S.2.04 ft.
S.2.55 ft.
S.2.72 ft.
0.00
31
32
33
34
35
36
A
B
D
(80)

FIG. 97.

TABLE I

AZIMUTHS OF THE SECANT, AND OFFSETS, IN FEET, TO THE PARALLEL

° Latitude,	AZIMUTHS AND OFFSETS AT							Deflection and Nat. tan to a Radius 66 Ft.
	0 Mile.	½ mile.	1 Mile.	1½ Miles.	2 Miles.	2½ Miles.	3 Miles.	
30	89° 58′.5 1.93 N.	89° 58′.7 0.87 N.	89° 59′.0 0.00	89° 59′.2 0.67 S.	89° 59′.5 1.15 S.	89° 59′.7 1.44 S.	90° (E. or W.) 1.54 S.	3′ 0″.2 0.69 ins.
31	89° 58′.4 2.01 N.	89° 58′.6 0.91 N.	89° 58′.9 0.00	89° 59′.2 0.70 S.	89° 59′.5 1.20 S.	89° 59′.7 1.50 S.	90° (E. or W.) 1.60 S.	3′ 07″.4 0.72 ins.
32	89° 58′.4 2.09 N.	89° 58′.6 0.94 N.	89° 58′.9 0.00	89° 59′.2 0.73 S.	89° 59′.5 1.25 S.	89° 59′.7 1.56 S.	90° (E. or W.) 1.67 S.	3′ 15″.0 0.75 ins.
33	89° 58′.3 2.17 N.	89° 58′.5 0.97 N.	89° 58′.8 0.00	89° 59′.1 0.76 S.	89° 59′.4 1.30 S.	89° 59′.7 1.62 S.	90° (E. or W.) 1.73 S.	3′ 22″.6 0.78 ins.
34	89° 58′.2 2.25 N.	89° 58′.5 1.01 N.	89° 58′.8 0.00	89° 59′.1 0.79 S.	89° 59′.4 1.35 S.	89° 59′.7 1.69 S.	90° (E. or W.) 1.80 S.	3′ 30″.4 0.81 ins.
35	89° 58′.2 2.33 N.	89° 58′.5 1.05 N.	89° 58′.8 0.00	89° 59′.1 0.82 S.	89° 59′.4 1.40 S.	89° 59′.7 1.75 S.	90° (E. or W.) 1.87 S.	3′ 38″.4 0.84 ins.
36	89° 58′.1 2.42 M.	89° 58′.4 1.09 N.	89° 58′.7 0.00	89° 59′.0 0.85 S.	89° 59′.4 1.46 S.	89° 59′.7 1.82 S.	90° (E. or W. 1.94 S.	3′ 46″.4 0.87 ins.
37	89° 58′.0 2.51 N.	89° 58′.3 1.13 N.	89° 58′.6 0.00	89° 58′.9 0.88 S.	89° 59′.3 1.51 S.	89° 59′.7 1.89S.	90° (E. or W. 2.01 S.	3′ 55″.0 0.90 ins.
38	89° 58′.0 2.61 N.	89° 58′.3 1.17 N.	89° 58′.6 0.00	89° 58′.9 0.91 S.	89° 59′.3 1.56 S.	89° 59′.7 1.95 S.	90° (E. or W. 2.08 S.	4′ 03″.6 0.93 ins.
39	89° 57′.9 2.70 N.	89° 58′.2 1.21 N.	89° 58′.6 0.00	89° 58′.9 0.94 S.	89° 59′.3 1.62 S.	89° 59′.7 2.02 S.	90° (E. or W.) 2.16 S.	4′ 12″.6 0.97 ins.
40	89° 57′.8 2.79 N.	89° 58′.1 1.25 N.	89° 58′.5 0.00	89° 58′.9 0.98 S.	89° 59′.3 1.68 S.	89° 59′.7 2.10 S.	90° (E. or W.) 2.24 S.	4′ 21″.6 1.00 ins.
41	89° 57′.7 2.89 N.	89° 58′.0 1.30 N.	89° 58′.4 0.00	89° 58′.8 1.02 S.	89° 59′.2 1.74 S.	89° 59′.6 2.17 S.	90° (E. or W.) 2.32 S.	4′ 31″.2 1.04 ins.
42	89° 57′.7 3.00 N.	89° 58′.0 1.35 N.	89° 58′.4 0.00	89° 58′.8 1.05 S.	89° 59′.2 1.80 S.	89° 59′.6 2.25 S.	90° (E. or W.) 2.40 S.	4′ 40″.8 1.08 ins.
43	89° 57′.6 3.11 N.	89° 58′.0 1.40 N.	89° 58′.4 0.00	89° 58′.8 1.08 S.	89° 59′.2 1.83 S.	89° 59′.6 2.33 S.	90° (E. or W.) 2.48 S.	4′ 50″.8 1.12 ins.
44	89° 57′.5 3.22 N.	89° 57′.9 1.45 N.	89° 58′.3 0.00	89° 58′.7 1.12 S.	89° 59′.2 1.93 S.	89° 59′.6 2.41 S.	90° (E. or W.) 2.57 S.	5′ 01″.0 1.16 ins.
45	89° 57′.4 3.33 N.	89° 57′.8 1.50 N.	89° 58′.3 0.00	89° 58′.7 1.16 S.	89° 59′.1 2.00 S.	89° 59′.5 2.49 S.	90° (E. or W.) 2.66 S.	5′ 11″.8 1.20 ins.
46	89° 57′.3 3.44 N.	89° 57′.7 1.55 N.	89° 58′.2 0.00	89° 58′.6 1.21 S.	89° 59′.1 2.07 S.	89° 59′.5 2.59 S.	90° (E. or W.) 2.76 S.	5′ 22″.8 1.24 ins.
47	89° 57′.2 3.57 N.	89° 57′.6 1.61 N.	89° 58′.1 0.00	89° 58′.6 1.25 S.	89° 59′.1 2.14 S.	89° 59′.5 2.67 S.	90° (E. or W.) 2.86 S.	5′ 34″.2 1.28 ins.
48	89° 5′.71 3.70 N.	89° 57′.5 1.66 N.	89° 58′.0 0.00	89° 58′.5 1.30 S.	89° 59′.0 2.22 S.	89° 59′.5 2.78 S.	90° (E. or W.) 2.96 S.	05′ 46″.2 1.33 ins.
49	89° 57′.0 3.82 N.	89° 57′.5 1.72 N.	89° 58′.0 0.00	89° 58′.5 1.34 S.	89° 59′.0 2.30 S.	89° 59′.5 2.87 S.	90° (E. or W.) 3.06 S.	5′ 58″.6 1.38 ins.
50	89° 56′.9 39.6 N.	89° 57′.4 1.78 N.	89° 57′.9 0.00	89° 59′.0 1.39 S.	89° 59′.0 2.38 S.	89° 59′.5 2.97 S.	90° (E. or W.) 3.17 S.	6′ 11″.4 1.43 ins.
° Latitude,	6 Miles.	5½ Miles.	5 Miles.	4½ Miles.	4 Miles.	3½ Miles.	3 Miles.	Deflection and Nat. tan to a Radius 66 Ft.
	AZIMUTH AND OFFSETS AT							

by observations on Polaris (Chapter XV), for the true meridian, and then laying off the azimuth found in table on page 200 under "0 miles." This line is then produced until the 6-mile point is reached, when a new secant line must be run. The angle to be deflected to the north from the former secant will be found in the last column of the table. The true direction of the meridian must be determined at least every 18 miles. The principal advantage of this method is due to the short offsets which reduce the amount of cutting and permit of noting the topography on the line as actually run. A higher degree of accuracy in turning off the deflection angles, as well as an independent check on this work is afforded, by the use of the linear measures given in the last column of the table. These deflection distances are computed for a distance of 66 feet and for the latitudes noted. In using this method, after producing the old secant forward by "double centring" 66 feet beyond the range line, the proper deflection distance taken from the table is carefully measured north of such 66-foot prolongation and the line of sight moved through the angle thus subtended. The direction so determined is then produced 6 miles as the new secant line.

160. The Tangent Method. "This method consists in laying off from the true meridian (see Fig. 92) established by an observation on Polaris at elongation, an angle of 90°, producing the direction thus determined, a distance of 6 miles, in a straight line, and measuring north therefrom, at half-mile intervals distances of correct length taken from Table II on page 203 (interpolated if necessary) for the given latitude, to attain other points on the latitude curve passing through the tangential or initial points.

"The azimuth or bearing of the tangent at successive mile points will be taken from the table to the nearest whole minute only, and will be inserted in the field notes, no interpolation being required except when test sights are taken. The true bearing between two points on a standard parallel will be derived from table II, by taking it in the column headed with one-half

THE TANGENT METHOD
T. 13 N., R. 23 E.
T. 13 N., R. 24 E.
Third Standard Parallel North, Latitude 45°34'.5 N.
Tangent Line
True Meridian
East
N. 90°00' E.
S. 89°59' E.
S. 89°58' E.
S. 89°57' E.
S. 89°56' E.
S. 89°55' E.
S. 89°54' E.
S. 89°53' E.
S. 89°52' E.
S. 89°51' E.
S. 89°50' E.
S. 89°49' E.
0.17 ft.
0.68 ft.
1.53 ft.
2.72 ft.
4.24 ft.
6.11 ft.
8.32 ft.
10.86 ft.
13.75 ft.
16.97 ft.
20.54 ft.
24.44 ft.
28.69 ft.
33.27 ft.
38.19 ft.
43.46 ft.
49.06 ft.
55.00 ft.
61.28 ft.
67.90 ft.
74.86 ft.
82.16 ft.
89.80 ft.
97.78 ft.
Regular Offsets from Tangent, Table V.

FIG. 92.

Table II.

ACUTE ANGLES WITH THE MERIDIAN, AND OFFSETS TO PARALLELS, AT POINTS ONE MILE APART ON A GREAT CIRCLE OR STRAIGHT LINE TANGENT TO THE PARALLEL AT THE INITIAL POINT.[1]

Latitude.	1 Mile from Tangent Point.		2 Miles.		3 Miles.		4 Miles.	
	Angle.	Offset.	Angle.	Offset.	Angle.	Offset.	Angle.	Offset.
°	° ′ ″	ft.	° ′ ″	ft.	° ′ ″	ft.	° ′ ″	ft.
30	89 59 30	0.39	89 59 00	1.54	89 58 30	3.47	89 58 00	6.17
32	89 59 28	.42	89 58 55	1.67	89 58 23	3.76	89 57 50	6.67
34	89 59 25	.45	89 58 50	1.80	89 58 15	4.05	89 57 40	7.20
36	89 59 22	.48	89 58 44	1.94	89 58 07	4.36	89 57 29	7.75
38	89 59 19	.52	89 58 39	2.08	89 57 58	4.69	89 57 18	8.33
40	89 59 16	.56	89 58 33	2.24	89 57 49	5.03	89 57 06	8.95
42	89 59 13	.60	89 58 26	2.40	89 57 40	5.40	89 56 53	9.59
44	89 59 10	.64	89 58 20	2.57	89 57 30	5.79	89 56 40	10.29
46	89 59 06	.69	89 58 12	2.76	89 57 19	6.20	89 56 25	11.04
48	89 59 02	.74	89 58 05	2.95	89 57 07	6.65	89 56 09	11.82
50	89 58 58	0.79	89 57 56	3.17	89 56 54	7.12	89 55 53	12.68

Latitude.	5 Miles.		6 Miles.		7 Miles.		8 Miles.	
	Angle.	Offset.	Angle.	Offset.	Angle.	Offset.	Angle.	Offset.
°	° ′ ″	ft.	° ′ ″	ft.	° ′ ″	ft.	° ′ ″	ft.
30	89 57 30	9.64	89 57 00	13.88	89 56 30	18.89	89 56 00	24.67
32	89 57 18	10.42	89 56 45	15.02	89 56 12	20.44	89 55 40	26.69
34	89 57 05	11.25	89 56 30	16.20	89 55 54	22.05	89 55 19	28.80
36	89 56 51	12.11	89 56 13	17.41	89 55 36	23.74	89 54 59	31.01
38	89 56 37	13.02	89 55 56	18.75	89 55 16	25.52	89 54 35	33.33
40	89 56 22	13.98	89 55 38	20.11	89 54 56	27.40	89 54 11	35.78
42	89 56 06	14.99	89 55 19	21.59	89 54 32	29.38	89 53 46	38.38
44	89 55 49	16.07	89 54 59	23.14	89 54 09	31.50	89 53 18	41.14
46	89 55 31	17.21	89 54 37	24.80	89 53 43	33.76	89 52 50	44.10
48	89 55 12	18.47	89 54 14	26.59	89 53 16	36.19	89 52 18	47.27
50	89 54 50	19.80	89 53 49	28.52	89 52 47	38.82	89 51 45	50.70

Latitude.	9 Miles.		10 Miles.		11 Miles.		12 Miles.	
	Angle.	Offset.	Angle.	Offset.	Angle.	Offset.	Angle.	Offset.
°	° ′ ″	ft.	° ′ ″	ft.	° ′ ″	ft.	° ′ ″	ft.
30	89 55 30	31.23	89 55 00	38.55	89 54 30	46.65	89 54 00	55.52
32	89 55 08	33.78	89 54 35	41.71	89 54 03	50.47	89 53 30	60.06
34	89 54 44	36.45	89 54 09	45.00	89 53 34	54.45	89 52 59	64.80
36	89 54 20	39.25	89 53 42	48.45	89 53 04	58.63	89 52 26	69.77
38	89 53 55	42.19	89 53 14	52.08	89 52 33	63.02	89 51 53	75.00
40	89 53 28	45.29	89 52 44	55.91	89 52 00	67.65	89 51 17	80.51
42	89 52 59	48.57	89 52 12	59.97	89 51 25	72.56	89 50 38	86.35
44	89 52 28	52.07	89 51 38	64.28	89 50 48	77.78	89 49 58	92.57
46	89 51 56	55.81	89 51 02	68.90	89 50 08	83.37	89 49 14	99.22
48	89 51 20	59.83	89 50 23	73.86	89 49 25	89.37	89 48 28	106.36
50	89 50 43	64.17	89 49 41	79.22	89 48 39	95.86	89 47 37	114.08

[1] From the "Manual of Instructions."

the distance between said points. The offsets at intervals of 1 mile are inserted in table; to obtain the length of offsets at the half-mile points, take one-fourth the offset corresponding to twice the distance of the half-mile point from the tangential point.

"This method is suitable for running standard parallels and latitude township lines in level open country, where no intersections with topographical features will be required; but in all cases the secant method will be found most convenient."

161. Initial Points. These points "will be selected with great care and with due consideration for their prominence and easy identification. They will be marked by an indestructible monument, preferably by a copper bolt set in rock and witnessed by rock bearings."

162. Base Lines. The base line is produced both east and west through the initial point by one of the methods described above, taking care to set all forward points by "double centring."

"In order to detect errors and insure accuracy in measurements, two sets of chainmen will be employed, one to note distances to intermedial points and to locate topographical features, the other to act as a check. Each will measure 40 chains, and in case the difference is inconsiderable, the proper corner will be placed midway between the ending points of the two measures; but if a discrepancy exceeds 8 links on even ground or 25 links on mountainous surface, the true distance will be found by careful re-chaining by one party or both."

"To obviate collusion between the sets, the second set should commence at a point in advance of the beginning corner of the first set, the initial difference in measurement being known only to the deputy."

As the line is run all township, section, quarter-section and meander corners are established at the proper distances and properly witnessed as provided by law, Art. 172.

163. The Principal Meridian. The principal meridian has its direction determined by the same methods as those used in the

base-line work. As the line is run all township, section, quarter-section, and meander corners are established at proper distances and all are witnessed.

164. Standard Parallels. The Manual provides for establishing the standard parallels, also called correction lines, both east and west of the principal meridian at intervals of four townships, or 24 miles apart. There are, however, many exceptions to this rule, especially in the older parts of the work.

Standard parallels, being true parallels of latitude, are to be run and measured in the same manner as provided for running base lines.

165. Guide Meridians. Guide meridians are extended north of the base line or standard parallel, at intervals of 24 miles east and west of the principal meridian, in the manner prescribed for running the principal meridian and with the same provisions for securing accuracy of alignment and measurement as those governing base lines and principal meridians.[1] The intersection of the meridian line with the correction line or standard parallel is marked as a corner of the section immediately south, and the distance is measured east or west, as the case may be, to the corner previously established on the correction line. The latter corner is to be considered belonging to the section north of the correction line. These two section corners are sometimes only a few feet apart.

All mile and half-mile distances on guide meridians are measured full 80 and 40 chains except the mile adjacent to the intersection of these lines with a correction line. In this last mile is thrown all the accumulated error of measurement of the entire 6 miles from the preceding township line. In fact, the error is all put in the last half mile of this distance.

166. Township Exteriors. Whenever practicable, the township exteriors in a block of land 24 miles square, bounded by standard lines, will be surveyed successively through the block, beginning with the southeast corner of the southwestern town-

[1] The limit of error is three chains.

ship. From this corner the surveyor runs a true meridian northward, placing a section corner or a quarter-section corner at every 40 chains, until at the end of 6 miles, the township corner is reached. From here a random line is run westward setting corners every 40 chains until the next guide meridian is intersected. Here the surveyor notes the distance north or south which the random line fails to meet the proper corner, and then returning back on the calculated true course, sets over on such corrected line all the temporary corners placed on the random line. It will be seen that this method puts all the deficiency due to the convergence of the meridian boundaries of the township, as well as any deficiency or excess in the westernmost half mile of the latitudinal boundary.

The range line is produced due north as before, another 6 miles, and a similar random line run westward and then corrected eastward as just described. This process is continued northward until the correction line is reached, where a closing corner is established at the intersection of the meridian and correction line.

The half-mile intervals along the range line are measured 40 chains for the entire 24 miles except the northernmost half mile. All the errors of measurement of the entire 24 miles are thrown into this last half mile. The remaining two range lines of the 24-mile square block, together with the latitudinal township lines are run and marked as described above. The only variation in this programme occurs on the last (easternmost) range line, from which random township lines are also run *eastward* to meet the limiting guide meridian, being then corrected westward in such a way that the excess or deficiency of such latitudinal measurement is all thrown into the westernmost half mile. The maximum allowable error in closing the random line upon the township corner provided in the Manual of Instructions is as follows: " If in running a random township exterior, such random exceeds or falls short of the proper length by more than three chains, allowing for convergency, or falls more than three chains

to the right or left of the objective point, it will be rerun, and if found correctly run, so much of the remaining boundaries of the township will be retraced, or resurveyed, as may be found necessary to locate the cause of misclosure."

167. Subdivision of Townships. The Manual provides for the following programme. The surveyor first determines the direction of the meridian at the southeast corner of the township to be subdivided. He also retraces the range line northward for 1 mile and the town line westward for 1 mile. This work gives a comparison of his own meridian observations and the length of his chain with that of the previous surveyor who ran the township lines.

From the southwest corner of section 36 the surveyor runs north parallel to the east boundary of the township, setting a

FIG. 93.—Closing Corners on a Correction Line.

quarter-section corner at 40 chains and a section corner at 80 chains (N. W. corner Sec. 36). He then runs a random line from this section corner east, parallel to the southern boundary of the township, and setting a temporary quarter corner at 40 chains, continues the line until the range line is intersected. If this intersection comes at the section corner the line is blazed back (westward) as the true line. If the random line intersects either north or south of the section corner, the correct bearing of the true line is calculated and the line rerun westward. The permanent quarter-section corner must then be established on the true base and at a point equidistant from its terminals. The limit of error allowed in the closing of the random line on the eastern section corner is 50 links or 33 feet. From the section corner just regained (N. W. corner Sec. 36), the survey is con-

tinued north between sections 26 and 25, the direction being changed if necessary to make it parallel to the east boundary of the township. At 40 and 80 chains on this line the quarter-section and section corners are set, and from the latter a random line is run east parallel to the south boundary of the township.

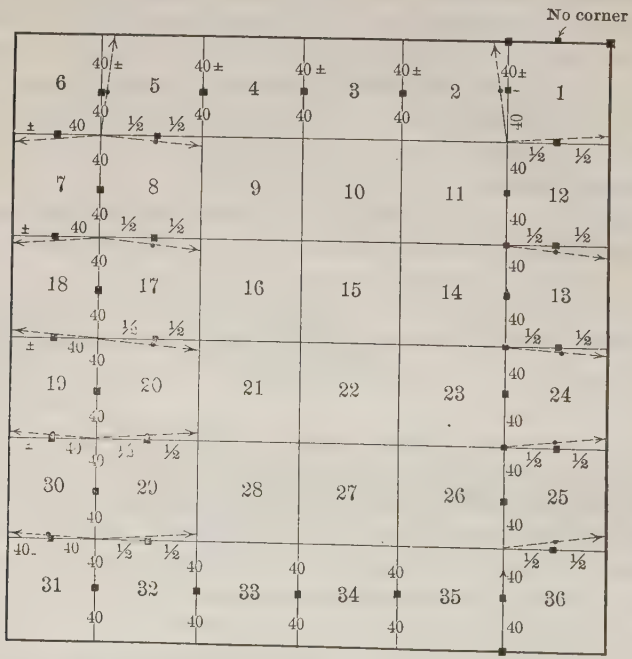


FIG. 94.—Manner of Subdividing a Section not adjacent to a Correction Line.

The return course is calculated, and rerun as before. This programme is continued until five sections have been surveyed. The method of surveying the remaining section will depend upon whether the north boundary of the township is a correction line or not. If the north boundary is not a correction line, from the section corner last established (S. W. corner of Sec. 1), a random

line is run north to the township boundary. The error in closing on the already established corner is noted, and the true course of the line calculated and rerun southward. If the township line

FIG. 95.—Part of a Government Township Plat.

is a correction line the point where the random line intersects it is established as the closing corner, and the distance from it to the adjacent standard corner is measured and recorded. In both cases the permanent quarter corner is placed at 40 chains

north of the initial corner, thus throwing all excess and deficiency into the last or more northern half-mile.

The second, third and fourth ranges are established in the same manner as described for the first. On the fifth range, however, random lines are run westward, also closing on the exterior township corners. In rerunning the corrected line astward the quarter corner is placed 40 chains from the inside corner, thus throwing all surplus or deficiency into the most western half mile. Fig. 95 shows the form of plat prepared from such a survey.

168. The Convergence of the Meridians is, in angular amount,

$$c = m \sin \tfrac{1}{2} (L + L');$$

where m = meridian distance in degrees, or difference of longitude, and L and L' are the latitudes of the two positions. In other words, the angular convergence of the meridians is the difference in longitude into the sine of the mean latitude.

The convergence in chains of two township-lines 6 miles apart, from one correction-line to another 24 miles apart, is

$$C = 24 \times 80 \times \sin c;$$

where c, in degrees, is obtained from the equation above. Thus in lat. 40° $c = \frac{6}{53} \sin 40°$, since one degree of longitude in lat. 40° = 53 miles. Thus $c = 4'.37$ for each 6-mile distance east or west in latitude 40°.

The following table gives the convergence of the meridians (to the nearest whole minute) from 1 to 5 miles apart. Because of being (theoretically) parallel to the eastern range line the meridional section lines will differ from the true meridian by the amounts shown in the table. As these lines are run with a needle compass rarely read closer than five minutes, the distinction would seem of little significance.

TABLE III[1]

CORRECTION FOR CONVERGENCY WITHIN A TOWNSHIP

Latitude.	Correction in minutes to be Applied to Bearing of Range Lines at a Distance of				
	1 Mile.	2 Miles.	3 Miles.	4 Miles.	5 Miles.
30° to 35°	1	1	2	2	3
35 to 40	1	1	2	3	3
40 to 45	1	2	2	3	4
45 to 50	1	2	3	4	5
50 to 55	1	2	3	5	6
55 to 60	1	3	4	5	7
60 to 65	2	3	5	7	8
65 to 70	2	4	6	8	10

[1] Taken from the Manual of Instructions, U. S. Land Office.

169. Requirements of the Work. If this field work conforms to the regulations given in the Manual, then,

(*a*) All interior meridional section lines are full 80 chains in length, except those immediately south of the north boundary of the township.

(*b*) All interior latitudinal section lines are full 80 chains in length, except those next to the west boundary of the township.

(*c*) All quarter corners are placed midway between the adjacent section corners except those on the north and west sides of the township.

(*d*) All meridional section lines are intended to be parallel to the eastern boundary of the township and all latitudinal section lines parallel to the southern boundary of the township.

(*e*) All deficiency in distance due to the convergence of meridians, as well as accumulative errors in measurements, are thrown in the most westerly half miles of the township.

(*f*) No quarter-section closing corners are established on correction lines for the sections adjacent to them on the south. The legal position for such corners, however, would be midway between the adjacent closing section corners of the section in

question, except in section 6, where the deficiency or surplus in measurement is all put in the westernmost half mile.

170. Fractional Sections. Where sections are made fractional by water, reservations, etc., the surveys should be made in a manner to give the same results as would have been secured had the section been full. This sometimes means producing the lines on ice or over the reservation. Where no opposite corners have been or can be fixed, the subdivision lines should be determined by running the lines from the established corners due north, south, east, or west, as the case may be, to the water-course, Indian reservation, or other external boundary line of the fractional section. While the law presupposes that the section lines, as marked in the field by the U. S. deputy surveyors, to be due north and south or east and west lines, they are in fact frequently found to vary considerably from their theoretical position. In such cases, in order to carry out the law, it will be necessary to run the subdivisional lines through fractional sections on mean courses, or, where there is no opposite corner, parallel to the section line. Neither these lot lines nor the quarter-section lines are ever run in the field, but are merely drawn on the government plats as a matter of convenience in expressing the areas.

A sample of the field notes in the form required by law is here given. It is to be commended for its accuracy and clearness.

171. Other Systems Used. The system of land survey here described has been in general use since 1846, but it should be noted that not infrequently conditions are found in the field which quite profoundly modify the system as here presented. Surveyors engaged in this work are furnished with a copy of the Manual, and special instructions for doing the work by the U. S. Land Office. Since the original law providing for a system of land survey was passed by Congress in 1785, Congress has amended the law in important particulars no less than seven different times. A brief statement of these changes may be found in Appendix I.

SPECIMEN OF FIELD NOTES

SUBDIVISIONS OF T. 6 N., R. 34 E.

Chains.	
	At cor. to secs. 10, 11, 14, and 15, by back sight on line bet. secs. 14 and 15. I find its mag. bearing to be S. 18° 35′ E., at 8 A.M., August 12.
	North, bet. secs. 10 and 11.
	Va. 18° 35′ E.
	Over table-land.
26.00	Foot of spur of high mountain, ascend abruptly over broken ground.
34.00	Head of ravine, course S. 70° E.
40.00	Set a sandstone, 20×8×4 ins., 15 ins. in the ground, for ¼ sec. cor., marked ¼ on W. face, and raised a mound of stone, 1½ ft. high, 2 ft. base, alongside. Pits impracticable.
42.00	Foot of sharp ridge, course E. and W.
43.25	Top of ridge, about 40 ft. high, and about 500 ft. above last sec. cor. Descend abruptly.
44.00	Foot, about 30 ft. below top, and ascend over broken ground.
51.10	Enter heavy pine timber.
80.00	A point about 900 ft. above last sec. cor. Set a sandstone, 24×6×4 ins., 18 ins. in the ground, for cor. to secs. 2, 3, 10, and 11, marked with 5 notches on S. and 2 notches on E. edges, from which
	A pine, 15 ins. diam., bears N. 67° E., 30 lks. dist., marked T. 6 N., R. 34 E., S. 2, B.T.
	A pine, 27 ins. diam., bears S. 23° E., 67 lks. dist., marked T. 6 N., R. 34 E., S. 11, B.T.
	A pine, 12 ins. diam., bears S. 47° W., 110 lks. dist., marked T. 6 N., R. 34 E., S. 10, B.T.
	A pine, 16 ins. diam., bears N. 50° W., 82 lks. dist., marked T. 6 N., R. 34, E. S. 3, B.T.
	Land, mountainous and broken.
	Soil, rocky; 4th rate.
	Timber, pine; 28.90 chs.
	54.00 chs. of line run over mountainous land.
	At 9.30 A.M. August 12, diurnal change has diminished the variation 2′.
	S. 89° 59′ E. on a random line bet. secs. 2 and 11.
	Va. 18° 33′ E.
40.00	Set temporary ¼ sec. cor.
80.00	Intersected N. and S. line at cor. to secs. 1, 2, 11, and 12. Thence I run N. 89° 59′ W. on a true line bet. secs. 2 and 11, with same va.
	Over rolling ground.
9.50	Foot of spur.
20.00	Stream, 4 lks. wide, course S.E., and enter timber.
40.00	Set a sandstone, 16×8×4 ins., 11 ins. in the ground, for ¼ sec. cor., marked ¼ on N. face, from which
	A pine, 18 ins. diam., bears N. 17° E., 48 lks. dist., marked ¼ S. B.T.
	A pine, 14 ins. diam., bears N. 40 W., 63 lks. dist., marked ¼ S., B.T.
50.00	Ascend abruptly.
55.65	Top of ascent.
58.90	Descend into deep ravine.
61.70	Bottom of ravine about 100 ft. deep, and ascend
68.00	Across ravine, thence over rough, broken ground through timber to
80.00	The cor. to secs. 2, 3, 10, and 11.
	Land, mountainous and broken.
	Soil, rocky; 4th rate.
	Timber, pine; 60.00 chs.
	72.08 chs. of line run over mountainous land.

172. Marking Corners. The kind of monument used has varied greatly in different States, depending upon the materials available or upon other local conditions. In general in timbered districts wooden posts have been used, while in the prairie States mounds of earth have sometimes been used. More care appears to have been taken in the more recent surveys, both in the direction of more accurate work of the survey and in its permanent marking. This is evidenced by the careful examination and checking of the contractor's work and by the more general use of stone monuments even in prairie country.

Where stones or posts are used, notches are cut on the edges, one for each mile, so as to show how far the monument is from each boundary of the township. On township lines the notches are cut on the sides and the monument set with the sides parallel to the points of the compass, instead of 45° from same. In addition to these marks, correction lines have their monuments marked "S. C." (standard corner), and "C. C." (correction corner), on their southern side. Quarter-section corners are marked $\frac{1}{4}$ S.

173. Witnessing Corners. The Manual provides that, wherever possible, every monument shall be witnessed by one or more objects. Thus, in a section post or stone may be witnessed by observing the bearing and distance to a tree. To identify the tree the surveyor cuts a deep blaze on the side of the tree adjacent to the section corner on which is inscribed the Section, Township, and Range, etc., thus:

T 7 N
R 10 E
S 22
B T

The letters B T are usually cut near the base of the stump so that it is frequently found after the tree is cut down.

At section corners four trees are marked, one in each section, while at quarter corners only two are marked. In prairie regions

rectangular pits are dug near the corner and with the earth so obtained a mound is made.

174. Witness Corners. Where the regular corners happen to fall in ponds or lakes, the Manual provides for the placing of so-called witness corners on the bank at its intersection with the line in question. Thus, in case a single-section corner falls in a lake, there would be four witness corners to it, corresponding to the shore end of the four lines leading to the corner. These corners are marked "W C." in addition to the regular section markings and are referenced the same as true corners.

175. Meander Corners. This is the name given to corners established at the intersection of section lines with streams of over three chains in width, or with other bodies of water. It is witnessed and marked in the provided-for $\frac{1}{4}$ section corners but in addition it is marked "M. C." on the side of the monument facing the lake. The prolongation of the line to the opposite bank is also marked by a meander corner.

176. Meandering. In order to plat the bodies of water and to facilitate the computation of the area of quarter sections or lots made fractional by lakes or rivers, meanders (or traverses) of the bank of river or lake are run connecting the adjacent meander corners. Court decisions make it clear that such a meander line is not to be understood as a boundary of the lot in question. Instead, ownership extends beyond such meander line as far as the high-water mark.

In addition to the above work the U. S. Manual of Instructions requires the following field notes:

177. Summary of Objects and Data Required to be Noted.[1]

1. The precise course and length of every line run, noting all necessary offsets therefrom, with the reason for making them, and method employed.

2. The kind and diameter of all bearing trees, with the course and distance of the same from their respective corners; and

[1] Taken from the Manual of Instructions, U. S. Land Office.

the precise relative position of witness corners to the true corners.

3. The kind of materials of which corners are constructed.

4. Trees on line. The name, diameter, and distance on line to all trees which it intersects.

5. Intersections by line of land objects. The distance at which the line intersects the boundary lines of every reservation, town site, duration claim, Indian allotment, settler's claim, improvement, or rancho; prairie, bottom land, swamp, marsh, grove, and windfall, with the course of the same at all points of intersection; also the distances at which the line begins to ascend, arrives at the top, begins to descend and reaches the foot of all remarkable hills and ridges, with their courses, and estimated height in feet, above the level land of the surrounding country, or above the bottom lands, ravines, or waters near which they are situated. Also, distance to and across large ravines, their depth and course.

6. Intersections by line of water objects. All rivers, creeks, and smaller streams of water which the line crosses; the distances measured on the true line to the bank first arrived at, the course down stream at points of intersection, and their widths on line. In cases of navigable streams, their width will be ascertained between the meander corners, as set forth under the proper head.

7. The land's surface—whether level, rolling, broken, hilly or mountainous.

8. The soil—whether rocky, stony, sandy, clay, etc., and also whether first, second, third, or fourth rate.

9. Timber—the several kinds of timber and undergrowth, in the order in which they predominate.

10. Bottom lands—to be described as wet or dry, and if subject to inundation, state to what depth.

11. Springs of water—whether fresh, saline, or mineral, with the course of the stream flowing from them.

12. Lakes and ponds—describing their banks and giving their height, and whether it be pure or stagnant, deep or shallow.

13. Improvements—towns and villages; houses or cabins, fields, or other improvements, with owners' names; mill sites, forges and factories, U. S. mineral monuments, and all corners not belonging to the system of rectangular surveying, will be located by bearing and distance, or by intersecting bearings from given points.

14. Coal banks or beds; peat or turf grounds; minerals and ores with particular description of the same as to quality and extent, and all diggings therefor; also salt springs and licks. All reliable information that can be obtained respecting these objects, whether they be on the line or not, will appear in the general description.

15. Roads and trails, with their directions, whence and whither.

16. Rapids, cataracts, cascades, or falls of water with the estimated height of their fall in feet.

17. Precipices, caves, sink holes, ravines, remarkable crags, stone quarries, ledges of rocks, with the kind of stone they afford.

18. Natural curiosities, interesting fossils, petrifactions, organic remains, etc.; also all ancient works of art, such as mounds, fortifications, embankments, ditches, or objects of like nature.

19. The magnetic declination will be incidentally noted at all points of the lines being surveyed, where any material change in the same indicates the probable presence of iron ores; and the position of such points will be perfectly identified in the field notes.

178. Original Notes; Where Kept. In the following States copies of the plats and notes may be obtained of the officers named:

Alabama: Secretary of State, Montgomery.
Arkansas: Commissioner of State Lands, Little Rock.
Illinois: Auditor of State, Springfield.
Indiana: Auditor of State, Indianapolis.
Iowa: Secretary of State, Des Moines.
Kansas: Auditor of State, Topeka.

Michigan: Commissioner of State Land Office, Lansing.
Mississippi: Commissioner of State Lands, Jackson.
Missouri: Secretary of State, Jefferson City.
Nebraska: Commissioner of Public Lands, Lincoln.
Ohio: Auditor of State, Columbus.
Wisconsin: Commissioners of Public Lands, Madison.

In other public-land States the original field notes are retained in the offices of the United States surveyors-general. In most, if

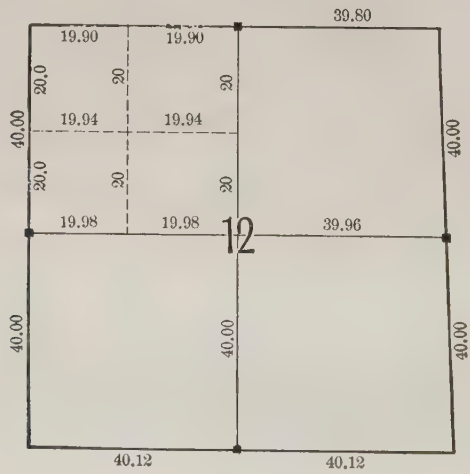


FIG. 96.

not all the States which have been subdivided into sections, copies of the original notes have been deposited with the proper officer of each county, the county surveyor or register of deeds.

179. Subdivision of Sections. The directions of the United States Land Office for doing this work will be found in full in Appendix I, and should be carefully studied. Where all four quarter-section corners are established, the work of dividing the section into quarter sections is simple, viz., connect the opposite quarter corners with straight lines. The intersection

of these straight lines gives the legal centre of the section, and it will be noted that this method holds in the fractional sections on the north and west side of the township. (See Fig. 96.)

In townships where the opposite quarter corners were never set, the quarter lines will be run from the established corners on *mean* courses where the section lines are not due lines. In exceptional cases such subdivisional lines may be run parallel to the nearest adjacent section line.

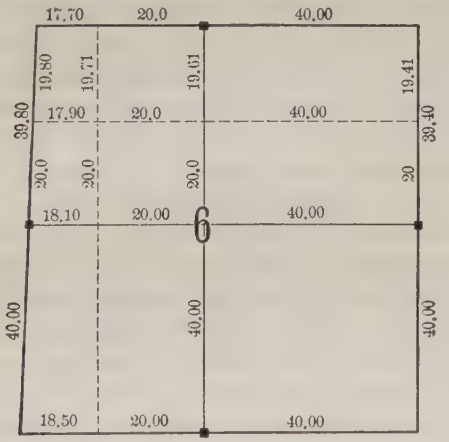


FIG. 97.—Sec. 6 T. N.R.E. as Surveyed by the U. S. Surveyors.

In further dividing the quarter section into quarters (40 acres) the quarter-quarter section corners will be established midway between the U. S. Government section and quarter section and midway between the quarter corners and the centre of the section, except on the last half mile of the lines closing on the west and north boundaries of the township, where they should be placed at 20 chains, proportionate measurement, to the north or west of the quarter-section corner. From such quarter-quarter section corners, determined as above, the subdivisional lines of

the quarter section will be run straight between opposite corresponding quarter-quarter section corners. The intersection of the lines thus run will determine the place for the corner common to the four quarter-quarter sections.

These directions can be best understood by a study of Figs. 96 and 97, the latter of which shows the most irregular section in the township because it is adjacent to both the north and west boundaries of the town.

180. Relocating Lost Corners. The general widespread indifference to the value and importance of preserving the original location of section and quarter-section corners has resulted in a multitude of disputes and expensive litigation regarding property lines. The location of so many public roads on section lines has frequently led to the careless displacement of the original monument, when the roads were improved for travel. Witness trees and line trees have been ruthlessly cut down and removed by the very landowners whose rights they preserved. Although the original monuments were not of much permanence, very little effort on the part of the early settlers was needed to have marked these locations in a very permanent manner. The value to the public of such preservation is gradually being realized, but at so late a period as to greatly increase the difficulties and cost of the work. A very large proportion of present land surveying consists in the relocation of section corners. The duties of the surveyor and the principles which must guide such work of relocation are fully discussed in Arts. 148–151. By act of Congress approved February 11, 1805, it is provided that the corners located in the field by the U. S. surveyors shall be established as the proper corners of the section or quarter section which they were intended to designate, *irrespective of whether they were properly located in the first place or not.* This law also provides [1] that "the boundary lines actually run and marked"

[1] The U. S. laws governing land subdivision are given in Appendix I and should be thoroughly studied in connection with this article.

in the field "shall be established as the proper boundary lines of the section or subdivisions for which they were intended, and the length of such lines as returned by the . . . surveyors aforesaid shall be held and considered as the true length thereof."

In a circular [1] of the General Land Office, Washington, D. C., dated October 16, 1896, certain principles and methods for the re-establishment of section corners are laid down. These methods have been so uniformly followed in court decisions as to have at present the force of law. The General Land Office makes an important distinction between an obliterated corner and a lost corner as follows:

"An *obliterated* corner is one where no *visible* evidence remains of the work of the original surveyor in establishing it. Its location, may, however, have been preserved beyond all question by acts of landowners, by the memory of those who knew and recollect the true site of the original monument. In such cases it is not a lost corner.

"A *lost* corner is one whose position cannot be determined beyond a reasonable doubt either from the original marks or reliable external evidence."

Examples of external evidence which would prevent the corner being lost are the following:

1. Monuments set by later surveyors referring to the original monument while it was still existent.

2. Bearing trees marked by county surveyors and duly recorded (when the U. S. B.T.'s were still recognizable).

3. Acts of landowners in placing fence lines on section lines while section corner was still existent.

4. Memory of old residents as to the original location of the corner.

Examples of the more common original marks are the following:

1. Original stake or other monument.

[1] See Appendix I for the full text of this circular.

2. One or more bearing trees or stumps of same.

3. Line or sight trees mentioned in the notes.

4. Field notes locating the corners by distances from natural objects as streams or ledges, etc.

A fatal but very common error made by the inexperienced surveyor results from a lack of sufficient care and labor in searching for evidence of the original marks of the survey. A hasty search may result in the surveyor's concluding that the corner is lost, when a diligent use of the pick and spade would have uncovered the monument or some of its witnesses. Such a mistake in judgment may result in the "lost corner" being located at a considerable distance from the easily found original monument. Such an error is akin to a gross error in the physician's diagnosis of his patient. In both cases the wrong medicine is worse than no medicine at all.

Not infrequently the testimony of non-interested old residents can be secured which will fix the location beyond a reasonable doubt. Or again, the acts of owners may perpetuate the original location of the corner beyond a reasonable doubt, although, in the meantime, all marks of the original survey are gone. An effective search for a lost or obliterated corner cannot be made without a copy of the original field notes.

Attention has been called (Art. 150) to the fact that *property lines* can be fixed quite independently of the actual section corners. This is due to a general legal principle, called adverse possession.

Surveyors have sometimes erred in their decision as to the proper way of restoring the corner if lost. For this reason surveyors should familiarize themselves with the laws of Congress and the instructions of the U. S. Land Office governing the particular survey in question. A careful study of the U. S. Land Office circular referred to above should be very helpful.

Finding the Area or Superficial Contents of Land when the Limiting Boundaries are Given

181. The Area of a Piece of Land is the area of the level surface included within the vertical planes through the boundary lines. This area is found in acres, roods, and perches, or, better, in acres only, the fractional part being expressed decimally. Evidently the finding of such an area involves two distinct operations, viz.: the field work, to determine the positions, directions, and lengths of the boundary lines; and the computation, to find the area from the field notes. There are several methods of making the field observations, giving rise to corresponding methods of computation. Thus, the area may be divided into triangles, and the lengths of the sides, or the angles and one side, or the bases and altitudes measured, and the several partial areas computed. Or the bearings and distances of the outside boundary lines may be determined and the included area computed directly. This is the common method employed. Again, the rectangular co-ordinates of each of the corners of the tract may be found in any manner with reference to a chosen point which may or may not be a point in the boundary, and the area computed from these co-ordinates. These three methods will be described in detail.

I. *Area by Triangular Subdivision.*

182. By the Use of the Chain Alone. In Fig. 98 let *ABCDEF* be the corner boundaries of a tract of land, the sides being straight lines. Measure all the sides and also the diagonals *AC*, *AD*, *AE*, and *FB*. The area required is then the sum of the areas of the four triangles *ABC*, *ACD*, *ADE*, and *AEF*. These partial areas are computed by the formula,

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)},$$

where s is the half sum of the three sides a, b, c in each case.

For a check, plot the work from the field notes. Thus, take any point as A and draw arcs of circles with A as the common centre, with the radii AB, AC, AD, AE, and AF taken to the scale of the plot. From any point on the first arc, as B, and with a radius equal to BC to scale, cut the next arc, whose radius was AC, giving the point C. From C and D with the measured distance CD, etc., until F is reached. Measure FB on the plot, and if this is equal to the measured length of this line, taken to the scale of the drawing, the field work and plot are correct

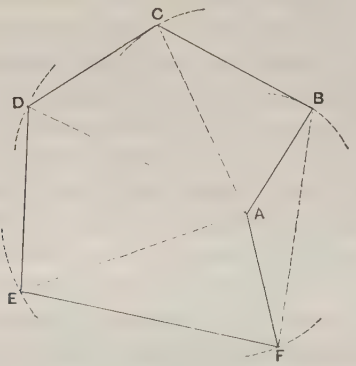


FIG. 98

It is evident the point A might have been taken anywhere inside the boundary lines without changing the method.

183. By the Use of the Compass, or Transit, and Chain. If the compass had been set up at A the outer boundaries could have been dispensed with, except the lines AB and AF. All that would be necessary in this case would be the bearings and distances to the several corners. We then have two sides and the included angle of each triangle given when the area of each triangle is found by the formula:

$$\text{Area} = \tfrac{1}{2}ab \sin C.$$

In this case there is no check on the chaining or bearings. The taking out of the angles from the given bearings could be checked by summing them. This sum should be 360° when A is inside the boundary line, and 360° minus the exterior angle FAB when A is on the boundary. If the boundary lines be measured also, then the area of each triangle can be computed by both the above methods and a check obtained.

184. By the Use of the Transit and Stadia.[1] Set up at A, or at any interior or boundary point from which all the corners can be seen, and read the distances to these corners and the horizontal angles subtended by them. The area is then computed by the formula given in the previous article. The distances may be checked by several independent readings, and the angles by closing the horizon (sum = 360°).

The above methods do not establish boundary lines, which is usually an essential requirement of every survey.

II. *Area from Bearing and Length of the Boundary Lines.*

185. The Common Method of finding land areas is by means of a transit and tape. The bearings and lengths of the boundary lines are found by following around the tract to the point of beginning. If the boundary lines are unobstructed by fences, hedges, or the like, then the compass is set at the corners, and the chaining done on line. If these lines are obstructed, then equal rectangular offsets are measured and the bearings and lengths of *parallel* lines are determined. In this case the compass positions at any corner for the two courses meeting at that corner are not coincident, neither are the final point of one course and the initial point of the next course, the perpendicular offsets from the true corner over-

[1] The stadia methods are described in Chapter VIII.

lapping on angles less than 180° and separating on angles over 180°.

The chaining is to be done as described in Art. 4, the 66-foot or Gunter's chain being used. Both the direct and the reverse bearing of each course should be obtained for a check as well as to determine the existence of any local attraction. For the methods of handling and using the compass see Chapter II.

186. The Field Notes should be put on the left-hand page and a sketch of the line and objects crossing it on the right-hand page of the note book. The following is a convenient form for keeping the notes. They are the field notes of the survey which is plotted on page 228. It will be seen that the "tree" was sighted from each corner of the survey and its bearing recorded. If these lines were plotted on the map they would be found to intersect at one point. If the plot had not closed, then these bearings would have been plotted and they would not have intersected at one point, the first line which deviated from the common point indicating that the preceding course had been erroneously measured, either in bearing or distance, or else plotted wrongly. In general such bearings, taken to a common point, enable us to locate an error either in the field notes or in the plot. The bearings of all division fences were taken, as well as their point of intersection with the course, so that these interior lines could be plotted and a map of the farm obtained. The "old mill" is located by bearings taken from corners *B* and *C*. The reverse bearings are given in parenthesis.

FIELD NOTES—COMPASS SURVEY. October 23, 1905.

No. of Course.	Point.	Bearing.	Distance along the Course.	Remarks.
	Bearing-tree	S. 76° 50′ E.	Ch.	True bearings given.
	Pasture fence	West	7.20	Variation of needle 5° 50′ east.
	Yard "	"	9.75	
1	" "	"	11.54	Henry Flagg, *Compassman.*
	Orchard "	"	13.90	
	Corner B	South	25.42	Peter Long, John Short, *Chainmen.*
Wt. = 1		(North)		
	B. T.	N. 54° 15′ E.		Courses 1 and 2 are along the centres of the highway.
	Old Mill	N. 58° E.		
2	Fence	North	12.50	
	"	"	24.10	
	Corner C	S. 89° 55′ E.	34.68	
Wt. = 1		(West)		
	B. T.	N. 22° 20′ W.		
	Old Mill	N. 26° 45′ W.		
	Fence	N. 61° 45′ W.	9.90	
3	Mill Creek		10.70	
	Fence	N. 64° W.	12.45	
	Corner D	N. 27° 40′ E.	24.00	
Wt. = 3		(S. 27° 45′ W.)		
	B. T.	S. 85° W.		
4	Corner E	N. 19° 10′ W.	7.40	
Wt. = 2		(S. 19° 15′ E.)		
	B. T.	S. 62° 30′ W.		
5	Fence	South	15.80	
	Corner F	N. 86° 50′ W.	25.58	
Wt. = 2		(S. 86° 45′ E.)		
	B. T.	S. 40° 15′ E.		
	N. bank Mill Creek		0.30	
6	S. " "		0.80	
	Corner G	S. 47° 30′ W.	1.50	
Wt. = 5		(N. 47° 30′ E.)		
	Fence	S. 32° E.	0.00	
	Offset, 0.40		0.00	
	" .60		3.00	
	" .80		6.00	
7	" .70		9.00	
	" .30		12.00	
	" .20		13.60	
	Corner H	S. 77° 45′ W.	13.60	
Wt. = 3		(N. 77° 45′ E.)		
8	Corner A	S. 89° W.	3.53	
Wt. = 1		(N. 89° E.)		

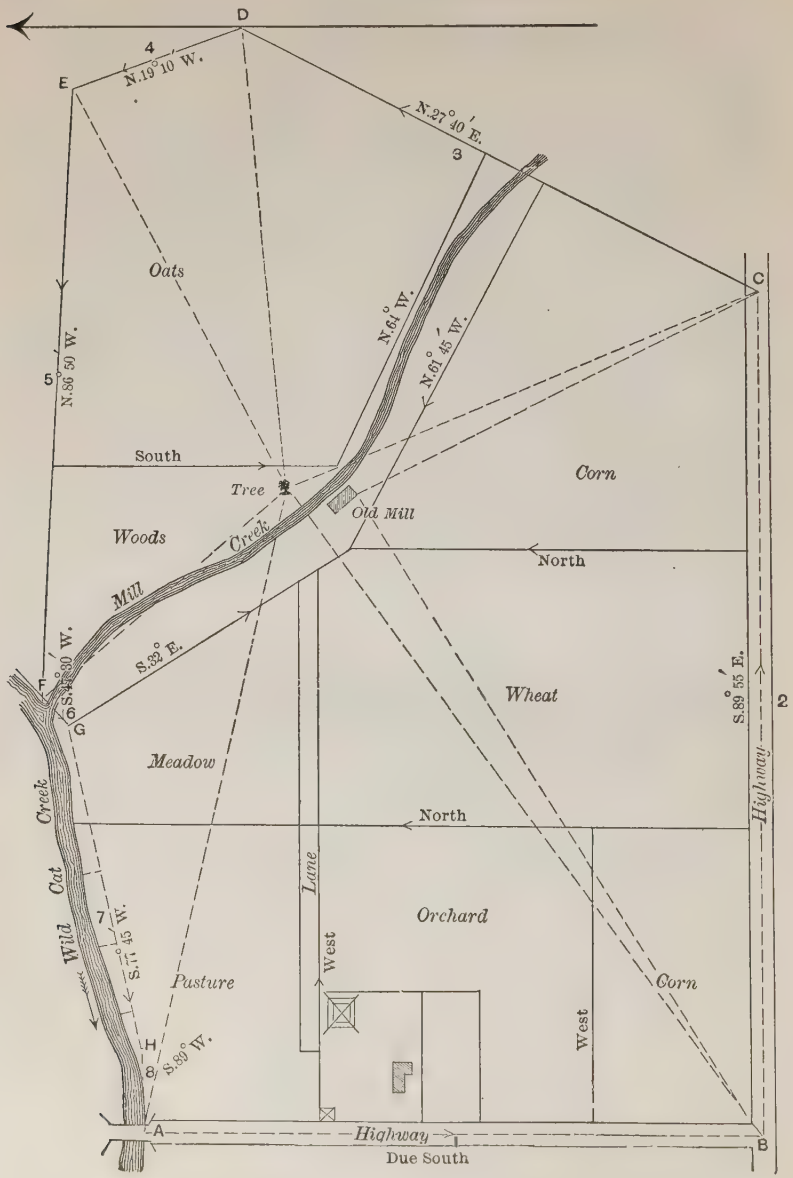


FIG. 99.—Plot of Farm Survey.

Computing the Area

187. The Method Stated. In Fig. 100[1] let $ABCDE$ be the tract whose area is desired. Let us suppose the bearings and lengths of the several courses have been observed. Pass a meridian through the most westerly corner, which in this case is the corner A. Let fall perpendiculars upon this meridian from the several corners, and to those lines drop other perpendiculars from the adjacent corners, as shown in the figure. Then we have:

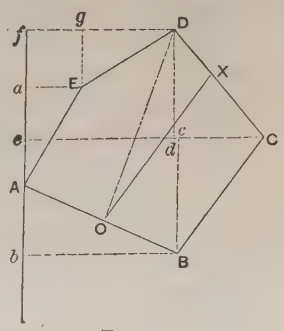


Fig. 100.

$$\text{Area } ABCDE = bBCDfb - bBAEDfb$$
$$= bBCe + eCDf - (bBA + AEa + aEDf). \quad . \quad . \quad (1)$$

Hence twice the area $ABCDE$ is:

$$2A = (bB + eC)Bc + (eC + fD)Dd$$
$$-(bB)Ab - (aE)Aa - (aE + fD)Eg. \quad . \quad . \quad . \quad (2)$$

We will now proceed to show that these distances are all readily obtained from the lengths and bearings of the courses.

188. Latitudes, Departures, and Meridian Distances. *The latitude* of a course is the length of the orthographic projection of that course on the meridian, or it is the length of the course into the cosine of its bearing. If the forward bearing of the course is northward its latitude is called its *northing*, and is reckoned positively; while if the course bears southward its latitude is called its *southing*, and is reckoned negatively.

The departure of a course is the length of its orthographic projection on an east and west line, or it is the length of the

[1] The lines OD and OX in this figure are used in Art. 207.

course into the sine of its bearing. If the forward bearing of the course is eastward its departure is called its *easting*, and is reckoned positively; while if its forward bearing is westward its departure is called its *westing*, and is reckoned negatively.

The meridian distance of a point is its perpendicular distance from the reference meridian, which is here taken through the most westerly point of the survey.

The meridian distance of a course is the meridian distance of the middle point of that course; therefore,

The double meridian distance of a course is equal to the sum of the meridian distances to the extremities of that course. The D. M. D.'s of the two courses adjacent to the reference meridian are evidently equal to their respective departures. The D. M. D. of any other course is equal to the D. M. D. of the preceding course plus the departure of that course plus the departure of the course itself, easterly departures being counted positively and westerly departures negatively. This is evident from Fig. 100.

Thus in Fig. 100 Dd is the latitude and dC is the departure of the course DC. If the survey was made with the tract on the left hand, then the latitude of this course is positive and the departure negative; while the reverse holds true if the survey was made with the tract on the right hand. In this discussion it will be assumed that the survey is made by going around to the left, or by keeping the tract on the left hand, although this is not essential. The D. M. D. of this course CD is $fD+eC$; or it is the D. M. D. of $BC+cC+(-dC)$.

In equation (2), Art. 187, the quantities enclosed in parenthesis are the double meridian distances of the several courses, all of which are positive, while the distances into which these are multiplied are the latitudes of the corresponding courses. If we go around towards the left the latitudes of the courses AB, DE, and EA are negative, and therefore the corresponding products are negative, while the latitudes of the courses BC and CD being positive, their products are positive.

We may therefore say that *twice the area of the figure is equal*

to the algebraic sum of the products of the double meridian distances of the several courses into the corresponding latitudes, north latitudes being reckoned positively and south latitudes negatively, and the tract being kept on the left in making the survey. If the tract be kept on the right in the survey, then the numerical value of the result is the same, but it comes out with a negative sign.

189. Computing the Latitudes and Departures of the Courses. Since the departure of a course is its length into the sine, and its latitude its length into the cosine, of its bearing, these may be computed at once from a table of natural or logarithmic sines and cosines. When bearings were (formerly) read only to the nearest 15 minutes of arc, tables were used giving the latitude and departure for all bearings expressed in degrees and quarters for all distances from 1 to 100. Such tables are called *traverse tables.* It is customary now, however, to read even the needle compass closer than the nearest 15 minutes; and if forward and back readings are taken on all courses, and the mean used, these means will seldom be given in even quarters of a degree. If the transit or solar compass is used, the bearing is read to the nearest minute. The old style of traverse table is therefore of little use in modern surveying. The ordinary five- or six-place logarithmic tables of sines and cosines are computed for each minute of arc, and these may be used, but they are unnecessarily accurate for ordinary land-surveying. For this purpose a four-place table is sufficient. If the average error of the field work is as much as 1 in 1000, then an accuracy of 1 in 5000 in the reduction is evidently all-sufficient, and this is about the average maximum error in a four-place table; that is, the average of the maximum errors that can be made in the different parts of the table.

Table III. is a four-place table of logarithms of numbers from 1 to 10,000, and Table IV. is a similar table of logarithms of sines and cosines, from 0 to 90°.

Since the log sine changes very fast near zero and the log cosine very fast near 90°, the table is made out for every minute

for the first three degrees from these points; for the rest of the quadrant it gives values 10 minutes apart, but with a tabular difference for each minute. It is very desirable to make the table cover as few pages as possible for convenience and rapidity in computation. In this table the zero point is south and angles increase in the direction SWNE, so that in the first quadrant both latitudes and departures are negative. In the second quadrant latitude is positive and departure negative, in the third both are positive, and in the fourth latitude is negative and departure positive. For any angle, falling in any quadrant, if reckoned from the south point in the direction here shown, the log sin (for departure) and log cosine (for latitude) may be at once found from Table IV. If these logarithms are both taken out at the same time and then the logarithms of the distance from Table III., this can be applied to both log sin and log cos, thus giving the log departure and log latitude, when from Table III., again, we may obtain the lat. and dep. of this course, giving these their signs according to the quadrant in which the azimuth of the line falls.

If Table IV. is to be used for *bearings* of lines as given by a needle compass, then enter the table for the given bearing, in the first set of angles, beginning at 0 and ending at 90°. If azimuths are read these should be reduced to bearings.

EXAMPLE

190. Compute the latitudes and departures of the survey plotted in Fig. 100, p. 229, by Tables III and IV. The following are the field notes as they would appear, first, as read by a transit and referred to the true meridian; and, second, as read by a needle compass.

Station.	Azimuth Referred to the South Point.	Compass Bearing.	Distance.
A	290° 45′	S. 69° 15′ E.	7.06
B	217° 15′	N. 37° 15′ E.	5.93
C	140° 30′	N. 39° 30′ W.	6.00
D	57° 45′	S. 57° 45′ W.	4.65
E	30° 00′	S. 30° 00′ W.	4.98

The following is a convenient form for computing the latitudes and departures:

	Course *AB* 4th Q.	Course *BC* 3d Q.	Course *CD* 2d Q.	Course *DE* 1st Q.	Course *EA* 1st Q.
log sin (dep.) =	9.9708	9.7820	9.8035	9.9272	9.6990
log dist. =	.8488	.7731	.7782	.6675	.6972
log dep. =	.8196	.5551	.5817	.5947	.3962
Departure =	+6.60	+3.59	−3.82	−3.93	−2.49
log cos (lat.) =	9.5494	9.9009	9.8874	9.7272	9.9375
log dist. =	.8488	.7731	.7782	.6675	.6972
log lat. =	.3982	.6740	.6656	.3947	.6347
Latitude =	−2.50	+4.72	+4.63	−2.48	−4.31

It is seen that Table III. gives the latitudes and departures to the fourth significant figure as well as to the third.

191. Balancing the Survey. If the bearings and lengths of all the courses had been accurately[1] determined, the survey would "close"; that is, when the courses are plotted successively to any scale the end of the last course would coincide on the plot with the beginning of the first one. Furthermore, the sum of the northings (plus latitudes) would exactly equal the sum of the southings (minus latitudes), and the sum of the eastings (plus departures) would exactly equal the sum of the westings (minus departures). It is evident that such exactness is not attainable in practice, and that neither the north and south latitudes nor the east and west departures will exactly balance, there always being a small residual in each case. These residuals are called the

[1] The error of closure simply shows a *want of uniformity* of measurement, for if all the sides were in error by the same relative amount, the survey would close just the same. For instance, if an erroneous length of chain were used, the survey might close but the area be considerably in error. See Arts. 195 and 197.

errors of latitude and departure respectively. The distribution of these errors is called *balancing the survey.*[1]

In the form for reduction of the field notes given below, wherein this example is solved, it is seen that the error of latitude is 6 links and the error of departure is 5 links. The distribution of these errors is made by one of the following Rules:

FORM FOR COMPUTING AREAS FROM BEARINGS AND DISTANCES OF THE SIDES

Stations.	Courses. Bearings.	Dist.	Dif. Lat. N. +	Dif. Lat. S. −	Departure. E. +	Departure. W. −	Balanced. Lat.	Balanced. Dep.	D. M. D.	+ Area.	− Area.
		Ch.									
A	S. 69° 15′ E.	7.06	.....	2.50	6.60	.....	−2.52	+6.61	6.61	.....	16.66
B	N. 37° 15′ E.	5.93	4.71	.....	3.59	.....	+4.71	+3.60	16.82	79.22	.....
C	N. 39° 30′ W.	6.00	4.63	.....	.....	3.82	+4.62	−3.81	16.61	76.74	.....
D	S. 57° 45′ W.	4.65	.....	2.48	.....	3.93	−2.49	−3.92	8.88	.....	22.11
E	S. 30° 00′ W.	4.98	.....	4.31	.....	2.49	−4.32	−2.48	2.48	.....	10.71
		28.62	9.35	9.29	10.19	10.24				155.96	49.48
			9.29			10.19				49.48	
			Error in lat. = .06		Error in dep. = .05					2) 106.48	

Area = 53.24 sq.ch.
= 5.324 *Acres.*

$$\text{Error of closure} = \frac{\sqrt{5^2+6^2}}{2862} = 0.0027$$
$$= 1 \text{ in } 366.$$

[1] It should be noted that these residuals come only from the accidental errors of the work. Any constant error like those due to a erroneous tape length will not appear in these residuals.

Rule - 1 and 3

Rules for Balancing a Closed Survey

192. The Compass Rule. Rule 1. *As the sum of all the distances is to each particular distance, so is the whole error in latitude (or departure) to the correction of the corresponding latitude (or departure)*, each correction being so applied as to diminish the whole error in each case.[1]

Rule 2. Determine the relative difficulties to accurate measurement and alignment of the several courses, selecting one course as the standard of reference. Thus, if the standard course would probably give rise to an error of 1, determine what the errors *for an equal distance* on the other courses would probably be, as $1\frac{1}{2}$, 2, 1, 0.5, etc. Multiply the length of each course by its number, or relative difficulty, as thus estimated. Then we would have:

As the sum of all the multiplied lengths is to each multiplied length, so is the whole error in latitude (or departure) to the correction of the corresponding latitude (or departure), each correction being so applied as to diminish the whole error in each case.

These two rules are based on the assumption that the error of closure is as much due to erroneous bearings as to erroneous chaining, which experience shows to be true in needle-compass work.

If, however, the bearings are all taken from a solar compass (or attachment) in good adjustment, or if the exterior lines are run as a traverse with a transit, so that the *angles* of the perimeter are accurately measured, then the above assumption does not hold, as it is highly probable that the error of closure is almost

[1] For finding the corresponding corrections to the lengths of the courses themselves, see note under tabular computations, p. 240. This was first suggested by Mr. Antonio Llano in *Eng. News*, Nov. 23, 1899. See also The Adjustment of a Compass Survey, by C. L. Crandall, Transactions Am. Soc. C.E., Vol. XLV, p. 453.

wholly due to erroneous chaining. Especially would this be highly probable if the azimuth is checked by occupying the first station on closing and redetermining the azimuth of the first course, as found from the traverse, and comparing it with the initial (true or assumed) azimuth of this course. If it thus appears that the traverse is practically correct as to angular measurements, it may be fairly assumed that the error of closure is almost wholly due to erroneous chaining. In this case use the following TRANSIT RULE 3:

193. *As the arithmetical sum of all the latitudes is to any one latitude, so is the whole error in latitude to the correction to the corresponding latitude,* each correction being so applied as to diminish the whole error in each case. Proceed similarly with the departures.[1]

In the solution given on p. 234 the first rule is applied. In ordinary farm-surveying it is not common to give the lengths of the courses nearer than the nearest even link or hundredth of a chain. In balancing, therefore, the same rule may be observed. In the more accurate work where the 100-foot tape is used, corrections should be made to nearest tenth of a foot.

194. Comparison of the Transit and Compass Rules for Balancing. If the compass rule should be used to balance a transit survey it is likely that the *adjusted* positions would be much more in error than those given by the original field notes. This is due to the fact that the application of the compass rule is certain to change the bearings of all lines, since the corrections in latitude and departure are all proportional to the *lengths of the courses.* If the sides of the farm are practically north and south and east and west, the use of the transit rule will be to change the *lengths* of the courses but not the bearings. This is

[1] It is evident that the courses could here be weighted for different degrees of difficulty in the chaining; but instead of multiplying the lengths of the courses by their weights, multiply the latitudes and departures by the weights of the corresponding courses, and then distribute the errors in latitude and departure by these multiplied latitudes and departures.

as it should be in transit work. If the sides of the farm are at right angles or nearly so, but *not parallel* with the cardinal points of the compass, all of the bearings should be changed by the constant angle which would reduce to such parallelism, *before* beginning the adjustment.

Where the bearings of the farm as surveyed by a transit are so irregular that they cannot be made approximately parallel and perpendicular to an assumed or true meridian, no rule is known for balancing the circuit which will not disturb the already proved true bearings and angles of the farm.

On the other hand, since it has been shown that the errors of compass surveying are quite as likely to be due to errors in the indicated bearings as in the chaining, the use of the compass rule for balancing such work should not lead to any distortion in the resulting bearings, because it puts the correction where the errors were most probably made.

The following application of both the compass and transit rules to balancing the fifth course of the problem on page 243 should make the above statement clear.

Applying the compass rule this course would be corrected in departure $\frac{1}{6}\frac{5}{5}\frac{4}{8}\frac{3}{4}$ part of the total error in departure, 17 feet, or about 4 feet. This correction in departure corresponds to a change in bearing or azimuth of this course of about 9 minutes. Hence we see the application of the compass rule to accurate transit work in this case changes the bearing of this *one* course by over *four* times as much as the total error made in running the entire six courses. It will be noted, however, that the transit rule does not change the bearing of this course at all, but instead puts all the correction, 2 feet, in the length.

195. The Error of Closure is the ratio to the whole perimeter of the length of the line joining the initial and final points, as found from the field notes. The length of this line is the hypotenuse of a right triangle of which the errors in latitude and departure are the two sides. Its length is therefore equal to the square root of the sum of the squares of these two errors.

This divided by the whole perimeter gives the error of closure, which ratio is usually expressed by a vulgar fraction whose numerator is one, being $\frac{1}{866}$ in the above example.

The error of closure for ordinary rolling country should not be more than 1 in 500. In city work it should be less than 1 in 5000, and should average less than 1 in 10000.

196. The Form of Reduction. On p. 240, the ordinary form of reduction is shown. Here the courses are not multiplied by different degrees of difficulty in chaining; and since it was a compass survey the effect of erroneous bearings is supposed to equal that from erroneous chaining, and so the first rule for balancing is used. The balanced latitudes and departures having been found, the double meridian distances are next taken out. In taking out these it is preferable to begin with the *most westerly corner*, whether this be the first course recorded or not. In the example solved on p. 234, it is the first corner occupied, but in that given on p. 243, it is not the first course. By beginning with the most westerly corner (which is equivalent to passing the reference meridian through that corner), all the double meridian distances will be positive; otherwise some of them may be negative. If attention be paid to signs we may begin at any corner to compute the double meridian distances.

A check on the computation of the D. M. D.'s is that, when computed continuously in either direction and from any corner, the numerical value of the D. M. D. of the last course must equal its departure with the opposite sign. This is a very important check and must not be neglected, as it proves the accuracy of all the D. M. D.'s.

We are now able to compute the double areas according to equation (2), Art. 187, since the terms entering in that equation have their numerical values determined. The several products, being the partial double areas, are written in the last two columns, careful attention being paid to the signs of these products. Thus, when the reference meridian is taken through the most westerly corner, then all the D. M. D.'s are positive and the

results take the sign of the corresponding latitude. If some of the D. M. D.'s are negative, then the signs of these partial areas are opposite to those of the corresponding latitudes. The algebraic sum of the partial double areas is twice the area of the figure, as shown in Eq. (2), Art. 187. If the distances are given in chains, then the area is given in sq. chains, and dividing by ten gives the area in acres. If the distances were given in feet, as it often is, being measured by a 100-foot chain or tape, then the area is in sq. feet, and this must be divided by 43560, the number of sq. feet in one acre, to give the area in acres. This is best done by logarithms, as shown in the example solved in Art. 189. It is preferable to express areas in acres and decimals rather than in roods and perches, as was formerly the custom.

On the following page is the reduction of the field notes given on p. 227. Here the several courses have been multiplied by the various degrees of difficulty in the chaining. Thus, the first and second courses were along the public highway and on even ground. These are taken as the standard and given the weight unity. The third course is on very uneven ground and is judged to give rise to about three times the error of courses one and two per unit's distance. It is therefore weighted three. The proper weight to give to the several courses is thus seen to depend on the character of the obstructions to accurate work, and represents simply the judgment of the surveyor as to the probable relation of these sources of error. The short course *FG* was very difficult to measure, as there were precipitous bluffs, and the course *GH* was also on very uneven ground.

Following the column of multipliers in the tabular reduction are the multiplied distances, the errors of latitude and departure are distributed according to the results in this column by Rule 2, Art. 192. This survey was also made with a needle compass and chain.

In the following example the transit was used, and the survey began at *A*. The azimuth of the line *AB* (Fig. 101) was found by a solar attachment, and then the other courses ran as a traverse,

Station.	Courses. Bearings.	Distances.	Relative Weighting.	Multiplied Distances.	Dif. Lat. N. +	Dif. Lat. S. −	Departure. E. +	Departure. W. −	Balanced. Lat.	Balanced. Dep.	D.M.D.	+ Area.	− Area.
A	South	25.42	1	25.42	.....	25.42	.....	.....	−25.42	0.000	0.000	.....	.....
B	S. 89° 55′ E.	34.68	1	34.68	.....	.05	34.68	.....	− 0.09	+34.69	34.69	.....	3.122
C	N. 27 40 E.	24.00	3	72.00	21.26	.....	11.15	.....	+21.19	+11.16	80.54	1706.643	.....
D	N. 19 10 W.	7.40	2	14.80	6.99	.....	.....	2.43	+ 6.98	− 2.42	89.28	623.174	.....
E	N. 86 50 W.	25.58	2	51.61	1.41	.....	.....	25.55	+ 1.36	−25.54	61.32	83.400	.....
F	S. 47 30 W.	1.50	5	7.50	.....	1.01	.....	1.10	− 1.02	− 1.09	34.69	.....	35.380
G	S. 77 45 W.	13.60	3	40.80	.....	2.88	.....	13.29	− 2.92	−13.28	20.32	.....	59.334
H	S. 89 W.	3.53	1	3.53	.....	.06	.....	3.53	− 0.06	− 3.52	3.52	.....	0.211
		135.71		249.89	29.66	29.42	45.83	45.90				2413.217	98.047
					29.42			45.83				98.047	

Error in lat. = 0.24 Error in dep. = 0.07

Error in closure $= \frac{\sqrt{24^2+7^2}}{1357^1} = 1$ in 543.

2)2315.170

Area = 1157.585 sq.chs.

= 115.76 acres.

NOTE.—To correct the *distances*, let C = length of course; L = latitude, and D = departure of that course; ΔC = correction to course; ΔL = lat. correction, and ΔD = departure correction. Then we have $C = \sqrt{L^2+D^2}$; and by the calculus $dC = \frac{LdL+DdD}{C}$. Now by substituting finite increments, we have $\Delta C = \frac{L}{C}\Delta L + \frac{D}{C}\Delta D$, which furnishes a rule for computing the corrections to the lengths of the courses. Care must be taken to pay attention to the signs of the corrections.

the horizontal limb of the transit being oriented by the back azimuth of the last course. The azimuths of the courses are all referred to the south point as zero, and increase in the direction SWNE. After the last course FA was run, the instrument was carried to A and oriented by a back sight on F and the azimuth of AB again determined. This agreed so well with the original azimuth of this course that the azimuths of all the courses were proved to be correct.[2]

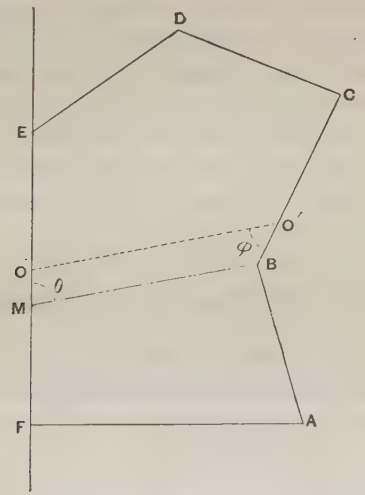


FIG. 101.[1]

The error of closure is therefore due to the chaining alone. A 100-foot chain was used so that the distances are all given in feet. The obstructions to chaining were about uniform, so the courses are all given equal weight. In balancing, Rule 3 must

[1] The lines MB and OO' in this figure are used in Art. 208.

[2] From the azimuth check here obtained, as compared to the errors in latitude and departure, decide whether the latter are due mostly to the chaining or whether the errors in azimuth have had an equal influence, and so determine whether to use rule 1 or rule 3 in balancing.

be used, since the errors are supposed to come only from the chaining.

If the errors in latitude and departure had been distributed by Rule 1, or in proportion to the lengths of the courses, the resulting area would have been 56.41 acres, a difference of 0.07 acres, or about one eight-hundredth of the total area.

197. Area Correction Due to Erroneous Length of Chain. If the measuring unit has not the length assigned to it in the computation, then the computed area will be erroneous. Such an error will not show in the balancing of the work or elsewhere, and hence an independent correction must be applied for this error. If the chain was too long by one one-thousandth part of its length, for instance, then all the courses are too short in the same ratio. And since similar plane figures are to each other as the squares of their like parts, we would have:

$$\text{true area} : \text{computed area} :: (1001)^2 : (1000)^2,$$

or $$\text{true area} = \tfrac{1002}{1000} \text{ computed area (nearly)};^1$$

or, in general, if l = length of chain and Δl = error in length, being positive for chain long and negative for chain short, and if Δl is small as compared with l, as it always is in this case. then if we let:

A = true area, A' = computed area,

C_A = correction to computed area,

and Δ = relative error of chain,

we have

$$A = \frac{l + 2\Delta l}{l} A' = (1 + 2\Delta) A';$$

whence,

$$A - A' = C_A = 2\Delta A'.$$

[1] The error in this approximation is one one-millionth in this case, and would always be inconsiderable in this class of problems.

Station.	Azimuth.	Dist.	Dif. Lat.		Departure.		Balanced.		D. M. D.	+ Areas.	− Areas.
			N. +	S. −	E. +	W. −	Lat.	Dep.			
A	164° 03′	838 ft.	805	.....	.....	230	+ 806	− 231	2539	2,046,434	.....
B	205 39	1004	905	.....	434	.....	+ 906	+ 432	2740	2,482,440	.....
C	112 12	896	338	.....	.....	830	+ 339	− 834	2338	792,582	.....
D	55 00	912	.....	523	.....	747	− 522	− 750	754	.....	393,588
E	0 04	1542	.....	1542	.....	2	−1540	− 2	2	.....	3,080
F	269 32	1392	11	.....	1392	.....	+ 11	+1385	1385	15,235	.....
A (check)	164° 05′	6584 ft.	2059	2065	1826	1809				5,336,691	396,668
				2059	1809					396,668	

Error in lat. = 6 17 = error in dep.

$$\text{Error of closure} = \frac{\sqrt{6^2 + 17^2}}{6584} = 1 \text{ in } 360.$$

(43560 ft sq. = 1A)

2)4,940,023

Area = { 2,470,012 sq. ft. or 56.70 acres.

That is to say, *the relative area correction due to erroneous length of chain is twice the relative error of the chain multiplied by the computed area, being positive for chain long, and negative for chain short.*

Finding the Area or Superficial Contents of Land when the Rectangular Co-ordinates of the Corners are Given with Respect to any Point as an Origin

198. Conditions of Application of this Method. Where many tracts of land, all bounded by straight lines, are somewhat confusedly intermingled, as is the case in many of the older States, and where the area of each tract over an extended territory is to be found, this method is greatly to be preferred to that by means of the boundary lines. In this case it is only necessary to make a general co-ordinate survey of the whole territory, as described in Chapter VIII., on Topographical Surveying, using the stadia for obtaining distances, and being careful to locate every corner of each tract. If areas alone are required, no attention need be paid to the obtaining of elevations for contour lines, and so the work is greatly facilitated. A transit and two or three stadia rods would be the instruments used. The survey would then be carefully plotted and the co-ordinates measured on the sheet, or they could be computed from the field notes. If the plotting is carefully done the former method is preferable. It is best to choose the origin of co-ordinates entirely outside the tract and so that the whole area falls in one quadrant, thus making all the co-ordinates of one sign.

Large tracts of mineral land are sometimes acquired by large companies, including perhaps hundreds of individual estates. In such cases a topographical map of the region is necessary; and when this survey is made, a little extra care to obtain all the "corners" of private claims will enable the areas of all such lots to be determined with great accuracy and at small additional cost. The method probably has no advantages when the area of but a single tract is desired.

199. The Method of Finding the Area from the Rectangular Co-ordinates of the Corners is as follows:

Let Fig. 102 be the same tract as that given in Fig. 100, and let the origin be one chain west of A and three chains south of B. Then, from the balanced latitudes and departures for this case, given on p. 234, we find the following co-ordinates of the

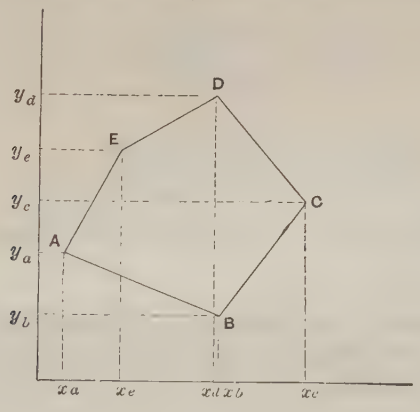


FIG. 102.

corners y_a, y_b, etc., denoting the latitudes of the corners A, B, etc., and similarly with x_a, x_b, etc., for departures:

$y_a = 5.52$, $y_b = 3.00$, $y_c = 7.71$, $y_d = 12.33$, $y_e = 9.84$.
$x_a = 1.00$, $x_b = 7.61$, $x_c = 11.21$, $x_d = 7.40$, $x_e = 3.48$.

The area of the figure $ABCDE$ is equal to the areas:

$$y_b BC y_c + y_c CD y_d - \{y_e ED y_d + y_a AE y_e + y_b BA y_a\};^{1}$$

or

$$A = \tfrac{1}{2}[(y_c - y_b)(x_b + x_c) + (y_d - y_c)(x_c + x_d) - (y_d - y_e)(x_d + x_e) - (y_e - y_a)(x_e + x_a) - y_a - y_b)(x_a + x_b)]. \quad (1)$$

[1] Here y_b, y_c, etc., are used to designate points and not ordinates. In the following equations they are ordinates.

By developing Eq. (1) we obtain:

$$A = \tfrac{1}{2}\,[y_a x_e - y_a x_b + y_b x_a - y_b x_c + y_c x_b - y_c x_a + y d x_c - y d x_e + y_e x d - y_e x_a].^{1} \quad (2)$$

From this we may obtain either of the following:

$$\left.\begin{aligned} A &= \tfrac{1}{2}\,[y_a(x_e - x_b) + y_b(x_a - x_c) + y_c(x_b - x_d) \\ &\qquad + y_d(x_c - x_e) + y_e(x_d - x_a)]; \\ \text{or} \\ A &= -\tfrac{1}{2}\,[x_a(y_e - y_b) + x_b(y_a - y_c) + x_c(y_b - y_d) \\ &\qquad + x_d(y_c - y_e) + x_e(y_d - y_a)]. \end{aligned}\right\} \quad . \quad . \quad (3)$$

From these equations we may obtain the following:

Rule for Finding the Area of a Closed Figure Bounded by Straight Lines from the Rectangular Co-ordinates of the Corners

Multiply the $\left\{\begin{matrix} abscissa \\ ordinate \end{matrix}\right\}$ *to each corner by the difference between the* $\left\{\begin{matrix} ordinates \\ abscissæ \end{matrix}\right\}$ *of the two adjacent corners, always making the subtraction in the same direction around the figure, and take half the sum of the products.*

The student will observe that this is simply a more general case of the former method of computing the area from the latitudes and double-meridian distances.

---

[1] If these co-ordinates be arranged thus:

$$\frac{x_a}{y_a} \times \frac{x_b}{y_b} \times \frac{x_c}{y_c} \times \frac{x_d}{y_d} \times \cdots \times \frac{x_a}{y_a}$$

then in accordance with formula (2), the area is equal to one-half the sum of the products of the quantities joined by the broken lines minus the sum of the products of the quantities joined by the full lines.

200. The Form of Reduction for this case is given below:

Corner.	Ordinates (y).	Abscissæ (x).	Difference between Alternate Abscissæ.	Double Areas.
A	5.52	1.00	− 4.13	−22.80
B	3.00	7.61	−10.21	−30.63
C	7.71	11.21	+ .21	+ 1.62
D	12.33	7.40	+ 7.73	+95.31
E	9.84	3.48	+ 6.40	+62.98

Plus areas = 159.91
Minus areas = 53.43

2)106.48

Area = 53.24 sq.chns.
= 5.324 acres.

This is the same result as found on p. 234, by the other method, as it should be, since the same balanced latitudes and departures were used in each case.

It is also evident that after the balanced latitudes and departures are obtained for the ordinary perimeter survey, the area may be computed by this form—from Eq. (3), p. 246, if preferred. Or, if the co-ordinates of the corners are taken at once from a map, or computed from traverse lines, the bearings and lengths of the courses joining such corners could readily be computed. Thus, the length of any course, as BC, is $BC = \sqrt{(x_c - x_b)^2 + (y_c - y_b)^2}$, while its bearing is the arc whose tan is $\frac{x_c - x_b}{y_c - y_b}$.

201. Supplying Missing or Erroneous Data. In any closed survey there are two geometric conditions that must be fulfilled, viz.:

1. The sum of all the latitudes must be zero.
2. The sum of all the departures must be zero.

These two conditions give rise to two corresponding equations. If l_1, l_2, l_3, etc., be the lengths of the several courses, and if

θ_1, θ_2, θ_3, etc., be their compass bearings, then our two geometric conditions give:

$$l_1 \sin \theta_1 + l_2 \sin \theta_2 + l_3 \sin \theta_3 + \text{etc.}, = 0. \quad . \quad . \quad (1)$$

$$l_1 \cos \theta_1 + l_2 \cos \theta_2 + l_3 \cos \theta_3 + \text{etc.}, = 0. \quad . \quad . \quad (2)$$

Since we have *two* independent equations, we can solve for *two* unknown quantities. These two unknowns may be any two of the functions entering in the above equations. Thus, if any two distances, any two bearings, or any one distance and any one bearing are missing, they may be found from these equations. Or, if but one bearing or distance is missing, it may be found from one of these equations and the other equation used for balancing either the latitudes or departures. When all bearings and distances are given, these equations are really used in balancing; but if they are both used to determine missing quantities, there can be no balancing of errors, for when the missing quantities are computed by these equations, both latitudes and departures will exactly balance. In other words, all the errors of the survey are thus thrown into these two quantities.

This artifice should therefore never be resorted to except where it is impracticable to actually measure the quantities themselves in the field.

There are four cases to be solved:

I. Where the bearing and length of one course are unknown.

II. Where the bearing of one course and length of another are unknown.

III. Where two bearings are unknown.

IV. Where two lengths are unknown.

The bearings will be reckoned from both north and south points around to the east and west points, as is common in compass surveying. Then the length of a course into the sin of its bearing gives its departure, and into the cos of its bearing gives its latitude. North latitude is plus and south latitude minus; east departure plus and west departure minus.

In every case let the sum of the departures of all known courses, taken with the opposite sign, be D, and the sum of their latitudes, taken with the opposite sign, be L. Then D and L are the departure and latitude necessary to close the survey.

CASE I. *Bearing and length of one course unknown.* The two condition equations here become:

$$\left.\begin{aligned} l_m \sin \theta_m &= D; \\ l_m \cos \theta_m &= L. \end{aligned}\right\} \quad \ldots\ldots \quad (3)$$

Whence,

$$\tan \theta_m = \frac{D}{L} \quad \ldots\ldots \quad (4)$$

Having found the bearing, find l_m from either of Eqs. (3). Particular attention must always be paid to the signs of D and L. Evidently sin θ_m (dep.) and cos θ_m (lat.) have the same signs as D and L respectively, whence the quadrant which includes the bearing may be determined and the proper letters applied.

CASE II. *The bearing of one course and the length of another unknown.*

In this case let a be the known bearing of the course whose length is unknown, and let l be the known length of the course whose bearing is unknown. Then we have:

$$\left.\begin{aligned} l_m \sin a + l \sin \theta_n &= D; \\ l_m \cos a + l \cos \theta_n &= L. \end{aligned}\right\} \quad \ldots\ldots \quad (5)$$

If we let sin $a = s$, and cos $a = c$, we have:

$$l_m = sD + cL \pm \sqrt{l^2 - (D^2 + L^2) + (sD + cL)^2}. \quad \ldots \quad (6)$$

Here there are two values of l_m which will satisfy the equation, and so there are two solutions to the problem. If the surveyor has no knowledge whatever of either the unknown length or

bearing, the problem is indeterminate. If he has seen the tract he could usually tell which length or which resulting bearing was the correct one, when the problem would become determinate. When l_m is found, substitute in one of equations (5) and find θ_n. Pay careful attention to the signs of the trigonometrical functions of all bearings. When the two unknown courses are nearly at right angles with each other the problem is impracticable.

The following trigonometric method of solving the above problem will be preferred by many because of its simplicity.

If the two unknown sides are not adjacent, make them adjacent, after which the length and bearing of the line which will close the circuit, leaving out the unknown sides, can be computed by Case I. This closing line and the two unknown sides form a triangle, one angle and two sides of which are known. The remaining side and angles can now be computed.

Case III. *When two bearings are unknown.*

Let l' and l'' be the known lengths of the courses whose bearings are unknown. Then the equations become:

$$\left.\begin{aligned} l' \sin \theta_m + l'' \sin \theta_n &= D; \\ l' \cos \theta_m + l'' \cos \theta_n &= L. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Whence,

$$\cos \theta_n = \frac{KL \pm D\sqrt{L^2 - K^2 + D^2}}{D^2 + L^2}; \quad . \quad . \quad . \quad . \quad (8)$$

Where,

$$K = \frac{l''^2 - l'^2 + D^2 + L^2}{2l''}.$$

The trigonometric solution as explained under Case II can be used here also and will be found to require less computation.

Both Case II and Case III are indeterminate unless the area is known or unless enough is known of the fie'd to show which of the two angles that correspond to a given sine or cosine is the correct one. Case III is indeterminate if the unknown sides are parallel.

CASE IV. *When the lengths of two courses are unknown.* Besides the algebraic solution, this case may be solved in the same way as Case II, by moving the two unknown sides until adjacent, and computing the length and bearing of the closing side of the field. This closing side, with the two unknown sides, will form a triangle all of whose angles and one side of which are known. The two remaining sides can be then computed.

The following method may also be used here as well as in Case II and III.

Consider the field to be revolved in azimuth so as to make one of the sides, whose length is unknown, the meridian, and change all the other bearings accordingly. If the latitudes and departures be now computed, the Σ departures, with the opposite sign, will be the departure of second unknown side. Knowing the bearing and departure of this side, its length and latitude are easily computed. The length of the remaining side is equal to the sum of the known latitudes with the opposite sign.

Unless the area is known, this case is indeterminate where the unknown sides are parallel.

202. To Relocate an Old Traverse when Two of Its Corners can be Identified. CASE I, where the known corners are adjacent.

This is discussed in Arts. 32 and 149.

CASE II, where the known corners are separated by two or more unidentified courses. Such a problem frequently arises when the surveyor is required to retrace the lines of angling roads or to retrace the boundaries of irregular tracts. In such cases the two chief sources of uncertainty concern the actual meridian and the true length of the chain used by the original surveyor. To determine both unknowns, the surveyor should assume a meridian, as close as possible to the one used in the original survey, and starting at a known corner, will then mark on the ground a random traverse, using the bearings and distances taken from the original field notes and continuing until the second known corner is reached.

If in the rare case the random line should happen to come out at this second known corner, it would of course show that the assumed meridian and the chain used in the resurvey were the

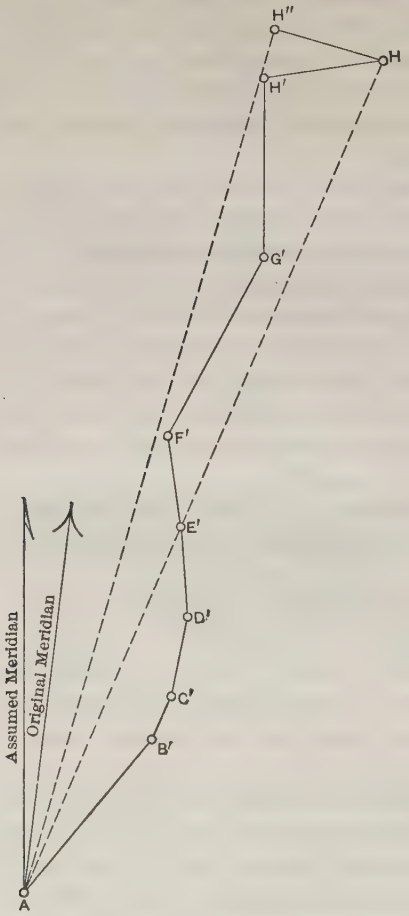


FIG. 103.—To Relocate an Old Traverse.

same as those used on the original survey. In such a case the random line and the true line coincide. Usually the last corner of the random will not agree with the actual known corner. In

such case if the length and bearing of the closing line to such corner be observed, the true positions of all intermediate corners can be computed by the following method.

Let $AB'C'D'E'F'G'H'$ in Fig. 103 represent the random line on assumed meridian (see Table I), and $ABCDEFGH$ the true original traverse of which only A and H can now be found. The closing line $H'H$ is observed to be N. 82° 32′, E. 63 feet. Draw AH' and AH and produce the former beyond H'.

Also with A as a centre and AH as a radius describe an arc intersecting AH' at H''. It is evident that if the random line erred in direction only, then the point H' would lie in the arc, while if the discrepancy ($H'H$) were due to the chain only, then H' would lie in AH or AH produced. This shows that the position of H' with reference to the arc and the diagonal line AH will determine the kind of correction as well as the direction in which it is to be applied. Thus $\frac{AH}{AH''}$, gives the length of the original chain in terms of the resurvey chain, while the distance HH'' measures the twist or error in azimuth of the random line.

Table I

SHOWING RANDOM, RUN ON ASSUMED MERIDIAN FOLLOWING ORIGINAL NOTES

Course.	Bearing.	Distance, Feet.	Latitude.	Departure.		Total Latitude.	Total Departure.
			Northing.	Easting.	Westing.		
AB′	N. 39° 30′ E.	2073.06	1599.6	1318.6	.....	1599.6	1318.6
B′C′	N. 24 15 E.	491.04	447.7	201.7	.....	2047.3	1520.3
C′D′	N. 9 45 E.	834.9	822.8	141.4	.....	2870.1	1661.7
D′E′	N. 4 0 W.	891.0	888.8	.....	62.2	3759.9	1599.5
E′F′	N. 9 15 W.	948.42	936.1	.....	152.4	4695.0	1447.1
F′G′	N. 31 20 E.	2132.50	1821.5	1108.9	.....	6516.5	2556.0
G′H′	North	1881.0	1881.0	.....	.....	8397.5	2556.0

(Note. Calculations in this article are to nearest tenths, by Geo. L. Teeple.)

From Fig. 103 and Table I it will be seen that the Bearing $H''H' = \tan^{-1}\frac{2556}{8397.5} =$ N. 16° 56′ E., while the bearing of $H'H$ is observed = N. 82° 32′ E. Hence the angle:

$$H''H'H = 65° 36'.$$

$$HH'' = 63 \text{ ft. } \sin 65° 36' = 57.4 \text{ feet.}$$

$$H'H'' = 63 \text{ ft. } \cos 65° 36' = 26.0 \text{ feet.}$$

$$AH' = \sqrt{(\Sigma \text{ Lat. of Random})^2 + (\Sigma \text{ Departure Random})^2}.$$

$$= \sqrt{(8397.5)^2 + (2556.0)^2} = 8777.9 \text{ feet.}$$

$$AH = AH' + H'H''$$

$$= 8777.9 + 26.0 = 8803.9.$$

(1) Ratio of $\frac{\text{Original chain}}{\text{Resurvey tape}} = \frac{8803.9}{8777.9} = 1.00296$, that is, the original 100-foot chain was 0.′296 too long.

(2) Error in the assumed meridian $= \tan^{-1}\frac{57.4}{8803.9} = 0° 22\frac{1}{2}'$.

From the figure it will be seen that $22\frac{1}{2}'$ is to be added to the random meridian, i.e., it is to be swung clockwise.

To correct the corners set on the random line to their true position, we refer the corrected line to the random line by adding the $22\frac{1}{2}'$ error in azimuth. This gives us the new bearings in Table II. Also we multiply each of the original distances by the ratio, 1.00296, and thereby derive the true distances in terms of the resurvey tape. These distances are put in Table II. We may now compute the new latitude and departure as well as the total latitude and departure of each corner on the *true* traverse.

The difference between the total latitude and departure of any corner on the random from Table I, and the total latitude and total departure of the corresponding corner on the true line from Table II, when applied to the corners on the random line will give the field locations for the true corners, that is, the corrections to the random corners.

Table II

CO-ORDINATES OF THE CORNERS OF THE TRUE LINE

Course.	Bearing.	Distance, Feet.	Latitude.	Departure.		Total Latitude.	Total Departure.
			Northing.	Easting.	Westing.		
AB	N. 39° 52½′ E.	2079.2	1595.7	1333.0	.....	1595.7	1333.0
BC	N. 24 37½ E.	492.5	447.7	205.2	.....	2043.4	1538.2
CD	N. 10 07½ E.	837.4	824.3	147.2	.....	2867.7	1685.4
DE	N. 3 37½ W.	893.6	891.9	.....	56.5	3759.6	1628.9
EF	N. 8 52½ W.	951.2	939.8	.....	146.7	4699.4	1482.2
FG	N. 31 42½ E.	2138.8	1819.5	1124.2	.....	6518.9	2606.4
GH	N. 0 22½ E.	1886.6	1886.6	12.4	.....	8405.5	2618.8

Table III has been prepared in a manner to show the corrections to each random corner. Thus corner D', when moved 23.7 feet east and 2 feet south of D' will give the original point D, as located in the original survey.

Table III

GIVING THE CORRECTIONS TO THE RANDOM CORNERS FOR THE TRUE CORNERS

Course.	Total Random Survey.	Easting Original Survey.	Correction to Random Corner.	Total Random Survey.	Northing Original Survey.	Correction to Random Survey
B	1318.6	1333.0	E. 14.4	1599.6	1595.7	S. 3.9
C	1520.3	1538.2	E. 17.9	2047.3	2043.4	S. 3.9
D	1661.7	1685.4	E. 23.7	2870.1	2867.7	S. 2.4
E	1599.5	1628.9	E. 29.4	3759.9	3759.6	S. 0.3
F	1447.1	1482.2	E. 35.1	4695.0	4699.4	N. 4.4
G	2556.0	2606.4	E. 50.4	6516.5	6518.9	N. 2.4
H	2556.0	2618.8	E. 62.8	8397.5	8405.5	N. 8.0

NOTE. The differences in the corrections in Table III at the end of Course 7 (E. 62.8 and N. 8.0) and the actual corrections observed at that point (E. 62.5 and N. 8.2) represent the accumulated mathematical errors due to use of tables and dropping all figures less than $\frac{1}{10}$ foot.

Plotting the Field Notes

203. To Plot a Compass Survey select a point for the initial station, and pass a meridian through it in pencil. By means of a semicircular protractor, such as is shown in Fig. 72, mark the bearing and draw an indefinite line from the station point. On this line lay off to scale the length of the course, thus establishing the next corner. Through this draw another pencil meridian, and proceed as before. If the plotting is perfect the length of the line joining the final with the initial point, taken to scale, is the error of closure of the survey; and the horizontal and vertical components of this line, taken to scale, should be the errors in departure and latitude respectively as obtained by the computation.

If preferred, the bearings of the successive courses may be so combined as to give the deflection angle at each station, and these laid off from the preceding course as already drawn. Errors are more likely to accumulate in the plot by this method, however, than by that first given.

Again, the rectangular co-ordinates of the several corners may be computed and these plotted from a pair of rectangular axes, but this is not a common practice.

For the plotting of transit surveys, especially where the stadia is used, see Chapter VIII.

The Areas of Figures Bounded by Curved or Irregular Lines

204. The Method by Offsets at Irregular Intervals. Where a tract of land is bounded by a body of water, as a stream or lake, it is customary to run straight lines as near the boundary as practicable and then to take rectangular offsets at selected intervals from these bordering lines to the irregular boundary. These small areas are then computed as trapezoids, the distance along the base line being the altitude and the half sum of the

adjacent offsets being the mean width. The offsets should therefore be run at such intervals as to make this method of computation sufficiently accurate. Such offsets were taken from the course *GH* in Fig. 99, the notes of which are given on p. 227.

The work of computation may be shortened by using a modified form of the method of areas from the rectangular co-ordinates of the corners, which, in this case, are the ends of the offset lines. Let Fig. 104 be an area to be determined from the offsets from

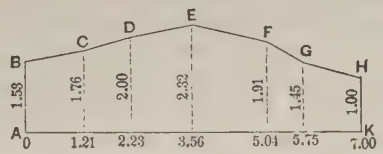


FIG. 104.

the line *AK*. The position and length of the offsets are given. Take the origin at *A* and let the distances along *AK* be the abscissæ, and the lengths of the offsets be the ordinates. Using the second of equations (3), p. 246, we have:

$$A = {}^{1}\tfrac{1}{2}\,[x_a(y_k - y_b) + x_b(y_a - y_c) + x_c(y_b - y_d) + x_d(y_c - y_e) + x_e(y_d - y_f) + x_f(y_e - y_g) + x_g(y_f - y_h) + x_h(y_g - y_k) + x_k(y_h - y_a)]. \quad (1)$$

But here x_a, x_b, y_a, and y_k are all zero; also $x_h = x_k$, hence this equation becomes:

$$A = \tfrac{1}{2}\,[x_c(y_b - y_d) + x_d(y_c - y_e) + x_e(y_d - y_f) + x_f(y_e - y_g) + x_g(y_f - y_h) + x_h(y_g + y_h)]. \quad (2)$$

From Eq. (2) we have the

[1] The plus sign is here used, since we have gone around the figure in a direction opposite to that followed in the general case.

Rule for Finding Areas from Rectangular Offsets at Irregular Intervals

Multiply the distance along the course of each intermediate offset from the first by the difference between the two adjacent offsets, always subtracting the following from the preceding. Also multiply the distance of the last offset from the first by the sum of the last two offsets. Divide the sum of these products by two.

The following is the numerical reduction for finding the area of the irregular tract shown in Fig. 104.

Offset.	Distance from A.	Length of Offset.	Differences.	Products.
	Ch.	Ch.	Ch.	
B	0.00	1.53		
C	1.21	1.76	−0.47	− 0.57
D	2.23	2.00	− .56	− 1.25
E	3.56	2.32	+ .09	+ .32
F	5.04	1.91	+ .87	+ 4.38
G	5.75	1.45	+ .91	+ 5.23
H	7.00	1.00	+2.45	+17.15

2)25.26

Area = 12.63 sq.chains.
= 1.263 acres.

It is evident that an area bounded on all sides by irregular or curved lines could have a base line run through it, and offsets taken from this line to both boundaries and the area computed by this method. Example 10, p. 269, should be so computed.

205. The Method by Offsets at Regular Intervals. If the intervals between the offsets, or ordinates, are all equal the computation is much simplified. On the assumption that the area is a series of trapezoids, we have the

Rule for Finding the Area from Rectangular Offsets at Regular Intervals

Add together all the intermediate offsets and one-half the end offsets, and multiply the sum by the constant interval between them.

The following rules for finding areas are found from the successive orders of differences in each case and may all be derived by a rigid development.[1] They assume that the bounding line is curved and that rectangular ordinates have been measured at uniform intervals from a base line traversing the figure.

Let the common interval between ordinates be d; let the lengths of the ordinates be h_0, h_1, $h_2 \ldots . h_n$; and let the number of intervals be N.

I. $N=1$, $A=\frac{d}{2}(h_0+h_1)$, Trapezoidal Rule.

II. $N=2$, $A=\frac{d}{3}(h_0+4h_1+h_2)$, Simpson's $\frac{1}{3}$ Rule.

III. $N=3$, $A=\frac{3d}{8}(h_0+3h_1+3h_2+h_3)$, Simpson's $\frac{3}{8}$ Rule.

IV. $N=4$, $A=\frac{2d}{45}[7(h_0+h_4)+32(h_1+h_3)+12h_2]$.

V. $N=6$, $A=\frac{3d}{10}[h_0+h_2+h_4+h_6+5(h_1+h_3+h_5)+h_3.]$

This is called Weddel's Rule. If a quadrant be computed by this rule, the result is $0.779r^2$ instead of $0.785r^2$, the true value.

If an area, bounded by a base line and two end ordinates, be divided by imaginary lines parallel to the end ordinates and

[1] See Appendix C.

equally spaced, as in Fig. 105, and if the middle ordinates of these partial areas be measured, then if d = common width of the partial areas and h_1, h_2, h_3, etc., their middle ordinates, a the first end ordinate, and b the last one, we have, approximately:

$$\text{I. } A = d\Sigma h,$$

where Σh signifies the summation of all the h's.

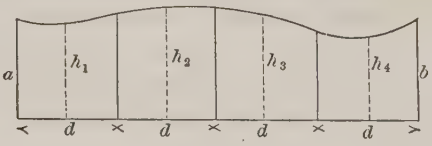


FIG. 105.

The following rules are, however, more accurate:

$$\text{II. } A = d\Sigma h + \frac{d}{12}(a - h_1 + b - h_n), \text{ Poncelet's Rule;}$$

or,

$$\text{III. } A = d\Sigma h + \frac{d}{72}(8a + h_2 - 9h_1 + 8b + h_{n-1} - 9h_n), \text{ Francke's Rule.}$$

The various rules above given are often used to determine areas of irregular figures such as steam diagrams, cross-sections of structural forms, streams, excavations, etc. The most ready and accurate means of determining all such areas, however, is by means of the planimeter.

THE SUBDIVISION OF LAND

206. The Problems arising in the subdivision of land are of almost infinite variety. All such problems are solved by the application of the fundamental principles and relations of geometry and trigonometry with which the student is supposed to be famil-

iar. There are, however, two classes of problems of such frequent application that they will be given in detail.

207. To Cut Off from a Given Tract of Land a Given Area by a Right Line, Starting from a Given Point in the Boundary. In Fig. 100, Art. 187, let O be the middle point on the line AB, from which a line is to be run in such a manner as to cut off three acres from the western portion of the tract. We may at once assume that the dividing line will cut the side DC in some point X, whose distance from D is to be found. First compute the area $OAED$, using the balanced latitudes and departures given on p. 234, we have the following:

Course.	Lat.	Dep.	D.M.D.	Double Areas.	
				+	−
	ch.	ch.			
AO	−1.26	+3.30	3.30	.....	4.16
OD	(+8.07)	(3.10)	9.70	78.28	.....
DE	−2.49	−3.92	8.88	.....	22.11
EA	−4.32	−2.48	2.48	.....	10.71
	(−8.07)	(−3.10)		Sums +78.28	36.98
				−36.98	
				2)41.30	
				Area = 20.65 sq.chns.	
				= 2.065 acres.	

Here the latitude and departure of the course OD are such as to make the latitudes and departures balance. The area is found to be 2.065 acres, leaving 0.935 acres to be laid off from OD by the line OX. It remains now to find the point X.

First compute the length and bearing of the line OD from Case I, Art. 201.

Thus we have:

$$\tan \theta = \frac{D}{L} = \frac{+3.10}{+8.07} = 0.384.$$

Whence $\theta = 21°$ from the table of natural tangents. From the table of natural sines, we find $\sin 21° = 0.358$.

Hence from Eq. (3), p. 249, we have:

$$l \sin \theta = D, \quad \text{or} \quad 0.358l = 3.10.$$

Whence,

$$l = 8.66 \text{ chains.}$$

The bearing is evidently N. 21° E.

We now have to find the distance DX such that the area ODX shall be 9.35 sq. chains. Since the area of any triangle is one-half the product of two sides into the sine of the included angle (another way of saying it is equal to half the base into the altitude), we have:

$$9.35 = \tfrac{1}{2}\,(8.66 \times DX) \sin ODX. \quad . \quad . \quad . \quad . \quad (1)$$

From the bearings of OD and DX we find the angle ODX to be 60° 30′, hence $\sin ODX = 0.870$, from which we find:

$$DX = 2.48 \text{ chains.}$$

The length and bearing of the line OX may be computed from its latitude and departure, the same as was done for the line OD above, or we may compute the angle DOX and length OX by solving the triangle DOX. The bearing of OX may then be found, and the line run from O. There will then be two checks on the work, viz.: the measured lengths of OX and DX must be equal to their computed values.

To find the angle DOX, let the three angles of the triangle be D, O, and X, and the sides opposite these angles be d, o, and x, respectively. Then we have:

$$\tan \tfrac{1}{2}\,(X - O) = \frac{x - o}{x + o} \tan \tfrac{1}{2}\,(X + O),$$

This equation gives the angle $(X-O)$, whence,

$$O=\tfrac{1}{2}(X+O)-\tfrac{1}{2}(X-O), \text{ and } X=\tfrac{1}{2}(X+O)+\tfrac{1}{2}(X-O).$$

Also,

$$d=OX=OD\frac{\sin D}{\sin X},$$

and,

$$o=DX=OD\frac{\sin O}{\sin X}.$$

We therefore have the following:

Rule for Cutting off a Given Area by a Line Starting from a Given Point in the Boundary

Having first surveyed the tract and plotted the same, join the given point on the plot with the corner which will give the nearest approximation to the desired area. Compute the length and bearing of this line, and of the area thus cut off. Subtract this area from the desired area, and the remainder is the area to be cut off in the form of a triangle, one side of which has bearing and distance given, and another side has its bearing alone given. From these data compute the lengths and bearings of the other sides, one of which is the line sought. This line may then be run, and its length measured, as well as the length of the portion of the opposite boundary cut off, for a check on the accuracy of the work.

208. To Cut Off from a Given Tract of Land a Given Area by a Right Line Running in a Given Direction. Let the problem be to cut off 30 acres from the northern portion of the tract shown in Fig. 101, p. 241, by a line whose bearing is N. 80° E., or whose azimuth is 260.[1]

[1] In this problem it would have shortened the operation somewhat if the meridian of the survey had been taken parallel to the dividing-line. The bearings could have all been changed to give angles from this meridian, and original computation made from these new bearings.

Pass a line parallel to the required line through the corner nearest to the probable position of the desired line. Let *MB*, Fig. 101, be such a line. Compute the lengths of the lines *EM* and *MB* by Case IV, p. 251.

From the computation, p. 243, we have the following:

Courses.	Azimuths.	Lengths.	Balanced Latitudes.	Balanced Departures.	D. M. D.'s	Double Areas.
BC	205° 39′	1004 ft.	+906 ft.	+ 432 ft.	2738	+2,480,628
CD	112 12	896	+339	− 834	2336	+ 791,804
DE	55 00	912	−522	− 750	752	− 392,544
EM	0 04	(926)	−926	− 1	1	− 926
MB	260 00	(1171)	+203	+1153	1153	+ 234,059
			(+723)	(−1152)		2)3,113,021

Therefore to close requires $L=-723$ and $D=+1152$.

Area = 1,556,510 sq.ft. = 35.73 ac's.

From Eq. (5), p. 249, we have

$$EM=\frac{D\cos 260°-L\sin 260°}{\sin 259°\ 56'}$$

$$=\frac{(+1152)(+.1736)-(-723)(+.9848)}{+.9846}$$

$$=\frac{200+712}{.9846}=926 \text{ ft.}$$

Whence from Eq. (9), we have

$$MB=\frac{D-EM\sin 4'}{\sin 260°}$$

$$=\frac{+1152-(926)(-.0011)}{+.9848}=1171 \text{ ft.}$$

Inserting these values of the lengths of the courses EM and MB, we can compute the area $BCDEM$. This is found to be 35.73 acres, or 5.73 acres too much. The problem now is to pass a line north of MB and parallel to it, so that the area included between the parallel lines and the intercepted portions of EF and BC shall be 5.73 acres, or 249,710 sq.ft. Let OO' be such a line. This line can be run when either MO or BO' is known. It is best, however, to compute both these distances, using one for a check. To find these distances,

Let x = perpendicular distance between the parallel lines MB and OO'.

Let angle $EMB = EOO' = \theta$,

and angle $OO'B = \phi$.

Then we have

$$\text{Area } MOO'B = \overline{MB} \cdot x - \tfrac{1}{2}x^2 \cot \theta + \tfrac{1}{2}x^2 \cot \phi$$
$$= \overline{MB} \cdot x + \tfrac{1}{2}x^2(\cot \phi - \cot \theta). \quad . \quad . \quad . \quad (1)$$

Since θ and ϕ are known angles, their cotangents are known quantities in any case. So, for simplicity, let

$$(\cot \phi - \cot \theta) = K;$$

also, let the distance $MB = D$,

and area $MOO'B = A$.

Then the equation becomes:

$$A = Dx + \tfrac{1}{2}Kx^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$x^2+\frac{2D}{K}x=\frac{2A}{K};$$

$$x=-\frac{D}{K}\pm\sqrt{\frac{2A}{K}+\frac{D^2}{K^2}}$$

$$=-\frac{D}{K}\pm\frac{1}{K}\sqrt{2AK+D^2};$$

$$=\frac{1}{K}(\pm\sqrt{2AK+D^2}-D). \quad . \quad . \quad . \quad . \quad . \quad (3)$$

That sign of the radical is to be used which will give a positive value to x. The other sign would give the value of x to be used in laying off the given area on the opposite side of MB, provided the sides OM and $O'B$ were continuous in that direction.

Using Eq. (3) for the problem in hand, we have

$\theta=79^\circ\ 56'$;
$\phi=54^\circ\ 21'$;
$A=249{,}710$ sq.ft.;
$D=1171$ ft.;
$K=0.7172-0.1775=0.5397$;

whence $$x=\frac{1}{0.5397}(\pm\sqrt{269{,}537+1{,}371{,}241}-1171)$$
$$=203.6 \text{ feet.}$$

We can now find MO and BO' from

$$MO=\frac{x}{\sin\theta} \quad \text{and} \quad BO'=\frac{x}{\sin\phi};$$

whence $MO=206.8$ feet and $BO'=250.6$ feet.

The length of the line OO' is

$$OO' = MB + x(\cot \phi - \cot \theta).$$

We may therefore write the following

Rule for Cutting off a Given Area by a Line Passing in Given Direction

Having first surveyed the tract and plotted the same, pass a line on the plot in the required direction through the corner which will give the nearest approximation to the desired area. Compute the lengths of the two unknown courses bounding this area, and then the area itself. Subtract this from the given area, and the remainder is the area which is to be cut off by a line parallel to the first trial line. This auxiliary area will always be a trapezoid, whose area, the length and bearing of one of the parallel sides, and the bearings of the remaining sides are known. The length of these sides may then be computed, one of the end lengths laid off, and the dividing line run. Measure the length of this line and also of the other end line for checks.

EXAMPLES

1. Compute the area, plot the survey, and determine error of closure from the following field-notes:

Station.	Bearing.	Distance.
A	S. 46° E.	20.00 ch.
B	S. 74½ E.	30.95
C	N. 33¼ E.	18.80
D	N. 56 W.	27.60
E	W.	21.25
F	S. 51¾ W.	13.80

Answer { Area = 104.4 ± acres.
Error of closure = 1 in 201.

This being a compass survey, the errors in latitude and departure must be distributed in proportion to the lengths of the courses, regardless of their bearings, or according to Rule 1, p. 235. If the errors in the bearings (or deflection angles) had been very small as compared with the errors in measuring the distances, as is the case when the deflection angles are measured with a transit, then Rule 3, p. 236, should have been used.

2. Find the area and error of closure from the following field notes:

Station.	Bearing.	Distance.
A	E.	130 rods.
B	N. 8° E.	137
C	N. 81 W.	186
D	S.	54
E	S. 36 W.	125
F	S. 45 E.	89
G	N. 40 E.	70

What would be the resulting difference in area from the use of Rules 1 and 3?

3. In Example 1, suppose the length and bearing of the first course were unknown. Let these be found as in Case I, Art. 201.

4. Suppose the length of Course *A* and bearing of *B* are unknown in same example. Compute by Case II.

5. Let the first two bearings be unknown. Compute them by Case III.

6. Let the lengths of the first two courses be unknown. Find them by Case IV.

7. Let it be required to cut off twenty-five acres from the west end of the tract given in Example 1 by a line passing through a point on the course *BC* at a distance of ten chains from *B*. Find the length and bearing of the division line and the other intersecting point on the boundary.

8. Let it be required to divide the tract given in Example 1 into three equal portions by north and south lines. Find the length and points of intersection of such lines with the boundary lines.

9. Compute the co-ordinates of the corners of the tract given in Example 1, taken with reference to a point 35 chains directly south of *A*, and then compute the area of the tract from these co-ordinates by the formula given in Example 1. This area should, of course, be the same as that obtained by any other method where the same balanced latitudes and departures are used.

10. An irregular tract of land has a straight line run through it and rectangular offsets taken to the boundary. Find the area of the tract from the following notes:

Distance.	Width.
ch.	ch.
0	2.35
10	8.42
14	12.60
20	11.38
25	10.75
28	6.15
30.50	0.00

Is it significant whether or not this tract lies on both sides or wholly on one side of the base-line?

11. Compute the area of the tract of which the following are the field notes. The rectangular offsets are taken on both sides of a straight axial line, *R* signifying right and *L* left.

Distances.	Side.	Width or Length of Offset.	Distances.	Side.	Width or Length of Offset.
ch.		ch.	ch.		ch.
0	R	4.23	18	R	15.80
0	L	0.00	20	L	5.00
5	R	7.16	25	R	12.20
7.50	L	3.45	30	L	2.62
10	R	12.68	30	R	6.48
10	L	6.00	30	L	0.00
12	R	10.75			

REFERENCES

1. Work of a United States Deputy Surveyor. Louis Ross. Harvard Engineering Journal, Jan., 1910. No. 11148 D.
2. Monuments for Surveys. Municipal Engineering, Oct., 1904.
3. A Property Survey and Its Cost. Arthur W. Tidd. Eng. News, Jan. 10, 1907. No. 81557.
4. The Elimination of Errors in Field Work, by Wm. Bouton, Jour. Asso. Eng. Soc., Vol. V, p. 151.
5. The Proper Mode of Subdividing a Full Government Quarter Section. 2A Enos. Jour. Asso. Eng. Soc., Vol. II, p. 196.
6. Riparian Ownership of Lands on Lakes and Rivers. Jour. Asso. Eng. Soc., Vol. XIII, 1894, p. 148.

7. A Method of Making a Farm Survey. B. Zahniser. Eng. News, March 29, 1900.
8. City, State and County Boundaries, by H. B. Wood, Jour. Asso. Eng. Soc., Vol. XXI, p. 219, 1898.
9. Practical Points on Surveying. Charles L. Hubbard. Mach, N. Y. March, 1908.
10. The History of Land Measurement in England. Wilfrid Airy. Engineering, March 12, 1909.
11. Principles Governing Resurveys with Cases, and Riparian Rights Hodgman's Land Surveying. P. 244.
12. Adverse Possession. Hodgman's Land Surveying. Pp. 254–5.
13. Adverse Possession. George W. Teeple. Engineering Contracting, Feb. 2, 1910.

LAND-SURVEYING INSTRUMENTS AND METHODS

1. Range Finders. Geo. Forbes Pam, Am. Supplement, March 22, 1902. See also Eng. News, Jan. 18, 1890, p. 54.
2. State, City and Town Boundaries. H. B. Wood, Jour. Asso. Eng. Soc., Nov., 1898.
3. How Massachusetts Boundary was Run without Random Lines. H. Stafford, Eng. Record, Dec. 10, 1904.
4. Pioneer Engineering. F. Shelford, London Engineer, May 8, 1898.
5. A Case of Jurisprudence in Land Surveying. A. E. Lehman, Pro. Eng. Club, Phil., July, 1898.
6. Surveying Conveniences. E. N. Millen, Eng. News, Oct. 28, 1909.
7. The Adjustment of a Transit Survey as Compared with that of a Compass Survey. C. L. Crandall, Am. Soc. C. E., Vol. XLV, p. 453.
8. Origin of the U. S. Land System. Jour. Asso. Eng. Soc., Vol. II, p. 282, and Vol. III, p. 275.
9. The Rectangular System of Surveying. W. A. Truesdale, Jour. Asso. Eng. Soc., April, 1904, and Nov., 1908.
10. How to Tell Township and Section Corners. Eng. News, Jan. 27 and Feb. 10, 1877.
11. The Early History of Instruments, etc. C. S. Howe, Jour. Asso. Eng. Soc., Vol. XVIII, p. 170–179.
12. The First Engineer. W. A. Truesdale, Jour. Am. Eng. Soc., Vol. XIX, July, 1897, p. 1.
13. A Brief Discussion of the Law of Boundary Surveys. Eng. News, Vol. XLVIII, pp. 141–144. W. E. Kern.

CHAPTER VIII

TOPOGRAPHICAL SURVEYING BY THE TRANSIT AND STADIA [1]

209. A Topographical Survey is such a one as gives not only the geographical positions of points and objects on the surface of the ground, but also furnishes the data from which the character of the surface may be delineated with respect to the relative elevations or depressions.[2]

210. There are three general methods of making such a survey.

First, with a compass (or transit) and chain, to determine geographical position, and with a level for obtaining relative elevations.

Second, with a plane-table, either with or without stadia rods.

Third, with a transit instrument and stadia rods.

The first method is very laborious, slow, and expensive. It is therefore not adapted to large areas. The second method has been more extensively used for this purpose than any other. The use of the plane-table is fully described in Chapter V. This method is giving place, however, to the third, which has been in use in America since about 1864, when it was officially adopted

[1] The word "stadia" is Italian and was originally used to designate the rod used by the inventor of the method. It is now too firmly established to be changed. On the U. S. Coast and Geodetic Survey the word "telemeter" is used in place of "stadia," but this, which very properly means *distance measurer*, has been appropriated for other appliances used for *measuring at a distance*, as temperature, for example. It would therefore seem that "stadia" is the better word to use.

[2] See also Appendix G.

on the United States Lake Survey. The system was first used by James Watt on the survey of a canal in 1770 or 1771.[1] In what follows, the third method will alone be described.

211. The Principle of the location of points by the transit and stadia, both horizontally and vertically, is that of polar co-ordinates. That is, the location of the point geographically is by obtaining its angular direction from the meridian through the instrument, which is read on the limb of the transit, and its distance from the instrument, which is read through the telescope on the stadia rod held at the point. This distance is found by observing what portion of the image of the graduated rod is included between certain cross hairs in the telescope. The farther the rod is from the instrument, the greater is the portion of the rod's image which falls between the cross wires.

For elevation, the vertical angle is read on the vertical circle of the transit, when the telescope is directed towards a point of the stadia rod as far from the ground as the telescope is above the stake over which it is set. The tangent of this angle of elevation, or depression, into the given horizontal distance, is the amount by which the point is above or below the instrument station.

In this way, both the chain and levelling instrument are dispensed with, and the slow and laborious processes of chaining over bad ground, and levelling up and down hill, are avoided. The horizontal distances are obtained as accurately, as by the chain; and the levelling may be done within a few tenths of a foot to the mile, which is amply sufficient for topographical purposes.

[1] See Bulletin University of Wisconsin Eng. Series, Vol. I, p. 354, by Prof. J. L. Van Ornum.

Theory of Stadia Measurements [1]

212. Fundamental Relations. In Fig. 106 let LS be any lens, or combination of lenses, used for the object glass of a telescope.

a and b represent the position of the two extra horizontal cross wires, spaced as near as may be at equal distances from the middle wire. F is a point in front of the object glass and at its principal focus. Now, when the telescope is focussed on the speaking or stadia rod AB, the cross wires a and b are seen projected on the rod at A and B respectively. In fact the object

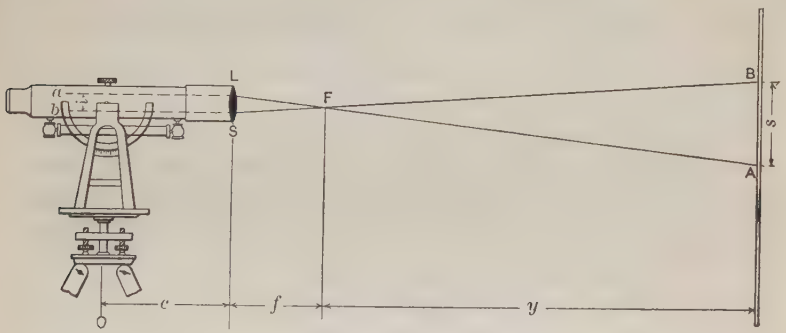


Fig. 106.—Theory of Stadia Measurements.

glass brings to a focus at a and b both of the solid cones of rays from A and B subtended by the object glass. Of these many rays, the two from A and B which pass through F, the principal focus, will traverse the telescope parallel to the optical axis of the telescope and hence parallel to each other.[2] Now since all

[1] For a good description of A New Prismatic Stadia see Jour. Asso. Eng. Soc, Vol. XIII, p. 43, Jan., 1894. One-half of the telescope objective is covered with a prism, which causes two portions of the rod, rays from which form a fixed angle at the objective, to coincide in the image, as in a sextant, thus dispensing with the use of stadia wires.

[2] This is a principle of physics easily verified.

the other rays of light from A and B come to a focus at the same points as the two rays passing through F, the effect is the same as though *all* rays came over these particular paths. Let i represent the distance between the cross wires, f the focal distance of the objective, LS the objective lens, and S the intercept of the stadia wires on the stadia rod, then in the similar triangles apexing at F, we have:

$$S : i :: y : f;$$

$$y = \frac{f}{i} S. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$\frac{f}{i}$ is a constant (K) for each instrument, and in case the wires are apart exactly $\frac{1}{100}$ part of the focal length of the lens, then K would equal 100.

Eq. (1) gives the distance which the rod is away from the fixed point F. If we let c stand for the distance from the object glass to the vertical axis of the transit, then the total distance D, from the rod to the transit plumb line (Fig. 106), is given by:

$$D = \frac{f}{i} S + (f + c), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

or,

$$D = KS + (f + c). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Eq. (3) is known as the stadia formula and K is called the stadia interval.

213. Determination of the Stadia Interval. The distances f and c can be measured directly on the transit in the following way. After focussing the objective on a distant point, measure with a rule the distance from the object glass to the centre of the horizontal axis of telescope for c, and the distance from the cross wires to the objective for f. The determination of the constant $K\left(=\frac{f}{i}\right)$ may be made in the following manner: Measure

a base line 500 to 1000 feet with a steel tape and place hubs at frequent intervals, as for example every 100 feet. Set up the transit a distance $f+c$ *back* of the base line. Then if the rod be held on the base-line hubs, the spaces, S, intercepted by the extreme wires, should bear a constant ratio to the corresponding base-line distances. Thus if S_1, S_2, S_3, S_n be the rod readings corresponding to the base-line distances D_1, D_2, D_3, D_n, then we would have from the Eq. (1) $Y=Ks$, $K=\frac{D_1}{S_1}=K=\frac{D_2}{S_2}=\frac{D_3}{S_3}=\frac{D_n}{S_n}$. Each of these equations will give a determination of the interval K. But as explained in Art. 217, on account of the effect of differential refraction these various values of K are not usually quite equal. The following table shows all the field and office work for determining the stadia interval as well as the upper and lower half intervals. The latter are useful in very long readings.

FIELD NOTES FOR DETERMINING STADIA INTERVAL

Distance.	Rod Readings for			Computed $1=\frac{D}{S}$.			Remarks.
	Full Interval.	Lower Half.	Upper Half.	Full Interval.	Lower ½ Interval.	Upper ½ Interval.	
200	1.91	0.95	0.96	104.71	210.53	208.33	Party: Transit
300	2.86	1.42	1.43	104.80	211.26	209.79	Observer, D. Show. No. 265.
400	3.81	1.90	1.92	104.98	210.52	208.33	Rodman, J. Jones.
500	4.77	2.38	2.39	104.82	210.09	209.25	Date, Hours | Weather,
600	5.72	2.85	2.87	104.90	210.53	209.06	Apr. 8. | 2–4 | Bright & still
700	6.68	3.32	3.35	104.79	210.85	208.95	
800	7.63	3.80	3.83	104.85	210.52	208.87	
900	8.59	4.29	4.30	104.77	209.79	209.30	
1000	9.55	4.77	4.80	104.71	209.64	208.33	
			Av.=	104.80	210.41	208.91	

The *average* of ten or more such values should give a much more reliable value than any single determination, however carefully made. Because of the importance of securing an accurate value for the interval, considerable pains and time are justified in securing an average value.

Instrument makers provide either fixed or adjustable stadia cross wires, but the former are usually preferred because of being less likely to accidental change.

214. To Set the Adjustable Wires So $K = 100$. This can be best done by the use of a level rod with two large targets. The transit is set up as before $(f+c)$ back of an accurately measured base line. With the rod held at the 100-foot point the lower wire is made to coincide with the lower target and then the upper wire adjusted until it bisects the upper target set at one foot higher. Such a test might be repeated at various distances, from 100 to 500 feet, as a check. Experience has shown that this method is not as easy or as accurate as the method discussed in Art. 215.

215. The Use of an Interval Factor. The practice of graduating a rod to fit the stadia wires of a particular transit has been found less accurate than the system which employs an interval factor with the rod graduated into standard units. The former method is objectionable, also, for the following reasons: First, the cost of regraduating and repainting the rod when the interval of the transit changes, as when new wires are inserted, or from other causes; second, such rods cannot be interchanged among other transits, or old rods used with new transits; third, such non-standard rods could not be used in levelling without computing laborious corrections.

All these objections are overcome by the use of rods graduated into standard units and the use of an interval factor $\left(K = \frac{f}{i}\right)$ to multiply all stadia readings on the standard rod to obtain the true stadia distance, viz., what the rod reading would have been if K had been equal to 100.

This method has failed of adoption because of the seemingly large amount of extra computation involved. By the use of a reduction table, however, the labor involved is very slight, while the advantages are many. Such a table would be prepared as follows: From the adopted interval compute the distance corre-

sponding to each stadia reading given in the columns marked Stadia in the following reduction table, and having added the $f+c$ to each of such computed distances, place the sums in second column opposite the proper stadia reading: $f+c$ should not be added to readings of less than 0.1.[1] Thus with an interval factor $K=104.80$, and $f+c=0^{m}.32$ the distance corresponding to a stadia reading of 0.1 would be $10.48+0.32=10.80$. Remembering that (leaving out the $f+c$) the distances are porportionate the stadia reading, the work of computing the table can easily be done in an hour's time.

It is assumed in this explanation that the interval was determined by measuring a base line whose zero was $f+c$ in front of the centre line of transit as explained on page 273. If the zero of the base line coincided with the centre line of the transit (plumb line), the construction of the table would be just the same, except that there would be no $f+c$ additions to the column headed Distance.

How to Use the Table. If, for example, the notes contained a stadia reading of $1^{m}.960$, to find from the table the true distance, viz., what the stadia would have read if the interval factor had been 100.0, you enter the table with 1.9 as an argument and take out the distance 199.44 corresponding to it; also opposite 0.06 find 6.29, which add to 199.44, giving 205.73 metres as the true distance (and including in it the $f+c$).

With a little practice it will be found that the field notes of an entire day may be reduced in this manner in fifteen minutes or less.

If the interval factor should happen to be nearly equal to 100, it would not be necessary to reduce the side shots, as on the usual scales of maps small differences would not be appre-

[1] This is assuming that stadia readings of less than 0.1 will never be taken except as a *part* of a much longer reading, as, for example, 0.06 forms a part of 1.96; so that if $f+c$ were added in the table both to the distances corresponding to 1.9 and 0.06 of the rod reading, the resulting sum would have $(f+c)$ added twice instead of once.

ciable on the map; but even in such a case it would be well to reduce the stadia shots of the *main traverse line*, because the omission of such corrections would introduce an *accumulative* error which might vitiate the accuracy of the entire map.

STADIA REDUCTION TABLE

$f+c=0^{m}.32$ $K=104.80$

Stadia Reading.	Distance.	Stadia Reading.	Distance.	Stadia Reading.	Distance.
.001	.10	.05	5.24	.9	94.64
.002	.21	.06	6.29	1.0	105.12
.003	.31	.07	7.34	1.1	115.60
.004	.42	.08	8.38	1.2	126.08
.005	.52	.09	9.43	1.3	136.56
.006	.63	.10	10.80	1.4	147.04
.007	.73	.2	21.28	1.5	157.52
.008	.84	.3	31.76	1.6	168.00
.009	.94	.4	42.24	1.7	178.48
.01	1.05	.5	52.72	1.8	188.96
.02	2.10	.6	63.20	1.9	199.44
.03	3.14	.7	73.68	2.0	209.92
.04	4.19	.8	84.16	etc.	etc.

216. A Simple and Accurate Way to Determine the Wire Interval of a Transit. In all topographic surveys extending over a considerable area a triangulation control is always found necessary. The accurately known distances computed from the triangulation can be made to give the very best determination of the stadia interval without any further field work for this purpose.

In taking topography by the transit and stadia method, it is usual to begin and end the network of traverse lines (upon which the topography is made to depend) at triangulation stations.

If the interval of the transit used be assumed to be 100, then the distance between the two adjoining triangulation stations, forming the terminals of the traverse, can be computed from the distances and bearings of the traverse. Then 100 times the

ratio of the true length, as determined by triangulation, to the distance so computed is the stadia interval of the transit; e.g., if the distance between two triangulation stations, as computed from the stadia traverse, should be 15,488 feet (assuming the stadia interval to be 100), and the true triangulation distance between the same points was 15,698 feet, then the assumed interval of the transit should be multiplied by the ratio of 15,698: 15,488 = 1.0135, giving a true interval of 1.0135 × 100 (the assumed interval) = 101.35.

Besides being extremely simple this method has the added advantage of having been determined by the person who did the instrumental field work of the survey,[1] and under the same conditions regarding time of day, weather, etc., as governed the topographic field work, thereby insuring a result free from any systematic errors. The resulting interval would also be free from accidental errors unless some blunder was made in the rod readings during the field work. To guard against such a possibility it would be well to make two or more computations of the interval, by the above method, using different lines in the triangulation. The agreement of the several determinations so made would insure freedom of error in the resulting interval.

NOTE. In the above method it is assumed that either there was no azimuth error in the traverse line, or that it had been duly corrected or distributed before the distance between triangulation stations had been computed.

217. How to Prevent Systematic Errors in Stadia Measurements. When the wire interval of a transit is accurately determined, stadia measurements are subject only to the accidental errors of reading the rod. According to a well-known law only the square root of such errors remains uncompensated.

As a matter of fact, the results of stadia surveys show much larger errors in measurement, in some constant direction, pointing conclusively to an incorrect interval determination or rod

[1] In order to eliminate a possible error from personal equation, which has sometimes been found to exist.

graduation. Professor L. S. Smith has shown [1] that the failure to secure a correct wire interval has been due to a lack of care in securing *as near as possible* the same conditions for the interval determination as are met in the field measurements. This requirement is important because, as he has experimentally proved, the effect of refraction is much greater near the ground in the strata of air traversed by the lower line of sight than it is in the strata a few feet above traversed by the upper line of sight (see Fig. 107). This causes the actual rod reading to differ from what it would have been had the air been homogeneous. This difference in the amount of refraction changes in amount at

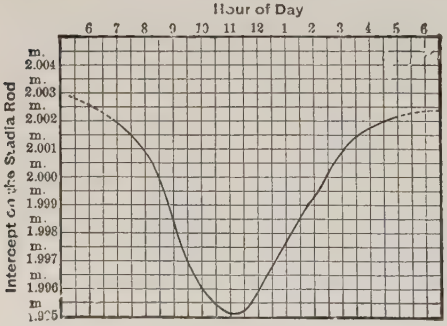


FIG. 107.

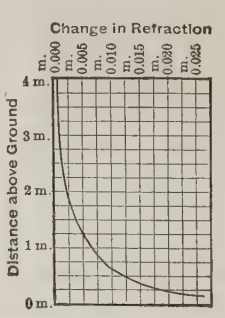


FIG. 108.

different hours of the day, giving a slightly different rod intercept for the same distance. In the typical curve shown in Fig. 107 it will be seen that the rod intercept is least during the middle of the day, and greatest in morning and evening.

If an interval was determined by rod readings taken on the base line near noon, smaller rod readings would result than the average readings for the day. As a result of the rod readings, s, being too small $\left(\text{since the interval } K \text{ equals } \frac{D}{s}\right)$, K would be

[1] See Bulletin of the University of Wisconsin, Engineering Series, Vol. I. No. 5, 1895; also Eng. News Vol. XXXIII, p. 364.

too large, and therefore measurements made with such a value of the interval (or with a rod divided under these conditions) would as a rule be excessive.[1]

Just the opposite effect would result with an interval made in the early or late part of the day, viz., the value of K obtained would be too small. However, if several determinations or tests be made, distributed over several hours of the field day, and better still on several days, the average of them all would give an interval comparatively free from systematic error.

As a result of these experiments the following rules for determining the wire interval or for graduating the rod become imperative for the most accurate work. These rules apply to any or all the methods previously described.

1. *Every instrument man should determine for himself his wire interval (or make the observations for graduating his rods).*

2. *Determine the wire interval for various distances (but only between the limits expected in the field work), and for several hours distributed through one or more days, on a base line which does not differ radically from the country to be surveyed.*

3. *For a radical change of field or season conditions, redetermine the wire interval or rod graduation.*

4. *Avoid reading the lower cross wire near the ground, either in the interval determination or in the field work, but the interval-determination readings should agree in this respect with the average field practice.*

It is the evident purpose of these rules to insure as far as possible that every condition obtaining during the test shall be as similar as possible to the conditions expected or planned for the field work. The experiments described in this article have unquestionably proved that if these rules be followed the accuracy obtainable will be very considerably increased and

[1] For a good example of such a case read the report of the St. Louis Topographic Survey, Journal of the Association of Engineering Societies, Vol. XII, p. 20. The average error of every sight on this survey was about $+1.5$ feet.

the stadia method thereby made even more valuable than it has been in the past.

218. Adaptation of Formulas to Inclined Sights. The discussion given in Art. 212 is applicable to horizontal sights only.

If the rod be held on the top of a hill, and the telescope pointed towards it, the reading on the rod will give the linear distance from instrument to rod, *provided the rod be held perpendicular to the line of sight.* As it would be inconvenient to do this, let the rod be held vertical in all cases. When the line of

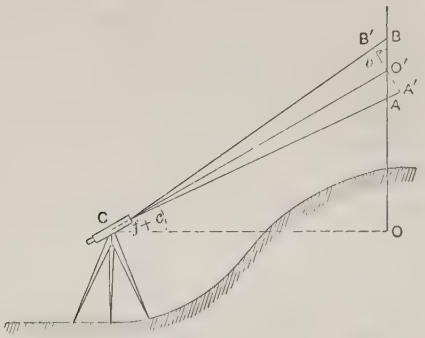


FIG. 109.

sight is inclined to the rod, the space intercepted is increased in the ratio of 1 to the cos of the angle with the horizon.

Thus, the space $A'B'$ (Fig. 109) for the rod perpendicular to the line of sight becomes AB for the rod vertical. But $A'B' = AB \cos v$ approximately.

Let $A'B' = r'$, the reading on the stadia for perpendicular position; and

Let $AB = r$, the actual reading obtained for a vertical position.

Then $r' = r \cos v$.

But in Eq. (1) we have $\frac{f}{i}s = Kr'$, and therefore $Kr' + c + f$ is

the distance CO'; whereas the distance on the horizontal, CO, is generally desired, and for this we have

$$CO = d = CO' \cos v = (Kr' + c + f) \cos v$$
$$= Kr \cos^2 v + (c+f) \cos v. \quad (7)$$

This is the equation for reducing all readings on the stadia to the corresponding horizontal distances.

The vertical distance of O' above O is equal to $CO' \sin V$.

But $$CO' = Kr' + f + c = Kr \cos v + f + c,$$

hence

$$OO' = h = Kr \cos v \sin v + (f+c) \sin v$$
$$= \tfrac{1}{2} Kr \sin 2v + (f+c) \sin v. \quad (8)$$

Eq. (8) is used for finding the elevation of the point on which the stadia is held above or below the instrument station.

219. Table V gives the values d and h computed from these formulas for a stadia reading of 100 feet (or metres, or yards), with varying angles up to 30°.

It will be noted that the second term in the right member of Eqs. (7) and (8) is always small, and its value depends on the instrument used. The values of this term are taken out separately in the table; and three sets of values are given of $(c+f)$, viz., 0.75 feet, 1.00 feet, and 1.25 feet. If the work does not require great accuracy, these small corrections may be omitted.

The use of the table directly involves a multiplication for every result obtained. Thus, if the stadia reads 460 feet, the angle of inclination 6° 20′, and we have $f+c=1$ foot, then

$$d = 4.60 \times 98.78 + 0.99 = 455.4 \text{ feet},$$

and $$h = 4.60 \times 10.96 + 0.11 = 50.53 \text{ feet}.$$

The table is not generally used for reductions for d when the angle of elevation is less than 3 to 5 degrees. When $v = 5° 44'$, this reduction amounts to just *one per cent.* When an error of 1 in 100 can be allowed, then the reduction to the horizontal would not be used under 6°. If the second term in $c+f$ be also neglected, these two errors tend to compensate; and if $c+f$ for the instrument used is 1 foot, and both these corrections be omitted, they do exactly compensate when the

stadia reading is		100	feet, vertical angle		5° 44′.
"	"	200	"	"	4° 04′.
"	"	300	"	"	3° 20′.
"	"	400	"	"	2° 52′.
"	"	500	"	"	2° 32′.
"	"	1000	"	"	1° 46′.
"	"	2000	"	"	1° 18′.

Therefore the reduction to the horizontal need *never* be made when v is less than 2°, and it generally may be neglected when v is less than 6°.

In obtaining the difference of elevation, h, the term in $c+f$ may be omitted for all angles under 6° if errors of 0.1 foot are not important. For elevations on the main line, however, this term should always be included.

In practice, therefore, the tables are mostly used to obtain the difference of elevation from the given stadia reading and angle of elevation.

The Instruments

220. The Transit.[1] That the transit may be best adapted to this work, there are certain features it should possess, though all of them are by no means essential. They will be named in the order of their importance.

[1] See Specifications for a Topographic Transit, Appendix F.

1st. The horizontal limb should be graduated from zero to 360°, preferably in the direction of the movement of the hands of a watch.

2d. The instrument should have a vertical circle rigidly attached to the telescopic axis, and not simply an arm that is fastened by a clamp-screw, and which reads on a fixed arc below. So much depends on the vertical circle holding its adjustment that its arrangement should be the best possible. Since the telescope is not transited, the vertical circle need not be complete. An extra level to control the zero of the vertical circle is desirable.

3d. The telescope should be inverting, for two reasons: *first*, in order to dispense with two of the lenses, and so obtain a better definition of image; and, *second*, that the objective may have a longer focal length, thus giving a flatter image and a less distorted field.

4th. The stadia wires should be fixed instead of adjustable, as in the latter case they are not stable enough to be reliable.

5th. The bubbles on the plate of the instrument should be rather delicate, so that a slight change in level may become apparent. They should also hold their adjustments well. This is very important, in order that the readings of the vertical angles may be reliable. It is also of great importance in carrying azimuth where the stations are not on the same level.

6th. The horizontal circle should read to thirty seconds; and there should be no eccentricity, so that one vernier-reading shall be practically as good as two.

7th. The instrument (or tripod) should have an adjustable centre, for convenience of setting over points.

8th. A solar attachment to the telescope will be found very convenient. In most regions the azimuth can be checked up by the reading of the needle, but in many places this is not reliable.

221. Setting the Cross Wires. The engineer should always have at hand a spider's cocoon of good wires, and a small bottle

of thick shellac varnish. If the dry shellac is carried it may be dissolved in alcohol. If no such cocoon is at hand a spider may be caught and made to spin a web. The small, black, out-door spider makes a good web for stadia purposes. A new wire should be allowed to dry for a few minutes, and an old one should be steamed to make it more elastic. The wires for stadia work should be small, round, and opaque. Some wires are translucent, and some are flat and twisted like an auger shank.

Scratches must be made across the face of the reticule where the wires are to lie. These must be made with great care, so as to have them equally spaced from the middle wire, parallel to each other, and perpendicular to the vertical wire. The distance apart of the extreme wires is to be computed by the equation $i = \frac{s}{D} f$ for any desired scale on the rod.

Take a piece of web on the points of a pair of dividers, by wrapping the ends several times about the points, which should be separated by about an inch; stretch the wire, by spreading the dividers, as much as it will bear; and lay the dividers across the reticule in such a way that the web comes in place. The dividers must be supported underneath, so that the points will drop just a trifle below the top of the reticule; otherwise they would break the web. Move the dividers until the web is seen, by the aid of a magnifying glass (the eye-piece will do), to be in exact position. Then take a little shellac on the end of a small stick or brush, and touch the reticule over the web, being careful to have no lateral motion in the movement. The shellac will harden in a few minutes, when the dividers may be removed. Shellac is not soluble in water.

222. Design of the Stadia Rod. The stadia rod is usually a board one inch thick, four or five inches wide, and twelve to fourteen feet long. Sometimes this is stiffened by a piece on the back. In Fig. 110 are shown several designs for stadia rods which have been long in use, and are found to work well. They are intended to be all in black on a white ground, though some

engineers prefer to use red at the foot or metre points. A most important consideration is to have the color in fairly large areas and devoid of long sharp points. This is because of the dis-

FIG. 110.—Design for Stadia Rods.

tortion of such shapes due to the unsteadiness or "boiling" of the air. If readings of less than 1000 feet are to be measured, the foot (see Figs. 110*A*) forms a convenient unit for subdivision.

For longer distances the yard and metre (see Fig. 110*B*) are much better units. All colors should be put on with turpentine so as to give a dull finish.

Until recent years, the method of graduating the stadia rod to fit the interval was in common use. At the present time, however, instrument makers have no difficulty in spacing the stadia wires very exactly $\frac{1}{100}$ part of the focal length of the objective. All side shots taken with such instruments could be platted by regarding the stadia constant 100. If in certain cases greater accuracy is required on the main traverse, an interval factor can be used in the manner explained in Art. 215.

223. The Following are some of the disadvantages of the old system of painting the rod to fit the interval, instead of using rods divided into *standard* units.[1]

1. Subsequent changes in interval require regraduating and repainting the rod.
2. Such rod cannot be interchanged among transits.
3. Old rods cannot be used with new transits.
4. Such rods cannot be used in levelling without computing necessary corrections.
5. Levelling rods cannot be used as stadia rods.
6. Observers with different personal equations cannot use the same rods without causing systematic errors in the measurements.

The wire interval should be tested every few months on a carefully measured base. Experience has shown that measurable changes in the wire interval result from radical changes in the climate or seasons, as well as from physical changes in the stadia wires themselves. If fixed wires sometimes change, no argument should be needed for avoiding adjustable wires.

It is sometimes recommended in texts that the field work of determining the wire interval be done on a "calm, cloudy day," or at some particular hour of the day which is supposed to

[1] From a paper by Professor John L. Van Ornum, Vol. XXXIV, Trans. Am. Soc. C.E.

give the most accurate value. This recommendation apparently overlooks the fact that the most accurate value sought must needs be the *mean* for the working day. It is not a safe plan to depend upon a single hour, or upon a certain kind of a day, although this might suffice for a survey of a small area.

General Topographical Surveying [1]

224. The Topography of a region includes not only the character and geographical distribution of the surface covering, but also the exact configuration of that surface with reference to its elevations and depressions. Thus any point is geographically located when its position with reference to any chosen point and a meridian through it is found, but to be topographically located its elevation above a chosen level surface must also be known. A topographical survey consists in locating by means of three co-ordinates a sufficient number of points to enable the intervening surface to be known or inferred from these. Evidently the points chosen should be such as would give the greatest amount of information. As for geographical outline, the corners, turns, or other critical points are chosen, so for configuration the points of changes in slope, as the tops of ridges and bottoms of ravines, or the brow and foot of a hill, are chosen as giving the greatest information.

225. Field Work. Let it be required to make a topographical survey of either a small tract, a continuous shore line, or of a large area, for the purpose of making a contour map.

In case of the small tract, any point may be taken as a point of reference, and the survey referred to it as an origin. In case of an extended region, a series of points should be determined with reference to each other, both in geographical position and in

[1] For a complete treatise on Photographic Surveying, by E. Deville, Surveyor General of Public Lands for the Dominion of Canada, apply to the Government Printing Bureau, Ottawa, Canada. The book contains 232 pages and many cuts.

elevation. These determined points should not be more than about three miles apart. The points of elevation or bench marks need not be identical with those fixed in geographical position. These last are best determined by a system of triangulation, and are called "triangulation stations."

First, a system of triangulation points is established, the angles observed, azimuths and distances computed, and the stations plotted to scale on the sheet which is to contain the map. This plotting is best done, for small areas, by computing the rectangular co-ordinates (latitudes and departures), and plotting them from fixed lines which have been drawn upon the map, accurately dividing it into squares of 1000 or 5000 units on a side. They may, however, be plotted directly from the polar co-ordinates (azimuth and distance) as given by the triangulation reduction. For this purpose, the sheet on which the map is first drawn, called the *field sheet*, should have *a protractor circle printed upon it, about twelve inches in diameter*. These *protractor sheets* of drawing paper can be obtained of most dealers in drawing materials, or the protractor circle may be printed to order on any given size or quality of paper. These protractor circles are very accurate, and are graduated to 15′ of arc. Plotting can be done to about the nearest 5′, if great care be taken.

Second, a line of levels is run, leaving B. M.'s at convenient points whose elevations are computed, all referred to a common datum. If the triangulation stations are not also B. M.'s, then a B. M. should be left in the near vicinity of each station. This is not essential, however.

Third, the topographical survey is then made, and referred to, or hung upon, this skeleton system of triangles and B. M.'s.

The topographical party should consist of the observer, a recorder, two or three stadia men, and as many axemen as may be necessary, generally not more than two.

The azimuth, preferably referred to the true meridian, is known for every line joining two triangulation stations, as well as the length of such line.

Set up the transit over a station, and set the horizontal circle (which should be graduated continuously from 0° to 360° in the direction of the hands of a watch) so that vernier A will read the same as the azimuth of the triangulation line by which the instrument is to be oriented. Clamp the plates in this position, and set the telescope to read on the distant station. Now clamp the instrument below, so as to fix the horizontal limb, and unclamp above. The azimuths of the triangulation lines are generally referred to the south point as the zero, and in small systems of this sort the forward and back azimuths are taken to be 180° apart. When the instrument has been set and clamped, all subsequent readings taken at that station are given in azimuth by the readings of vernier A on the horizontal limb. For any pointing, therefore, the reading of this vernier gives the azimuth of the point referred to the true meridian, and the rod reading gives the distance of the point from the instrument station. These enable the point to be plotted on the map. To draw the contour lines, elevations must also be known.

If the elevation of the station is known, measure the height of instrument (centre of telescope) above it with the stadia,[1] as soon as the instrument is levelled up over that station. Suppose this comes to the 4.12-unit mark. Write in the note book, as a part of the general heading for that station, "Ht. of Inst. = 4.12." Then, for all readings from that station for elevations, bring the middle horizontal wire to the 4.12-unit mark on the rod, and read the vertical angle. From this inclination and distance, the height of the point above or below the instrument station is found. If the rod be graduated symmetrically with reference to the two ends, one need not be careful always to keep the same end down, and so errors from this cause are avoided.

The record in the note book consists of—

1st. *A Description of the Point*, as "N.E. cor. of house,"

[1] Or, if preferred, a light staff, about 5 feet long, may be carried with the instrument for this purpose, it being graduated the same as the stadia rods for this instrument.

"intersec. of roads," "top of bank," etc. "△ 16" for "stadia station 16," etc.

2d. *Reading of Ver. A* or *azimuth.*

3d. *Stadia Reading.*

4th. *True Distance.*

5th. *Vert. Angle.*

These five columns are all that are used in the field. There should be three additional columns on the left-hand page, for reductions, viz.:

6th. *Difference of elevation*, corresponding to the given vertical angle and distance, and which is taken from a table or diagram.

7th. *Height of instrument.*[1]

8th. *Elevation*, which is the true elevation of each point referred to the common datum.

The right-hand page should be reserved for sketching.

It will be found most convenient to let the sketching proceed from the bottom to the top of the page, as in this case the recorder can have his book properly oriented as he holds it open before him, and looks forward along the line. The notes may advance from top to bottom, or *vice versa*, as desired. If there are many "side shots" from each instrument station, one page will not usually contain the notes for more than two stations, and sometimes not even for one.

The sketch is simply to aid the engineer when he comes to plot the work, and should rarely be omitted altogether. The sketches should be made with care. The observer usually makes his own sketches and plots his own work, but sometimes the addition of a recorder increases the progress.

After the instrument is oriented over a station, and its height taken on the stadia, the stadia men go about holding the rods at all points which are to be plotted on the map, either in position or in elevation, or both. The choice of points depends altogether on the character of the survey; but since a single holding

[1] This is frequently omitted.

of the rod gives the three co-ordinates of any point within a radius of a quarter of a mile, it is evident the method is complete, and that all necessary information can thus be obtained. For very long sights, the partial wire intervals (intervals between an extreme and the middle wire) may be read separately on the stadia, and in this way twice as great a distance read as the rod was designed for. The limit of good reading is, however, usually determined by the state of the atmosphere, rather than by the length of the rod. When the air is very tremulous, good readings cannot be made over distances greater than 500 feet; while, when the atmosphere is very steady, 2000 feet may be read with equal facility.

Before the instrument is removed from the first station, the forward stadia man selects a suitable site for the next instrument station (generally called *stadia station*, and marked △, and drives a peg or hub at this point. This peg is to be marked in red chalk, with its proper number, and should have a taller marking-stake driven by the side of it. The peg for the △ should be large enough to be stable; for it must serve as a reference point, both in position and elevation, during the period of the survey. It is often desirable to start a branch line, or to duplicate some portion of the work, with one of these stations as the starting-point; and, since each △ is determined in position and elevation, with reference to all the others, one can start a branch line from one of these as readily as from a triangulation station. It is not usually necessary to put a tack in the top, but the centre may be taken as the point of reference. The stadia man first holds his stadia carefully over the centre of this △ *with its face towards the instrument.* The observer now reads the distance and puts the middle horizontal wire at a point on the rod equal to the present *HI*, or if this is not convenient, he notes and records the height on the rod to which he sights the middle cross wire. At a given signal from the instrument man the rodman now turns the edge of the rod toward the observer so that a more accurate azimuth can be secured.

It is advisable, in good work, to reorient and relevel the instru-

ment just before reading to the forward △. The transit is very apt to get out of level after being used for some time, with more or less stepping around it, and the limb may have shifted slightly on the axis, both of which might be so slight as to make no material difference for the side readings, but which would be important in the continued line itself. It is best, therefore, to level up again, and reset on the back station, before reading to the forward one. If immediately after orienting the transit the azimuth to some prominent point be taken and recorded, the instrument can be reoriented on this point without sending the rodman to the back sight. This will save considerable time.

When the instrument is moved, it is set up over the new station, and the new height of instrument determined and recorded. After reading the distance and setting the horizontal wire for the vertical angle on the back sight the rear stadia man holds his rod, edgewise, on the station just left; and by this the observer orients his instrument, *making vernier A read* 180° *different from its previous reading on this line.*[1] Clamping the plates at this reading, the telescope is turned upon the rod on the back station, and the lower plate clamped for this position. The circle is now oriented, so that, for a zero reading of vernier *A*, the telescope points south.

It will be noted that the telescope is never reversed in this work.[2]

226. Check Readings. The *distance* and *vertical angle* should both be reread, on this back reading, for a check. If the vertical circle is not in exact adjustment, this second reading of the vertical angle will show it, for the numerical value of the angle should be the same, with the opposite sign. If they are not the same, then the numerical mean of the two is the true angle of elevation, and the difference between this and the real readings is the index error of the vertical circle. This error

[1] For the manner of setting the vernier on back sight see Art. 91, p. 87.

[2] The reversal would introduce errors due to lack of adjustment, without any substantial gain.

may be corrected in the reduction, or the vernier on the vertical circle may be adjusted.

The second reading of the vertical angle on the stadia stakes is thus seen to furnish a constant check on the adjustment of the vertical circle, and should therefore never be neglected. If the circle is out of adjustment by a small amount, as one minute or less, in ordinary work it would not be necessary either to adjust it or to correct the readings on side shots, for the elevations of contour points are not required with such extreme accuracy. The mean of the two readings on stadia stakes would still give the true difference of elevation between them, so that there would be no continued error in the work.

The work proceeds in this manner until the next triangulation station is reached. In coming to this station, it is treated exactly as any other stadia station, viz., the forward reading to it, and the back reading from it, are identical with those of any two consecutive stations. Having thus occupied the second triangulation station, and having oriented the instrument on the last instrument station, turn the telescope upon some other triangulation station whose azimuth from this one is known. The reading of vernier A for this pointing should be this azimuth, and the difference between this reading and the known azimuth of the line is the accumulated error in azimuth due to carrying it over the stadia line. This error[1] in good work should not exceed five minutes in the course of two or three miles.

The check in distance is to be found from plotting the line, or from computing the co-ordinates of the single triangulation line, and also of the meandered line, and comparing the results. The elevations are checked by computing the elevation of the new triangulation station from the stadia line, and comparing this with the known elevation from the line of levels. In case the elevations of the triangulation stations are not given, but only certain B.M.'s in their vicinity, then the check can be made on

[1] To secure as accurate work as this the bubble and horizontal axis adjustment should be carefully made, especially in hilly country.

these just the same. Thus, in starting, read the stadia on the neighboring B.M., and from this vertical angle compute the elevation of the △ over which the instrument sets, and then proceed as before. In a similar manner, the check for elevation at the end of the line may be made on a B.M. as well as on the stadia △.

227. Azimuth Checks. After the instrument is oriented at each new station, the needle should be read as a check as follows: After orienting make the A vernier read 180° or true north, when the needle should read the declination of the place. To enable the observer to locate large errors in azimuth or distance, or both, it is a good practice to take azimuth readings to a common object from a series of consecutive stations, if such be possible. If the plot does not close, go back and plot in these azimuths; and if there has been no error in azimuth or distance between △'s, and no error in reading the azimuths for these stations, then all these lines will meet in a common point on the plot. If all but one intermediate line meet at a point, then the error probably was in reading the azimuth of this pointing alone. If several of the first pointings intersect in a point, and the remaining pointings of the set taken to this object intersect in another point, then it is highly probable that the error was in reading the azimuth or distance of the line connecting these two sets of △'s; and the relative position of the points of the intersection will enable the topographer to decide whether the error was in azimuth or distance, and about how much. If, in this way, the error be located, the instrument can be taken to this point, and the readings retaken.

228. Reducing the Field Notes. The following order of making the office computations is a convenient one:

1. (Unless the stadia interval is 100.) Reduce stadia readings to stadia distance by use of an interval table (see page 278).

2. Compute the horizontal and vertical components of the stadia distances, using a diagram or some approved stadia slide rule,[1] and record in proper column.

[1] Colby's, Webb's or Cox's can be bought of instrument makers, and are fully described in the manufacturers' catalogues.

3. Compute the elevations of all instrument stations in the circuit (H. I.'s may be included) and record the *adjusted* values in red ink.

4. Compute the elevations of all side shots based upon the adjusted elevations of the turning points.

5. Tabulate the survey and balance the latitudes and departures.[1] Compute the co-ordinates of each turning point. The last step is frequently omitted. In this case the points on the traverse are plotted by polar co-ordinates, the check in the closure of the traverse being considered sufficient.[2]

Page 299 is a sample of the field notes and office reductions. The use of a stadia slide rule or a stadia diagram results in a great saving in time and effort. The former solves one or both of the following equations for horizontal and vertical components,

$$d = KS \cos^2 V$$

and

$$h = \frac{KS \sin 2V}{2}.$$

229. Stadia-Reduction Diagrams.[3] The stadia-reduction diagram can be purchased in the market or constructed as follows: Omitting any consideration of the $f+c$ correction in the stadia formula Eqs. (7) and (8), p. 283, the horizontal component of any stadia distance for any given vertical angle is proportional to the distance. Obviously the corrections for horizontal will likewise be proportional to the distance. To construct a diagram which will give the corrections for horizontal a distance equal to the longest probable shot, say, 1000 feet, is laid off to scale on cross-section paper, as in Fig. 111. Some angle, as 10°, is then selected, and then from Table V, p. 846, is taken the correction for this angle and distance which will reduce

[1] On the Great Lakes Survey any traverse circuit showing a greater error than 1 in 750 is rerun.

[2] This method is used on the Mississippi and Missouri River Commission.

[3] See note bottom page 319.

the stadia distance to the true horizontal. This correction, 30.2 feet, is then laid off to scale at right angles to the distance line at *B*. Let *C* be the plat of the point. Then if *C* be connected with *A* by a straight line, the correction for any other distance on this angle will be given by the ordinate to the line *CA* at the proper distance on *AB*. Similar lines should be drawn for each degree within the required limits.

In the same manner a diagram may be constructed to give differences of elevations (see Fig. 112). Here, however, it will

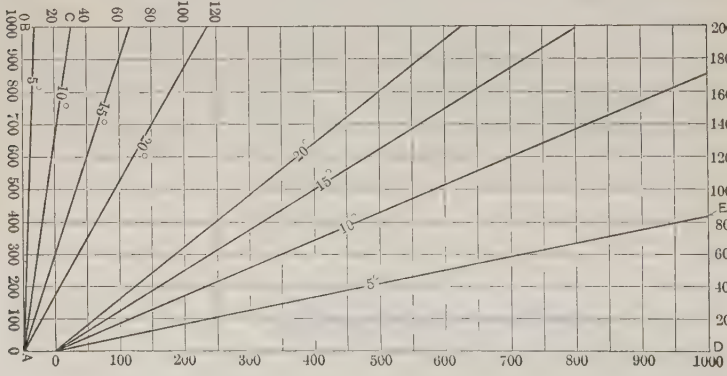


FIG. 111.

FIG. 112.

Stadia Reduction Diagrams.

be necessary to draw not only the degree lines, but also the ten-minute divisions, in order that the elevations may be estimated with sufficient exactness. Thus from Table V, p. 846, for a distance of 1000 feet and a vertical angle of 5° we take 86.8 feet as the difference of elevation. This is laid off at *D* to the scale of the diagram, giving point *E*, which is then connected with *O* by a straight line. Below 5° the ten-minute lines can be interpolated between the even degrees without appreciable loss of accuracy.

All stadia slide rules are accompanied with full explanations and directions for use. A few minutes of practice is sufficient to insure a full understanding of their use.

The accuracy which may be expected in the closure of elevations in a given circuit is given in Art. 235. If the closing error in levels is distributed over the circuit, the adjusted values should greatly improve the work.

Sample Field Notes

LINE PARKER No. 2

May 27, 1910. PARKER, Instrument.
JONES, Rodman.

∠ Occu.	∠ Sighted.	Azimuth.	Stadia Reading	Distance.	Vertical Angle.	Vertical Comp.	E. I.[1]	Elevation.
				(94)	(10)			
.....	42	297° 30′	.99	100	−14° 10′	33.73	.....	70.09
				(196)	(10)			
.....	41	111 00	1.98	200	+ 8° 10′	+18.12	.....	121.94
				(393)				
∠ 40	∠ 38	60 10	4.01	405	− 9° 57′	68.93	103.82[1]	29.92
H. I.=5				(392)				
.....	∠ 40	240 10	4.00	404	+ 9° 58′	68.88	.....	98.82
				(285)				
.....	39	161 00	2.97	300	+12° 54′	65.28	.....	95.20
∠ 38	∠ 31	343 25	2.05	207	.....	8.05	34.67[1]	26.63
H. I.=4.75								
.....	∠ 38	163 25	2.05	207	.....	1.38	.....	29.92
					H. I.			
.....	35	139 20	2.67	270	+ 1° 58′	9.20	.....	35.83
.....	32	64 10	1.40	141	.....	11.30	.....	20.00
∠ 31	∠ 18	65 45	4.46	450	.....	4.490	31.29	26.80
H. I.=4.66								

Elev. ∠ 31 = 26.63 Notes begin at bottom of page

Explanation of Field Notes. On the right-hand page a sketch showing the location of each numbered stadia shot should be drawn. It will very frequently be convenient to have the notes recorded from the bottom of the page up. The reason for this is that if not done, the note book must be held upside down in

[1] These elevations of instrument, 8th column, are based on the average of the *two* determinations of the difference of level, viz., on B. S. and F. S.

order to make the points in the sketch correspond with the points in the ground. This would apply in the case of a survey of a road, for example. When the traverse is closed, and the entire sketch can be completed on a single page, the notes may as well increase downward. The first method is shown in the sample notes, but because of lack of room the sketch has been omitted. The sketch should show what each shot locates, a building, a road, etc.; and in addition the contours should be roughly sketched in order that the full topographical significance of each location may be seen. In general, time will be saved if the maximum and minimum elevations are located with only such additional locations as may determine the changes in slopes. The form of notes [1] shown is suitable for a comprehensive survey, and might be modified by omitting the first and eighth columns. The field work makes use of columns 1, 2, 3, 4, and 6 only. The stadia rod readings when reduced to distance by the use of an interval table are recorded in column 5. The figures in parentheses, column 5, are the horizontal components of the stadia distances. Column 7 contains the vertical components, while column 8 shows the height of instrument above the datum plane. If the distance of the horizontal axis above each △ station is recorded in column 1, it will be seen that the notes provide for two entirely independent lines of levels.

In practice, except in cases of obvious error, the two differences of level are averaged to compute the elevation of the foresight. Where possible, level readings should be taken instead of vertical angles, because these subtracted from the H. I. (column 8) give at once the elevations of the points sighted. Such level readings are recorded in column 7.

In wooded country it is frequently impossible to sight on the rod at a point equal to the H. I. of the transit (see shots 41 and 42). In such cases the point sighted (as 10 feet) by the middle cross wire is recorded just above the vertical angle. In

[1] This form of field book can be purchased of the University Co-operative Co., Madison, Wis.

a + vertical angle if this distance be subtracted from the computed vertical component, such remainder, when added to the H. I. (column 8), gives the elevation of the point direct. Thus in shot No. 41, the Ver. Comp. is 28.12, this less 10 gives the true vertical distance between the H. I. and the *bottom* of the rod (i.e., the ground), 18.12. This added to 103.82, the H. I., gives 121.94, the elevation. With the vertical angle of $-14°$ 10′ (shot No. 42), the computed vertical component is 23.73, to which we add 10, giving 33.73 as the true difference of level between the ground and the H. I. of the instrument. Then 103.82 minus 33.73 gives 70.09 for the ground elevation.

In general, however, when reading vertical angles, it saves time to sight at the point on the rod equal to the H. I. above the station occupied. The vertical components of vertical angles so computed are to be combined directly with the *elevation* of *the station occupied* to get the elevations of the ground.[1] Crowded notes are rarely clear. It will be found that, if every other line only be used, like the sample notes on page 299, much clearer notes will result.

Beginners are likely to underestimate the importance of a sketch, but the experienced man never slights this part of the work. After a little practice it can be drawn rapidly. A good sketch with a few well-chosen locations makes a more truthful map and at less cost than a poor sketch with many more locations. In many cases of land surveying the entire notes may be recorded most clearly on a carefully drawn sketch.

230. Plotting of the Stadia Line. It is customary to first plot the stadia traverse alone, from one triangulation station to the next, to find whether or not it checks within reasonable limits. This part of the work should be done with extreme care, so that if it does not check it cannot be attributed to the plotting. In case it does not check within the desired limit,

[1] The student should hand in diagrams showing the geometrical proof of the above statements.

then the line of investigation will be about as follows until the error is found.

1st. Replot the stadia line.

2d. Recompute and replot the triangulation line.

3d. By examining the discrepancy on the plot, try to decide whether the error is in azimuth or distance, and, if possible, where such error occurred, and its amount.

4th. Examine the note book carefully, and see if there is any evidence of error there.

5th. If there is a large probability that the error is of a certain character, and that it occurred at a certain place, take the instrument to that station, set it up, and redetermine the azimuths or distances which seem to be in error.

6th. If there is no high probability of any certain errors to be examined for in this way, then go back and run the line over, *taking readings on* △'s *only*. If the elevations had been found to check, the vertical angles may be omitted on this duplicate line; and, on the other hand, if the plot came out all right, but the elevations could not be made to check, then a duplicate line must be run to determine this alone; and in this case the vertical angles between △'s are all that need be read. In cases of this kind it will be found a great help to have the △'s so well marked that they can be readily found.

With reasonable care in reading and in the handling of the instrument, it will never be necessary to duplicate a line entire, for all readings between △'s are checked. The vertical angles and distances are checked by reading them forward and back over every stadia line; and the azimuth is checked by the needle readings, and also when the second triangulation station is reached.

If, in the progress of the work, the readings on the back △ for distance and vertical angle do not fairly agree with these quantities as read from the previous station, the recorder should note the fact: and the observer should then re-examine these readings; and, if found to be right, the first readings, taken from

the other station, should be questioned, and the mean not taken in the reduction.

For plotting the stadia lines a parallel ruler (moving on rollers) is very desirable; otherwise, triangles must be used. The plotting is done by setting the parallel ruler or triangle on the proper azimuth as found from the protractor printed on the sheet, moving it parallel to itself to the station from which the point is to be plotted, and drawing a pencil line in the right direction. Then, with a triangular scale,—or, better, with a pair of dividers and a scale of equal parts,—lay off the correct distance on this line; and this gives the point.

If the instrument was oriented in the field for a zero reading for a south pointing, then the protractor on the sheet must have its south point marked zero, and increase around to 360° in the same direction in which the limb of the instrument increases, preferably in the direction of the movement of the hands of a watch.

231. Plotting the Side Readings. Having plotted the stadia traverse and made it check, the next step is to plot the side readings. This can be done by using any of the paper protractors in the market. Perhaps the cheapest and most convenient paper protractor is one devised by Professor David A. Molitor in 1908 and illustrated in Fig. 113. This protractor is printed on cardboard which should be cut out on the marked lines, leaving a small triangular piece adjacent to the centre. The protractor swings on a fine sewing needle, the hole for which may be cut out and replaced by a piece of transparent celluloid. If other scales should be needed, flat boxwood scales can be glued on over the printed scale. The protractor is 6 inches in diameter.

The manner of use is very simple. A meridian line is drawn through the plot of the station from which the side shots were taken, and the protractor centred over the same point. The protractor is then rotated about this centre, until the meridian reads the recorded azimuth. The protractor arm is now pointing

along this azimuth so that the point can be plotted at the proper distance, which is read directly on the arm.

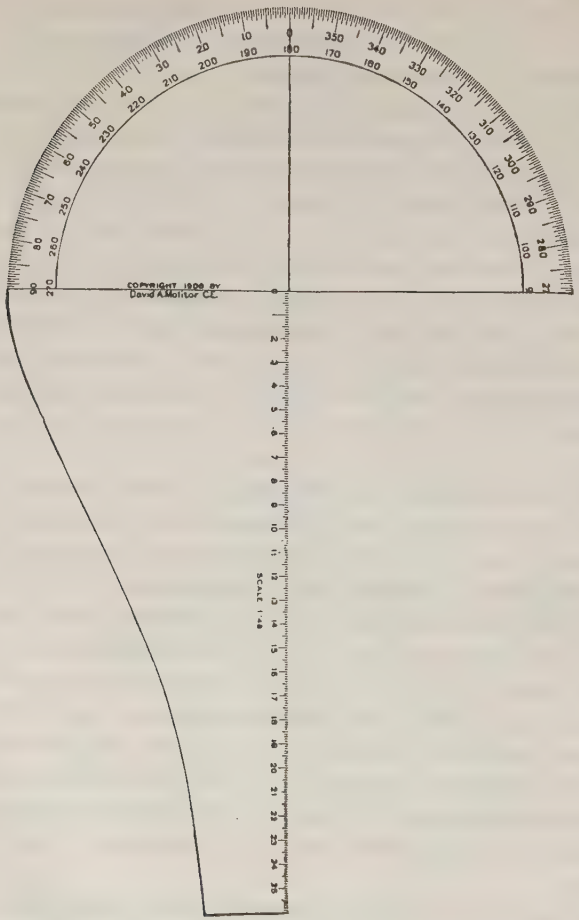


FIG. 113—Molitor's Protactor.

A special form of protractor, shown in Fig. 114, has also been used with great success in France and on the Mississippi River surveys.

It is essentially a semicircular protractor, provided with a needle-pointed pivot at its centre, and having the straight edge

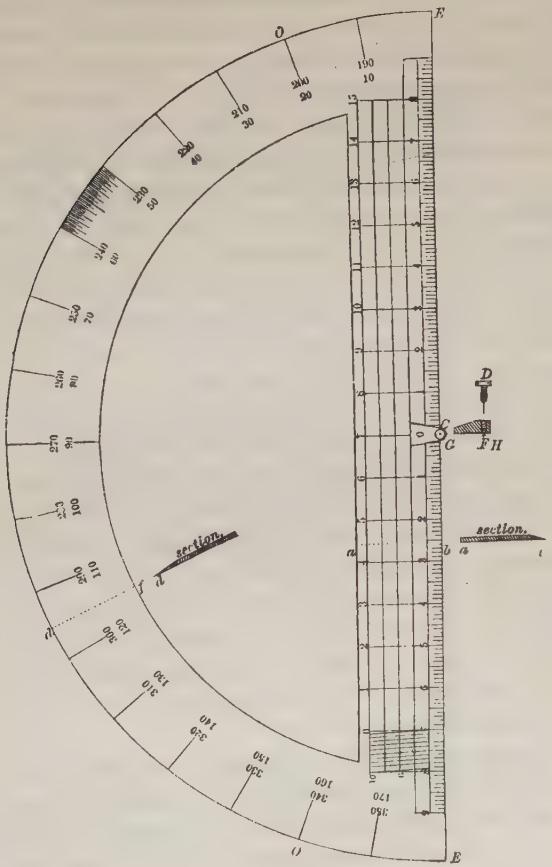


FIG. 114.

graduated so that distances can be measured off each way from the pivot; the angular deflection is given by the graduated circle, reading from a point marked on the paper. The bottom of the

plate is flush with the bottom of the protractor, and the hole *F* is at the centre, and should be only large enough to admit a fine needle. The screw *D* has a hole drilled in its axis to admit the needle point. It is also split, so that when it is screwed down it will clamp the needle firmly. If the latter is broken, it can readily be replaced by a new one. In addition to the scale on the bevelled edge, a diagonal scale is also provided as shown. This instrument combines all the requisites for rapid and accurate plotting of points located by polar co-ordinates or by intersections.

In using this protractor the needle point is placed at, say, the first station, and pressed firmly down. A meridian line is then decided upon, and a point is marked on it at the outer edge of the protractor circle. This will be the initial point from which the angles will be read. As azimuth is read from the south around by the west, it is plain that the circle, numbered as shown and revolved about the pivot till the proper reading coincides with the meridian line, will give the direction of the required point along the graduated diameter, while from the latter the distance can be pricked off. A point can be plotted in any direction without lifting the protractor from its position.

The price of this protractor, $12.50, is quite in striking contrast with the paper protractor illustrated in Fig. 113. It is difficult not to soil the drawing in using this or any other metal protractor. On the other hand experience indicates that practically as good work can be done with the cheap paper protractor as with the expensive metal one.

Still other methods are employed for plotting the side shots, such as solid half-circle protractors, of paper or horn, weighted in position, with their centres over the station. This is oriented on a meridian drawn through the point, and then all the points plotted whose azimuth falls between 0° and 180°, when the protractor is laid over on the other side, and the remaining points plotted. In this case the ruler is laid across the protractor, with some even division at the station. This method is more troublesome, less rapid, and defaces the drawing more, than the other

methods given above. The plotter should have an assistant to read off to him from the note book. When all the elevations have been plotted, the contour lines are sketched in.

The plotting should keep pace with the field work as closely as possible, being done at night and at other times when the field work is prevented or delayed. In difficult ground the map could be carried into the field and the contours sketched in on the ground. At least the stadia lines should be plotted up and checked before the observer leaves the immediate locality. Where the elevations are checked on B.M.'s, these checks should be immediately worked out. This much, at least, could be done each evening for that day's work.

232. Contour Lines. In engineering drawings the configuration of the surface is represented by means of contour lines. A contour line is the projection upon the plane of the paper of the intersection of a level surface with the surface of the ground. These cutting level planes are taken, 5, 10, 20, 50, or 100 feet apart vertically, beginning with the datum plane, which is usually taken below any point in the surface of the region. (See Fig. 115.) Mean sea level is the universal world's datum which should always be used when a reasonably accurate connection with the sea can be obtained.[1] Such contour lines are also shown on Plate II. The proper drawing of these contours requires some accurate knowledge of the surface to be depicted, aside from the elevations of isolated points plotted on the map. This knowledge may consist of a vivid mental picture of the ground, derived from personal observation, or it may be gained from sketches made upon the ground. Even with this knowledge the draughtsman must keep vividly in mind the true geometrical significance of the contour line, in order to properly depict the surface by this means. The ability to draw the contour lines accurately on a field sheet is the severest test of a good topographer. They are first sketched

[1] See in Chapter XVII, "Precise Levelling," Art. 494.

and adjusted in pencil and then may be drawn in ink. Fig. 116 shows a landscape and the corresponding sketch.

A few fundamental principles may be stated that will asist the young engineer in mastering this art.

1. All points in one contour line have the same elevation above the datum plane.

2. Where ground is uniformly sloping the contours must be equally spaced, and where it is a plane they are also straight and parallel.

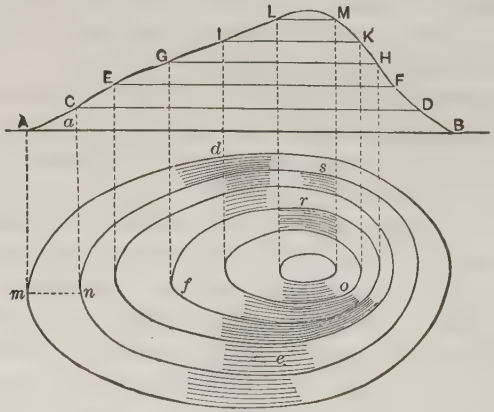


Fig. 115.—Contour Projection.

3. Contour lines never intersect or cross each other, except in case of a cave in a side hill.

4. Every contour line must either close upon itself or extend continuously across the sheet, disappearing at the limits of the drawing. It cannot have an end within these limits (an apparent exception, though not really one, is the following).

5. No contour should ever be drawn directly across a stream or ravine. The contour comes to the bank, turns up stream, and disappears in the outer stream line. If the bed of the stream, or ravine, ever rises above this plane, then the contour crosses it; but in the case of a stream the crossing is never actually

shown. In the case of a ravine the crossing is shown, if points have been established in its bed.

6. Where the contour closes upon itself, the included area is either a hilltop or a depression without outlet. If the latter, it would in general be a pond or lake. In other words, such

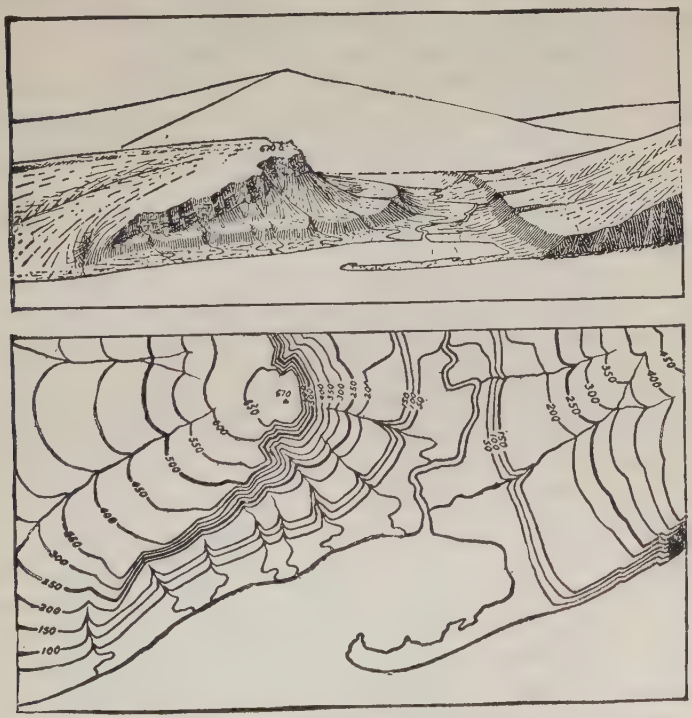


FIG. 116.—Contour Sketch. (Wilson's "Topographic Surveying.")

contours enclose either maximum or minimum points of the surface.

7. If a higher elevation seems to be surrounded by lower ones on the plot, it is probably a summit; but if a lower elevation seems to be surrounded by higher ones, it is probably a ravine,

or else an error; otherwise it is a depression without outlet, in which case there would probably be a pool of water shown.

8. Contour lines cut all lines of steepest declivity, as well as all ridge and valley lines, at right angles.

9. Maximum and minimum ridge and valley contours must go in pairs; that is, no single lower contour line can intervene between two higher ones, and no single higher contour line can intervene between two lower ones.

10. Vertical sections, or profiles, corresponding to any line across the map, straight or curved, can be constructed from a contour map, and conversely a contour map may be drawn from the profiles of a sufficient number of lines.

11. Each contour is designated by its height above the datum plane, as the 50-foot contour, the 60-foot contour, etc. In flat country, where the contour lines are few and wide apart, always put the number of the contour on the *higher* side, otherwise it sometimes may be impossible to tell on which side is the higher ground.

12. In taking surface elevations for determining contour lines, points should always be taken on the ridge and valley lines, and at as many intermediate points as may be desirable. There are two general systems of selecting these points. By one system points are chosen approximately in lines or sections cutting the contours about at right angles, the critical points being the tops and bottoms of slopes; while by the other system points are selected nearly in the same contour line,—that is, on the same horizontal plane,—the critical points being the ridge and valley points, these being the points of maximum and opposite curvature in the contour lines themselves. By the second method one or two principal contours may be followed continuously, the points being taken as nearly as may be on these contour lines. If such principal contours are 50 feet apart, then when these are accurately drawn on the map, any desired number of additional contours may be interpolated between the principal ones.

Some of the above principles are illustrated in the contour map[1] shown in Fig. 117.

233. The Final Map.[2] The field sheets are drawn as

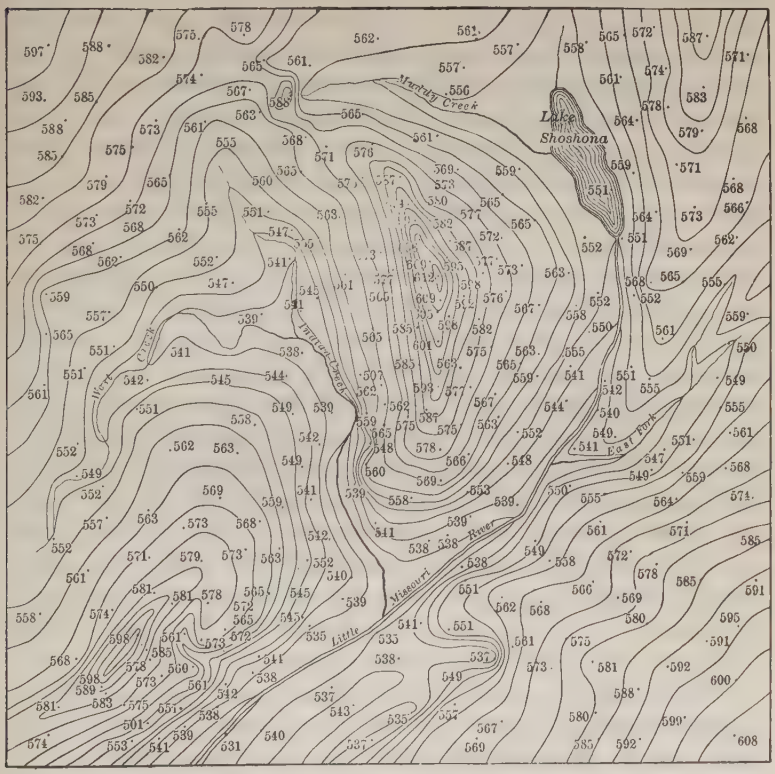


FIG. 117.—A Contour Map.

described above, in pencil, or partly in pencil and partly in ink, or wholly in ink, according to the use to be made of them. If

[1] This has been prepared from data given in a civil serivce examination conducted for the U. S. G. S.

[2] See also Appendix G.

they are simply to serve as the embodiment of the field survey, to be used only for the construction of the final maps, they are usually left in pencil, a 6-H pencil being used. The field sheets are usually small, about 18x24 inches. The final sheets may be of any desired size. Usually several field sheets are put on one final sheet, which will be worked up wholly in ink, or color, the scale remaining the same. The work on the field sheet is then simply transferred to the final sheet by the most convenient means available. Tracing paper (not linen) may be used. This is carefully tacked or weighted down over the field sheets, and the principal features, such as triangulation stations, stream and contour lines, roads, buildings, fence lines, etc., are traced in ink. The tracing paper is then removed and laid upon the final sheet, orienting it by making the triangulation stations on the tracing coincide with the corresponding stations on the final sheet, where they have been carefully plotted from the triangulation reduction. All the matter on the tracing may now be transferred to the paper beneath by passing over the inked lines with a dull point, bearing down hard enough to leave an impression on the paper below. If preferred, the tracing may have its under surface covered with plumbago (soft pencil scrapings), after the tracing is made, and then with a very gentle pressure of the tracing point will leave a light pencil line on the final sheet. In either case, when the tracing is removed, these lines may be inked in on the final sheet.

If the map is to be photo-lithographed it must be drawn wholly in black, as given in Plate II. If not, it is best to use some color in its execution, Plate III. The water lines may be drawn in blue, and the contours in brown on arable land, and in black on barren or rocky land. In this way the character of the surface may be partly given. Where the slopes are very steep the contour lines become nearly coincident, but to further emphasize the uneven character of the ground, cross hatching, or hachures, may be employed on slopes greater than 45° from the horizontal. All these conventional practices are illustrated

on Plate II, except the use of colors, this map having been drawn for the purpose of being photo-lithographed. Where large areas of the same symbols are to be mapped, they are sometimes printed on the *original* map by means of specially constructed hand rollers.

234. Topographical Symbols are more or less conventional, and for that reason given forms should be agreed upon. The forms given in Plate II were used on all the Mississippi River surveys made under the Commission, and are recommended as being elegant and fairly representative or natural. Evidently the rice, cotton, sugar, and wild-cane symbols would find no place in maps of higher latitudes. The cypress-tree symbols may be used for pine to distinguish them from deciduous growth, and the sugar-cane symbol could be used for corn if desired. It is not important to distinguish between different kinds of cultivated crops, since these are apt to change from year to year, but it is sometimes desirable to do so to give a more varied and pleasing appearance to the map. The grouping of the trees in a large forest is also varied simply for the appearance, to prevent monotony. Colors are sometimes used in place of pen-drawn symbols, but these are necessarily so very conventional as to require a key to interpret them, and besides, unless well done, it makes the map look cheap and unprofessional.

235. Accuracy of Stadia Measurements. The attainable accuracy depends upon the care taken in graduating the rod (or in determining the stadia constants), and upon the ruggedness of the country surveyed. Results of surveys have shown that a higher degree of accuracy is attainable in moderately rugged country where the line of sight passes at a considerable distance from the ground, thus avoiding the excessive and irregular errors of differential refraction. In general it may be said that under the most unfavorable circumstances an accuracy 1 in 500 is easily attained, while under favorable circumstances this accuracy may be increased to 1 in 2000 or

more.[1] The results obtained in the U. S. Lake Survey are perhaps a fair average for various conditions. On that survey any circuit with a larger closure error than 1 in 750 is usually rerun. The later stadia work of this great survey shows an average closure error of less than 1 in 1000. In 1905–6 11 circuits averaging 2.8 miles in length and using an average course distance of 275 metres gave an error of 1 in 1520 in distance and 34″ per mile in azimuth. On the resurvey of Niagara River 18 stadia circuits averaging 1.3 miles in length and employing an average length shot of 255 metres gave an average closing error of 1 in 1450 in distance and 1′ 25″ per mile in azimuth.

On the Mexican Boundary Survey the transit and stadia method was used for taking topography over 1750 square miles, as well as in all measurement of the entire boundary line. The conditions under which this work was done were most unfavorable for accuracy, but in a trial measurement of 100 miles by the stadia and the chain the former was found, by a comparison with the true triangulated distance, to be far more accurate than the chain. The following table will show the degree of accuracy of both the stadia and the chaining work. The heading "corrected chain," as given because the stadia detected the dropping or addition of a chain length six times in this distance. The error was always found to be in the chain and the record was corrected as here given. This work was on a *flat level mesa.*

Triangulation checks on stadia measurements were also obtained on this survey along a stretch very unfavorable to accuracy, viz., a succession of high ridges and deep cañons, necessitating large vertical angles or sights of excessive length. The following table gives the results of these checks.[2]

[1] For a statement of the effect of length of sight upon the accuracy, see Bulletin of the University of Wisconsin, Engineering Series, Vol. I, No. 5, p. 127, 1895. Also Eng. News, Vol. XXXIII, p. 364, by Leonard S. Smith.

[2] From paper by Professor John L. Van Ornum, Vol. XXXIV, Am. Soc C. E., Topography on the U. S.–Mexican Boundary.

TABLE OF DISTANCES AS DETERMINED BY STADIA, TRIANGULATION AND CHAIN

Error.	Stadia.	Triangulation.	Corrected Chain	Error.
$-\frac{1}{600}$	13171	13193	13193	0
$+\frac{1}{19969}$	19970	19969	19965	$-\frac{1}{4992}$
$-\frac{1}{796}$	21473	21500	21474	$-\frac{1}{827}$
$-\frac{1}{698}$	21598	21629	21605	$-\frac{1}{901}$
$-\frac{1}{3803}$	22813	22819	22804	$-\frac{1}{1521}$
$+\frac{1}{639}$	49959	49879	.....	.....
$-\frac{1}{21284}$	148982	148989	.....	.....

TABLE OF DISTANCES AS DETERMINED BY TRIANGULATION AND STADIA

Triangulation.	Stadia.	Error.	Accumulated Error.
223.6	222.7	$-\frac{1}{248}$	$-\frac{1}{248}$
411.7	412.7	$+\frac{1}{412}$	$+\frac{1}{6350}$
757.6	755.0	$-\frac{1}{291}$	$-\frac{1}{557}$
785.0	785.3	$+\frac{1}{2617}$	$-\frac{1}{989}$
943.8	945.5	$+\frac{1}{555}$	$-\frac{1}{6243}$
983.3	985.2	$+\frac{1}{518}$	$+\frac{1}{2937}$
1273.2	1272.8	$-\frac{1}{3183}$	$+\frac{1}{5378}$
1346.6	1349.0	$+\frac{1}{561}$	$+\frac{1}{1977}$
1788.0	1789.3	$+\frac{1}{1375}$	$+\frac{1}{1810}$
4433.0	4446.0	$+\frac{1}{341}$	$+\frac{1}{731}$
73095.5	73056.6	$-\frac{1}{1879}$	$-\frac{1}{4095}$
86041.3	86020.1	$-\frac{1}{4059}$	.....

Unlike a triangular check, the closure error of a complete stadia circuit is a check only on the accidental or compensating errors. While for this reason, the following table gives no measure of the constant or non-compensating errors of the linear measurements, they are valuable as showing the errors to be expected in similar field work.

Number of Lines	Aggregate Length of Lines in Metres.	Average Length of Courses in Metres.	Average Number of Courses per Line.	Average Error in Distance on Closing Equals 1 in.	Average Azimuth Error on Closing per Kilometre of Line Run.
29	111823.2	253.0	15.2	553	1′ 55″
49	280706.8	356.7	16.1	782	1 04
28	290633.9	437.7	23.7	817	0 43
12	143352.0	580.4	20.6	786	0 37
118	826515.9	386.2	18.1	752	0 59 .6

236. Accuracy of Levels Run by the Stadia and Transit. The errors in carrying levels by means of the stadia and vertical angles is a function of the average vertical angle employed. This may be seen from the following tabulation of field work taken from the report of the Mexican Boundary Survey, 1891–6.

Number of Lines. (Circuits.)	Aggregate Length of Lines.	Sum of the Vertical Components of Courses.	Average Vertical Angle of Lines.	Error in Elevation on Closing per Kilometre of Line Run.		Error in Distance on Closing Equals 1 in.
	Metres.	Metres.	° ′	Feet	Metres.	
14	182960	1262.4	0 24	0.17	0.053	
55	338132	11706.1	1 59	.37	.111	741
28	186426	12139.8	3 43	.49	.150	
17	112025	11930.6	6 05	.59	.181	842
114	819543	37038.9	3 3	.40	.123	

From the above table it will be seen that on 14 circuits, averaging 13,070 metres (8.2 miles), run in rolling country and employing a small vertical angle, the closing error in elevation

was 0.17 feet per kilometre; also that the closing errors increase quite rapidly as the average vertical angle increases, until, in the most rugged country requiring an average vertical angle of 6° 5′ 17 circuits averaging 6590 metres (4.1 miles), have a closing error of 0.59 feet per kilometre.

The relation of error to vertical angle, as deduced from the 512 miles of closed stadia lines comprising the above table, is best shown by the graphic curve in Fig. 118.

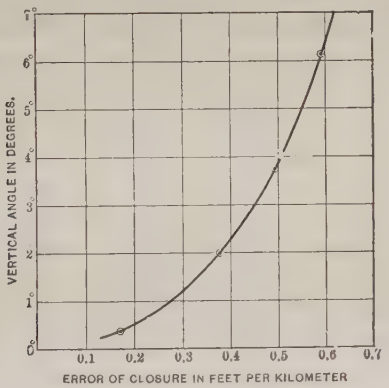


FIG. 118.

A good example of the use of the transit and stadia method in running levels in city topographic surveys is found in the recent topographic survey of St. Louis. In this survey a transit and stadia line was run over 40 miles long. At twenty-four points along this circuit the line checked on triangulation points and precise level bench marks with the results shown in the following table.

The average length of the lines between check points was 1.7 miles, and the average error for this distance was 0.24 of a foot or 0.18 of a foot per mile of line.

It should be noted that while the total accumulated error in elevation for the entire 40 miles was but 0.64 of a foot, at

a point on the line distant 20 miles from the beginning the error in elevation was zero, while in 7 miles more it was over twice the error at the end of the circuit, thus emphasizing the fact that the errors in such work tend to compensate.

RESULTS OF LEVELLING BY THE STADIA METHOD,
as obtained on the St. Louis Topographical Survey.

Station.	Azimuth Error.	Accumulated Error in Elevation in Feet.	Distance in Miles from the Starting-point.	Error in Horizontal Measurement.	Error of Closure in Elevation between Check-points. Feet.
2547	0° 1′ 20″	+0.42	2.0	+1: 387	0.42
2803	0 50	+ .46	4.1	+1: 619	.04
2777	2 20	+ .17	6.2	+1:1177	.29
1332	2 00	+ .09	7.8	+1:1149	.08
1393	2 00	+ .50	9.2	+1: 987	.41
774	3 33	+ .52	10.9	+1:1000	.02
400	8 13	+ .09	12.3	+1:1084	.43
389	7 06	+ .08	14.6	+1:1203	.01
1839	9 32	+ .25	16.3	+1:1025	.17
1871	9 52	− .01	18.4	+1: 965	.26
2008	10 42	+ .14	20.5	+1: 836	.15
2067	11 42	+ .37	22.4	+1: 877	.23
41	12 12	+ .39	23.8	+1: 961	.02
2292	11 42	+ .77	25.2	+1:1063	.38
2304	10 42	+1.37	27.0	+1:1139	.60
1699	10 42	+1.00	28.9	+1:1484	.37
566	9 40	+1.23	30.3	+1:1644	.23
1500	9 05	+1.03	31.8	+1:1724	.20
1488	10 00	+0.68	33.5	+1:2267	.35
937	12 35	+0.94	34.9	+1:3291	.26
958	9 18	+0.98	36.2	+1:3945	.04
1115	9 30	+0.64	37.9	+1:5174	.34
2476	8 20	+0.36	39.8	+1:6420	.28
124	8 20	+0.64	40.4	+1:6332	.28

The following data regarding the accuracy of stadia surveys made under the Mississippi River Commission are given as a fair sample of the results obtained under that organization. In 1896, thirty-six stadia circuits, with an average length of 2750

metres (1.7 miles), were run over a certain rough, hilly country, between Dubuque, Ia., and Prairie du Chien, Wis., ranging in elevation from 100 to 300 feet, with an average error in elevation of 0.77 foot per circuit or 0.59 foot per mile of line run. The maximum error in this work was 1.50 feet on a circuit 4000 metres (2.5 miles) long, and the minimum error in elevation was 0.0 foot on a circuit 2100 metres (1.4 miles) long. Seventy-four other circuits, averaging 2900 metres (1.8 miles), were run in a nearly level river bottom, with a resulting error in elevation of 0.43 foot per circuit or 0.31 foot per mile of line run. The maximum error in this work was 2.5 feet on a circuit 4300 metres (2.7 miles) long, while the minimum error in elevation on the same work was 0.03 foot on a circuit 4400 metres long. As indicating the general law of compensating errors, it may be said that of the above 74 circuits 38 gave too high and 29 circuits too low elevations, while on 7 circuits the error was zero.

It should be noted that *after such closing errors have been properly adjusted among the various points of the circuit*, which is the usual practice, the probable error in elevation of any such adjusted values is very much smaller than the closing error of the circuit, and for all mapping purposes it is far within the limits of accuracy required. It may then be concluded that for all kinds of accurate topographic work, with the possible exception of special very large-scale surveys, the transit and stadia and the plane table and stadia method are by far the best and most economical methods to use.

NOTE.—For reducing stadia readings for "difference of elevation" and also for "correction for horizontal distance" an excellent diagram has been put on the market by A. H. Abbott & Co., of Chicago, Ill. This was devised by Mr. Morris K. Trumbull, and is described by him in the Jour. W. Soc. Eng., Vol. III, p. 1399. This diagram is 24 ins. by 30 ins. in size, has blue lines on a white ground, on heavy paper, and gives angles up to 8°, distances to 1600 feet, and elevations to 100 feet. It also shows the "correction for horizontal distance" at a glance, while taking out the "difference of elevation." This is the most convenient diagram the authors have ever seen. It is said to give results as rapidly as the Colby slide-rule. The price is $1.00 postpaid.

237. The Cost of Topographic Maps. The cost of topographic work depends upon such a variety of conditions as to preclude any general statements. In the following table will be found a statement of the unit costs of a number of representative surveys including both detail and geographic surveys. The economics of this subject have not been sufficiently considered in preparing plans for topographic work. As a result, excessive costs have followed unnecessary field work, i.e., more field data than the scale of the final map and its contour interval could show. Again sufficient field locations may be lacking for the map.

SCALE AND COST, ETC., OF DETAIL TOPOGRAPHIC SURVEYS

Name.	Area.	Contour Interval.	Scale.	Cost per Square Mile.
		ft.		
Philadelphia Water Department. .	446	10	1:4800	$145.00
Connellsville Coke Region	168	10	1:7200	40.00
District of Columbia.....	48	5	1:4800	2500.00*
Croton Watershed.....	400	20	3 in. = 1 mi.	18.00
British Ordnance Survey	.....	.....	1:2500	.....
St. Louis Survey	44	3	1:2400	740.00
Baltimore Survey	32	5	1:2400	1800.00

* Field work cost $1360 per square mile.

SCALE, COST, ETC., OF SMALL SCALE MAPS

Name.	Area.	Contour.	Scale.	Cost per Square Mile.
U. S. Geol. Survey	.....	20 ft.	1:62500	$10.00
		100 ft.	1:125000	4.00
The Great Lakes Survey	3000	.....	1:10000	125.00
Mississippi River Commission....	1300	5 ft.	1:10000	70.00
U. S.–Mexican Boundary.....	1700	10 m.	1:30000	10.00*
State Massachusetts	8900	20 ft.	1:30000	13.00
Great Britain	.....	Hachur's + contours	1 in. = 1 mi.	184.00

* Cost of instrument work only.

Such conditions can be prevented by preparing specifications and directions for the field, based upon ample study of the purposes for which the maps are made and the conditions under which the work must be done.

REFERENCES ON TOPOGRAPHIC SURVEYS

1. A Topographic Map of Butte, Mont. R. H. Chapman. Eng. and Mining Jour., May 9, 1896.
2. History of the British Ordnance Survey. J. J. Amos, Surveyor, March 25, 1910; also Feb. 12, 1909.
3. Cost and Methods in Different Countries, 1893, II, p. 266; also 1894, II, 252–254.
4. Scales of Various Countries. Eng. News, I, p. 470, June 7, 1894.
5. Topographic Maps of the U. S. Geol. Survey, Eng. News, 1893, II, p. 152.
6. Topographic Surveys by Ogden, Am. Soc. C. E., Oct., 1893.
7. Geology in Its Relation to Topography. J. C. Banner, Trans. Am. Soc. C. E., Vol. XXXIX, pp. 53–78.
8. Topographic Methods used on the Vanderbilt Estate at Biltmore, N. C. Howard, Jour. Asso. Eng. Soc., April, 1897.
9. Topographic Surveying by U. S. C. and G. S., and U. S. G. S. officers. Am. Soc., C. E., Vol. LIV, Part 3.
10. Topography on the Survey of the Mexican–U. S. Boundary. J. L. Van Ornam, Am. Soc. C. E., Vol. XXXIV, p. 259.
11. Topographic Surveys. W. C. Bannel, Soc. Western Engr's, Oct. 1903.
12. Topographic Work in Grand Canon of the Gunnison. I. W. McConnell, Eng. Record, Oct. 3, 1903.
13. Photographic Methods on Canadian Topog. Survey. A. O. Wheeler, Eng. Record, Feb. 18, 1905; also Sept. 25, 1909.
14. Photographic Surveying. W. Griffith, Mines and Mining, June, 1907.
15. Topographic Surveying. A Text by H. M. Wilson. Published by John Wiley & Sons, New York.

CHAPTER IX

HYDROGRAPHIC SURVEYING [1]

238. Hydrographic Surveying includes all surveys, for whatever purpose, which are made on, or are concerned with, any body of still or running water. Some of the objects of such surveys are the determination of depths for mapping and navigation purposes; the determination of areas of cross-sections, the mean velocities of the water across such sections, and the slope of the water surface; the location of buoys, rocks, lights, signals, etc.; the location of channels, the directions and velocities of currents, and the determination of the changes in the same; the determination of the quantity of sediment carried in suspension, of the volume of the scour or fill on the bottom, or of the material removed by artificial means, as by dredging.

A hydrographic survey is usually connected with an extended body of water, as ocean coasts, harbors, lakes, or rivers. The fixed points of reference for the survey are usually on shore, but sometimes buoys are anchored off the shore and used as points of reference. All such points should be accurately located by triangulation from some measured base whose azimuth has been found. The buoys will swing at their moorings within small circles, these being larger at low tide than at high, but the errors in their positions should never be sufficient to cause appreciable error in the plotted positions of the soundings. Where soundings need to be located with great exactness, buoys could not be relied on. The triangulation work for the location of the fixed points of reference differs in no sense from that for a topographical survey. In fact, a hydrographic survey is usually

[1] Revised for the 17th edition by Professor Leonard S. Smith.

connected with a topographical survey of the adjacent shores or banks, the triangulation scheme serving both purposes. It is not uncommon, however, to make a hydrographic survey for navigation purposes simply, wherein only the shore line and certain very prominent features of the adjacent land are located and plotted. This is the practice of the U. S. Hydrographic Office in surveying foreign coasts and harbors. In this case the work consists almost wholly in making and locating soundings for a certain limiting depth, as 100 fathoms, or 100 feet, inward to the shore, and along the coast as far as desired. The length and azimuth of a base-line are determined and the latitude observed by methods given in Chapter XV. The longitude is found by observing for local time, and comparing it with the chronometer time which has been brought from some station whose longitude was known. Whenever telegraphic communication can be obtained with a place of known longitude, the difference between the local times of the two places is found by exchanging chronographic signals. No special description will be here given of the methods used in this part of the work, as they are all fully described in Chapters XIV and XV.

The Location of Soundings

239. Methods. The location of a sounding can be found with reference to visible known points by (1) two angles read at fixed points on shore; (2) by two angles read in the boat; (3) by taking the sounding on a certain range, or known line, and reading one angle either on shore or in the boat; (4) by sounding along a known range, or line, taking the soundings at known intervals of time, and rowing at a uniform rate; (5) by taking the soundings at the intersections of fixed range lines; (6) by means of cords or wires stretched between fixed stations, these having tags, or marks, where the soundings are to be taken; (7) by transit and stadia. These methods are severally adapted to different conditions and objects, and will be described in order.

240. Two Angles Read on Shore. If two instruments (transits or sextants) be placed at two known points on shore, and the angles subtended by some other fixed point, and the boat be read by both instruments, when a sounding is taken, the intersection of the two pointings to the boat, when plotted on the chart containing the points of observation duly plotted, will be the plotted position of the sounding. If three instruments are read from as many known stations, then the three pointings to the boat should intersect in a point when plotted, thus furnishing a check on the observations. The objections to this method are that it requires at least two observers, and these must be transferred at intervals, as the work proceeds, in order to maintain good intersections, or in order to see the boat at all times. While an observer is shifting his position the work must be suspended. If there are long lines of off-shore soundings to be made and there are no fixed points or stations on shore of sufficient distinctness or prominence to be observed by the sextant from the boat, then this method must be used. When the angles are read on shore, signals should be given preparatory to taking a sounding, and also when the sounding is made. If, however, the soundings are taken at regular intervals the preparatory signal may be omitted, and only the signal given when the sounding is taken. This usually consists in showing a flag. The instrument may be set to read zero when pointing to the fixed station. This reading need only be taken at intervals to test the stability of the instrument.

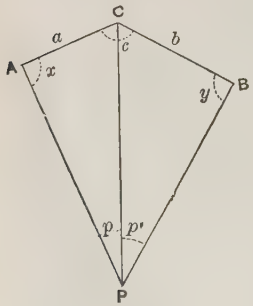


FIG. 119.

241. By Two Angles Read in the Boat to three points on shore whose relative position are known. This is called the "three-point" problem. Let A, C, and B be the three shore points, being defined by the two distances a and b and the angle C. Let the two angles

P and P' be measured at the point P. The problem is to find the distances AP and BP.

(*a*) *Analytical Solution.* Let the unknown angle at A be x and that at B be y. Then we may form two equations from which x and y may be found. For,

$$PC = \frac{a \sin x}{\sin P} = \frac{b \sin y}{\sin P'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Also, $$x + y = 360° - (P + P' + C) = R. \quad . \quad . \quad . \quad . \quad (2)$$

From (2), $$y = R - x,$$

and $$\sin y = \sin R \cos x - \cos R \sin x.$$

Substitute this value of sin y in (1), reduce, and find

$$\cot x = \frac{a \sin P' + b \sin P \cos R}{b \sin P \sin R}$$

$$= \cot R \left(\frac{a \sin P'}{b \sin P \cos R} + 1 \right). \quad . \quad . \quad . \quad . \quad (3)$$

When x and y are found, the sides AP and BP are readily obtained. This is perhaps the simplest analytical solution of the problem.

(*b*) *Geometrical Solution.* The following geometrical solution is of some interest, though it is seldom used:

Let A, C, and B be the fixed points as before, and P and P' the observed angles. Having the points A, B, and C plotted in their true relative positions, draw from A the line AD, making with AB the angle P' (CPB), and from B the line BD, making with AB the angle P (APC), cutting the former line in D. Through A, D, and B pass a circle, and through C and D draw a line cutting the circumference again in P. The point P is the

plotted position of the point of observation from which the angles P and P' were measured.

For P must lie in the circumference through ADB by construction, otherwise ABD would not be equal to APD, as they are both measured by the same arc AD. The same holds for the angle P'. Also, the line PD must pass through C, otherwise the angle APC would be greater or less than P, which cannot be. The point P is therefore on the line CD, and also on the circumference of the circle through ADB, whence it is at their intersection.

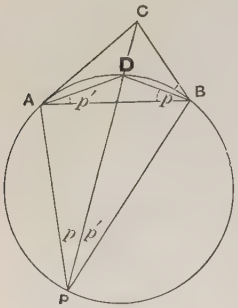


FIG. 120.

This demonstration is valuable as showing when this method of location fails to locate, and when the location is poor. For the nearer the point D comes to C the more uncertain becomes the direction of the line CD, and when D fails at C—that is, when P is on the circumference of a circle through A, B, and C—the solution is impossible, inasmuch as P may then be anywhere on that circumference without changing the angles P and P'. This is also shown by Eq. (3), above; for if A, C, B, and P all fall on one circumference, then $x+y=R=180°$; whence $\cot x=\infty\times 0$, which is indeterminate. For $\cot R=-\infty$, and $\cos R=-1$. Also $a\sin P'=b\sin P$, both being equal to the perpendicular from C on AB. The equation then becomes

$$\cot x=\infty\,(1-1)=\infty\times 0.$$

(*c*) *Mechanical Solution.* If the three known stations be plotted in position and the two observed angles be carefully set on a three-armed protractor,[1] then when the three radial edges coincide with the three stations, the centre of the protractor circle corresponds to the position of the point of obser-

[1] For description, with cut, see p. 169.

vation. With a good protractor this method gives the position of the point as closely as the nature of the observations themselves would warrant. It is the common method of plotting soundings when two sextant angles have been read from the sounding boat.

(*d*) *Graphical Solution.* The angles may be laid off on tracing paper or linen by lines of indefinite length, and this laid on the plot and shifted in position until the three radial lines coincide with the three stations, when their intersection marks the point of observation. This is the most ready method of plotting such observations when no three-armed protractor is available.

The advantages of this method of locating soundings are that it requires but one observer, no time is lost in changing stations, and the party are all together, and hence there can be no misunderstandings in regard to the work. If the soundings are made in running water, so that the boat cannot be stopped long enough to read two sextant angles, two sextants are sometimes used with one observer, he setting both angles and reading them afterwards; or two observers may be employed in the same boat and the angles taken simultaneously.

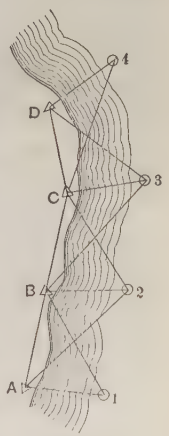


FIG. 121.

242. By One Range and One Angle. The range may be two stations or poles set in line on shore, or it may consist of one point on shore and a buoy set at the desired position offshore. If buoys are used they must be located by triangulation from the shore stations. A triangulation system along a rocky or wooded coast may consist of one line of stations on shore and a corresponding line of buoys. The angles are read only from the shore stations, two angles in each triangle being observed. If the buoys are well set and the work done in calm weather, the results will be good enough for topographical or hydrographical

purposes. The stations and buoys should be opposite each other, as in the figure, and readings taken to the two adjacent shore stations and to the three nearest buoys from each shore station. If the length of any line of this system be known, the rest can be found when the angles at *A*, *B*, *C*, and *D* are measured. In such a system the measured lines should recur as often as possible, ordinary chaining being sufficient.

243. Buoys, Buoy-Flags, and Range-Poles. A convenient buoy for this purpose may be made of any light wood, 18 inches to 3 feet long in tideless waters, and long enough to maintain an erect position in tide waters. It should be from 6 to 10 inches in diameter at top, and taper towards the bottom. If the buoy is not too long, a hole may be bored through its axis for the flag-pole, which may then project 2 or 3 feet below the buoy and as high above it as desired. The buoy rope is then attached to the bottom end of the pole and made of such length as to maintain the pole in a vertical position in all stages of the tide. The anchor may be any sufficiently heavy body, as a rock or cast-iron disk. If the buoys are liable to become confused on the records, different designs may be used in the flags, as various combination of red, white, and blue, all good colors for this purpose.

The range-poles should be whitewashed so as to show up against the background of the shore. The ranges are designated by attaching to the rear range poles slats (barrel-staves would serve) arranged as Roman numerals when read up or down the pole. If range-poles are relied on, they must be very carefully located and plotted, in order to establish accurately a long line of soundings from a very short fixed base.

The observed angle may be either from the boat or from a point on shore. In either case any other range post of the series may be used either for the position of the observer, if on shore, or for the other target point if the angle is read from the boat.

244. By One Range and Time-Intervals. This is a very

common and efficient method, and quite satisfactory where soundings need not be located with the greatest accuracy and where there is no current. A boat can be pulled in still water with great uniformity of speed; and if the soundings be taken at known intervals with the ends of the line of soundings fixed, the time intervals will correspond almost exactly with the space intervals. If the ends of the line of soundings are not fixed by buoys or sounding stations on shore, but the line simply fixed by ranges back from the water's edge, the positions of the end soundings may be fixed by angle readings and the balance interpolated from the time intervals.

245. By Means of Intersecting Ranges. This method is only adapted to the case where soundings are to be repeated many times at the same places. When the object of the survey is to study the changes occurring as to scour or fill on the bottom it is very essential that the successive soundings should coincide in position, otherwise discrepant results would prove nothing. Such surveys are common on navigable rivers and in harbors. Many systems of such ranges could be described, but the ingenious engineer will be able to devise a system adapted to the case in hand.

246. By Means of Cords or Wires. In the case of a fixed but narrow navigable channel, having an irregular bottom, or undergoing improvement by dredging, it may be found advisable to set and locate stakes on opposite sides of the channel, to stretch a graduated cord or wire between them, and to locate the soundings by this. By such means the location would be the most accurate possible.

Making the Soundings

247. The Lead is usually made of lead, and should be long and slender to diminish the resistance of the water. It should weigh from five pounds for shallow, still water, to twenty pounds for deep running water, as in large rivers. If depth only is

required, the lead may be a simple cylinder something like a sash weight for windows. If specimens of the bottom are to be brought to the surface at each sounding, the form shown in Fig. 122 may be used to advantage. An iron stem, *I*, is made with a cup, *c* at its lower end. The stem has spurs cut upon it, or cross-bars attached to it, and on this is moulded the lead which gives the requisite weight. Between the cup and the lead is a leather cover sliding freely on the shank and fitting tightly to the upper edges of the cup. When the cup strikes the bottom, it sinks far enough to obtain a specimen of the same, which is then safely brought to the surface, the leather cover protecting the contents of the cup from being washed out in raising the lead. A conical cavity in the lower end of the lead, lined with tallow, is often used, and it is found very efficient for indicating sand and mud. It is often very essential to know whether the bottom is composed of gravel, coarse or fine, sand, mud, clay, hard-pan, or rock, and this knowledge can be obtained with the cup device described above.

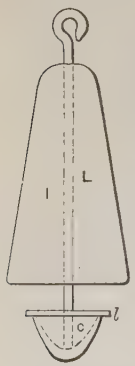


FIG. 122.

248. The Line should be of a size suited to the weight of the lead, and made of Italian hemp. It is prepared for use by first stretching it sufficiently to prevent further elongation in use after it is graduated. Probably the best way to stretch a line is to wind it tightly about a smooth-barked tree, securely fasten both ends, wet it thoroughly, and leave it to dry. Then rewind as before, taking up the slack from the first stretching, and repeat the operation until the slack becomes inappreciable. It may now be graduated and tagged. Sometimes it is fastened to two trees and stretched by means of a "Spanish windlass," and then wet. It is quite possible to stretch the line too much, for sometimes sounding lines have shortened in use after being stretched by this method. Soundings at sea are taken in fathoms. On the U. S. Lake Survey all depths over twenty-four feet (four

fathoms) were given in fathoms, and all depths less than that limit were given in feet. On river and harbor surveys it is common to give depths in feet. Channel soundings on the Western rivers made by boatmen are given in feet up to ten feet, then they are given in fathoms and quarters, the calls being "quarter-less-twain," "mark-twain," "quarter-twain," "half-twain," "quarter-less-three," "mark-three," etc., for depths of $1\frac{3}{4}$, 2, $2\frac{1}{4}$, $2\frac{1}{2}$, $2\frac{3}{4}$, 3, etc., fathoms respectively.

If the line is graduated in feet, leather tags are used every 5 feet, the intermediate foot-marks being cotton or woollen strips. The 10-foot tags are notched with 1, 2, 3, etc., notches for the 10-, 20-, 30-, etc., foot-points, up to 50 feet. The 50-foot tag may have a hole in it, and the 60-, 70-, 80-, etc., foot-marks have tags all with one hole and with 1, 2, 3, etc., notches. The intermediate 5-foot points have a simple leather tag unmarked. Sometimes the figures are branded on the leather tags, but notches are more easily read. The zero of the graduation is the bottom of the lead. The leather tags are fastened into the strands of the line; the cloth strips may be tied on. The line should be frequently tested, and if it changes materially a table of corrections should be made out and all soundings corrected for erroneous length of line. For deep soundings, piano wire is frequently used.

249. Sounding Poles should be used when the depth is less than about 15 feet. The pole may be graduated to feet simply, or to feet and tenths, according to the accuracy required.

250. Making Soundings in Running Water. The sounding-boat should be of the "cutter" pattern, with a sort of platform in the bow for the leadsmen to stand on. If the current is swift, six oarsmen may be required and two observers and one recorder. One of the observers may act as steersman. If the depth is not more than 60 or 80 feet, the soundings are made without checking the boat, the leadsman casting the lead far enough forward to enable it to reach bottom by the time the line comes

vertical. When the depth and the current are such as to make this impossible, the boat is allowed to drift down with the current and soundings taken at intervals without drawing up the lead. The boat is then pulled back up stream and dropped down again on another line, and so on.

In still water a smaller crew and outfit may be used, as the boat may be stopped for each sounding if necessary.

The record should give the date, names of observers, general locality, number or other designation of line sounded, the time, the two angles, the stations sighted, and the depth for each sounding, and the errors of the graduated lengths on the sounding line.

251. The Water-Surface Plane of Reference. In order to refer the bottom elevations to the general datum plane of the survey, it is necessary to know the elevation of the water-surface at all times when soundings are taken. In tidal waters the elevation of "mean tide" is the plane of reference for both the topographical and hydrographical surveys, and then the state of the tide must be known with reference to mean tide. This is found from the hourly readings of a tide-gauge (provided it is not automatic), the elevation of the zero of which, with reference to mean tide-water, has been determined. All soundings must then be reduced to what they would have been if made at mean tide before they are plotted.

If the soundings are made in lakes, the datum is usually the lowest water-stage on record; and here also gauge-readings are necessary, as the stage of the water in the lake varies from year to year. In this case the gauge need only be read twice a day.

In rivers of variable stage the datum is either referred to mean or low-water stage, or else to the general datum of the map. If the stage is changing rapidly the gauge should be read hourly when soundings are taken, otherwise daily readings are sufficient. If the soundings are to be referred to the general datum of the map, then the *slope* of the stream must be taken into account. If they are referred to a particular stage of water

in the river, then the slope does not enter as a correction, as the slope is assumed to be the same at all stages, although this is not strictly true.

252. Lines of Equal Depth correspond to contour lines in topographical surveys; but to draw lines of equal depth with certainty the elevations of many more points are necessary than are needed for drawing contour lines, because the bottom cannot be viewed directly, while the ground can be. Where the ground is seen to be nearly level no elevations need be taken, while for a similar region of bottom a great many soundings would be required to prove that it was not irregular.

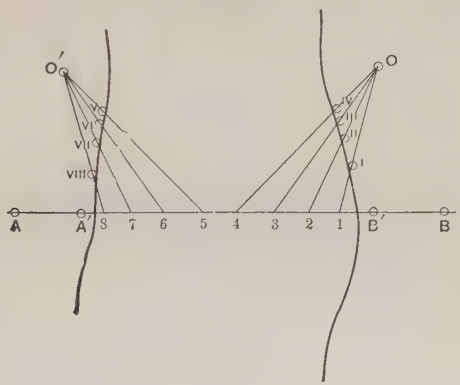


FIG. 123.

253. Soundings on Fixed Cross-Sections in Rivers. Where the same section is to be sounded a great many times, and especially when it is desirable to obtain the successive soundings at about the same points, it is best to fix range posts on the line of the section (on both sides if it be a river) and then fix one or more series of intersecting ranges at points some distance above or below the section on one or both sides of the river. The soundings can then be made at the same points continuously without having to observe any angles at all. Such a system of ranges is shown in Fig. 123. AA' and BB' are range poles on the

section line. *O* and *O'* are tall white posts set at convenient points on opposite sides of the river, either above or below the section. I, II, III, etc., are shorter posts set near the bank in such positions that the intersection of the lines *O*-I, *O*-II., etc., with the section range *BB'* will locate the soundings at 1, 2, etc., on this section line. The posts in the banks should be marked by strips nailed upon them so as to make the Roman numerals as given in the figure. Such a system of ranges as the above is useful also for fixing points on a section line, for setting out floats, or for running current meters for the determination of river discharge.

254. **Areas of Cross-Section** are obtained by plotting the soundings on cross-section paper, the horizontal scale being about one-tenth or one-twentieth of the vertical. The horizontal line representing the water surface is drawn, and the plotted soundings joined by a free-hand line. The enclosed area is then measured by the planimeter. If the horizontal scale is 50 feet to the inch and the vertical scale 5 feet to the inch, then each square inch of the figure represents 250 square feet of area. The planimeter should be set to read the area is square inches, and the result multiplied by 250.

Areas of cross-section are usually taken in running water, and here great care must be taken to get vertical soundings, and to make the proper sounding-line corrections. They should be taken near enough together to enable the bottom line to be drawn with sufficient accuracy.

Bench-Marks, Gauges, Water-Levels, and River-Slope

255. **Bench-Marks** should be set in the immediate vicinity of each water gauge, and these connected by duplicate lines of levels with the reference plane of the survey. If the gauge is not very firmly set, or if it is necessary to move it for a changing stage, its zero must be referred again to its bench mark by duplicate levels, whenever there is reason to suspect it may have been

disturbed. Such bench marks as these are usually spikes in the roots of trees or stumps.

256. Water Gauges are of various designs, according to the situation and the purpose in view. For temporary use during the period of a survey, a staff gauge is best, consisting of a board painted white, of sufficient length, graduated to feet and tenths

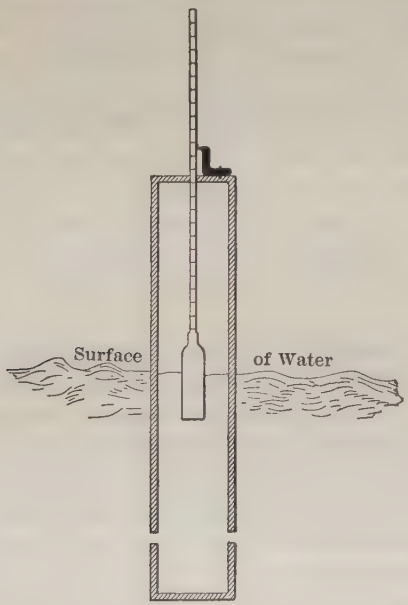


FIG. 124.—A Bottle Water Gauge.

in black. Sometimes it is graduated to half-tenths, but this is useless unless in still water, and there is never any need of graduation finer than this. The gauge may be read to hundredths of a foot if the water is calm enough. It should be nailed to a pile or to a stake driven firmly near the water's edge. It is read twice a day, or oftener, if the needs of the service require.

The gauge shown in Fig. 124 is simple to construct and well designed for rough water. It consists of a wooden box, 6 inches

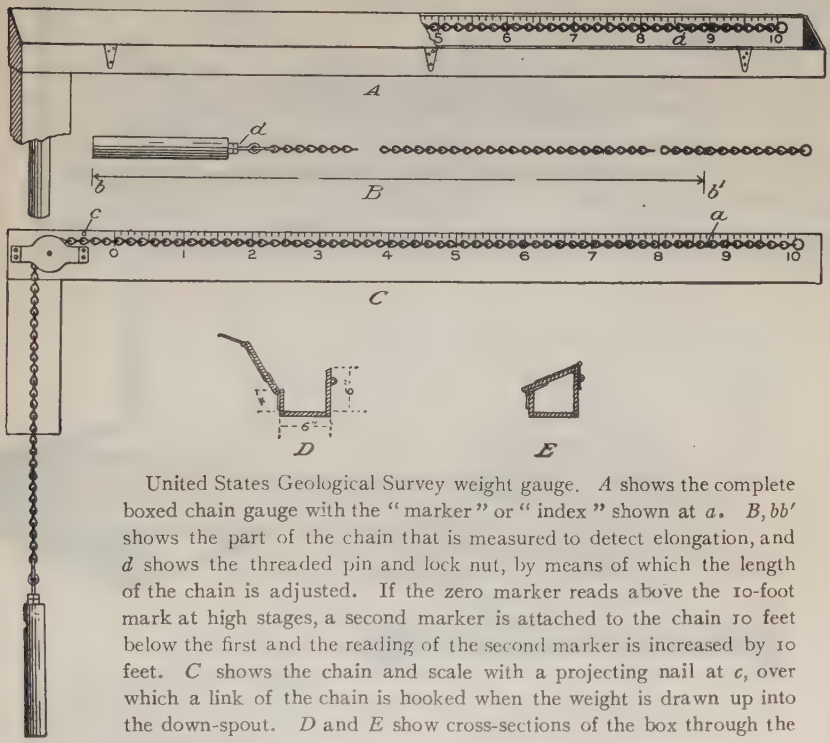
square, with a closed bottom but with one or more holes bored low down in its sides or in the bottom to admit the water. An empty bottle with a graduated wooden staff one inch square fastened tightly in its mouth, floats in the water. The staff moves freely up and down through a hole in the top of the box, recording every change in the height of the water. The gauge may be read at the top surface of the box or a special pointer can be provided for this purpose.

The staff is graduated in feet and tenths from the line of flotation, the numbers increasing upward. If the tops of the gauge boxes be connected by spirit levels, the water height at any point would be determined by subtracting the gauge reading from the elevation of the corresponding box. If the staff was so graduated as to increase downward, it would be possible to so paint the staff that it would read directly the true elevation of the water. In practice the first method is generally found most convenient.

257. Chain Gauges. The chain gauge is often used on high bridges and at points where the banks are unfavorable to the maintenance of a staff gauge. A view of such a gauge as used by the U. S. Geological Survey is shown in Fig. 125.

It consists of a graduated board held horizontally and at least as long as the extreme fluctuations in the river height. At the end of the board over the water is attached a pulley over which a chain carrying about a 12-pound weight, passes. When not in use the weight is secured by a ring attached to the other end of the chain. To make a reading the weight is lowered until its bottom just touches the water surface, when the position of the index point of the chain is read on the graduated board. If the elevation of the water at some particular stage is determined by spirit levels, then any future elevation is determined by the corresponding gauge reading. It has been found advisable to set permanent bench marks near by so as to check any changes in the height of the gauge as a whole. Any changes in the length of the chain can be detected by testing it with a steel tape. The

great advantage of the chain gauge is its freedom from disturbance by ice or logs.



United States Geological Survey weight gauge. *A* shows the complete boxed chain gauge with the "marker" or "index" shown at *a*. *B*, *bb'* shows the part of the chain that is measured to detect elongation, and *d* shows the threaded pin and lock nut, by means of which the length of the chain is adjusted. If the zero marker reads above the 10-foot mark at high stages, a second marker is attached to the chain 10 feet below the first and the reading of the second marker is increased by 10 feet. *C* shows the chain and scale with a projecting nail at *c*, over which a link of the chain is hooked when the weight is drawn up into the down-spout. *D* and *E* show cross-sections of the box through the hasp and lock.

FIG. 125.—Chain Gauge. (From Hoyt and Grover's "River Discharge.")

258. Self-registering Gauge. Where very accurate and continuous records of the fluctuations of water surfaces are required, some form of self-registering gauge is used like that shown in Fig. 126.

The record of such a gauge (shown in Fig. 127) is made as follows: The centre pin is connected to a float on the surface of the water in such a way that its motion is proportional to the

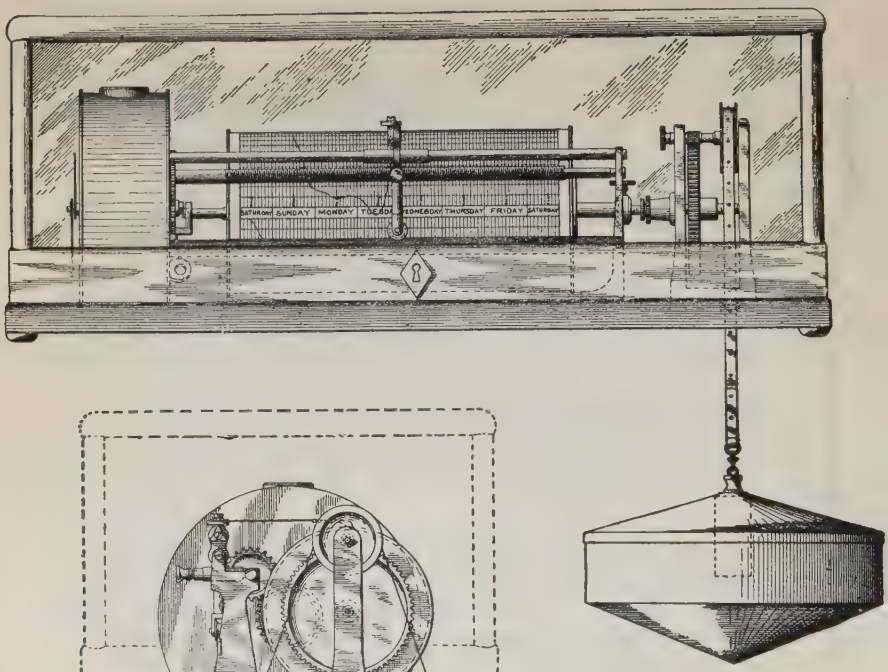


FIG. 126.—Frieze gauge, front view with float and counterweight in position; cover case in place and sample of an actual record for four days (Saturday to Tuesday, both inclusive). Fig. 126*a*, End view of same, from the right.

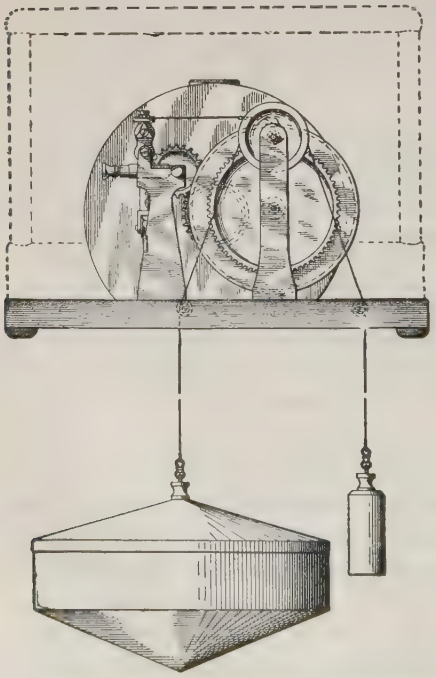


FIG. 126*a*.—End View.

change in elevation of the float. The pin is made to trace this motion on a continuous roll of paper made to move uniformly by a clock escapement and rolling up on one roll as fast as it unrolls on another. The two outer pencils record the time. The elevation of the surface at any time is given by the corresponding ordinate of the curve. Thus at 15 o'clock (3 p.m.) the elevation was 579.0 feet.

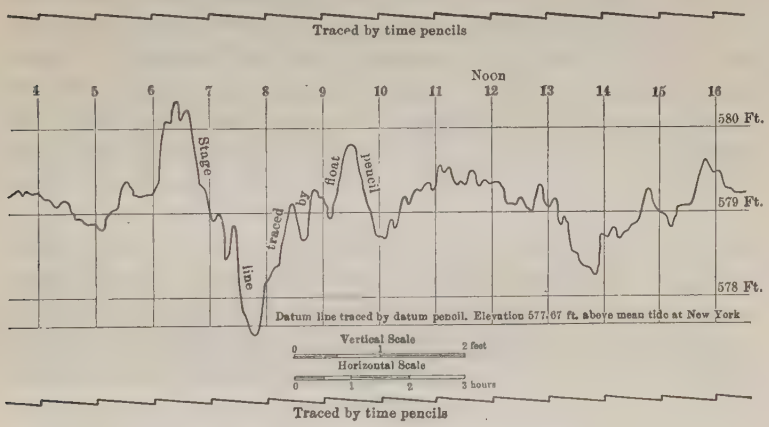


FIG. 127.—Automatic Record of Gauge.

259. Water Levels. The surface of still water is by definition a level surface. This fact is used to great advantage on the sea-coast, on lakes, ponds, and even on streams of little slope or on such as have a known slope. Thus, in finding the elevations of the Great Lakes above the sea level, the elevation above mean tide water of the zero of a certain water gauge at Oswego, N. Y., on Lake Ontario, was determined. Then the relative elevations of the zeros of certain gauges at Ports Dalhousie and Colborne, at the lower and upper ends of the Welland Canal respectively, were found by levelling between them, thus connecting Lake Ontario with Lake Erie. Lakes Erie and Huron were joined in a similar manner by connecting a gauge at Rock-

wood, at the mouth of the Detroit River with one at Lakeport, at the lower end of Lake Huron. Lakes Michigan and Huron were assumed to be of the same level on account of the small flow between them and the very large sectional area of the Straits of Mackinac. Finally, a gauge at Escanaba, on Lake Michigan, was joined by a line of levels with one at Marquette, on Lake Superior. This completed the line of levels from New York to Lake Superior, when sufficient gauge-readings had been obtained to enable *water levels* to be carried from Oswego to Port Dalhousie, on Lake Ontario; from Port Colborne to Rockwood, on Lake Erie, and from Lakeport, on Lake Huron, to Escanaba, on Lake Michigan. It was found that these water levels were very accurate. Relative gauge readings were compared for calm days, as well as for days when the wind was in various directions, and a final mean value found which in no case had a probable error as great as 0.1 foot.[1] More recent records of the Great Lakes Survey show even more accurate results, which in fact are better than the best precise levels.

A line of levels run along a lake shore or canal in calm weather should be checked at intervals by reading to the water surface, and in a topographical survey the stadia rod should frequently be held at the water surface, even when the body of water is a stream with considerable slope, as it gives a check against large errors even then, and at the same time gives the slope of the stream. Mean sea level at all points on the seacoast is universally assumed to define one and the same level surface. It is probable, however, that this is not strictly true. Wherever a constant ocean current sets steadily against a certain coast, it would seem that the water here must be raised by an amount equal to the head necessary to generate the given lost motion. If the current flows into an enclosed space, as the equatorial current into the Gulf of Mexico, or the tides into the Bay of Fundy, the water

[1] See Professional Paper No. 24, Primary Triangulation of the U. S. Lake Survey, 1882.

surface may rise much higher. There is some evidence that the elevation of mean tide in the Gulf of Mexico is two or three inches higher than that of the Atlantic at Sandy Hook. The evidence on this point is as yet insufficient to warrant any certain conclusion, however.

260. River Slope is a very important part of a river survey. Sometimes it is desirable to determine it for a given stretch of river with great care, in which case it is well to set gauges at the points between which the slope is to be found and connect them by duplicate lines of accurate levelling. The gauges are then read simultaneously every five minutes for several hours and the comparison made between their mean readings. For ordinary purposes the river slope may be determined with sufficient accuracy by simply reading the level or the stadia rod at water surface as the survey proceeds, daily readings of stage being made at permanent gauges at intervals of twenty miles or less along the river, depending on the accuracy desired. The water surface is then reduced to some particular stage.

In all natural channels the local slope is a very variable quantity. It is frequently negative for short distances in certain stages, and over the same short stretch of river it may vary enormously at different stages, and even for the same stage at different times. This is determined by the local channel conditions, and these are constantly changing in streams flowing in friable beds and subject to material changes of stage. Great caution must therefore be exercised in introducing it into any hydraulic formulæ for natural channels. It is usually expressed as a fraction, being really the natural sine of the angle of the surface to the horizon. That is, if the slope is one foot to the mile it is $\frac{1}{5280} = 0.000189$.

261. Transit and Stadia Locations. For locating detail soundings in a lake or river, the transit and stadia method has been found both cheap and accurate.[1] If the river is wider than

[1] See paper by Professor John L. Van Ornum, C.E., Jour. Asso. Eng. Soc., Vol. XIV, March, 1895.

1500 feet a party can be placed on each side of the river. Occasionally the transitmen can secure a check on the work by exchanging azimuths, each transitman also reading the distance between instruments. Each transit party would consist of the transitman and the recorder. The same party with the addition of rodmen could, if desirable, take the topography adjacent to the river, or this work and the setting of sounding stations might properly be given to a separate party. The boat party would consist of a recorder, steersman, leadsman, stadia-rodman, and two or more oarsmen. The leadsman is stationed in the bow of the boat with the rodman close by him, the latter always holding his rod vertical and facing the transitman. The leadsman takes the soundings and calls them out to the recorder at the same time giving a signal to the transitman. The latter in the case of a river survey would be able to read the stadia distance and at the same time measure the azimuth to the sounding pole or line, thus fixing the position of the sounding by polar co-ordinates. In order to identify the soundings with the proper locations, the transitman and leadsman exchange signals with different colored flags when a location and sounding are made, and this fact is made a part of the record. Time may be taken, but usually only at the beginning and end of each line is found necessary. The supporters of this method claim the following advantages for the transit stadia system:

(1) Economy.

(2) Elasticity or adaptation to conditions.

(3) More numerous locations of soundings.

(4) More accurate location than reading angles by two transits or by the use of a sextant in a boat.

(5) Adequate instrumental oversight of the work at the time.

262. Sounding by "Sweeping." In comparatively shallow water where the mean bottom is only a few feet greater than the draft of the larger boats, and in water where the bottom is extremely and abruptly irregular, there is no certainty that any system of sounding described above has developed all small

areas dangerous to navigation. This fatal defect has led in recent years to the introduction of a new system of "sweeping" the entire area.[1] At first "sweeping" was done chiefly with bars of iron or chains suspended at known depths from the ship. The weights of these bars limited the length of the sweep to about 100 feet, and the resistance of the bar was so great as to make the process a slow one. This led to the introduction of the wire sweep, 1200 feet long,[2] held up by a system of buoys (Fig. 128) and propelled by three launches. The outside

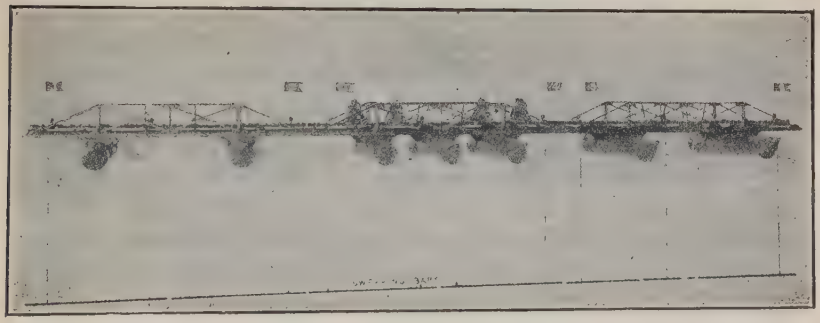


FIG. 128.—Sounding by Sweeping.

launches steer a sufficiently divergent course to keep the wire taut. Provision is made for instantly detecting any existing dangerous shoals, which are promptly marked in a manner to warn navigators. With one of these outfits, the U. S. Lake Survey has covered 26 square miles in one month.

[1] For details of this method, see Report of Chief Eng. U. S. A., for 1903, p. 2763, and also Report U. S. Coast and Geodetic Survey, 1905, p. 285.

[2] Designed by Mr. F. C. Shenehon, U. S. Asst. Engineer.

CHAPTER X

STREAM MEASUREMENT

The Discharge of Streams

263. General Statement of the Problem. Measurements of river discharge are usually made in connection with water supply problems, but sometimes they are made for other purposes, as for the location of bridge piers or harbor improvements. In the case of bridge piers the direction of the current at different stages must be known, so that the piers may be set parallel to the direction of the current. In finding the discharge of the stream or other channel the object may be:

(1) To obtain an approximate value of the discharge at the given time and place.

(2) To obtain an exact value of the discharge at the given time and place.

(3) To obtain a general formula from which to obtain subsequent discharges at the given place, or to test the truth of existing formulæ, or to determine the relative efficiency of certain appliances or methods.

It will be assumed that the second object is the one sought, and modified forms of the methods used to accomplish this may be chosen for other cases.

The *mean velocity* of a stream is by definition the total discharge in cubic feet per second divided by the area of the cross-section in square feet. This gives the mean velocity in feet

per second. Evidently this is the mean of the velocities of all the small filaments (as of one square inch in area) on the entire cross-section. If the velocities of these filaments could be simultaneously and separately observed and their mean taken, this would be mean velocity of the stream. It is quite impossible to do this; but the nearer this is approached, the more accurate is the final result. If, however, we could obtain by a single observation the mean velocity of all the filaments in a vertical plane, the number of necessary observations would be diminished without diminishing the accuracy of the result.

264. Factors Controlling the Accuracy of Stream Measurement. The determination of the rate of flow past a certain section of a stream is termed a "discharge measurement." This rate is the product of two factors, the cross-section and its mean velocity. The first factor can easily be measured with all needful accuracy. The mean velocity is a function of the surface slope, wetted perimeter, roughness of bed, etc., and is very difficult to measure accurately. This is chiefly due to the fact that the velocity at every point changes slightly even at a time when the stream has a constant discharge and hence constant mean velocity.[1] These changes in water currents, when closely observed, appear very complex. The water can be seen to boil and swirl, the movement being upward here, downward there, and at still other places diagonally across the stream.

If, therefore, only a few single measurements of velocity are made, there will be grave danger that their mean will not correctly represent the average of the entire cross-section.

Again, the accuracy of a stream measurement will depend in large measure upon a proper choice of gauging station. In selecting a permanent gauging station as many of the following features should be secured as possible:

(*a*) A straight reach of about 500 feet both above and below the station.

[1] This applies to smaller streams especially.

(*b*) The river bed should be permanent but not stony.

(*c*) The banks should be of sufficient height to contain the floods.

(*d*) The section should be free from the influence of bridge piers and mill dams.

Again, experience has shown that *fluctuations* in the height of the water during a measurement influences the accuracy of the work.

The appreciation of the importance of this work has, in recent years, brought about marked improvements in both the instruments and the methods used in stream measurement. These methods fall naturally into three classes:

(1) Velocity method. (2) Slope method. (3) Weir method.

265. Methods of Measuring Mean Velocity. The two most common methods of observing the velocity of water is by the use of rod floats and by current metres. Because of floating in the water, the rod float gives the velocity directly. The current metre gives the velocity only indirectly. The impact of the water causes the current metre wheel to revolve, but in order to compute the velocity of the water it becomes necessary to calibrate the current metre, i.e., to find the ratio between the velocity of the wheel and the corresponding velocity of the water.[1]

266. Current Metre Method. A current metre for measuring the velocity of water comprises two essential parts: 1st, a wheel which when suspended in the water is made to revolve by the pressure of the water, and 2d, a device for recording or indicating the number of revolutions of the wheel. Current metres may be divided into two classes, direct action and differential action, depending upon whether the water, in revolving the wheel, does or does not exert a force which tends to retard the motion of the wheel. Wheels of the first class consist of flat or warped surface vanes, set on a horizontal axis. The

[1] There are other methods of obtaining the velocity of water, but all have little application in stream measurement.

Haskell and Fteley metres are good examples of this class. (See Figs. 129 and 130). In this class of metre the friction of the wheel on its bearing increases as the velocity decreases, hence for extreme slow velocities it is not as well suited.

The wheels of differential metres are made up of a series of cups which revolve on a vertical axis. Because of the resistance of the water on the convex side of the cups this type of metre revolves slower than the first.

The Price and Ellis metres, shown in Figs. 131, 132, 133, and 134, are good examples of this class. The essentials of a good current metre are:

(*a*) Simplicity of construction, with no delicate parts likely to get out of order; (*b*) small area of resistance to the velocity of the water; (*c*) a simple and effective device for indicating the number of revolutions; (*d*) adaptable for use under all conditions.[1]

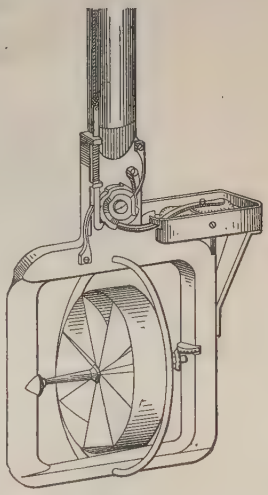


FIG. 129.—Fteley Metre.

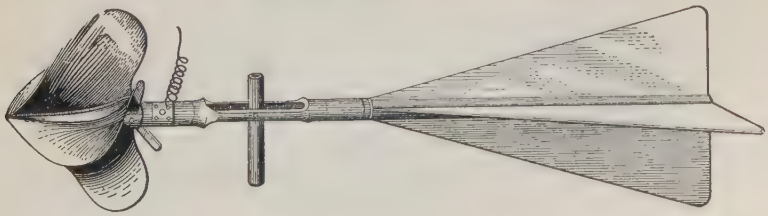


FIG. 130.—Haskell Metre.

267. Rating the Metre. A metre is rated by conducting it through water over a measured base, and noting the time and

[1] The above classification and description of metres is condensed from a paper by Mr. J. C. Hoyt, C.E., Proceedings Am. Soc. C.E., August, 1909.

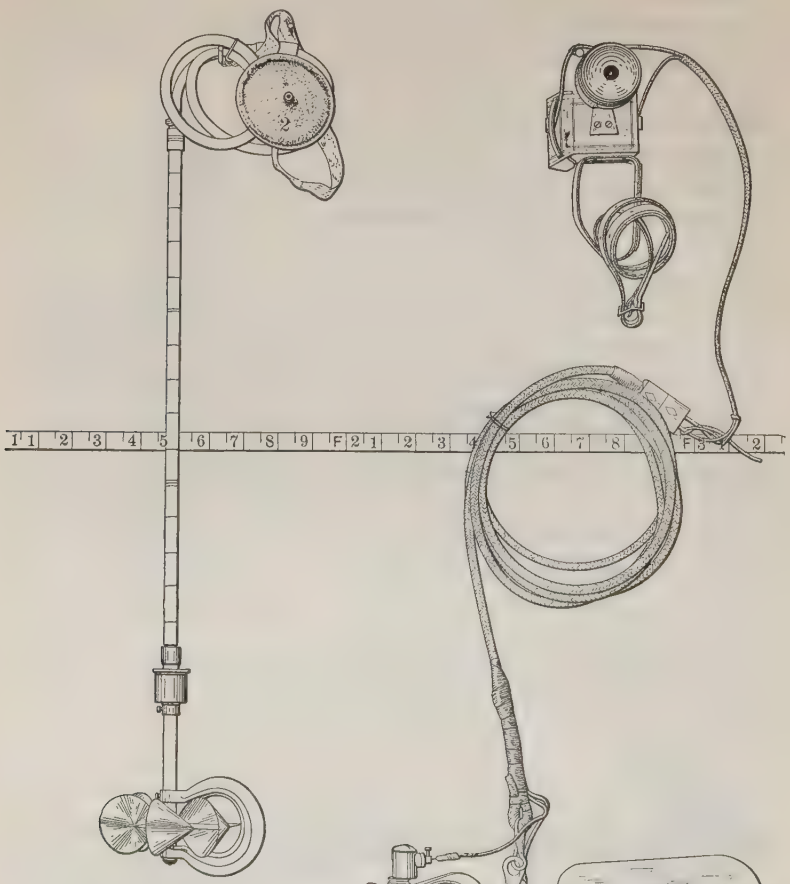


FIG. 131.—Price Acoustic Metre.

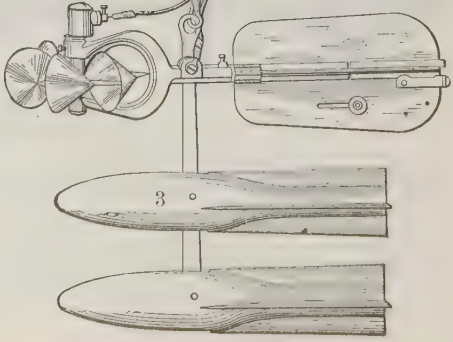


FIG. 132.—Price Electric Metre.

the number of revolutions. From these data the velocity in feet per second and the revolutions per second are computed for each trial. Many trials are made at speeds varying from the least which will turn the wheel, to five or more feet per second.

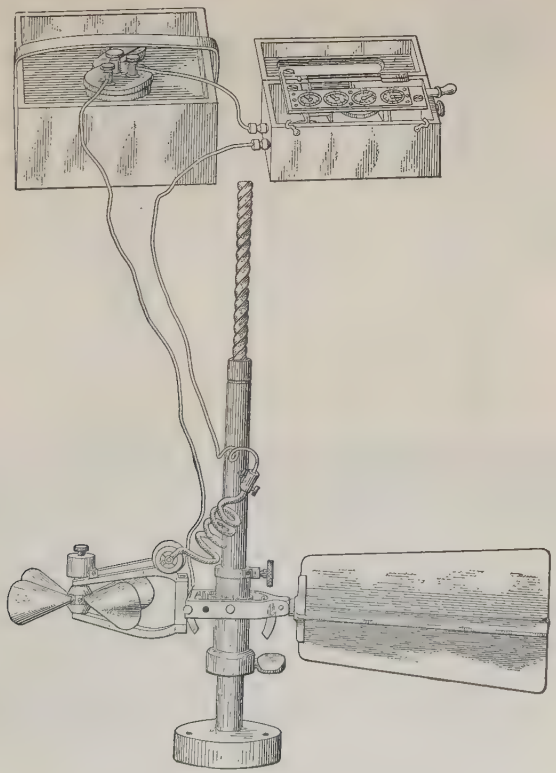


FIG. 133.—Ellis Metre.

Commonly the metre is mounted to the bowsprit of a boat, as shown in Fig. 135, which is then towed along the base line at varying speeds. More accurate results may be obtained by suspending the metre from a car which moves on a permanent

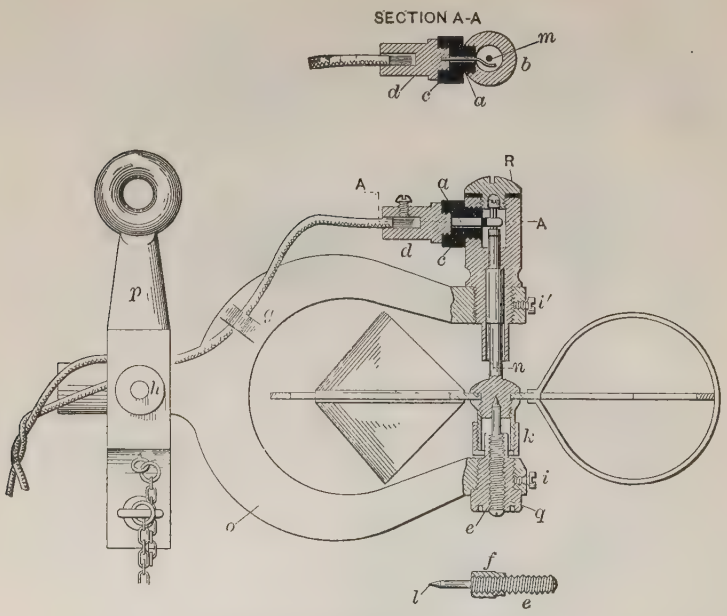


FIG. 134.—Price Metre Cross-section.

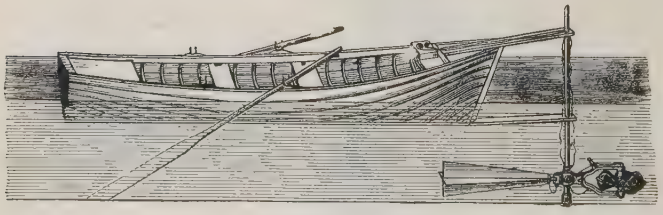


FIG. 135.

track. (See Fig. 136.) The following table gives the record of a metre rating, and will explain itself: [1]

[1] Made by C.E. students in the University of Wisconsin Summer School of Surveying.

FIELD NOTES FOR RATING PRICE CURRENT METRE, No. 768, JUNE, 1907, PORTAGE, WIS.

Trial.	Time, Seconds.	Number Revolutions.	Velocity per Sec. Feet.	Revolutions per Second.		Remarks.
1	183.1	34.25	0.546	0.187		Price metre No. 768
2	178.4	37.0	.562	.207	R_m=0.197	Portage, June, 1907
3	202.6	39.25	.492	.193	V_m=0.534	Base line 100 ft.
4	186.5	37.5	.537	.201		U. S. Canal Lock
5	60.0	36.75	1.660	.611		
6	51.7	42.0	1.934	.812	R_m=0.726	Party:
7	56.4	43.0	1.773	.762	V_m=1.776	Chief, L. S. Smith.
8	57.6	41.5	1.736	.720		Timer, Larson.
9	40.0	42.5	2.50	1.062		Recorder, Milstein.
10	28.9	43.5	3.46	1.505	R_m=1.410	Ass't, Robinson and
11	29.3	43.0	3.41	1.467	V_m=3.265	Swenholt.
12	27.1	43.5	3.69	1.605		
13	19.7	43.0	5.07	2.183		
14	20.2	43.0	4.95	2.123	R_m=2.190	
15	19.0	44.0	5.26	2.316	V_m=5.062	
16	20.1	43.0	4.97	2.139		

FIG. 136.—Current-metre Rating Station at Los Angeles, Cal.

268. The Graphical Solution of a Metre Rating. There are two ways of finding the constants of the metre—the graphical and the analytical.

The first is much simpler than the second and can be made quite as accurate by the use of the following plan:

Plot the velocities in feet per second as abscissas and revolutions per second as ordinates of the rating curve, and draw a mean curve through the platted points.[1] For velocities of two feet or less per second, use a scale of 5 inches per foot and for velocities above 2 feet per second, use 1 inch per foot. The scale for revolutions per second, for velocities of 2 feet or less should be 10 inches per revolution per second, while for velocities greater than 2 feet, it should be 2 inches per revolution per second.[2] Fig. 137 shows a rating curve, but very much reduced in size. A velocity table can be prepared from such a curve or the curve may be used directly to reduce revolutions per second into velocity in feet per second.

269. The Analytical Solution. An examination of rating curves shows that with few exceptions they are either straight lines or may be considered as made up of two or more straight lines, the equation of which is

$$V = aR + c, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where V is the velocity of the water and R represents the revolutions of the wheel per second. The coefficient a is the ratio of the revolutions per second to the corresponding velocity in feet per second. The constant c represents the velocity which will just overcome the friction of the wheel. If the observed velocity and the corresponding observed revolutions per second be substituted for V and R in Eq. (1) we have an obser-

[1] If the observations are taken in groups, as shown in the preceding field notes, the mean curve can be drawn through the mean of each group with great accuracy.

[2] This requires the construction of two curves, one for high and one for low velocities. See Fig. 137. The curve on the right is for low velocities.

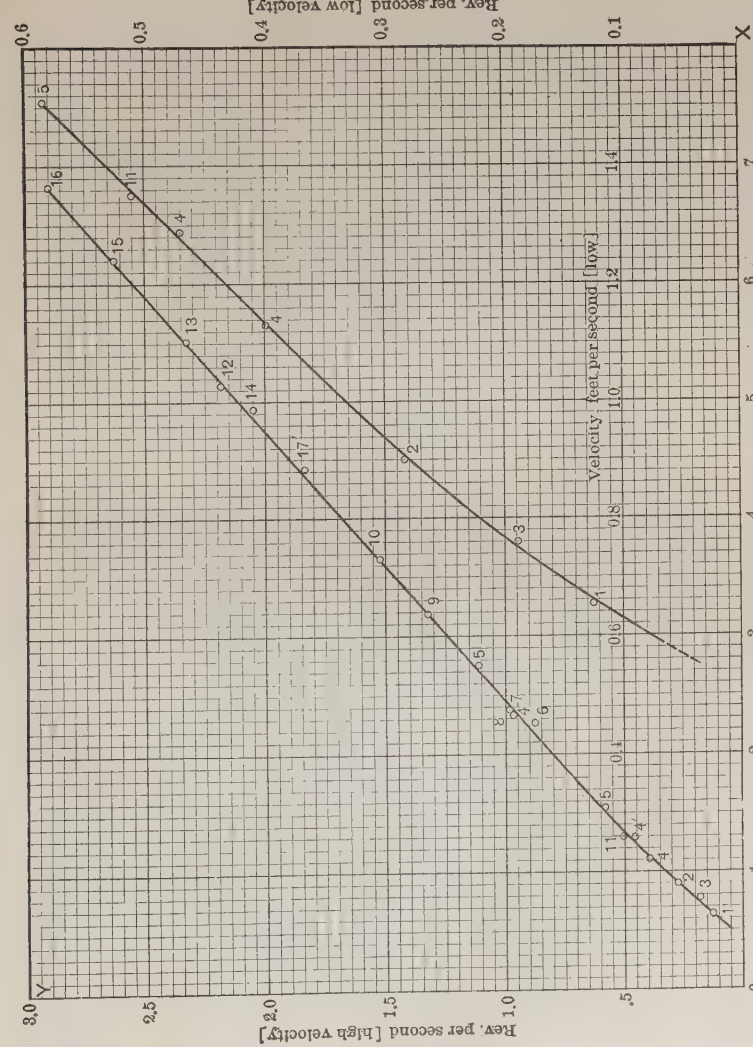


FIG. 137.—A Rating Curve of a Current-metre.

vation equation containing only two unknown quantities a and c. The substitution of a second set of observed values for V and R will give a second and simultaneous equation. From these two equations values for the constants a and c can easily be determined.

By reference to the field notes on page 351 it will be noted that the observations have been taken in four groups corresponding to mean velocities of $V=0.534$, $V_2=1.776$, $V_3=3.265$, $V_4=5.062$ (feet per second). The corresponding mean revolutions of the wheel per second are $R=0.197$, $R_2=0.726$, $R_3=1.410$, $R_4=2.190$. Substituting these observed values of V and R in Eq. (1) we have the following observation equations:

$$0.534=0.197\,a+c, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$1.776=0.726\,a+c, \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$3.265=1.410\,a+c, \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$5.062=2.190\,a+c. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Solving Eqs. (2) and (3) we obtain values of a and c for slow velocities while Eqs. (3) and (4) give values for the middle and (4) and (5) values for the upper part of the curve.

A velocity rating table can now be prepared in which the argument may be either the revolutions of the wheel per second, or the times of 100 revolutions. An example of the first method is the rating table on the following page.

270. **Variations of Velocity in a Given Cross-section.** The velocity varies at different depths in any given vertical section and the mean velocity also varies with its distance from the shore. The variations in the velocity at different depths have been found not to vary with the stage of water except between narrow limits. This fact makes possible an accurate determination of the mean velocity of the stream with only a comparatively small number of velocity measurements, provided that the latter are properly distributed in the cross-section.

RATING TABLE FOR METRE No. 264 [1]

From observations made June 22, 1909, at Portage, Wis. by C. W. Esau, computations made by C. W. E. Description of metre—Price Electric Metre submerged 3 feet.

Revolutions per Second.	Velocity per Second.	Revolutions per Second.	Velocity per Second.	Revolutions per Second.	Velocity per Second.	Revolutions per Second.	Velocity per Second.	Revolutions per Second.	Velocity per Second.
0.00	....	0.50	1.15	1.00	2.38	1.50	3.55	2.00	4.67
.05	0.9	.55	1.27	1.05	2.50	1.55	3.67	2.05	4.78
.10	.21	.60	1.39	1.10	2.61	1.60	3.79	2.10	4.89
.15	.32	.65	1.51	1.15	2.72	1.65	3.90	2.15	5.00
.20	.45	.70	1.63	1.20	2.84	1.70	4.01	2.20	5.11
.25	.54	.75	1.75	1.25	2.95	1.75	4.12	2.25	5.22
.30	.68	.80	1.87	1.30	3.07	1.80	4.23	2.30	5.33
.35	.80	.85	1.99	1.35	3.19	1.85	4.34	2.35	5.44
.40	.92	.90	2.14	1.40	3.31	1.90	4.45	2.40	5.55
.45	1.03	.95	2.26	1.45	3.43	1.95	4.56	2.45	5.66

271. Velocity in the Vertical Plane under Open Conditions. The variation of the velocity in the vertical under *open* conditions can be best understood by a study of Fig. 138 [2] which is based on actual measurements. In these curves the measured velocities, expressed in per cent of the mean, are platted as abscissas, and the depths in per cent of the total depth are platted as ordinates. The three curves represent rivers with (*a*) smooth, (*b*) rough beds and (*c*) under general conditions. It will be noted from Fig. 138 that the mean of 78 vertical curves under general conditions locates the mean velocity almost exactly at .6 of the depth, while the curves taken in rivers with smooth or rough beds vary only 2 or 3 per cent from this condition.

Again these curves show that the average of the velocities observed at .2 and .8 of the depth, even more closely gives the mean velocity of the vertical section. And in general these curves

[1] Taken from the records of the University of Wisconsin Summer School of Surveying.

[2] Taken from Water Supply Paper No. 76, U. S. G. S., by H. A. Pressey.

prove that only a single measure of velocity is necessary to compute the mean of the vertical section, provided the proper coeffi-

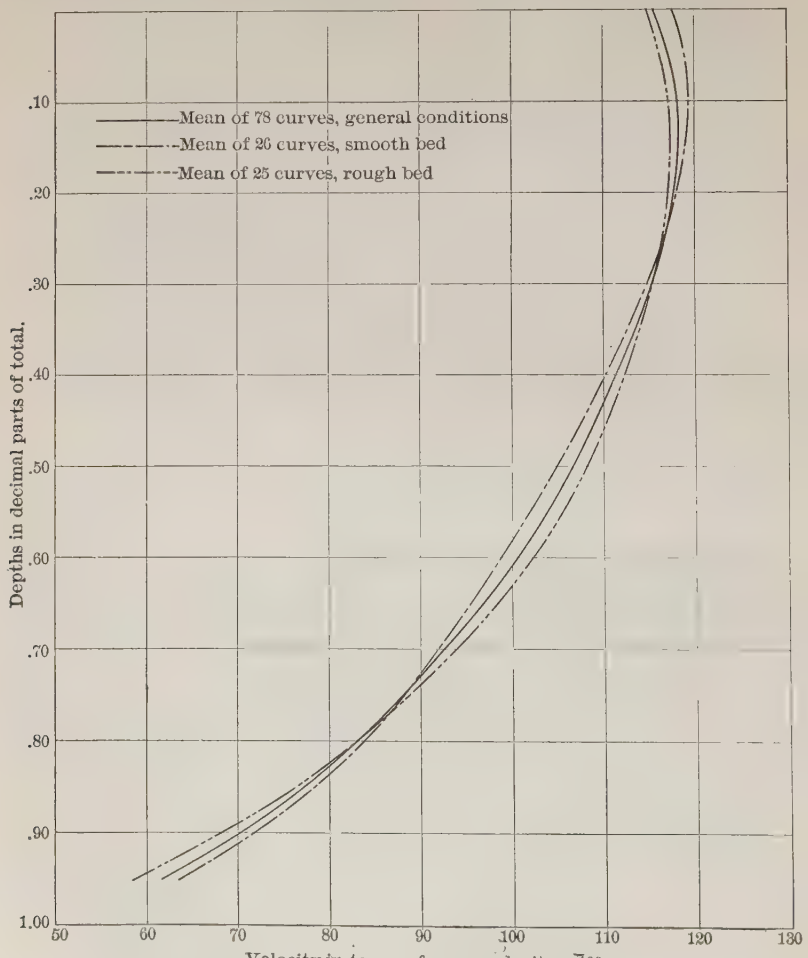


FIG. 138.—Velocities in the Vertical Plane.

cient be known. Thus to reduce mid-depth velocities to mean velocities, we multiply the former by .94. See also Fig. 143.

272. Velocity in the Vertical Plane under Ice Conditions. When a river channel is covered with ice, the above relations are changed, because the increased friction between ice and water retards the velocity of the water adjacent.

Fig. 139 shows in a striking manner the effect of ice cover in retarding the velocity near the surface. A study of Fig. 140[1] which shows a normal vertical curve both with ice cover and

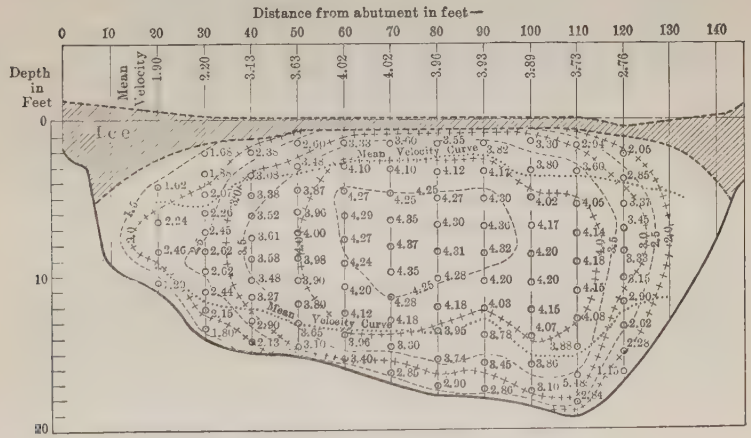


FIG. 139.—Velocity with Ice Cover. From U. S. Geological Survey Water Supply, Paper No. 76.

without, will assist in understanding the effect of this increased friction. In both cases it will be seen hat the average of the velocities at .2 and .8 depth gives the mean of the section. In general it may be stated that the work of measuring an ice-covered river is more difficult and less accurate than under open conditions.

273. Velocity from Bank to Bank. Not only does the velocity of the water vary at different depths as shown in Arts. 270 and 271, but it varies with the distance from the shores or

[1] Taken from U. S. Geological Survey Bulletin, No 76.

bridge piers. This is shown in Figs. 141 and 142, where the distances from the shore are platted as abscissas and the mean velocity for the corresponding points are platted as ordinates of the horizontal mean velocity curve. In measuring the flow of ordinary-

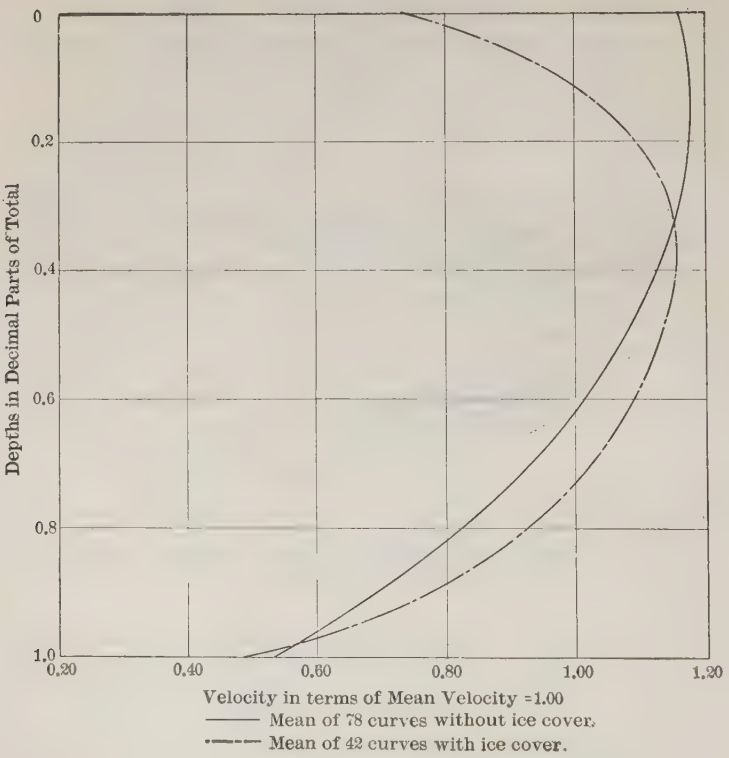
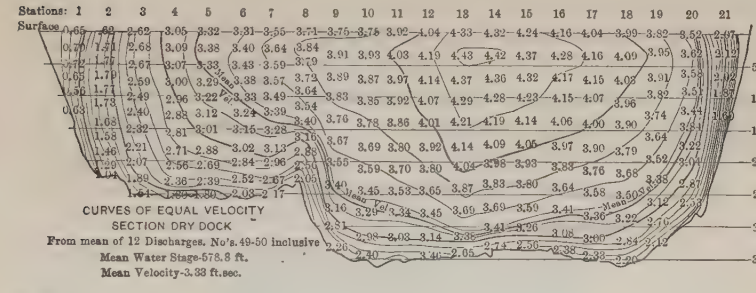


FIG. 140.—Velocity in Vertical Plane.

sized rivers, no attempt is made to find any one section whose mean velocity will be the mean of the entire river. In the case of very wide rivers like the St. Lawrence and St. Clair, the usual methods fail because of the length of time required to cover the

FIG. 141.—Observed Velocities in the Cross-section.


Stations: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
Surface
5 Ft.
10 "
15 "
20 "
25 "
30 "
35 "
CURVES OF EQUAL VELOCITY
SECTION DRY DOCK
From mean of 12 Discharges. No's. 49-50 inclusive
Mean Water Stage-578.8 ft.
Mean Velocity-3.33 ft.sec.

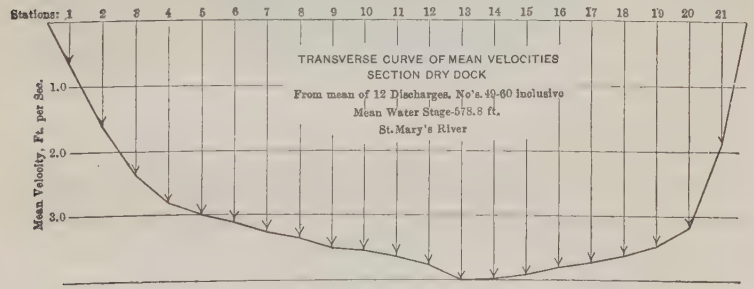

Stations: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
TRANSVERSE CURVE OF MEAN VELOCITIES
SECTION DRY DOCK
From mean of 12 Discharges. No's. 49-60 inclusive
Mean Water Stage-578.8 ft.
St. Mary's River
Mean Velocity, Ft. per Sec.
1.0
2.0
3.0

FIG. 142.—Mean Horizontal Velocity Curve.

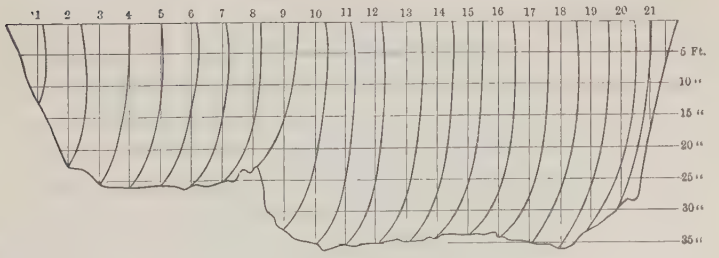


FIG. 143.—Vertical Velocity Curves. (Velocity in Vertical Section.)

entire river cross-section, and other methods must be adopted based upon the horizontal mean velocity curve. See Art. 281.

Rod Floats

274. The Use of Floats in Determining Velocity and Discharge. Where a single or only occasional measurement of the flow of a river is to be made the use of floats is deemed preferable. Floats are of three general types: surface, subsurface, or double floats and tube or rod floats. The general method of their use is as follows: A suitable site is selected on a long straight reach having a fairly uniform cross-section and free from eddies or cross currents. A base line 50 to 300 feet in length is then measured on the bank parallel to the stream, which, if not too wide, is spanned by two parallel wires conspicuously divided by tags into equal parts of from 5 to 25 feet, depending upon the size of the river. These wires are drawn from both ends of the base line and perpendicular to it. Careful soundings to tenths of a foot are then taken along the wires at each tag. If the river is too wide to use this method, then the soundings can be made and located by some one of the various methods explained in Arts. 239–246. The floats are then dropped from a boat into the stream at least 50 feet above the upper cross-section. An observer in the boat notes the station on each cross-section under which the float passes and returns the float to the upper section. A second observer with stopwatch notes the time taken for the float to travel the measured distance between the cross-sections. Surface floats consisting of chips are frequently used in reconnaissance work, but not even approximate results can be expected from such a plan if the wind be blowing.

275. By Subsurface Floats. The ideal subsurface float consists of a large intercepting area maintained at any depth in a vertical position by means of a fine cord joined to a surface float of minimum immersion and resistance, which bears a signal-flag. As good a form as any, perhaps, for the lower float, or

intercepting plane, consists of two sheets of galvanized iron set at right angles, and intersecting in their centre lines, as shown in Fig. 144. There are cylindrical air cavities along the upper edges and lead weights attached to the lower edges of the vanes. These serve to give the desired tension on the connecting cord and to maintain the float in an upright position, even though the cord is drawn out of the vertical by faster upper currents.

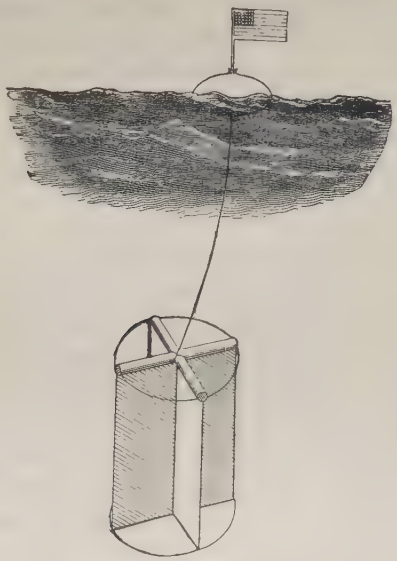


FIG. 144.—Submerged Float.

The vanes should be from 6 to 15 inches in breadth and from 8 to 20 inches high, according to the size of the stream. The circular ribs serve simply to hold the vanes in place. The upper float is hollow, cylindrical in plan, and carries a small flag. The tension on the cord should be from one to five pounds, according to the size of the floats. The cord itself should be of woven silk and as small as possible, so as to exercise a minimum influence on the motion of the lower float. Wire

is not suitable for this purpose, as it kinks badly in handling. The theory is that the lower float will move with the water which surrounds it, and that the upper float will be accelerated or retarded according as the surface current is slower or faster than that at the submerged float. The velocity of the current at any depth can thus be determined by running the lower float at this depth and observing the time required for the upper float to pass between two fixed range lines at right angles to the direction of the current about 200 feet apart. The floats are started about 100 feet above the upper range line, and picked up after having passed the lower range. Two transits are usually used for locating and timing the floats, one being set on each range. When the float approaches the upper range the observer on this line sets his telescope on range and calls "ready" as the float enters his field of view. The other observer then clamps his instrument and follows the float with the aid of the slow-motion or tangent screw. When the float crosses the vertical wire of the upper instrument he calls "tick," and the lower observer reads his horizontal angle. He then sets his telescope on the lower range while the upper observer follows the float with his telescope, and the operation is repeated to obtain an intersection on the lower range. One or two timekeepers are needed to note the time of the two "tick" calls, the difference being the time occupied by the float in passing from the upper to the lower range line. Both these signals are sometimes transmitted telegraphically to a single timekeeper. When the angles are plotted the path of the float is also obtained.

The chief objection to double floats is the uncertainty as to whether the cord is vertical, and the consequent uncertainty as to the position of the subsurface float. Another objection is the modifying effect of the surface float and the cord upon the velocity of the lower float. In general, better and cheaper results can be obtained by the use of rod or tube floats.

276. Rod or Tube Floats. These consist of long cylindrical tin tubes or light wooden poles, 1 to 2 inches in diameter, and of

various lengths, weighted at the bottom, so that they will float vertically with only 2 or 3 inches exposed above the water surface. Such rods will integrate the velocities in a vertical filament and provided they extend nearly to the bottom can be made to give reliable results. Care should be taken to use the longest float possible at each station. This length should not be less than .9 of the depth.

277. Computation of Discharge by the Rod-Float Method. The first step is to compute the area in square feet of each subdivision of the cross-section. Each of these is multiplied by the mean velocity of such partial cross-section as given by the corresponding floats. This product represents the discharge in cubic feet per second through the respective partial cross-section and the sum of all such products gives the total discharge of the stream.

In extensive experiments made by James B. Francis [1] and others it was found that the measurement of velocities by rod floats were nearly always large as compared with measurements made by a standard weir. This was due to the fact that the rod, because of not reaching the bottom, was not affected by the extreme low velocity near the stream bed, and hence indicated too great a velocity. He found that this effect could be corrected by multiplying the discharge as obtained with the floats, by a coefficient C as follows:

[2] $Q = CQ'$, in which,

Q = actual discharge,

Q' = discharge as determined by rod floats.

C = coefficient = $1 - 0.116\ (\sqrt{D} - 0.1)$ and

D = ratio of $\dfrac{\text{distance of bottom of float from bottom of stream}}{\text{depth of stream}}$.

[1] See "Lowell Hydraulic Experiments," by J. B. Francis, pp. 146–208. See also Water Supply Paper No. 95, by U. S. G. S., by E. C. Murphy, p. 54.

[2] This correction should be applied to each partial discharge.

If preferred, the same computation can be made graphically as follows: (See Fig. 145.) Plat the two cross-sections of the stream over each other and draw the average section between them. Also draw the length of all floats in their average positions on the mean cross-section. Under each float lay off the velocity, corrected by the above coefficient, as ordinates, and draw a mean velocity curve through all such ordinates. Finally multiply each ordinate of the velocity curve by the corresponding ordinate of the mean cross-section and plat the products as ordinates to the discharge curve. Draw through such ordinates a smooth curve and its area will represent to scale, the discharge of the stream in cubic feet per second.

278. The Advantages of Rod Floats are as Follows:[1] (1) They interfere less with the natural motion of the water; (2) they measure velocity directly; (3) they can be used in streams of any size; (4) they are not affected by silt or weeds; (5) they measure only forward velocity; (6) they are easily made and cheap.

Their disadvantages are:[2] (1) they are affected to some extent by wind; (2) they no not give mean velocity, but velocity of impulses; (3) they cannot be used in very deep streams or in streams of rough or irregular bed; (4) they are expensive to operate.

279. Discharge Measurements with Current Metre. Discharge measurements of a river may be made from a bridge, cable, boat or by wading, but the same method is used for all. A section is first selected at right angles to the stream, which is then divided into a convenient number of partial areas. These division points, 2 to 20 feet apart, are marked and referenced so they can be recovered later. Accurate soundings are then made at each division point. If made from a bridge the lead is lowered until the bottom is reached and a point is marked

[1] See Proc. Inst. Civil Engineers, Vol. LXXI, p. 1.

[2] Water Supply Paper No. 64, U. S. G. S., p. 17.

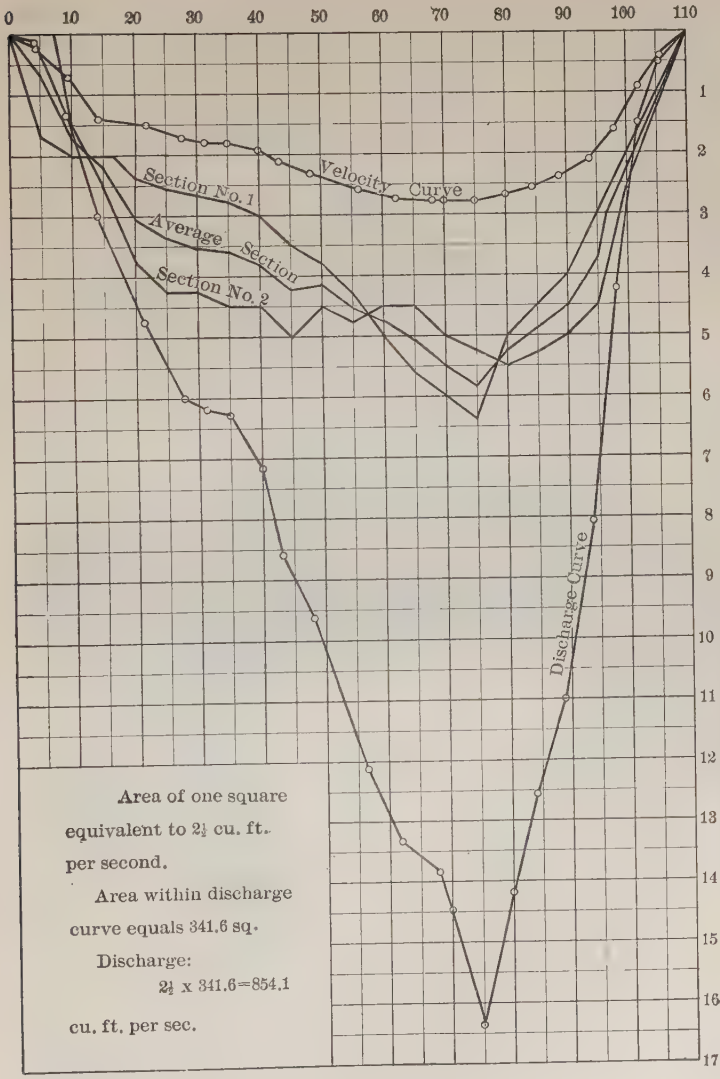


FIG. 145.—Graphic Solution.

on the cable opposite a fixed point on the bridge; the weight is then raised until it just touches the water, when the length of cable that has passed the fixed point is measured, this being the depth of the water. The metre is now lowered into the water and the mean velocity of the vertical filament determined by one of the following methods:

(*a*) Vertical velocity curve method; (*b*) two-tenths and eight-tenths method; (*c*) six-tenths depth method and (*d*) sub-surface method.

(*a*) Here the velocity is measured just below the surface, at each fifth or tenth of the depth and at or as near the bottom as

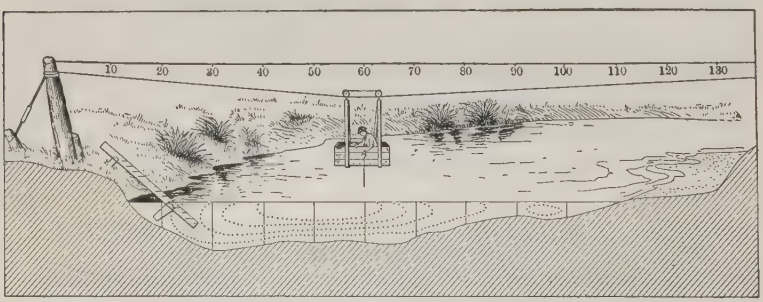


FIG. 146.—River Gauging from a Cable.

possible. From these measurements when platted, the mean velocity of each section can be computed. Figs. 141–142 show the results of such measurements. This method, although accurate, is not applicable to large rivers because of the time consumed in making such measurements, and is chiefly valuable in determining the coefficients required to reduce values obtained by shorter methods to the true result.

(*b*) In this method the velocities at each section are observed only at 0.2 and 0.8 of the depth. A study of many hundreds of vertical velocity curves shows that the mean of the velocity at 0.2 and 0.8 of the depth gives very uniformly the mean velocity of the section and this is true both in open and in frozen con-

ditions. This method is generally used by the U. S. Geological Survey. See Fig. 140.

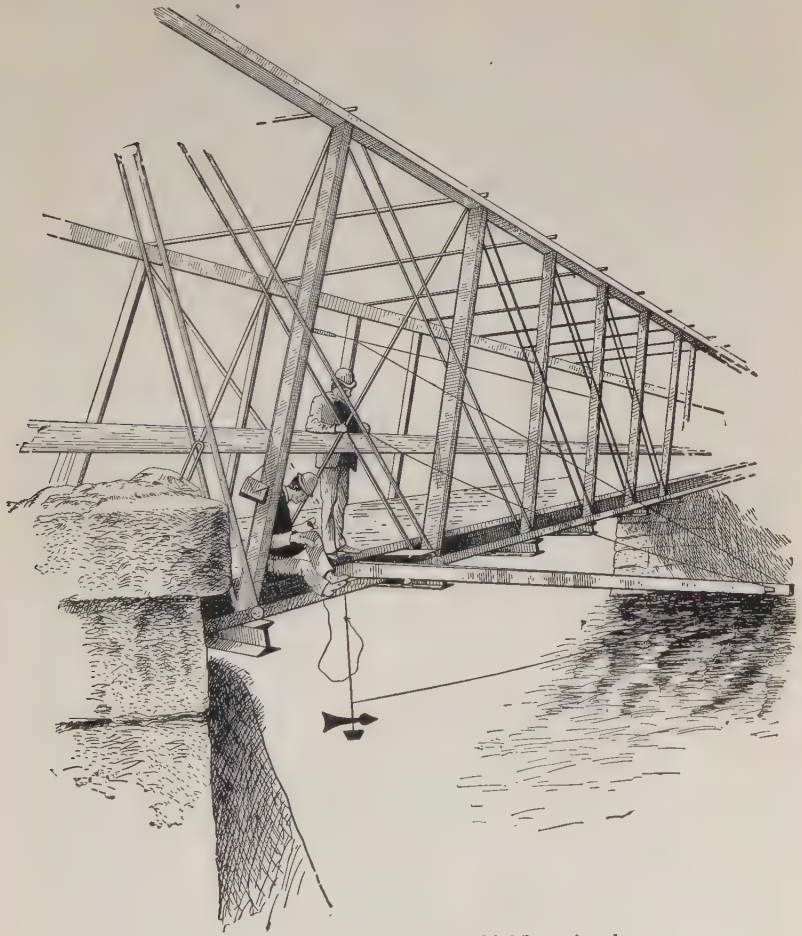


FIG. 147.—Use of Stay Line to hold Metre in place.

(*c*) This method provides for a single observation at each section and that at 0.6 of the depth. Its justification rests upon

the fact that in the several hundred of vertical velocity curves examined, the mean depth of the mean velocity was found almost exactly 0.6 of the depth. While much faster than (*b*) this method is not quite so accurate.

(*d*) The metre in this method is held from half to one foot below the surface and the resulting velocity multiplied by some coefficient to reduce it to the mean velocity. The method is commonly used at times of a freshet when the high velocities preclude the taking of the velocity much below the surface. The coefficient varies from 78 to 98 per cent, depending upon the depth and velocity of the stream. For average streams in moderate freshets, a coefficient of 90 per cent may be used; in floods a coefficient of 90 to 95 per cent; and for streams at ordinary stages from 85 to 90 per cent.[1]

280. Computations of Current Metre Measurements. These computations are based upon the following field work, soundings at measured intervals across the stream and the determination of the mean velocity in the vertical at each sounding point. The mean velocity, area and discharge for each partial area between soundings are computed independently, the total discharge being the sum of such partial discharges. A very simple and accurate formula for making these computations is the following:

$$\text{Discharge} = l_1\left(\frac{d_0+d_1}{2}\right)\left(\frac{V_0+V_1}{2}\right) + l_2\left(\frac{d_1+d_2}{2}\right)\left(\frac{V_1+V_2}{2}\right) + l_n\left(\frac{d_{n-1}+d_n}{2}\right)\left(\frac{V_{n-1}+V_n}{2}\right).^2$$

where d_0, d_1, d_2, d_n are the depths and V_0, V_1, V_2, V_n are the velocities at the measuring points, and l_1, l_2, l_n are the measured distances between such points. It will be noted that the prod-

[1] Paper by J. C. Hoyt, Proceedings Am. Soc. C.E., August, 1909.

[2] For proof of accuracy see paper by J. C. Stevens, Eng. News, June 25, 1908.

uct of l and the mean depth gives the partial cross-section and that the product of this by the mean of the corresponding observed velocities gives the partial discharge.

On page 370 will be found the field notes and office computations [1] of a discharge measurement of a river made from a bridge. A simultaneous measurement by the rod-float method gave a result about 5 per cent in excess of the meter measurement.

281. Current Metre Measurement of Large Rivers. The usual methods of using current meters are not adapted to very wide rivers like the St. Clair, Detroit or Niagara rivers because of the excessive time required. The following method [2] gives accurate results in a fraction of the time. Two metres at least are required, one of which makes a record at a so-called index point, located near the centre of gravity of the subdivision of the river cross-section, say 100 feet or more wide, while the second one determines the mean velocity of frequent vertical sections on either side of the index point. From this field work the proper coefficient can be determined which will reduce the velocity observed at the index point to the mean velocity of the entire width of 100 feet or more. In this way a river 2600 feet wide can afterwards be accurately measured by observing the velocity at the 26 index points and reducing same by the measured coefficients. If the cross-section remained the same the coefficients would need determining only once. Usually they have been found to change only slightly for moderate changes of surface level.

282. Slope Method of Measuring Stream Flow. This method involves an accurate measurement of the slope of the water surface and the cross-section of the stream together with a knowledge of the roughness of the sides and bottom.

---

[1] Made by C. E. students in the University of Wisconsin Summer School of Surveying.

[2] See Report of Chief Engineer, U. S. A., for 1900, p. 5341; also Jour. W. Soc. Eng., p. 451, 1899.

CURRENT METRE NOTES

Date June 28, 1909, 2.M. Stream, Baraboo River.
Party, Jessup and Beasley. Locality, Upper Bridge.
Metre No. 265. Gauge height, beg. 3.76 end 3.78, mean 377.
Total area, 299.8. Mean velocity, 2.02. Discharge, 606.5.

Observations.					Computations.						
						Velocity.					
Dist. from Initial Point.	Depth	Depth of Observation.	Time in Secs.	Revolutions.	Revolutions per Second.	Mean in Vertical.	Mean in Section.	Mean Depth.	Width.	Area.	Discharge
0	0										
							.72	1.25	5	6.2	4.5
5	2.5	1.5	50	30	.6	1.43					
							2.04	3.25	5	16.2	33.2
10	4.0	2.4	50	56	1.12	2.66					
							2.88	3.88	5	19.4	55.9
15	3.75	2.25	50	65	1.30	3.10					
							3.00	3.88	5	19.4	58.2
20	4.0	2.4	50	61	1.22	2.90					
							3.00	4.00	5	20.0	60.0
25	4.0	2.4	50	65	1.30	3.10					
							2.98	3.75	5	18.8	55.8
30	3.5	2.1	50	61	1.20	2.86					
							2.93	3.50	5	17.5	51.3
35	3.5	2.1	50	63	1.26	3.00					
							2.69	3.60	5	18.0	48.4
40	3.7	2.2	50	50	1.00	2.38					
							2.36	3.65	5	18.2	43.1
45	3.6	2.1	50	49	.98	2.37					
							2.12	3.75	5	18.8	39.8
50	3.9	2.2	50	40	.80	1.96					
							1.80	3.95	5	19.8	35.6
55	4.0	2.4	50	36	.72	1.71					
							1.62	4.00	5	20.0	32.4
60	4.0	2.4	50	32	.64	1.52					
							1.45	3.88	5	19.4	28.1
65	3.75	2.1	50	29	.58	1.38					
							1.25	3.87	5	19.4	24.2
70	4.0	2.4	50	23	.47	1.12					
							.93	3.75	5	18.8	17.4
75	3.5	2.1	50	16	.31	.74					
							.73	3.00	5	15.0	10.9
80	2.5	1.5	50	15	.30	.73					
							.62	2.12	5	10.6	6.6
85	1.75	1.0	50	11	.22	.52					
							.26	.87	5	4.3	1.1
90	0										
										299.8	606.5

Comp. by Beasley, Chk. by Jessup.

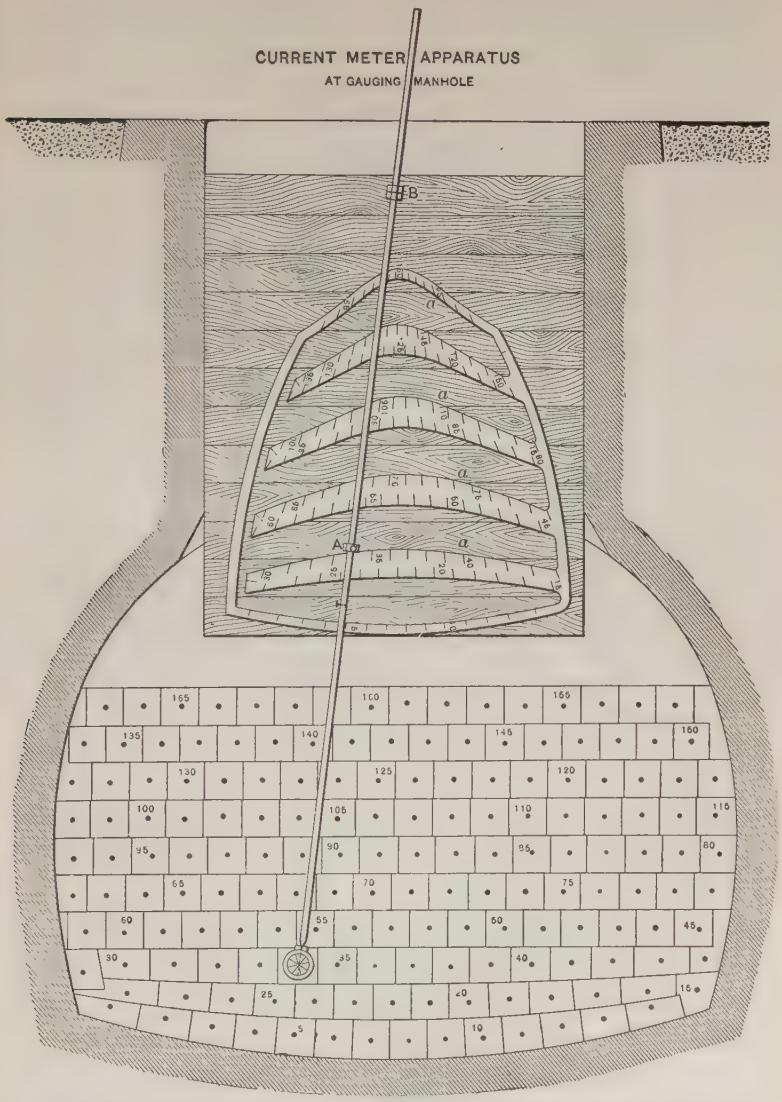


FIG. 148.—Method of Measuring the Flow in a Sewer by a Current Metre.

The most commonly used formula is the following, devised by Kutter in 1869:

$$V = \left[\frac{41.6 + \frac{1.811}{n} + \frac{.00281}{s}}{1 + \frac{(41.6 + .00281)}{s} \frac{n}{\sqrt{Rs}}} \right] \sqrt{Rs}.$$

In this formula V = velocity, in feet per second; s = surface slope $= \frac{h}{l}$, h being the fall of the surface in the distance l; R = hydraulic radius $= \frac{F}{w}$, F being the cross-sectional area and w the wetted perimeter;[1] and n a ratio whose value varies from 0.009 for a well-planed wooden channel to 0.035 for a channel overgrown with weeds or covered with stones.

This is sometimes condensed in the equation $V = C\sqrt{RS}$.

In determining the slope, precise levels are run between hook gauges (Art. 289) located at cross-sections, as far apart as practicable. The slope is determined on both sides of the river and the mean taken as the slope of the river. Sufficient cross-sections must be taken between the gauges to give an accurate mean of the area and the hydraulic radius.

The disadvantages of the method are due to the difficulty of determining the slope with sufficient accuracy and of selecting the proper value of n in the formula. The method should not be used when either rod floats or current metres are available. The method is sometimes the only one which can be used at times of extreme flood.

283. General Conclusions. In all forms of stream measurement, it should be kept in mind that only by taking the greatest care in the selection of the gauging section, and in the planning and executing of the field work, can reliable results be secured. Fortunately except in rare cases an error of 5 per cent or even more would not be significant for the purposes of the work.

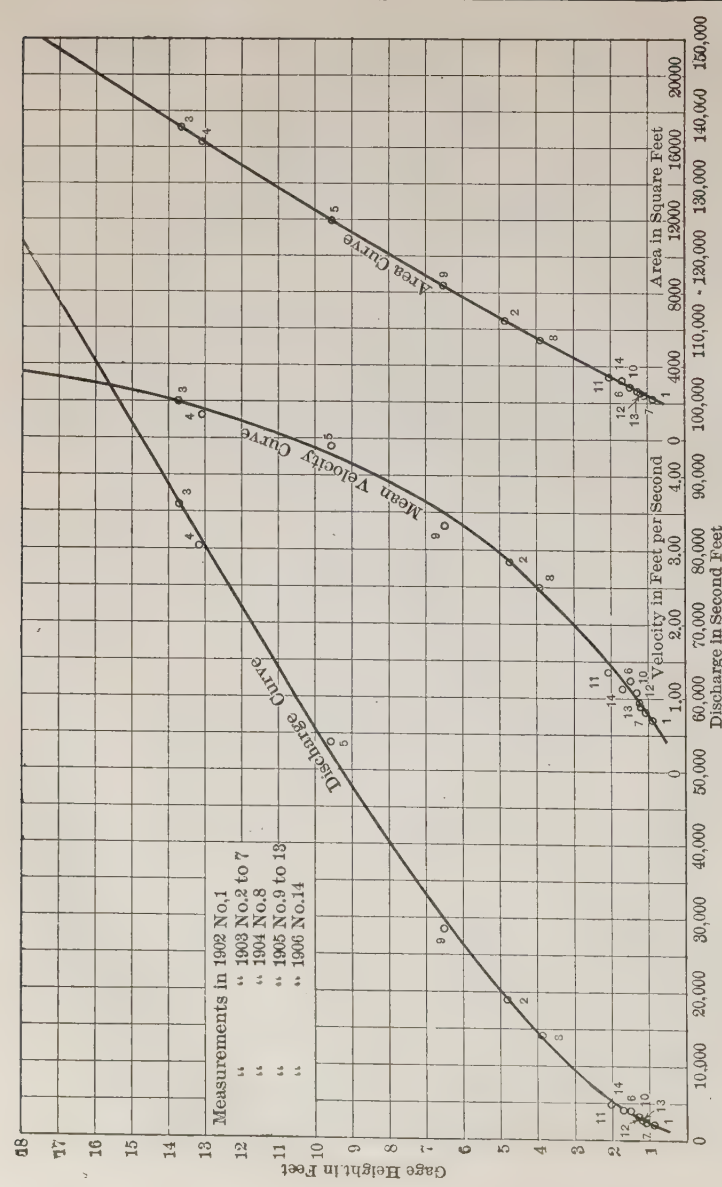
[1] The wetted perimeter of a channel is that part of its cross-section which is in contact with the water.

Where stream measurements are to be made, only occasionally, the rod-float method will probably continue to be the most widely used. Under these circumstances it is likely to prove more reliable than the current metre because of the failure to either properly calibrate or operate the metre. But where systematic stream measurements are to be made the current metre can easily be made the cheapest and most reliable method, while under frozen conditions it furnishes the only practical method of stream measurement.

284. Application of Stream Measurement. When the stream bed is fairly permanent and otherwise complies with the requisites for a good gauging station (see Art. 264) a comparatively few stream measurements only are required to enable an accurate estimate of the daily flow, provided care is taken to establish a gauge and to read it once or more daily.

The stream measurements should be taken at various stages from low to extreme high water. With such data a rating curve of the river can be made by platting the gauge heights as ordinates and the corresponding discharges as abscissa. An example of such a curve is shown in Fig. 149. This figure also shows the corresponding mean velocity and area curves. These curves are all platted as functions of the gauge heights. The mean velocity at any given gauge height is obtained by dividing the measure discharge by the measured area of the corresponding stream cross-section. The area curve can be best measured by careful soundings taken at low water, supplemented by measurements with tape and level of that part of the cross-section between such low-water stage and the highest stage expected. When the cross-section is platted, the area corresponding to each additional foot of gauge height can be calculated and the area curve drawn. Such an area curve is of use specially at high-water stages, when accurate soundings, especially the bottom half, are very difficult to secure.

The curve of mean velocity shows the relation between gauge height and the mean velocity of the current. An effort should be



From Hoyt and Grover's "River Discharge," by permission.

FIG. 149.—Discharge Curve of a River.

made to measure the mean velocity at enough gauge heights to accurately determine the curve. Sometimes, however, it becomes necessary to determine the curve from limited or discordant values of the velocity.[1] It is plain that the area curve and the

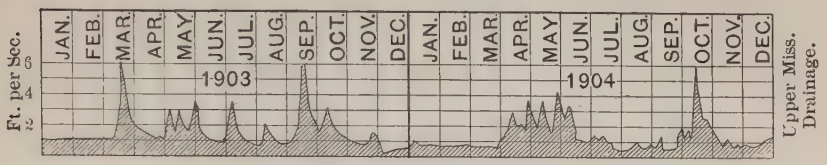


FIG. 150.—Hydrograph of a River.

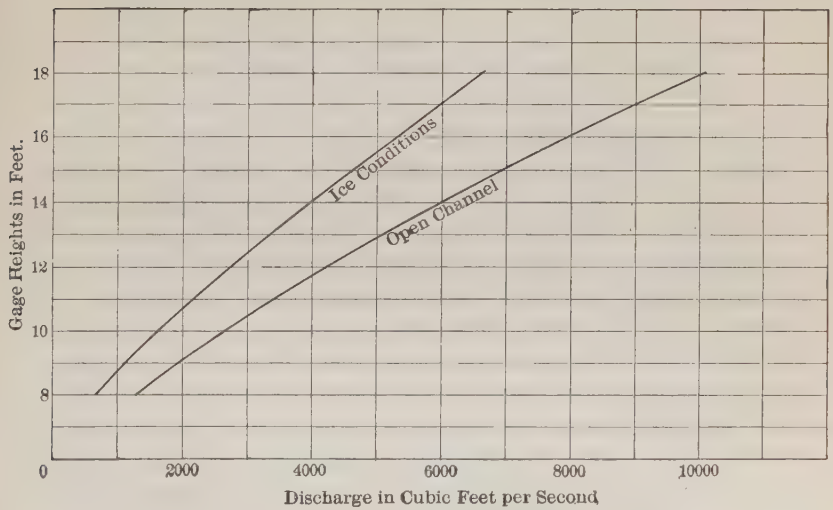


FIG. 151.—Rating Curves for Open and Ice Conditions.

curve of mean velocity will give the discharge at any gauge height.

If preferred, a rating table for the station can be prepared whose argument is the gauge height. By the use of such a rating

[1] For a full discussion of this whole matter see Hoyt and Grover's "River Discharge." p. 84, published by John Wiley & Sons.

curve or table, a hydrograph of the river can be constructed by platting the daily discharges as ordinates and the days of the month as abscissas. Such a hydrograph is shown in Fig. 150.

On account of the reduced cross-section and the increased surface friction in rivers under ice condition, the discharge at such times is not the same function of the gauge height as in open conditions. In fact, the discharge at any particular gauge height varies as the thickness of the ice changes. For this reason it is very difficult to secure accurate rating tables of northern rivers during the winter. Fig. 151 shows clearly how great a change in rating curve is sometimes caused by ice cover.

Weir Method of Measuring Stream Flow

285. General Statement. For measuring the flow of a small stream, a weir furnishes the most accurate method. Such weirs are usually rectangular and are made sharp-crested, that is of a form which allows the water to touch the inside edge only. The back of the weir should be a vertical plane surface. If the ends of the weir are placed at some distance from the sides of the channel, see Fig. 152, the stream of water will be contracted laterally. For complete contraction this distance should be at least three times the height of the weir. If the ends are flush with the sides of the channel, see Fig. 153, there will be no end contractions. The only variables concerned in either form of weir are the head on the weir and the velocity of approach, both of which can be accurately and easily measured. The disadvantages of the weir method are:

(1) The cost of constructing the weir, or the difficulty of finding one in use whose shape closely resembles one whose coefficient is known, and

(2) The limited range of values of the coefficient of a weir of any shape.

Weirs are either sharp crested or wide crested.

286. Sharp-Crested Weirs.[1] A sharp-crested weir is one which is entirely cleared by the water in passing over it, as in Fig. 154. A wide-crested weir shown in Fig. 155.

Fig. 154.

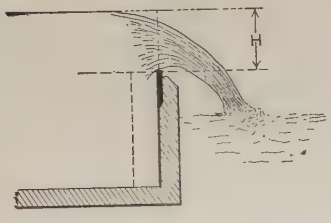
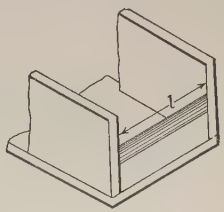
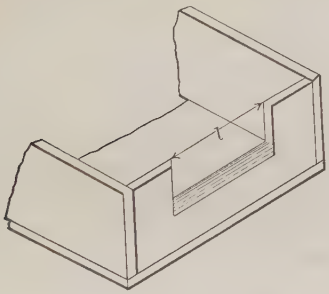
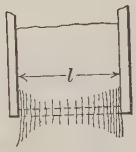


Fig. 153.—Without Contractions.







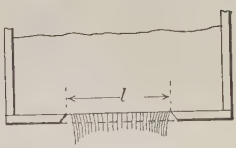


Fig. 152.—With Contractions.

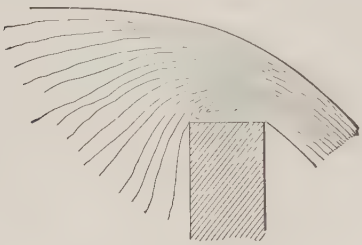


Fig. 155.

[1] The following discussion on weirs, Art. 286 to Art. 288, is taken by permission from Turneaure and Russell's text, "Public Water Supply."

The formula for discharge from a rectangular weir is

$$Q = c \cdot \tfrac{2}{3} l \sqrt{2g} H^{\frac{3}{2}},$$

where l = length of weir and H = height of water on the weir. If the channel is small, the velocity of approach will have an appreciable effect upon the discharge. If h = head due to this velocity $= \frac{v^2}{2g}$, then this factor is commonly taken account of by the use of the equation

$$Q = c \cdot \tfrac{2}{3} l \sqrt{2g} (H + nh)^{\frac{3}{2}},$$

in which n is a coefficient varying from 1 to 1.5. The velocity of approach can be estimated with sufficient accuracy by first determining the value of Q with the term nh omitted, then using this value of Q to determine V, and then a more accurate value of Q, etc. From a careful analysis of the experiments of Francis, Fteley and Stearns, and others, Hamilton Smith has adopted values for n of 1.4 for weirs with end contractions (see Fig. 152), and $1\frac{1}{3}$ for weirs with contractions suppressed (see Fig. 153). From his study of the experiments referred to he has also derived the values for the coefficient c as given in the two tables on the following page.

Mr. James B. Francis from his elaborate series of experiments at Lowell [1] on a weir 10 feet long, and operating under heads of 0.4 to 1.6 feet derived the formula for weirs without end contractions,

$$Q = 3.33 l H^{\frac{3}{2}},$$

and with contractions,

$$Q = 3.33 (l - 0.2H) H^{\frac{3}{2}}.$$

[1] "Lowell Hydraulic Experiments," New York, 1871.

TABLE I

TABLE OF COEFFICIENTS FOR CONTRACTED WEIRS—(SMITH)

Effective Head in Feet.	Length of Weir in Feet.						
	0.66	1	2	3	5	10	19
0.1	0.632	0.639	0.646	0.652	0.653	0.655	0.656
0.15	.619	.625	.634	.638	.640	.641	.642
0.2	.611	.618	.626	.630	.631	.633	.634
0.2	.605	.612	.621	.624	.626	.628	.629
0.3	.601	.608	.616	.619	.621	.624	.625
0.4	.595	.601	.609	.613	.615	.618	.620
0.5	.590	.596	.605	.608	.611	.615	.617
0.6	.587	.593	.601	.605	.608	.613	.615
0.7	.....	.590	.598	.603	.606	.612	.614
0.8	.....	.....	.595	.600	.604	.611	.613
0.9	.....	.....	.592	.598	.603	.609	.612
1.0	.....	.....	.590	.595	.601	.608	.611
1.2	.....	.....	.585	.501	.507	.605	.610
1.4	.....	.....	.580	.587	.504	.602	.609
1.6	.....	.....	.....	.582	.591	.600	.607

TABLE II

TABLE OF COEFFICIENTS FOR SUPPRESSED WEIRS—(SMITH)

Effective Head in Feet.	Length of Weir in Feet.						
	19	10	7	5	4	3	2
0.1	0.657	0.658	0.658	0.659			
0.15	.643	.644	.645	.645	0.647	0.649	0.652
0.2	.635	.637	.637	.638	.641	.642	.645
0.25	.630	.632	.633	.634	.636	.638	.641
0.3	.626	.628	.629	.631	.633	.636	.639
0.4	.621	.623	.625	.628	.630	.633	.636
0.5	.619	.621	.624	.627	.630	.633	.637
0.6	.618	.620	.623	.627	.630	.634	.638
0.7	.618	.620	.624	.628	.631	.635	.640
0.8	.618	.621	.625	.629	.633	.637	.643
0.9	.619	.622	.627	.631	.635	.639	.645
1.0	.619	.624	.628	.633	.637	.641	.648
1.2	.620	.626	.632	.636	.641	.646	
1.4	.622	.629	.634	.640	.644		
1.6	.623	.631	.637	.642	.647		

If there be but one contraction, $0.1H$ is to be used instead of $0.2H$. To take account of the velocity of approach, the formulas become,

$$Q = 3.33\, l[(H+h)^{\frac{3}{2}} - h^{\frac{3}{2}}],$$

and

$$Q = 3.33\, (l - 0.2H)[(H+h)^{\frac{3}{2}} - h^{\frac{3}{2}}],$$

in which $h = \frac{v}{2g}$. In these formulas the unit of length is the foot.

The most elaborate series of experiments on weirs ever made is that which has been carried out by Bazin, and which is reported in the Annals des Ponts et Chausseés for the years 1888–98. The formula deduced by him for sharp-crested weirs with no end contractions is in foot units,

$$Q = \left(.405 + \frac{.00984}{h}\right)\left(\left(1 + .55\frac{h}{p+h}\right)^2\right) lh\sqrt{2gh},$$

in which p = height of weir above the bottom of the channel, and h is the actually observed head on the weir. The velocity of approach is taken account of in the formula. This formula gives slightly larger values for the discharge than the Francis formula.[1]

287. Submerged Weirs. A submerged weir is one at which the level of the water below the weir is above its crest, there being, however, a certain definite fall in passing the weir, as shown in Fig. 156. As the result of an analysis of the experiments of Francis and of Fteley and Stearns, Mr. Herschel [2] adopts the following formula for discharge, for submerged, sharp-crested weirs without end contractions:

$$Q = 3.33 l(nH)^{\frac{3}{2}},$$

[1] See a valuable paper by G. W. Rafter, including exhaustive discussion, in Trans. Am. Soc. C.E., Dec., 1900.

[2] Trans. Am. Soc. C.E., 1885, Vol. XIV, p. 194.

where l = length of weir;

H = height on the weir on the upstream side;

n = a coefficient, depending on the ratio of the head on the downstream side H' to the head H.

The values of n are given in Table III.

TABLE III

VALUES OF n FOR SUBMERGED WEIRS—(HERSCHEL)

$\frac{H'}{H}$	n	$\frac{H'}{H}$	n	$\frac{H'}{H}$	n	$\frac{H'}{H}$	n
.00	1.000	.20	.985	.45	.912	.70	.787
.02	1.006	.25	.973	.50	.892	.75	.750
.05	1.007	.30	.959	.55	.871	.80	.703
.10	1.005	.35	.944	.60	.846	.90	.574
.15	0.996	.40	.929	.65	.819	1.00	.000

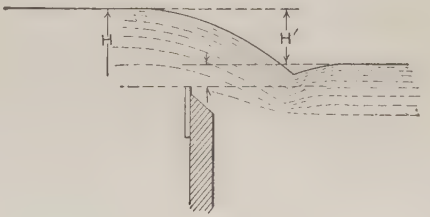


FIG. 156.—Submerged Weir.

288. Weirs of Varying Section. In many cases it is desirable to determine the flow of a stream by measurements taken of the height of water flowing over some dam or weir; and, on the other hand, in the design of waste-weirs some method of estimating their capacity is essential. The law of flow over such weirs is very complicated, and the only accurate way of determining the constants for any particular case is by means of experiments on a section of the same form as the one in question. If this is impossible, the best substitute for it is to use constants which have been determined for a weir agreeing as closely in form as may be to the one under consideration.

Here again Bazin's work is of the greatest value. He employed in his experiments weirs of a great variety of form. The heights used were one metre and one and one-half metres; and the heads employed reached a maximum of about one-half metre. The end contractions were suppressed in all cases, and below the weir the sides of the channel were continued so that in general there was not perfectly free access of air beneath the sheet of water. In the following Table IV are given values of the coefficient C in the formula $Q = Clh^{\frac{3}{2}}$, for several forms of weirs, as deduced from Bazin's experiments. The head h is in all cases the actually observed head on the weir. The coefficients as given were obtained from curves plotted from Bazin's results.[1]

More recently Mr. G. W. Rafter and Professor G. S. Williams have made for the United States Board of Engineers on Deep Waterways an important series of experiments at the Cornell University Hydraulic Laboratory.[2] In these experiments the height of water on the weir was carried to a value of about 5 feet, the weir being 6.58 feet long and from 4.6 to 4.9 feet above the bottom of the channel. Free access for the air beneath the sheet of water was provided. In Table V are given coefficients for several of the forms experimented on, for use in the formula $Q = Clh^{\frac{3}{2}}$, where h is again the actually observed head on the weir. These values were deduced from the data and discharge curves given in Professor Williams' discussion of Mr. Rafter's paper above referred to.

It is important to note that, on account of the difference in conditions with respect to the entrance of air below the sheet of water in the two series of experiments above described, it appears that in those forms having a steep downstream face the coefficients from Bazin's experiments are much the higher. For slopes greater than one-to-one the differences are small. Note

[1] See Rafter's paper in Transactions Am. Soc. C.E., Dec., 1900, for a compilation of Bazin's coefficients.

[2] Trans. Am. Soc. C.E., Dec., 1900, pp. 266, 316.

TABLE IV

VALUE OF THE COEFFICIENT C IN THE FORMULA $Q = Cl^{\frac{3}{2}}$, FROM BAZIN'S EXPERIMENTS

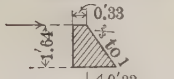
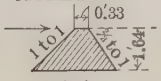
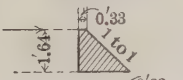
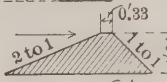
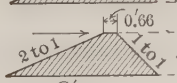
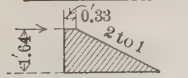
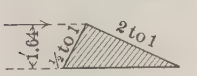
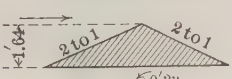
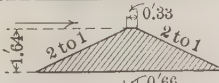
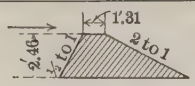
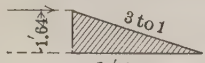
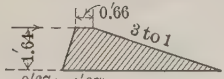
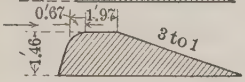
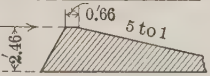
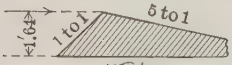
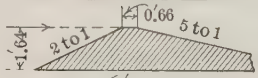
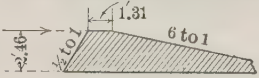
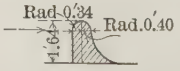
Number	SECTION OF WEIR	Height of Weir in Feet.					
		0.4	0.6	0.8	1.0	1.2	1.4
1		4.06	4.20	4.20	4.18	4.15	4.12
2		3.40	3.70	3.88	3.98	4.05	4.10
3		3.18	3.37	3.54	3.68	3.80	3.88
4		3.26	3.70	4.06	4.31	4.00	3.80
5		3.41	3.74	4.00	4.20	4.18	4.24
6		3.20	3.68	4.02	4.26	4.42	4.20
7		3.47	3.72	3.87	4.01	4.13	4.23
8		3.20	3.38	3.56	3.74	3.85	3.95
9		3.14	3.44	3.67	3.83	3.83	3.86
10		2.68	2.90	3.13	3.35	3.52	3.64
11		3.74	3.81	3.88	3.93	3.98	4.02
12		3.86	3.87	3.92	4.02	4.08	4.15
13		3.44	3.64	3.80	3.94	4.04	
14		3.18	3.34	3.49	3.65	3.79	3.90

TABLE IV—*Continued*

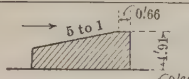
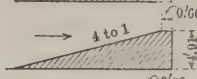
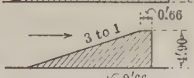
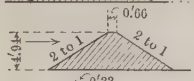
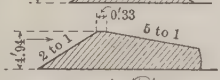
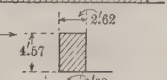
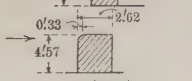
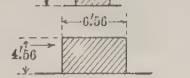
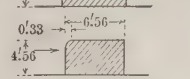
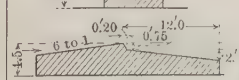
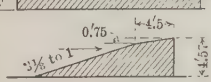
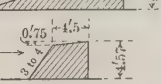
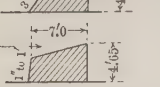
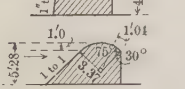
Number	SECTION OF WEIR	Height of Weir in Feet.					
		0.4	0.6	0.8	1.0	1.2	1.4
15		2.77	2.83	2.92	3.05	3.16	3.29
16		3.08	3.33	3.49	3.54	3.60	3.64
17		3.24	3.40	3.60	3.76	3.86	3.97
18		3.04	3.12	3.19	3.26	3.36	3.48
19		3.06	3.10	3.14	3.18	3.20	3.23
20		2.83	3.00	3.15	3.27	3.38	3.47
21		3.56	3.54	3.57	3.62	3.66	3.70
22		3.12	3.23	3.34	3.47	3.56	3.64
23		2.80	2.85	2.90	2.97	3.05	3.14
25		2.86	2.88	2.93	2.96	2.98	3.00
26		3.40	3.76	4.08	4.36	4.56	4.56

also that weirs with wide crests give lower coefficients than those with narrow crests; also that low weirs give higher coefficients than high weirs.

289. Depth of Water on the Weir. This is the principal function of the discharge; it is the difference of elevation between

Table V

VALUES OF THE COEFFICIENT C IN THE FORMULA $Q = Clh^{\frac{3}{2}}$.
(FROM THE CORNELL UNIVERSITY EXPERIMENTS.)

NUMBER	SECTION OF WEIR	HEAD ON WEIR IN FEET							
		1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
1		3.51	3.37	3.33	3.31	3.29	3.23	3.16	3.14
2		3.51	3.37	3.33	3.31	3.29	3.25	3.20	3.21
3			3.76	3.68	3.68	3.70	3.75	3.83	
4			3.68	3.71	3.81	3.90	4.00	4.06	
5		3.81	3.61	3.68	3.65	3.72	3.80	3.93	
6			2.85	2.89	3.03	3.16	3.30	3.50	3.71
7			3.12	3.20	3.31	3.45	3.63	3.78	3.92
8				2.45	2.49	2.51	2.56	2.59	2.67
9		2.93	2.85	2.82	2.90	2.90	2.92	2.95	
10		3.29	3.23	3.34	3.42	3.45	3.47	3.52	
11		3.51	3.52	3.47	3.55	3.62	3.68	3.72	
12		3.59	3.62	3.54	3.52	3.57	3.55	3.69	
13		3.81	3.61	3.57	3.63	3.62	3.67	3.71	3.80
14		3.37	3.20	3.50	3.60	3.70	3.84	3.95	

the top of the weir and the surface of the water at a distance above the weir equal to about $2\frac{1}{2}$ times the height of the weir above the bottom of the channel. Evidently this is a quantity which cannot be directly measured. The best way of measuring this quantity is as follows: At a convenient point arrange a closed vertical box which connects by a free opening with the channel at about mid-depth at a point some 6 feet above the weir. The water will then stand in this box at its normal elevation, unaffected by the slope towards the weir. The elevation of this water surface is determined by means of a *hook gauge*, Fig. 157, which consists of a metallic point turned upwards and made adjustable in height by means of a thumbscrew. When the point of the hook comes to the surface of the water it causes a distorted reflection. The elevation of the water surface can be found in this way with extreme accuracy. The difference of elevation between the point of the hook and the crest of the weir can then be determined with a level and rod. This difference is H in the preceding formulas.

Fig. 157.

290. The Use of Dams as Weirs. Dams which answer the following requirements[1] afford an accurate and most practicable method of stream measurement.

First. The dam or weir must have sufficient height so that back water will not interfere with the free fall over the same; otherwise the dam will be available for purposes of measurement only during stages when no such interference exists.

Second. The dam or weir body must be so constructed that no leak of appreciable size will occur during the time when it is utilized for measuring purposes.

[1] From Prof. D. W. Mead's "Water Power Engineering."

Third. The abutments of the dam or sides of the weir must be so constructed as to confine the flow over the dam at all stages; otherwise the weir will be useless for measurements during flood conditions.

Fourth. The crest of the weir must be level and must be kept free from obstructions caused by floating logs or ice.

Fifth. The crest of the dam or weir must be of a type for which coefficients for use in the ordinary weir formula have been determined. (See Art. 288.)

Sixth. If the dam has an adjustable crest, great care must be used to prevent leakage along such crest and to keep a complete and detailed record of the condition of the crest during the time of the observations.

Seventh. If water is diverted around the dam, which is usually the case when a dam is built for power purposes or for navigation the diverted water must be measured or estimated and added to the amount passing over the dam. Such diverted water can sometimes be measured by a weir or current meter.

When such water is used in water wheels, an accurate record of the gate opening of the wheels can be kept, from which the amount of water used in the wheels can be estimated if the wheel's discharge has been calibrated or if the wheel is of some well-known type.[1] The conditions for the accurate determination of weir discharge should be such as not to involve the use of low heads of less than 6 inches over broad crested dams.

Measurements by means of a weir or dam have the general advantage of continuity of record during the periods of ice and flood and the disadvantage of uncertainty of the coefficient to be used in the weir formula, of complication by the diversion of water around the dam, and the interference of flow by the occasional lodgement of material, or of injury to the crest.

[1] See Water Supply and Irrigation Paper No. 180, Turbine Water Wheel Tests and Power Tables, by R. E. Horton.

291. The Miner's Inch. This is an arbitrary standard both as to method and as to volume of water discharged. It rests on the false assumption that the volume discharged is proportional to the area of the orifice under a constant head above the top of the orifice. Its use grew out of the necessities of frontier life in the mining regions of the West, and should now be discarded in favor of absolute units. The miner's inch is the quantity of water that will flow through an orifice 1 inch square, under a head of from 4 to 12 inches, according to geographical locality. Even if the head above the top of the orifice be fixed, and a flow of 144 miner's inches be required, the volume obtained would be 3.3, 4.2, or 4.7 cubic feet per second, according as there were 144 holes each 1 inch square, one opening 1 inch deep and 144 inches long, or one opening 12 inches square, the tops of all the openings being 5 inches below the surface of the water. This simply illustrates the unreliable nature of such a unit. In some localities the following standard has been adopted: An aperture 12 inches high by $12\frac{3}{4}$ inches wide through one one- and one-half-inch plank, with top of opening 6 inches below the water surface, is said to discharge 200 miner's inches. By this standard the miner's inch is 1.5 cubic feet per minute, or 2160 cubic feet in twenty-four hours. Other standards vary from 1.39 to 1.78 cubic feet per minute. When the miner's inch can only be defined as a certain number of cubic feet per minute, it is evidently no longer of service and should be abandoned. The method by weirs is more accurate, and could almost always be substituted for the method by orifices.

LITERATURE

I. References on General Hydraulics

1. Francis, Jas. B. Lowell Hydraulic Experiments. New York. D. Van Nostrand. 1883.
2. Fanning, J. T. Hydraulic and Water Supply Engineering. New York D. Van Nostrand & Co. 1886.

3. Smith, Hamilton, Jr. The Flow of Water through Orifices, over Weirs, and through Open Conduits and Pipes. New York. Wiley & Sons. 1886.
4. Church, Irving P. A Treatise on Hydraulics. New York. Wiley & Sons.
5. Merriman, Mansfield. Treatise on Hydraulics. New York. Wiley & Sons. 1903.

II. References on Stream Gauging

1. Ganguillet, E, and Kutter, W. R. A General Formula for the Uniform Flow of Water in Rivers and Other Channels. Trans. by Hering and Trautwine. New York. Wiley & Sons. 1893.
2. Humphreys, D. C. Discharge Measurements. Jour. Asso. Eng. Soc., Vol. XV, No. 5, p. 187. 1895.
3. Stewart, Clinton B. Discharge Measurements of the Niagara River at Buffalo, N. Y. Jour. W. Soc. Eng., Dec. 1899.
4. Murphy, E. C., Hoyt, J. C., and Hollister, G. B. Hydrographic Manual of the U. S. G. S. Water Supply and Irrigation Paper, No. 94, 1904.
5. Murphy, E. C. A Method of Computing Flood Discharge and Cross-Section Area of Streams. Eng. News, April 6, 1905.
6. Barrows, H. K. Work of the Hydrographic Branch of the United States Geol. Sur. in N. E. and a Discussion of the Methods used for Estimating Stream Flow. Jour. Asso. Eng. Soc., July, 1905.
7. Hoyt, J. C., and Grover, N. C. River Discharge. New York. John Wiley & Sons. 1907.
8. Hoyt, J. C. Current Meter Measurement of Stream Flow. Proc. Am. Soc. C. E., Aug., 1909.

III. References on Flow of Water over Weirs

1. Francis, J. B. Experiments on Submerged Weirs. Trans. Am. Soc. C. E., Vol. XIII, p. 303. 1884.
2. Herschel, Clemens. Problem of the Submerged Weir. Trans. Am. Soc. C. E., Vol. XIV, p. 189. 1885.
3. Hind, R. H. Flow over Submerged Dams. Proc. Inst. C. E., Vol. CXXXV, p. 307. 1886.
4. The Flow of Water over Dams and Spillways. Eng. Rec., June 2, 1900.
5. Frizzel, J. P., and Herschel, Clemens. Flow over Wide Horizontal Top Weirs. Eng. News, 1892, Vol. II, pp. 290, 440, 446; 1895, Vol. I, p. 75.

6. Johnson, T. T., and Cooley, E. S. New Experimental Data for Flow over a Broad-Crest Dam. Jour. W. Soc. Eng., Jan., 1896.
7. Wide-Crest Weirs. Bazin's Formula. Eng. News, 1890, Vol. I, p. 162; Vol. II, p. 577; 1896, Vol. I, p. 26.
8. Rafter, Geo. W. The Flow of Water over Dams. Proc. Am. Soc. C. E., March, 1900.
9. Mead, D. W. Water Power Engineering. McGraw Pub. Co., 1908, pp. 65–77.
10. Horton, Robt. E. Weir Experiments, Coefficients and Formulas. W. S. Paper, U. S. G. S., No. 150.
11. Molitor, David A. Hydraulics of Rivers, Weirs and Sluices. New York. John Wiley & Sons.

CHAPTER XI[1]

MINING SURVEYING

Introductory

The subject of mining surveying as treated in this chapter will include the surveys necessary in obtaining title to the mineral lands of the United States, the surveys for the location of the surface improvements and boundary lines, as well as the underground surveying necessary in making connections, laying out work, determining the relation of the underground workings to the surface lines, points, etc., and measuring the ore removed or still in the mine. For definitions of mining terms, see p. 441.

Surveying Mining Claims

292. Title to Mining Claims. The public lands of the United States have been divided into mineral and agricultural lands, the former being subject to appropriation under the statutes as lode claims, placer claims, mill sites, and tunnel sites. The title becomes initiate by discovery; this is followed by location and record which completes the possessory title. This title can be afterward, except in the case of tunnel sites, perfected by patent proceedings so that a fee simple deed to the claim is obtained from the government.

[1] This chapter has been rewritten for the fifteenth edition (1900) by Professor Robert S. Stockton, E.M., of the Colorado State School of Mines, and by Mr. Edward P. Arthur, Jr., E.M., U. S. Deputy Mineral Surveyor, of Cripple Creek, Colo.

293. Fundamental Conditions Governing the Location of Mining Claims. In the case of a *lode claim* there are five essentials to a valid location: (1) discovery of lode or vein; (2) posting of location notice; (3) sinking of discovery shaft; (4) marking the boundaries of claim; (5) the making and filing of the location certificate. Lode claims are limited by the U. S. statutes (R. S. 2320) to 1500 feet along the vein or lode and to 600 feet in width, i.e., 300 feet on each side of vein. The end lines must be parallel, and the side lines usually are. The different States, and originally the different districts, had the right to limit the width, at the same time keeping within the congressional limits. In Colorado the width has been reduced to 300 feet, except in the counties of Gilpin, Summit, Clear Creek, and Boulder, where it is only 150 feet. North Dakota and South Dakota fix the width at 300 feet, allowing the counties to reduce this within the congressional limits. All the other States have adopted the width of 600 feet except where limited by the old district rules.

In locating a *placer claim*, the discovery of auriferous gravel or any of the substances designated by law as subject to placer location is followed by the posting of a location notice, marking the boundaries, and making and filing the location certificate. One person or corporation may appropriate 20 acres, or an association of at least 8 persons may secure 160 acres, of placer ground. The ground appropriated may conform to the government land subdivisions if on surveyed lands, or it may be in any shape bounded by straight lines.

A mill site may be located on non-mineral land by posting a location notice, marking the boundaries, and filing a location certificate. The area of a mill site is limited by the U. S. statutes to 5 acres, and may be further limited by district rule. It may be taken in any desired form bounded by straight lines. A very convenient size is 726 feet by 300 feet.

A tunnel-site location appropriates the right of way for a tunnel 3000 feet long, and may in addition appropriate ground for a dump up to an area 250 feet square. The location is

accomplished by posting a notice, marking the line of tunnel and the corners of the dump area by stakes properly inscribed, and filing the location certificate, which must in this case be acknowledged before a notary public. The tunnel site differs from the other locations mentioned in that it is not subject to patent. The veins cut by the tunnel are located as lode claims and may be patented as such, the title dating back to the location of the tunnel site. Under recent decisions the tunnel site practically withdraws from the public domain all the lodes within an area of 3000 feet square that were not located prior to the tunnel site.

The possessory title to a tunnel site involves what the law calls continuous working. This has been held to mean no cessation of work for more than six months.

294. Location Surveys. A surveyor is generally called upon to lay out the claim, mark the boundaries, and fill in the location certificate ready for the owner to file with the county or district recorder. The law, however, allows the location survey to be made by the miner himself without an instrument, i.e., the boundaries may be laid down very roughly, if with honest intent, and hold in law, but locators have found it profitable to have the claims surveyed to avoid the losses resulting from gross blunders so often made when unskilled persons have attempted to lay out the claim. The survey should be made with sufficient care so that when the ground is surveyed for patent the corners will be identical with, or come within, the location corners, and also fulfil other necessary conditions, as will be shown under the discussion of patent work.

This class of surveying will be discussed under four heads, viz.: lode claims, placer claims, mill sites, and tunnel sites.

295. Shape of Lode Claims. The *simplest form* of a lode claim resulting from the conditions laid down in Art. 293 is a rectangle 300×1500 feet. The two things which should govern the exact direction of the lines are the direction of the vein and the position of the discovery. The usual method is to set up

the instrument at or near the point of discovery. The direction of the vein is determined by the outcrop or any other available data, but the vein line is often run so as to take in any vacant ground. The line is then run out, its bearing being taken from the true or magnetic meridian. The distances are measured with a steel tape either horizontally or by taking the slope distance and slope angle and reducing to the horizontal, this work being done with about the same accuracy as is used in ordinary land surveying. The lode or vein line is run both ways from the discovery, the only condition being that the total distance is not more than 1500 feet. From the ends of the lode line, *in this case* the centre line, and at right angles to it, and distant 150

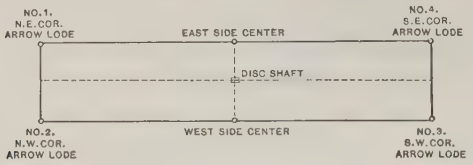


FIG. 158.

feet, are set the corner stakes. From the centre of the claim the side centre stakes are also set. In Fig. 158 is shown a diagram of the claim and also the markings on the corner stakes. The dotted lines show those actually run on the ground. The corner stakes should be 3 or 4 inches square, and $3\frac{1}{2}$ feet long, set in the ground or in a mound of stone. They should also be blazed and marked on a side toward the claim with the number of the corner and the name of the claim.

One or more of the corners should be tied in by bearing and distance to some permanent objects, as a government survey corner, or the corners of an official survey of some other mining claim, or, failing these, to bearing trees and rocks. A practice is in vogue of connecting claims to mountain peaks by bearing only, but it is not to be recommended except as a last resort.

It may be very often advisable to mention in the description of the claim the name of the mountain on which it lies, or the names of adjacent roads, streams, etc., as a help to identification.

Another simple form of a lode claim is where the end lines are not perpendicular to the lode lines and side lines, the resulting

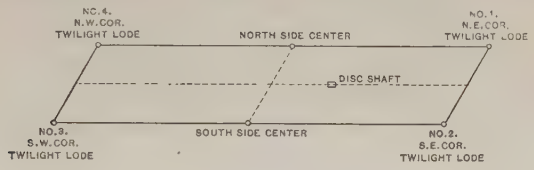


FIG. 159.

figure being a parallelogram as shown in Fig. 159. This condition generally results from the claim abutting on a fixed line.

The surveying is just the same as in the first case except that the distance measured from the lode to the side lines along the end lines, or a line parallel to them at the middle point, is greater than 150 feet, and is found by dividing 150 by sin α, where α is the angle between the end line and lode line as shown in the figure.

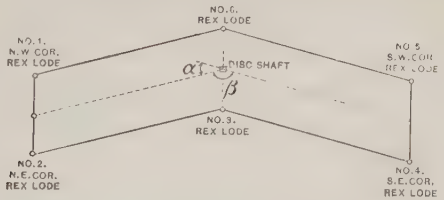


FIG. 160.

If the vein bends, the lode line is made to follow it, thus forming a crooked claim. Two forms are shown in Figs. 160 and 161.

In Fig. 160 let β represent the angle between the two directions of the lode line, then the line from Cor. No. 3 to Cor. No. 6 will bisect this angle if the side lines are parallel and at equal

distances in both parts of the claim. The distance from the discovery to Cors. Nos. 3 and 6 will be 150 divided by sin $\frac{1}{2}\beta$. If the end lines are not fixed as to direction by some outside condition they may be made parallel to line 3–6, and will by the geometric conditions laid down be exactly equal to it. The lode line is run and corners set as before except that the side centres may be omitted if the bend is near the centre of the claim.

In Fig. 161 let β represent the angle between the two parts of the lode line, then the line from Cor. No. 3 to Cor. No. 6 will bisect the angle β as in Fig. 148. Now if one end line, as line 4–5, is to be perpendicular to the lode line, it will be 300 feet long and not parallel to the line between 3 and 6, while line 1–2 is

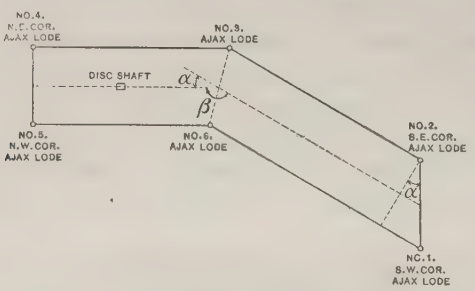


FIG. 161.

parallel to 4–5, but has a length of 300 divided by cos α. The changes in the surveying will be made sufficiently clear by a glance at the figure.

The location certificate is generally made out by the surveyor, printed forms being used with blanks for the name of the lode, the name of the locator, the date of the location, the number of feet claimed on each side of the discovery, and a general description giving the length and the bearing of the boundary lines and of the tie lines.

The location survey for a placer claim or a mill site does not differ fr m an ordinary land survey to establish boundaries. The corners are like those of lode claims and are marked with

the name of the placer or mill site and the number of the corner on the side toward the claim. The tie lines are also the same as in the lode claims. The location also covers about the same points and is made out in a manner very similar to the lode certificates.

As *a tunnel site* is not subject to patent, but is held by possessory title only, the survey is properly a little more accurate than the location surveys. The true meridian should be determined as in patent work. The survey consists in running out the line of the tunnel, placing stakes close enough together so that one can be seen from the other and not further than 300 or 400 feet apart as a maximum distance. The manner of setting stakes varies with the different surveyors, but the above method seems to be the best practice. These stakes are marked by the number and the name of the tunnel site. The dump area is also marked by a stake at each corner. The location certificate is rather an uncertain document, several forms being used. A very good form is given in Morrison's "Mining Rights."

296. Patent Surveying. In order to complete the title to any mining claim by obtaining a U. S. patent, it is necessary to have the boundaries carefully surveyed and substantial monuments set. The monuments are connected with some permanent object in order to fix the position of the claim. This work is known as *patent surveying*, and can only be undertaken by a U. S. Deputy Mineral Surveyor holding a commission from the government. This commission may be obtained by passing an examination under the Surveyor-General of the State in which it is desired to practise, and by filing a bond for $10,000. For full instructions for making patent surveys, see Appendix B.

Patent work follows the same general plan as location surveying in the running of the boundaries and setting the corners, but is done with much more accuracy. In patent work all conflicting prior official surveys are connected with the claim under survey, and improvements of all kinds are located. The following list of apparatus, while not absolutely necessary for this

work, has been found by those familiar with the best practice to be most convenient. It consists of: (1) A light mountain transit, mounted on an adjustable tripod, and an adjustable plumb-bob. The plates and vertical arc or circle of the transit are $4\frac{1}{2}$ to $5\frac{1}{2}$ inches in diameter and read to minutes. (2) A steel tape, preferably 500 feet long, usually graduated every 5 or 10 feet. (3) A short ribbon tape graduated to hundredths of a foot, together with a note book, field book of tables, axe, nails, chisel, and timber scribe. Some surveyors carry a rod and also some red cloth for making stations. A good pick and shovel are necessary for setting corners.

297. Instrumental Work. In this kind of surveying the compass needle is never used except as a rough check, all angles being read on the plate circle by means of the verniers. There are two general methods in vogue in the mining districts of carrying the meridian from one station to another. These will be designated as the Azimuth and the Angle methods. The better method is by azimuths, as described in this book, or its modification, where the bearings are carried on the horizontal limb in the same way as the azimuths. It is well for beginners to check the readings of all important sights at least. If the instrument has been oriented and a sight taken and recorded, this can be checked by setting on the back azimuth reading, and sighting to see if the cross wires are on the point; then loosen the alidade and set on the recorded forward reading, and again sight to see if the forward point is caught; if so, the recorded reading is correct. In the angle method the vernier is always set on zero for the backsight and the angle read to right or left, as the case may be, to the forward point. A nice check may then be made by loosening the lower motion and turning on to the back station, then loosening the alidade and turning again on to the forward point. The angle now read should be just twice the original one for a check. This method is much in use by old railroad men, and although it has a good number of followers it cannot be recommended. Care should be taken in

reading the vertical angles, as it seems easier to make a mistake in reading these than in reading the horizontal angles. With a complete vertical circle the best check is to reverse the telescope and turn on the point and read again. With a vertical arc a good check may be had by resetting the recorded angle on the arc and then sighting to see if the horizontal wire cuts the point whose angle is being checked.

Too short a back sight is to be avoided. It is often possible before leaving a station to obtain a distant fore sight or back sight, and this should be done when practicable.

298. Stations and Methods of Measuring. Stations are generally stakes driven into the ground, the height above the surface being determined by the nature of the country; the exact station point is marked by the head of an eightpenny or a tenpenny wire finishing nail, driven about half way into the stake.

Stations may be taken on fallen logs or tree stumps by putting in a nail as before. In winter weather, if a clear sight can be obtained, a fortypenny spike driven directly into the frozen ground will make a good station. It is usually a good plan to put a piece of white paper on the station nail in order that it may be more easily seen.

In measuring, pins are not used, nor is there any special effort made to have the tape horizontal, the measurement being taken from the horizontal axis of the instrument directly to the nail-head, and the vertical angle read to the same point, thus enabling the horizontal distance to be calculated.

The readings are made to hundredths of a foot. Some surveyors measure back and forward from each station as a check, but this is hardly necessary with careful work.

The distance between stations is limited by the contour of the country and the length of the tape. With a tape of the ordinary length the distance between stations would be less than 500 feet.

In making a measurement care should be taken that the tape is straight and subjected to a pull depending on the dis-

tance that the tape is unsupported. Then get the point on the tape opposite the axis of the instrument, and with the small tape measure along from this point toward the zero end of the long tape to the nearest graduation. This will give the number of feet and fractions to be added to the reading at the graduation mark, and the sum is recorded as the slope distance.

299. Field Work and Adjustment of Claim. The field work is generally accomplished with the aid of one assistant, who carries the forward end of the tape, puts in stations, clears out the lines, assists in setting corners, etc. In a heavily timbered country an extra axeman is of advantage. The field work consists in the determination of the true meridian by astronomical observation, as explained in Chapter IV, finding the position of the corners of the location survey, the discovery shaft, and all improvements on the claim, whether made by the claimant or not. The corners of conflicting prior official surveys must be located, and a tie made to a corner of the government land survey, if one exists within two miles, or, if such does not exist, the tie is taken to a U. S. L. monument as described in the Manual of Instructions for the Survey of the Mineral Lands of the U. S. prepared by the Commissioner of the General Land Office. This work will be alluded to hereafter as the Manual of Instructions. It is reprinted in full in Appendix B and must be studied in connection with all that is written here in regard to patent work.

Before the deputy can have official authority to survey the claim the applicant must send his application for an order for the survey, accompanied by a certified copy of the "location certificate," to the Surveyor-General, and deposit the required fees (see Manual of Instructions). The Surveyor-General will then issue to the surveyor mentioned an order for survey, designating the survey number to be used. A copy of the certified copy of the location certificate is also enclosed.

It is well, however, to tie up the claim before the claimant makes application for the survey order, and, if the location does

not agree with the description given in the location certificate, to make out and file an amended certificate giving the correct description. This may save some time and a little expense. The Land Office at the present time is very particular and will allow but slight variation between the field notes turned in by the deputy and the location certificate on which the order for survey was based.

The next step, after tying up the claim, is to compute or adjust the lines of the final claim to be used in the application for the patent.

300. Conditions Controlling the Patent Survey. The conditions are imposed by the fundamental principle that monuments hold over descriptions and by the law and Land Office regulations on this subject. These conditions are summarized as follows:

1. The final claim must lie wholly within the location stakes.
2. The length along the vein or lode must not exceed 1500 feet.
3. The end lines must be parallel.
4. The distance from lode line to side line must at no point exceed half of the statutory width.
5. The lode line must pass through the discovery shaft.
6. The end lines must be free, as explained in the Manual of Instructions.
7. The bearings must be given from the true meridian.

Having the lines of the final claim computed in accordance with the above, the patent corners are set as described in the Manual of Instructions.

The meridian used should never vary more than 2 or 3 minutes from the true meridian. If the variation with some conflicting survey is more than this, these surveys must be reported as disagreeing, as provided in the Manual of Instructions.

It is now the common practice of deputies to find the meridian by direct solar observations, as explained in Art. 98, although some still use one of the various solar attachments.

Below is given an actual example of the notes, etc., of the Jewel lode in the Cripple Creek Mining District in Colorado. This was selected as a typical example from a large number of surveys with which the writers have been connected both in this and other districts. The survey was made with a light mountain transit and a 500-foot tape, etc., as described. The field work included the following operations: The true meridian was determined by direct observation, the image of the sun being thrown on a card held behind the eye-piece. The symmetrical sets of sun observations were taken, one before and one after noon. A traverse was run to tie up the location corners of the Jewel lode, and also the corners of patent surveys, Nos. 9689, 8882, 9134, 9122, 9190, 8927, the improvements, and a corner of the government survey. In this case the order for survey had been received by the deputy, 10264 being the survey number assigned.

301. The Field Notes. These are shown on p. 403. The first section of the notes taken on January 20 represent the tying-up survey, as it is called. When this was finished, the notes were taken to the office, where the slope distances were reduced to the horizontal, the sun observations calculated, and the true meridian determined. The corrected azimuths were inserted in column 3 of the notes, and the horizontal distances in column 6 as shown. The work was also platted as shown by Fig. 162, the transit lines as run in the field being dotted and the claim lines shown unbroken. The bearing and lengths of the lines of the claim were calculated from the data obtained from the notes. It will be seen by Fig. 162 that the north end of the Jewel lode was taken in by Survey No. 9190, Ida, No. 1 lode, a patent claim of prior location. This necessitated the cutting back of the Jewel lode until it had a free end centre off the Ida No. 1 lode (as shown in Manual of Instructions). Calculations were made from which to set the patent corners to conform with the above requirement. A traverse was calculated from Station 1 to Corner No. 1.

SURVEY FOR PATENT No. 10264, JEWEL. Jan. 20, 1896.

Left-hand page of Note-book. Right-hand page of Note-book.

Station.	Assumed Azimuth.	Correct Azimuth.	Slope Distance.	Vertical Angle.	Horizontal Distance.	Object Sighted.	Remarks.
N.E. cor. Jewel	159° 57′	160° 24′	.....	.....	.....	Flag-pole.	Victor school-house.
"	320° 17½′	320° 44½′	.....	+ 20° 46′	9.35 A.M.	Sun's centre.	Observations for meridian.
"	320° 40′	321° 7′	.....	+ 21° 00′	"	"	
"	320° 55′	321° 22′	.....	+ 21° 08′	"	"	
"	255° 26′	255° 53′	118.26	11° 18′	115.97	4/9689.	
"	75° 27′	75° 54′	.....	.....	.....	4/8882.	
"	23° 10′	23° 37′	195.30	− 8° 23′	193.21	1/9134 Pine Ridge.	
"	38° 37′	39° 04′	230.15	− 8° 22′	227.70	1/9122 Lorena.	
"	350° 45′	351° 12′	265.85	− 2° 32′	265.39	3/9689.	
"	90° 01′	90° 28′	301.60	− 2° 59′	301.12	N.W. cor. Jewel lode, cor. lying down.	
"	293° 00′	293° 27′	260.32	+ 9° 00′	257.04	S. ¼ cor. Sec. 32 T. 15 S. R. 69 W., 6th Prin. merid.	
"	358° 30′	358° 57′	462.20	+ 2° 31′	461.81	⊙ 1.	
⊙ 1	42° 40′	43° 07′	129.58	− 6° 22′	128.78	1/9190. Ida No. 1.	
"	16° 20′	16° 47′	82.00	level	82.00	Shaft 3½×8×30.	
"	358° 30′	358° 57′	316.55	+ 4° 55′	315.39	⊙ 2.	
⊙ 2	358° 30′	358° 57′	203.25	+ 1° 26′	203.18	⊙ 3.	
⊙ 3	112° 25′	112° 52′	274.15	− 1° 40′	274.04	4/9151. Anniversary.	
"	358° 30′	358° 57′	321.95	− 3° 21′	321.40	⊙ 4.	
⊙ 4	319° 23′	319° 50′	283.20	− 7° 08′	281.01	3/8927.	
"	7° 30½′	7° 57½′	194.90	− 10° 17′	190.45	S.E. cor. Jewel.	
"	44° 36′	45° 03′	.....	+ 17° 10′	3.15 P.M.	Sun's centre.	Observations for meridian.
"	44° 51′	45° 18′	.....	+ 16° 59′	"	"	
"	45° 04½′	45° 31½′	.....	+ 16° 49½′	"	"	
S.E. cor. Jewel	150° 20′	150° 47′	185.00	+ 6° 15′	183.90	Shaft 5×7×13.	
"	134° 15′	134° 42′	210.50	level	210.50	" 5×8×18 discovery Jewel.	
"	119° 10′	119° 37′	335.00	"	335.00	" 4×6×12.	
"	89° 30′	89° 57′	300.50	"	300.50	S.W. cor. Jewel.	
1/9134	17° 11′	17° 38′	231.72	+ 3° 38′	231.25	1/9690. Divide.	
"	92° 47′	93° 14′	.....	.....	.....	4/9134.	

SURVEY No. 10264, JEWEL. FEB. 6, 1896.

⊙ 1	.....	7° 46½′	69.6	− 5° 53′	69.23	To 1/10264 granite 28×12×10, set 18″ grd. md. stone.
1/10264	.....	90° 06′	303.90	− 5° 01′	302.74	− 2′.74 to 4/10264 pine post 3′ long, 6″ sq., 22″ grd. md. stone.
"	.....	115° 36′	5.45	level	5.45	Spruce tree 5″ diameter B. T.
"	.....	208° 56′	.....	.....	.....	¼ cor.
4/10264	.....	300° 28′	51.10	level	.....	Pine tree 16″ diameter B. T.
2 10264 is	identical	with S.E.	cor. Jewel			Granite 26×16×10, set 18″ grd. md. stone.
2/10264	.....	90° 06′	300.00	level	300.00	3/10264 granite 28×12×10, set 17″ grd. md. stone.

The notes of February 6, p. 403, show the work of setting the corners, Fig. 163. Cor. No. 1 was set from Station 1, and Cor.

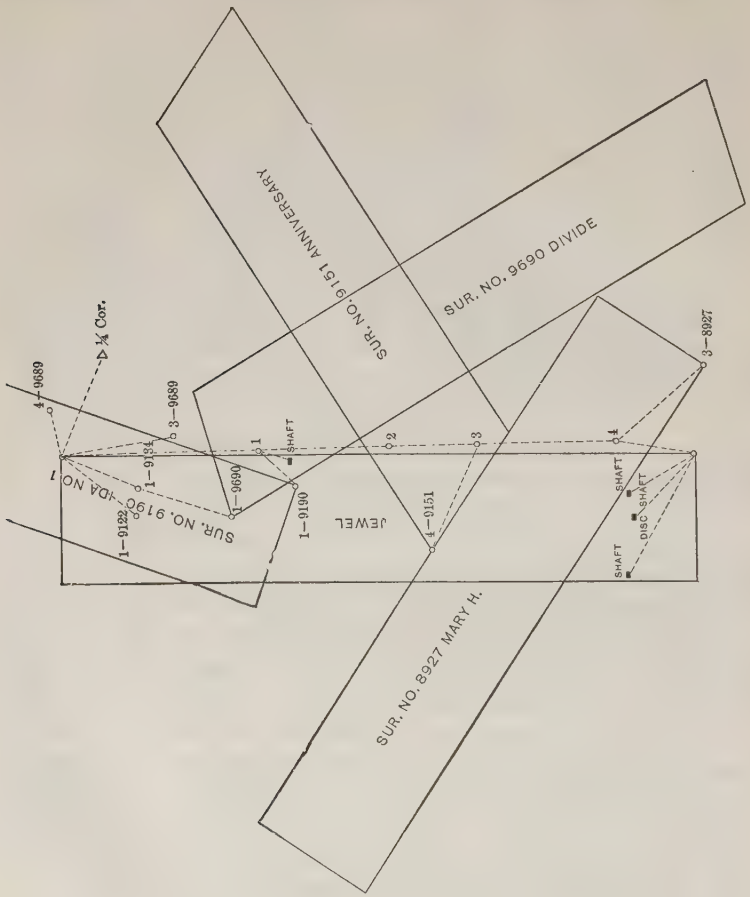


Fig. 162.—Preliminary Survey.

No. 4 was set from Cor. No. 1. Cor. No. 2 was identical with the S.E. corner of the location, and Cor. No. 3 was set from it.

The method of measuring a given distance on sloping ground

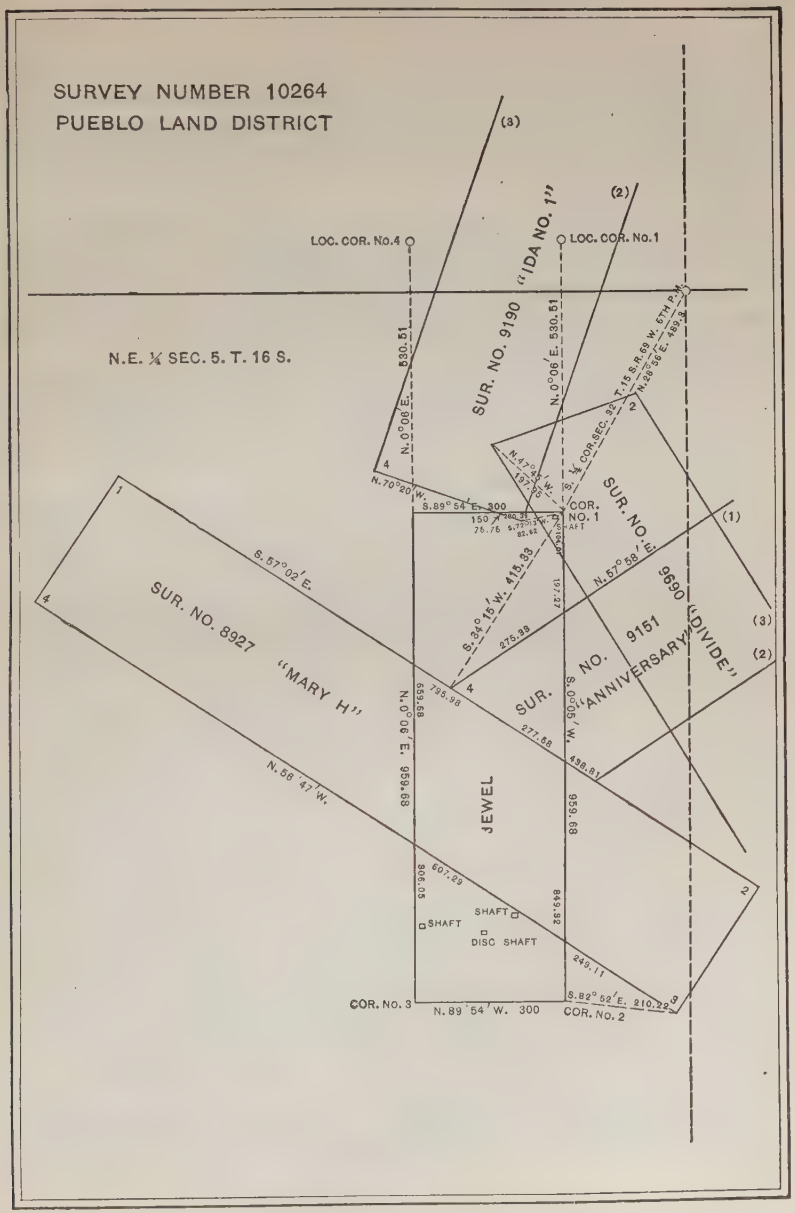


FIG. 163.—Preliminary Plat.

is best done in the following way: Set a point on line within a foot or two of the given distance, measure the vertical angle and calculate the horizontal distance. The remaining small distance is measured either backward or forward horizontally to the required point.

The style and markings of the corners are described in the Manual of Instructions. With a stone corner the exact point is marked by a cross chiselled into the stone. Post corners are preferably more than 4 inches square, the exact point being marked by a nail driven in.

In making the calculations a seven-place table of logarithms is largely used, although a six-place table would serve. A traverse table reading to minutes is very convenient. Some surveyors use calculating machines.

The areas are computed by dividing the figure into triangles and quadrilaterals or by double meridian distances. A planimeter is much used for checking the areas.

All calculations should be carefully checked and should be made in a computation or scratch book so that they can be kept for future reference.

After the corners are set and the field work completed the notes are taken to the office and worked up, i.e., a traverse is calculated as from Cor. No. 1 to the S. $\frac{1}{4}$ Cor. of Sec. 32 and a tie calculated from some corner of the Jewel to a corner of each prior conflicting official survey. The intersections were calculated so that the distance from a particular corner to the line intersected could be given as well as the distance from the intersection to the interior or nearest corner on the intersected line. Closing traverses, including some convenient lines of conflicting surveys, were calculated in order to see how the Jewel checked with these surveys. Finally the areas are computed, the notes written up, and the preliminary plat made as shown in the Manual of Instructions.

302. Preparing the Plat. Fig. 162 shows the working plat. It will be seen that on the preliminary plat (Fig. 163) the claim

is considerably shorter than the original location. Several official surveys that conflicted with the location did not do so with the final survey. Of the shafts tied up two belong to the claim and two were other improvements. The more northerly shaft was left out when the claim was cut back.

It will be noticed that the Jewel claim is cut off at the exact point where the assumed lode of the claim enters the Ida No. 1 lode, at 150 feet from each of the north corners. The present and better practice is to leave a little free ground for the assumed lode line to pass through before entering the claim for which it is cut off, as in this case the distance along the north end line of the Jewel from its N.W. corner to the intersection of the Ida No. 1 line would be 151 feet instead of 150 feet, as it now is.

The field notes are written on regular blanks furnished by the Surveyor-General and must follow the form given in the Manual of Instructions. They include a description of the claim by bearing and distance, also the intersections and areas, and the certificate of $500 worth of improvements, if these are completed at the time. If they are not, the certificate may be filed at a later date (see Manual of Instructions, Sec. 46). The present Land Office ruling is that the affidavits of labor and improvements filed subsequent to the notes must show that the work was completed prior to the expiration of the period of publication. It also requires that when two or more locations are embraced in one application, the value of the labor and improvements must equal $500 for each and every location embraced.

It is required to give the distance and bearing along the lode line on each side of the discovery shaft. This last statement is usually put in the notes just before the statement of areas, though not shown in the specimen field notes in the Manual of Instructions.

The preliminary plat is generally made on tracing cloth to a scale of 200 feet to 1 inch, and shows the claim with its conflicts and improvements. It must give the bearing and length of all lines used in the notes. The notes with the affidavits of

the deputy and his assistants, the copy of the location certificate received, with the order from the Surveyor-General, and any report the deputy may have to make, together with the preliminary plat, are returned to the Surveyor-General's office, where they are examined and the work checked; if found correct, they are approved and the Surveyor-General's certificate of approval attached.

A copy is then made of the approved notes, attached to which is the Surveyor-General's certificate that it is a true copy of the notes, and that the proper amount has been expended in labor and improvements, in case the affidavit has been filed. The approved notes, together with a number of copies of the approved plats one greater than the number of locations embraced in the claim, are now returned to the deputy who made the survey. The a proved plats are much the same as the preliminary plat made by the deputy, except that the Surveyor-General's certificate of approval and of expenditure is attached.

This properly finishes the work of the surveyor, who now turns the notes, plats, etc., over to an attorney. The surveyor should keep a record of all official correspondence and of all notes returned to the Surveyor-General's office; also a copy of all plats.

303. Placer Claims. The patent survey of a placer claim is much the same as that of a lode claim except that it is not limited to such a great extent. Like a lode claim, however, it must coincide or lie within the location boundaries. The lines of a placer claim cannot lap over other claims, and the area in conflict be excluded as with lode claims. All known lodes must be excluded. Placers are often patented with lodes under one survey number. Unlike lode claims, the deputy must submit under oath to the Surveyor-General's office a description of the soil, vegetation, etc., on the ground included within the placer claim (see Manual of Instructions). Two disinterested persons are also necessary to make a corroborative affidavit. When placer ground is patented by legal subdivision

no survey is necessary, but a descriptive report must be made.

304. Mill Sites. A mill site may be patented on non-mineral land not contiguous to vein or lode, i.e., not joining the end line of a lode claim, nor including within its boundaries known lodes. A mill site when located in connection with a lode claim may be patented under the same survey number as the lode and without additional development work. It must not contain more than 5 acres, and is surveyed and the corners are set at the same time as the lode claim. The patent survey must not extend over the location lines. One corner is tied to a corner of the government survey, and to a corner of the lode claim with which it is to be patented. A mill site may be patented, not connected with a lode claim, if a mill is actually built or in process of construction.

305. Amended Surveys. These surveys are ordered by special instructions from the General Land Office, and no rule can be given for the work (see Manual of Instructions). A very common cause for ordering these surveys is that claims applying for patent after the survey is made lose the ground on which one or both end lines are situated. Therefore, after all suits are settled and before a patent is issued, an amended survey is ordered to cut back the end lines to free ground. The amended survey must keep within the original survey. The survey retains the same number, but must be mentioned in the notes, etc., as an amended survey; as, "Sur. No. 9463 Am." Amended surveys are also ordered when a material error has been made in the original patent survey and for a variety of other causes.

306. Adverse Surveys. When the owner of a mining claim applies for a patent it often happens that ground is included that is claimed by other parties as belonging to another claim. These parties must file a protest in the local land office in order to protect themselves against the issuance of the patent to the conflicting ground. The protest is known as "an adverse," and must be accompanied by a plat showing the conflict claimed

to exist and a description of the conflict by bearing and distance. It is required that this plat and survey be made by a U. S. Deputy Mineral Surveyor under oath, who also makes affidavit as to the labor and improvements on the adversing claim. The field work on an adverse survey is much the same as the tying-up survey in patent work, except that it is not necessary to connect with a corner of the government survey, nor to tie in the corners of any conflicting official survey other than those directly concerned. The adverse plat must show the whole of the adversing claim and that part of the adversed claim necessary, also the position of all improvements.

The conflict between the claims is generally colored on the plat. The description should begin at a corner of the adversed claim or at a point connected by bearing and distance to such a corner. The area of the conflict is given. If the adversing claim has also been surveyed for a patent, no field work will be necessary in making the adverse plat.

Underground Surveying

307. General Statement. Underground surveying has for its object the determination of the position of the various workings of a mine with relation to themselves and to the surface boundaries and improvements, or with relation to a system of reference planes. The information gathered by means of the survey is used to determine the proximity of property lines, the neighborhood of other workings, to give points for running connections to locate and map any geological features that are important, to calculate the ore stoped out or the probable amount in sight, etc. All of these uses are not necessarily required of one survey, but it is true that no mining property of any magnitude can be worked systematically and intelligently without surveys. The survey is shown by means of one or more maps. These maps include a plan, and generally the projection of the workings on two vertical planes at right

angles to each other; also, in many cases, sections at particular points. The survey is generally begun by determining the true meridian, tying in the boundary monuments and surface workings, and establishing points near the entrance to the mine. The meridian must then be carried into the shaft incline, adit, or tunnel, and the main parts of the mine. This is the most particular part of underground surveying, as the special difficulties encountered make great care necessary to avoid serious error. Having the meridian underground the workings are run out if a general map is wanted, or if some special problem is to be solved the data necessary can be obtained.

308. Instruments. For this work a light mountain transit similar to the one described for patent work is generally used. It has, in addition to the points mentioned, an auxiliary telescope on either the top or the side of the main telescope, for vertical or highly inclined sightings, and for careful work should have either a complete vertical circle or else have a reversion level bubble attached to the telescope. In some cases, for the most particular work, an instrument with eccentric bearings and striding levels may be advisable. Where long sights can be taken, a 300- or 500-foot tape, with a short one for reading the fractions, may be used. For short sights, which are the majority of those taken underground under ordinary circumstances, a 100-foot ribbon tape, divided to hundredths, will be most advantageous. Besides a variety of things to be found at the mine, the surveyor will need wire for plumbing shafts, heavy plumb-bobs, extra blumb-bobs for station sights, screw eyes, nails, an axe, notebook, pocketbook of tables, etc. Figs. 164 and 165 show the two types of mining transits that are most used.

309. The Use of Top Telescope. In underground work we wish to find the bearings of the lines, their lengths, and the differences of elevation between the stations. The survey is then a traverse to find the three co-ordinates of the points occupied. When the sights become so steep that they cannot be taken with the main telescope on account of the interference

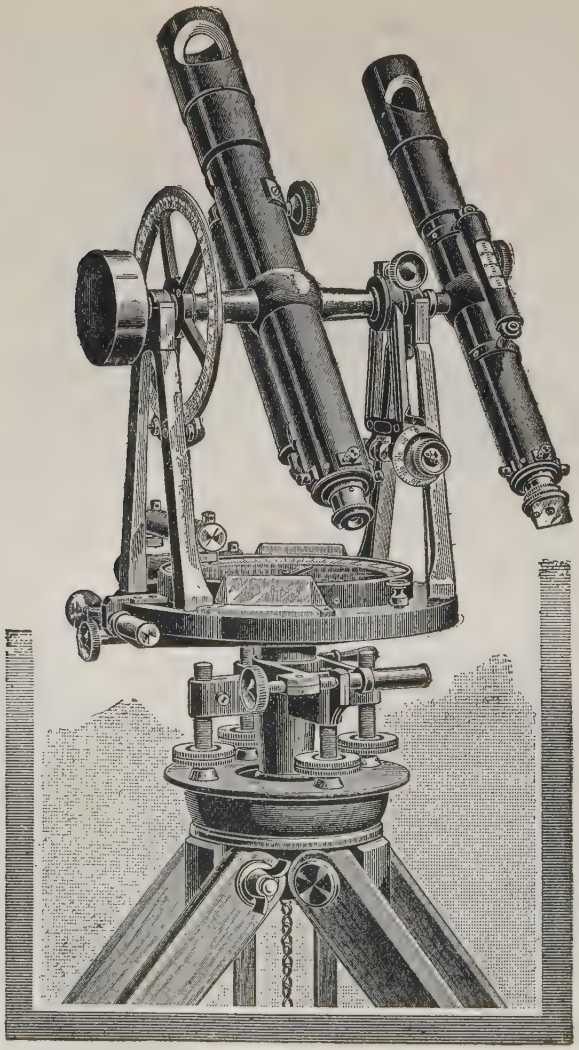


Fig. 164.

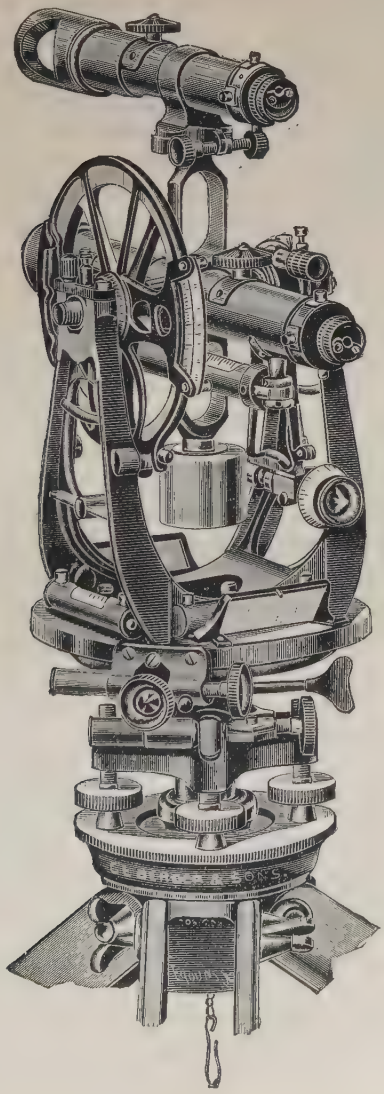


FIG. 165.

with the plate, then, unless the instrument is provided with eccentric bearings, either a top or a side telescope is a necessity. In using the top telescope, it must be adjusted so that its line of sight is parallel to the line of sight in the main telescope. If the Saegmuller type of top telescope (Fig. 47) is used, it is adjusted as for solar work, and it is only necessary to level both telescopes and bring the vertical wire of the top telescope into the same vertical plane as the vertical wire of the main telescope. This is done by sighting on a point some distance away with the main telescope and swinging down till the point comes into view in the top telescope, then move the top telescope by its lateral tangent screws until the cross wires cut the point sighted. The instrument is now ready for use, but has the disadvantage of low power in case of the Saegmuller solar form, and is less stable under trying circumstances than a telescope mounted directly on the axis.

The adjustable telescope is mounted on a central axis with trivet plates and adjusting screws. With this form the line of sight is adjusted to the optical centre by rotating in wyes as with a wye level. In this case wooden wyes may be made to serve. When this has been done the vertical wires of the two telescopes are brought into coincidence as above, using the lateral tangent motion of the top telescope. Then to bring the horizontal wire of the top telescope to a position such that the two lines of sight are parallel, measure the vertical distance between the telescopes and draw two horizontal lines at this distance apart on a piece of white paper. Sight the horizontal wire of the lower telescope on the lower line on the paper at a distance of 50 feet or more from the instrument; then bring horizontal wire of top telescope to coincide with upper mark by means of the trivet adjusting screws. This form permits of very accurate work, ranking next to the special mining transits with eccentric bearings.

The ordinary form of top telescope (Fig. 165) is fixed rigidly in place so that the adjustment given above must be made by moving the cross wires. The rigid type of top telescope is

mounted by some makers on two pillars fixed to the main telescope.

If the measurement is taken from the axis of the top telescope to the next station as shown in Fig. 166, the reductions are made as follows: Let d equal the slope distance; r, the distance between the telescopes; α, the angle of depression or elevation; then the horizontal distance is equal to $d \cos \alpha + r \sin \alpha$. The difference in elevation is equal to $d \sin \alpha \pm r \cos \alpha \pm$ H. I.

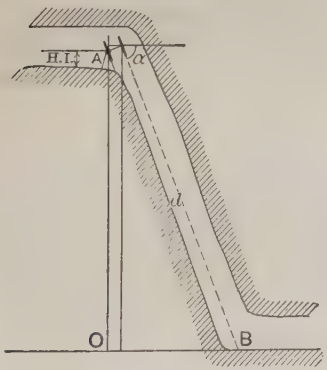


Fig. 166.

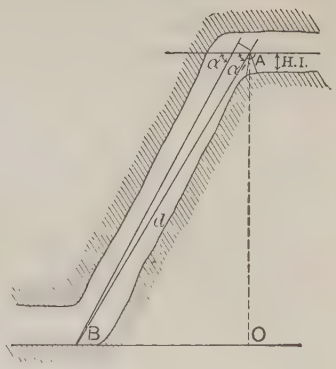


Fig. 167.

If the top telescope is mounted on standards, or if for some other reason it is desirable to measure from the horizontal axis of the main telescope, the reductions are as follows: In Fig. 167 let d equal the slope distance; r, the distance between the telescopes; and α, the angle of depression or elevation read. The real angle of depression or elevation of the measured line is α', α' being equal to $\alpha \pm \sin^{-1}\left(\frac{r}{d}\right)$, taking minus sign for correction to angle of depression, and plus for angle elevation read. Then horizontal distance is equal to $d \cos \alpha'$. Difference in elevation is equal to $d \sin \alpha' \pm$ H. I.

310. Use of Side and Top Telescope. In using a side telescope the lines of sight in the two telescopes should be made parallel in a manner very similar to that used for the top telescope. The side telescope screws firmly to the end of the horizontal axis of the transit; it is generally provided with tangent screws for movement in a vertical plane, and may have adjusting screws for lateral movement; this would be called an adjustable side telescope.

To use the adjustable side telescope, first adjust the line of sight to the line of collimation, using the method by means of wyes as before, then adjust the horizontal wire by sighting a point 200 or 300 feet away through the main telescope, swinging on the vertical axis of the transit until the point comes into view in the side telescope, and moving this telescope up or down by tangent screws until the horizontal wire cuts the point. For the vertical wire measure the distance between the telescopes and draw vertical lines at this distance apart on a paper set a distance of 50 or 100 feet from the instrument. Now bring the vertical wire of the main telescope to coincide with one line, and then bring the vertical wire of the side telescope to coincide with the second mark by means of its adjusting-screws. In case of a non-adjusting side telescope make the correction by moving the reticule. In a very common form the vertical wire must be brought to the proper position by means of the movement of the reticule, and the horizontal wire is adjusted by means of tangent screws moving the side telescope. The measurements are now made directly from the horizontal axis of the instrument to next station and the H. I. measured. Then the horizontal distance is equal to the slope distance times the cosine of the angle of elevation or depression, and the difference in elevation is equal to the sine of the angle of elevation or depression times the slope distance, plus or minus the H. I. When the side telescope is used and only one forward sight taken to each station, the azimuth read must be corrected by an angle whose sine is the distance between the telescopes divided by the horizontal distance between

the stations. The correction can be obviated by sighting to a double rod, the two parts being separated by a distance equal to the eccentricity of the side telescope, but this is not recommended.

It is to be noted that if both the fore and back sights on any course be taken with the side telescope, the instrument remains properly oriented at each succeeding new station. Consequently the field notes need show no correction, but in the office those sights taken with the side telescope must have their azimuths increased or diminished according as the side telescope is on the right- or left-hand side.

One of the great advantages of the side telescope is that by reversion the errors in adjustment of side telescope and transit

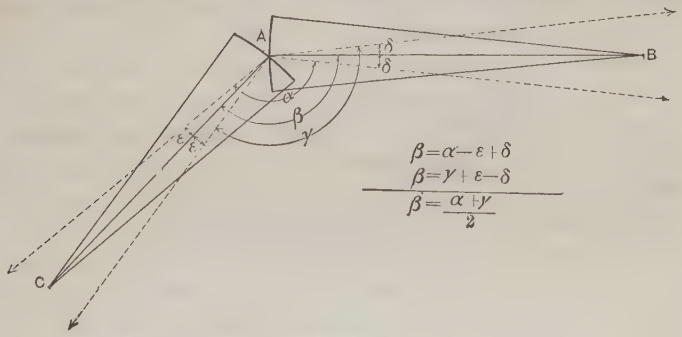


FIG. 168.

can be eliminated, provided the instrument is carefully levelled. To accomplish this two sights and readings are taken for each forward and backward sight, as follows:

The instrument being at station *A* Fig. 168, and the telescope direct, station *B* is sighted with the back azimuth properly set off and the forward azimuth Z', and the vertical angle read to station *C*. Then with telescope reversed station *B* is sighted, using the same back azimuth and the azimuth Z'', and the vertical angle again read to station *C*.

From Fig. 168 it is plain that the average of the two azimuths

read (the second reading being corrected by 180° as telescope was reversed) will be the true azimuth Z between the stations, thus eliminating the effect of the side telescope if the latter maintains a fixed relation to the main telescope; also eliminating the effect of an inclined horizontal telescope axis, which is such an important factor where steep sights are taken. The average of the two vertical angles read will give the true vertical angle. For an absolute check repeat the whole operation.

311. Stations. Underground stations may consist of overhead or floor stations, and may be either permanent or temporary. When it is advisable to establish a station in a tie or in a stake or plug driven in the floor, the head of a wire nail is good. For a permanent station in a rock floor a hole 3 to 6 inches deep is drilled and a wooden plug set, in which a nail can be driven. Some writers mention the use of a $\frac{3}{8}$ inch boiler rivet sunk in a hole $\frac{1}{2}$ inch diameter and $1\frac{1}{2}$ inches deep. The rivet is split for an inch and has a wedge started in the split; this wedge strikes the bottom of the hole first, spreads the rivet, and holds it firmly in solid rock.

For permanent overhead stations screw eyes set into caps or stulls, from which a plumb-line can be suspended, are the most convenient; where timbers are not available, a hole is drilled in the rock from 3 to 6 inches, and a wooden plug driven in and the screw eye set as before. Horseshoe nails with a hole punched in the head have been used in place of screw eyes, but it is an unnecessary trouble to have them made. A three-inch wire nail, bent in the form of a staple, makes an excellent station, as the sharp corner allows the plumb-bob string to always take the same position.

In a large mine the stations should be carefully numbered. If the mine is worked by levels, a good system is to number sations on 1st level 101, 102, etc., those on 2d level 201, 202, etc., and so on for all the levels, thus avoiding mistakes and saving time in hunting up notes. The numbers may be painted or scribed on the adjacent timbers or rock. A very reliable way

is to use round-headed nails, as ⁘ for 324, a washer serving for zero. The very best method, however, is to use brass or zinc tags marked with stamps. This tag is nailed to the plug or timber according to the character of the station.

It is very important in extensive workings to have the stations legibly and systematically numbered and referenced; the latter precaution is especially necessary where the station is in a timber, as witness the mishap of the man who ran a connection from a station in a stull, which the miners had turned end for end! Also in some places, notably in coal mines, the miners have a habit of removing or tampering with the stations, thus causing endless trouble unless they are properly referenced and tested before use.

Stations in shafts or inclines for temporary use may consist of nails driven into the edge of the shaft timbers, or in a sprag, and are usually set at an angle such that they can be sighted from above and below. Stations are very often made of a piece of tin about 3×4 inches with a cross cut in it. This is covered with tracing cloth and nailed to a plank over a hole or notch so that the cross, when illuminated, can be sighted from both directions.

Stations are illuminated for sighting by holding a light close behind them. The best method is by holding a piece of tracing cloth or oiled paper between the station and the light, thus presenting a comparatively large illuminated surface on which the station and cross wires may be seen. Plummet lamps have been used; they are simply ordinary mine lamps made in the form of a plummet and swung by a bail so that the flame hangs true. It is sometimes necessary to illuminate the cross wires in taking sights, as well as the stations. It is generally sufficient to hold a candle near enough in front of the object glass to throw light into the telescope tube; in other cases a metal or paper reflector is used, while some mining transits have a hollow axis through which light may be directed. The difficulties of properly lighting the stations and cross wires vary with the length of the

sight, the condition of the air, and the character of the rock. In coal mines it is much more difficult to get sufficient light owing to the absorption of light by the black walls, etc.; those coal mines that are not fiery use oil lamps and torches. In metal mines candles are much used, except in very wet places, where the falling water would extinguish them. A good candle, where it can be used, makes the best light for the surveyor, giving a clear light, without soot or smoke, and suitable for reading and sighting. Electric mine lamps have been devised, but have not yet come into general use.

312. To Carry the Meridian into the Mine. By Means of the Transit. If the mine is entered by adit, tunnel, or slope, the meridian is carried in by ordinary underground traverse, to be described later.

If the mine is entered by an incline or a crooked shaft, the line is run in with a transit provided with an auxiliary telescope. This method is of wide application and is much used by engineers. It is adapted to that class of mines working more or less vertical veins, where the shaft or incline follows the vein on its dip. One shaft surveyed by the authors was so crooked that in 600 feet eight sights were necessary, yet the bottom of the shaft was only a few feet away from where it would have been if the shaft had been vertical. In this case the bends in the shaft limited the length of the sights and at some points necessitated stagings between the levels. When the incline is straight, as in the case of some of the largest mines, the length of the sights is only limited by the length of the tape, the condition of the atmosphere, etc.

It is easier to run down an incline than up, on account of the greater ease in measuring the distances. In any event it is more difficult to see down than up, except in very wet places, on account of the greater difficulty in lighting the stations.

Great care should be taken in reading the vertical angle, as a small error in such a case changes appreciably the horizontal distance between the stations. The accuracy diminishes as the

steepness of the sights increases, because the azimuth is determined by the horizontal projection of the line of sight, and this base line becomes very short with steep sights. A steep sight implies a great difference in elevation between fore sight and back sight; thus an error in the adjustment of the transit is greatly magnified, and must be very carefully eliminated in important cases, by reversal, striding levels, etc. A very small error in sighting to, or setting over, stations marking the extremities of a short base line will be material: e.g., suppose the shaft will allow only a 5-foot base line; then if the sights were 100 feet long, the vertical angle would be about 87° 8′. Now, if it is admitted that there is a probability of the combined error of setting and sighting being .003 feet at right angles to the plane through the stations, the error would be $\tan^{-1}\left(\frac{.003}{5}\right)$ equals 2 minutes. This is a compensating error, consequently the square root of the total number of errors probably remains uncompensated. Thus it is seen that it would take great care to run, say, 400 feet down the shaft with an error not more than 3 or 4 minutes. From the above discussion, and also from the results of experience, it follows that for a vertical shaft the meridian may be carried down more accurately and more quickly by plumb-lines than by use of the mining transit. This statement holds with more force with a deep shaft. No definite rule can be given in regard to this work, as the details in almost every case are different and much depends on the ingenuity of the surveyor and assistant.

313. By Use of Plumb-Lines.[1] The plumb-bobs used are of lead or iron, and weigh from 5 to 20 lbs., according to the depth of the shaft and the condition of the same. Up to perhaps 1000 feet the weight might be from 8 to 12 lbs. with No. 22 copper

[1] The operator must guard against constant air currents, however slight, as these may cause a constant deviation in deep shafts, even with very heavy bobs. See Eng. and Min. Jour., April 26, 1902, p. 578.

or soft steel wire; for shorter distances bobs weighing from 5 to 8 lobs., with No. 24 wire, will do; while for greater depths or heavier bobs it may be advisable to use No. 20 wire. Copper wire is generally to be preferred, although it stretches more. The plumb-bobs should swing in a bucket of oil, mud, or other liquid that will retard the vibrations. The point of the bob in swinging traces an ellipse. If we do not choose to wait for the bobs to stop swinging, a board may be placed close to each wire and at right angles to the plane through the two wires and the vibrations read for some time by means of a scale, then a mark is made opposite the mean position and the sight taken. Otherwise the surveyor waits until the vibrations have stopped or have nearly done so, and then takes the sight. With this method the observations on the wire must be continued long enough to make sure that the line is stationary and not merely at some point in its slow vibration. There is in use in the Pennsylvania coal mines a bob made of two flat disks of iron connected by ribs and weighing 20 lbs.; this is specially designed so that it may be left until it has assumed a stationary position.

There are several different conditions met with in plumbing shafts, which naturally divide the subject for systematic discussion. These cases will be taken up in order as follows:

314. When There are Two Shafts. Hang a plumb-line in each shaft at A and B, Fig. 169, then on the surface run a traverse from A to B, from which the true azimuth and length of the direct course AB can be calculated. Underground assume the bearing of the first course (1) to A and run a traverse to the second plumb-line at B and calculate the azimuth and length of the underground line AB. The difference in the azimuths found by the two surveys will give the correction to be applied to the underground azimuths to bring them to the true meridian. For a check on the work the horizontal distances resulting from the calculation of the two traverses should agree.

In Fig. 169 it is assumed that the surface survey determined the azimuth of AB as 270° 10′, and that the azimuth as determined

from the underground survey with an assumed meridian was 275° 20′, from which it follows that all the azimuths of the underground survey should have 5° 10′ subtracted from them to read from the true meridian. This method, or a modification of it, should be used whenever possible, on account of the accuracy easily attainable by it. The base-line on which the meridian depends being in this case the length of the line between the

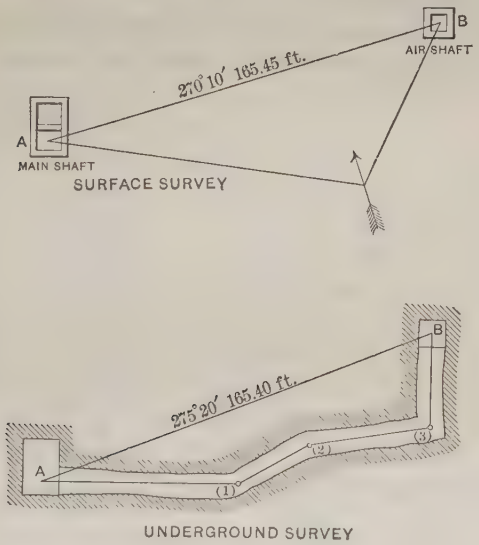


FIG. 169.

plumb wires, corresponding to *AB* in Fig. 169. This method is often used for running from one level to another in a mine where the main shaft and a winze or manway can be used.

315. When One Vertical Shaft is the Only Entrance to the Mine. In this case two plumb-lines are hung in the shaft as far apart as possible. As *AB*, Fig. 170. They are placed in this way because the longer the base-line the less a slight discrepancy will affect the meridian determined from it. Before letting

down the wires it is usually well to make an examination of the shaft to determine the most advantageous way of hanging them; keeping in mind the desirability of a long base-line, but arranging the wires so as to be most available, at the different levels, for taking off the line. It often happens that the head works limit to some extent the position in which the wires can be hung. This part of the work requires good judgment and experience on the part of the engineer.

In the larger mines, with shafts of three or more compartments, it is, as a rule, easy to get a good base-line, but it is often

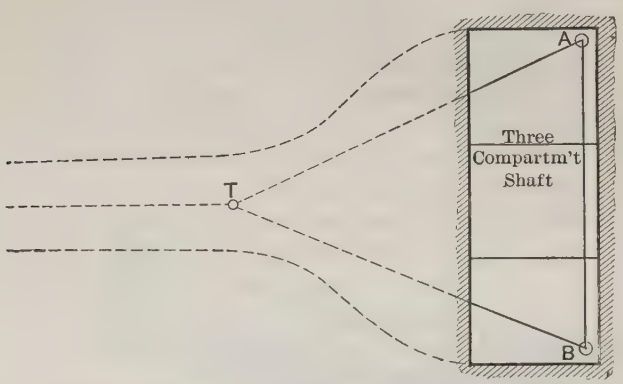


FIG. 170.—Carrying Azimuth down a Single Vertical Shaft by Plumb Lines.

necessary to carry a meridian into shafts much smaller but of considerable depth, and where the several levels run off at different horizontal angles from the shaft. In some cases the operation of plumbing will have to be carried out two or more times in order to connect with the different levels. See Fig. 170.

After the preliminary examination the wires are let down the shaft with a small plumb-bob or other weight, the heavy bobs being attached at the bottom. The transit is set up on the surface so as to be in line with the position of the wires determined upon. One wire is then sighted directly, and the other brought into line.

The wires may be suspended from heavy spikes driven into the timbers or into a cross-beam put in for that purpose, or any other convenient way that furnishes a solid support. A very important point is to make sure that the wires hang perfectly free. This may be determined by passing a light slowly around the wire at the bottom and observing the same from the top. Sometimes it is necessary to climb down the shaft and inspect the wires in detail, or the support may be given a measured movement and the wire watched to see if a corresponding movement takes place below. In a well-timbered shaft using a bucket, a man can be lowered slowly without disturbing the wires, and their position thus fully determined.

In order to get the instrument below, the wires and supports are so arranged, if possible, that they will allow of the bucket or cage passing down while the wires are swung temporarily against the timbers. If this cannot be done, and only one transit is available, it will have to be carried down the ladders. At the various levels of the shaft, the transit must be set in line with the two wires at a distance of from 5 to 15 feet or more from them, according to the exigencies of the case. This requires great care, especially where currents of air or falling water prevent the wires from becoming absolutely stationary. The vibration is so slow in many cases that there is danger of taking the wire in a wrong position. A plumb-bob swung with a 500-foot cord would require over 12 seconds to vibrate in air, and if retarded by swinging in oil or water it may take several times as long. As soon as the transit is in place its position should be marked by a permanent station and another one put ahead, so that at least two stations are available for orientation. It often happens that it is not convenient to establish a permanent station where the instrument sets, after transiting in, but one may be set ahead and a back-sight station fixed at some convenient point on line.

When this has been done at each level where the azimuth is to be taken off, the plumb-lines are removed from the shaft

and the different levels run out at the surveyor's convenience.

If, in order to get a long base-line, the wires are placed so that they are not in line with the levels, it becomes necessary to set the instrument at one side in the level and tie in to both wires, thus obtaining the azimuth by the solution of a traverse or triangle as in the first case, where there are two shafts. This method is sometimes necessary, but as a general rule it is not to be recommended, unless a much longer base-line can be had by its use.

The depth of the shaft can be determined by measuring directly from level to level, or from point to point down the shaft; or it may be found by bringing the wire up over a pulley, under constant tension, and measuring it with a tape as it is drawn up.

With the short base-lines that are necessary in most shafts, every precaution must be taken to insure good results. With a 10-foot base-line an error of 0.006 foot, or less than one sixteenth of an inch, if at right angles to the line, would mean an error in azimuth of 2 minutes. This, in a distance of 1 mile, would throw the position of a point about 3 feet out of place. On the other hand, a case is mentioned in the Trans. Am. Inst. Min. Eng., where shafts 200 feet deep, but so twisted as to give base lines 6 to 18 inches long, were plumbed and connections made. The connections were, however, in no case far from the bottom of the shaft.

316. When it is Necessary to Carry a Meridian Down a Shallow but Steeply Inclined Working, and an Instrument with an Auxiliary Telescope is not Available. This can be done by stretching a wire down the incline and by suspending one or two plumb-bobs near the top and bottom of the incline, thus enabling the transit to be lined in both above and below and the meridian transferred. When it is inconvenient to hang the plumb-bobs on the wire, they may be hung tangent to it.

317. By Alignment of a Wire from the Surface. If the bottom of the shaft or incline is not too far away, the method used in the Severn tunnel may be employed. A wire is stretched 100 feet or more into the drift at the bottom, the ends passing over screws with which the wire may be accurately aligned by the transit, for the few feet visible from the top of the shaft. This is quite an accurate method, where applicable, if a heavy transit with a large telescope is used.

The work of carrying the meridian into the mine should, in very important cases, be gone through with, independently, at least twice, besides using every precaution and check in measuring the angles and distances in each case.

This part of the survey is generally the most expensive for the mine, as it stops work in the shaft. It also generally presents more difficulties and requires more judgment on the part of the engineer than any other part of the survey.

318. Underground Traversing. This is ordinarily carried on much the same as on the surface, with the exception of a number of details due to darkness and other conditions peculiar to underground work. The measurements are usually made on the horizontal in running out an ordinary level or entry. If for any reason the elevations of the stations are required with a fair degree of accuracy, it will be necessary to read the vertical angles and the height of instrument at each station. For very exact work, as in running out grades, a levelling rod should be used.

The H. I. is generally recorded positive when the instrument is above the station, and negative when it is below. The necessity for taking the H. I.'s can be avoided by measuring the distances and vertical angles both ways from the alternate stations occupied. This method carries the elevation forward and gives the elevation of stations measured to.

The three-tripod method of traversing underground makes use of two target lamps mounted on an axis similar to the one on the transit, so that the cross of the target is the same distance

above the levelling head as the cross wires of the transit. This involves the use of the short-axis transit, so that it can be lifted off above the levelling head and be exchanged with the lamp target.

The instrument being set up over any station, one tripod with lamp is set up over the back-sight station, and the other over the forward station. The sights are now taken to the target lamps, and the measurement made from the horizontal axis of the instrument to the centre of the target. The vertical angle is read and the H. I. measured. Now the instrument-head is lifted out of its socket and carried to the forward station, replacing the target lamp at that point, the target lamp being taken back to the former instrument station. The rear target and tripod is next moved up to the forward station, and the work proceeds. The three-tripod method, it is claimed, is expeditious and accurate; its disadvantages are the extra cost of apparatus and the difficulty of carrying so much around. It would seem to be adapted to the needs of a surveyor who had a large mine to look after, and where all the workings were horizontal or not steep enough to require ladders. These conditions are more often present in coal mines than in metal mines.

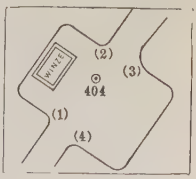
On p. 429 is shown a good arrangement for the notes of an underground traverse and connection to the surface. Traverses are usually run through all the main workings of the mine. Starting at each level from the meridian previously brought in, as many permanent stations and bench marks are established as may seem desirable. The position of these stations should be noted, and also any irregularities in the drift to enable it to be platted correctly. For very careful work, offsets from the traverse lines are taken at short intervals. In some cases it is necessary to set reference points near the stations so that these can be replaced if destroyed.

Where winzes or manways extend from one level to another, it is a good plan, when convenient, to run connections through them, as a check on the work if for no other purpose. The work

UNDERGROUND SURVEY

(Left-hand page.) March 16, 1899. (Right-hand page.)

Station.	Azimuth.	Slope Distance.	Vertical Angles.	Horizontal Distance.	Vertical Distances.	Object Sighted.	Remarks.
⊙ 2	325° 50′	236.06	+ 9° 02′	233.13	.....	⊙ 31.	
"	203° 00′	314.45	− 3° 30′	313.86	.....	⊙ 3.	
⊙ 31	15° 04′	292.41	− 3° 06′	291.98	.....	4/7912.	
"	321° 50′	335.88	− 8° 22′	332.30	.....	1/7912.	
⊙ 3	246° 49′	193.49	level	193.49	.....	⊙ 4.	
"	65° 54′	98.70	+ 2° 32′	98.60	.....	⊙ 15.	
⊙ 4	176° 17′	350.90	− 6° 39′	348.54	.....	⊙ 43.	
⊙ 43	177° 22′	331.14	−10° 44′	325.36	.....	2/7912.	
2/7912	86° 12′	300.50	+ 0° 31′	300.49	.....	3/7912.	
⊙ 15	217° 33′	17.94	level	17.94	.....	⊙ 16.	
⊙ 16	146° 30′	7.22	"	7.22	.....	S.E. wire depth of shaft to 4th level 340 ft.	
"	150° 00′	6.00	"	6.00	.....	S.E. corner shaft.	
"	124° 45′	12.00	"	12.00	.....	S.W. " "	
⊙ 18	326° 30′	7.66	"	7.66	.....	S.E. wire ⊙ 18 is underground.	
"	198° 51′	14.56	"	14.56	.....	⊙ 19.	
⊙ 19	186° 24′	43.85	"	43.85	.....	⊙ 401, 4.5 ft. N.W. centre drift 14′ wide.	
⊙ 401	238° 05′	90.25	"	90.25	.....	⊙ 402.	
"	235° 33′	57.00	"	57.00	.....	Centre of drift.	
⊙ 402	232° 37′	53.09	"	53.09	.....	⊙ 403.	
⊙ 403	240° 11′	72.64	"	72.64	.....	⊙ 404. H. I. above floor = 4.15.	
"	189° 21′	16.18	"	16.18	.....	⊙ 405.	
"	333° 39′	34.51	"	34.51	.....	⊙ 406.	
⊙ 404	152° 28′	10.00	"	10.00	.....	W. side room.	
"	75° 28′	8.40	"	8.40	.....	Angle (1).	
"	207° 58′	5.60	"	5.60	.....	" (2).	
"	247° 42′	43.80	"	43.80	.....	Centre breast.	
"	307° 38′	10.50	"	10.50	.....	Angle (3).	
"	340° 28′	15.50	"	15.50	.....	E. side room.	
"	19° 28′	14.00	"	14.00	.....	Angle (4).	
"	114° 39′	8.20	"	8.20	.....	N.W. cor. winze.	
"	162° 38′	6.60	"	6.60	.....	N.E. cor. winze. 4′.7 wide.	
"	158° 56′	74.84	−51° 51′	46.23	58.85	⊙ 40 H. I. = 1′.8. In winze 2½′ W. centre 8′ long.	
⊙ 40	206° 38′	120.32	−48° 10′	80.25	88.65	⊙ 41 H. I. = 2′.7. In winze 2′ W. centre 8′ long.	
⊙ 41	223° 04′	30.90	−16° 23′	29.64	8.72	⊙ 501 N. Centre drift, permanent = 516 Hills.	
⊙ 501N.	263° 17′	13.23	level	13.23	.....	⊙ 502 N. permanent.	
"	259° 23′	34.22	"	34.22	.....	Centre breast.	
"	56° 39′	43.50	"	43.50	.....	" "	



when completed gives data for a general map of the mine, but supplies no detailed information in regard to the stopes. When the latter is necessary a line can be run into the stopes from the most convenient reference station. Stope work consists generally in determining approximately the amount of ground broken and the extent of the workings; therefore if one or two points in the stope have been connected with a permanent station the rest of the measurements may often be taken with the tape without reading angles.

For such surveying work or for any approximate work, especially in confined spaces, the German swinging compass and clinometer is very good. It is hung on a cord or wire stretched from one station to another. The cord corresponds to the line of sight between stations, as its direction and angle of elevation or depression is read directly by compass and clinometer. One of the chief advantages of this method is the ease with which co-ordinates can be measured directly from the cord, at any point, to the walls or boundaries of the confined space. The hand compass and clinometer is much used in measuring up stopes, and is often sufficiently accurate for use in the examination of mine and ore bodies by an engineer making a preliminary report.

319. Underground Levelling. For underground levelling the ordinary wye level is used. It should be provided with a heavy adjustable tripod in addition to the ordinary one. In case a level is not available the transit can be used unless extreme accuracy is wanted. A convenient rod is one 5 feet long and capable of being extended to 9 feet. The rod can be read by target directly when visible, otherwise a small steel rod like a knitting needle may be soldered on to the target at the zero line so as to project 2 or 3 inches, then a paper and light held behind it properly will enable the target to be set. Considerable care should be used in underground levelling where connections are to be made by tunnels or drifts, as for good haulage and drainage it is necessary that the grades connect well.

320. Mapping the Survey. If the mine workings lie in one bed, horizontal or nearly so, as in many coal mines, one plan of the workings may be sufficient. In ordinary metal mines, where the workings extend more or less vertically downward, the survey is properly shown by at least three maps: first, the plan or projection on a horizontal plane; second, the longitudinal section, generally the projection on a vertical plane coinciding as near as may be with the general direction of the levels; third, the transverse section or projection on a vertical plane at right angles to the longitudinal one. (See Plate IV.) In addition to these three maps it may be desirable to make true sections as special points, and sometimes a separate map of the surface survey is made, although this is usually combined with the plan. The maps should have a title giving the name of the mine and location by mining district, county, and State; also there should appear the name of the surveyor, date of survey, meridian used, and scale. The map should be on cloth-backed paper or tracing cloth, and may show on the plan the position, number, and elevation of the permanent stations with the bearing and length of the lines joining them, or the co-ordinates of the stations, according to the system used. The advisability of showing these data on the map depends on the use that is to be made of them and must be decided according to the nature of the case.

It is often a good plan when the different levels of the mine are in nearly the same vertical plane to plat them in with inks of different colors, in order to prevent confusion. (See Plate IV.) There is an objection to this when blue prints are to be made, that the colored inks do not print well. The number and arrangement of the maps vary greatly among the different surveyors even when the same conditions are to be fulfilled. A very convenient way of making the maps is to have a main map on heavy paper on which all the workings are platted and kept up to date; a tracing is made, and kept up with the main map, from which blue prints can be taken as desired. The scale used varies from

RECORD NOTES

Instrument Station.	Azimuth.	Bearing.	Slope Distance.	Vertical Angle.	Horizontal Distance.	Vertical Distance.	Height Instrument.	Elevation.	Latitude.	Latitude Co-ordinate.	Departure.	Departure Co-ordinate.	Station.
404	158° 56′	N. 21° 4′ W.	74.84	− 51° 51′	46.23	58.85	4.15	342.81	+ 43.14	+ 1884.16	− 16.62	987.40	40
40	206° 38′	N. 26° 38′ E.	120.32	− 48° 10′	80.25	88.65	1.80	288.11	+ 71.73	1867.30	+ 35.97	970.78	41
41	223° 04′	N. 43° 4′ E.	30.90	− 16° 23′	29.64	8.72	2.70	201.26	+ 21.65	1939.03	+ 20.24	1006.75	501N.

10 to 200 feet per inch, depending on the extent of the workings and the use to which the maps are to be put. A very common scale for working maps of the metal mines in Colorado is 20 feet to 1 inch. Such a map should be accurately drawn, the angles being laid off by means of a large metal protractor or by latitudes and departures, natural tangents, chords, etc. It can then be used as an exact check on all work and calculations; the distances scaled off will be close enough for running air connections and for use in problems of similar importance.

Where the workings are very extensive the use of co-ordinates in platting and in keeping the records may be advisable. In such a case the simplicity and regularity of the method gives it a great advantage. It also allows of using the formulas of analytic geometry in the solution of problems.

The co-ordinate method is used in the big mines of South Africa, not only on account of the advantages mentioned, but also because the maps that must be filed with the officials at stated intervals are required to be platted in this way, each point being given by its x and y co-ordinates in metres. The reduction of the notes should be in a book kept for that purpose and may be arranged somewhat as shown in this Article.

To explain the method of constructing the maps, let AB, Fig. 171, be any course whose length, azimuth, and angle of depression are known. For the plan multiply the length AB by the cosine of the vertical angle which projects it into the length OB in the figure. Suppose the plane of the longitudinal section has the direction OL, making a known angle with the meridian. Change the bearing of the course so as to refer to OL as a meridian. Now the latitude[1] of AL, the course as it appears in the longitudinal section, will be AO, the difference in elevation between A and B, and the departure[1] OL, the projection of OB upon the assumed plane. In the same way it is evident that AT is the course as it appears in the transverse section, and that it is given by its projections AO, the difference in elevation of the two stations, and OT, the projection of OB upon the assumed plane. The student could readily compute these values from the given data.

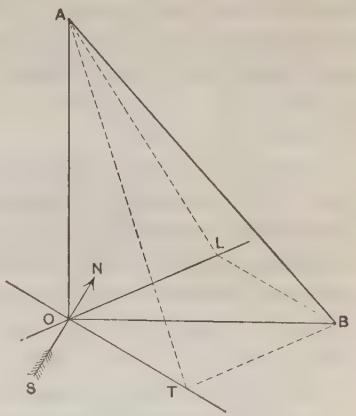


FIG. 171.

321. The Problems of Underground Surveying. To make a connection, i.e., to find the bearing of a line joining two given points and the horizontal distance and difference in elevation between them. This is one of the most important problems, as it includes all cases where connections are to be made from one part of the mine to another by sinking shafts or driving cross-cuts, winzes, etc., along the course figured, and it requires more accuracy than is necessary for mapping or similar purposes. This problem can always be solved directly whenever it is possible to run a traverse from one given point to the other,

[1] This is a new use of these terms, since formerly they have always referred to a *horizontal* plane, not a vertical plane.

no matter how devious or roundabout the route taken. For on reducing the traverse lines and calculating the lost line the bearing and horizontal projection of the required line are known, while the difference in elevation of the two known points gives the other projection, and consequently the grade between the points. If this principle is understood, the surveyor can turn attention to the real difficulties—the liability to error brought about by the necessity for short and steep sights, the interference of water, bad air, steam, the lack of light, and the cramped places. The young surveyor is advised in such a case to pick out the most unreliable sights, assume a probable error, and figure out what difference it would make in the connection. If not an allowable error, the survey should be gone over with more care if possible, and in any event getting average values that should be nearer the truth.

Too much stress cannot be laid on the importance of the care to be exercised in running connections, as there is nothing the mining surveyor's reputation depends on more directly than his uniform success in this matter. In fact, a failure in such a case may involve a large loss to his employer, or if he has guaranteed the work the cost comes out of his own pocket. On the other hand an error in many cases cannot be remedied, but results in a permanent injury to the mine.

322. Example: Showing the Method of Making an Underground Connection. It is required to give points in cross-cut on second level to start a vertical upraise to connect with shaft at first level. The method is as follows: Set up in the cross-cut on the first level and carefully tie in two corners of the shaft and run to station (2) in the level, thence to station (3) at the top of the winze, then down the winze and along the second level to the cross-cut, and thence to the breast of cross-cut. Permanent points are put in here: station (7) near the breast, and station (6) further back. These points will serve to orient by, for the latter work. Plat the data thus obtained and find how much further the cross-cut should be run and where the station

is to be cut. Calculate a traverse from one corner of the shaft to station (7), and the bearing and length of one side of the shaft. When the station has been cut out set a point for one corner of the shaft from station (7); as figured then from this corner the points for the other corners of the shaft may be set. A satisfactory way of establishing these corners is to set wooden plugs in holes in the rock below, with nails to mark the exact points. Plumb-lines can be suspended over these nails and the upraise thus kept in line. It should be distinctly stated whether the points given are inside or outside the timbers. Generally it is best to put them inside the timbers and let the miners cut out as much more as necessary.

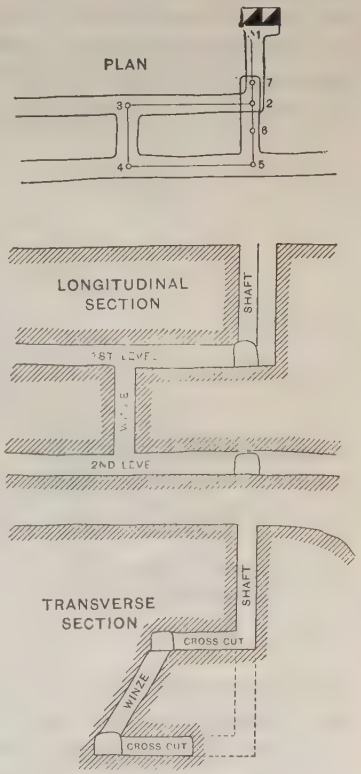


FIG. 172.

The difference in elevation between the top of the station at the end of the cross-cut and the bottom of the shaft will be the length of the upraise. Great care must be taken in this work if the shaft is to be timbered up. Timbering both ways in a shaft is to be avoided if possible, because an error of 2 or 3 inches at the point of meeting is bad, whereas if not timbered until connection is made, that error would not be material on account of the leeway that must be allowed for the timbers.

In case it is required to run a drift in a certain direction for a connection, two plugs on the line will be a good guide for the

miners. If plumb-lines are hung on these plugs, the line may be followed fairly well.

323. To Establish Property Lines Underground. This problem is probably of more frequent occurrence than any other, and is also very important. In many cases the boundary line passes through a rich ore shoot, and the necessity for care in establishing it may be readily seen. It is not uncommon for such a line to be determined underground by a joint survey conducted by the surveyors of the adjoining properties. The following example, Fig. 173, will serve to illustrate the method of solution.

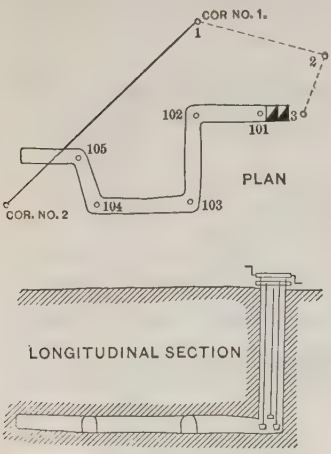


FIG. 173.

It is required to establish underground the line joining Cor. No. 1 with Cor. No. 2. This may be done as follows: Determine the course of the line and run from the corner or some convenient point on the line to the shaft, thence down to the level by the most convenient method, in this example by plumb-lines. From the station at the bottom of the shaft run out the drift to station 105, and from this station take the bearing of the drift toward the line. Now calculate from the traverse a direct course from Cor. No. 1 to station 105. Having calculated the length and bearing of this line, and observed the bearing of the boundary line and of the drift, solve a triangle which will give the distance from station 105 to the boundary line. If it is desired to have the exact point marked, a plug may be put in on line. Sometimes the line cuts diagonally across the drift, or in any case where there is room enough it may be desirable to set up over the station on line

and place two plugs or marks on the boundary line, one in each wall.

324. To Find the Bearing and Length of a Line to Intersect a Vein of Known Dip and Strike. This includes problems involving the consideration of the vein as a plane with a known dip and strike. This class of problems can generally be solved only for approximate results, because it is very rarely that a vein is a true plane, as, even when not faulted, the variations from a constant dip and strike may be considerable. It can only be said in such cases that if the vein continues on a constant strike and dip it will be intersected at a certain point.

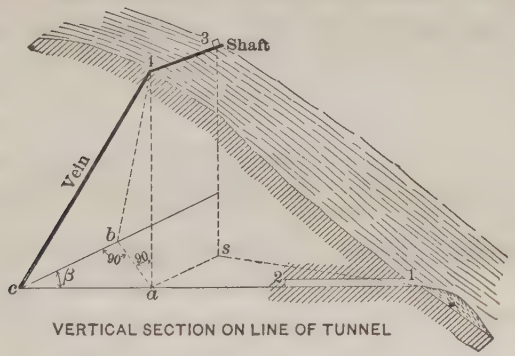


FIG. 174.

Although this uncertainty exists, it is nevertheless most important to know at what point to expect the vein.

As an example, let it be required to find the distance that the tunnel (1) to (2), Fig. 174, must be run to intersect the vein (3)–(4)–*b*–*c*, there being an incline at (3) and an outcrop of the vein.

Set up at station (1) and sight to (2); this determines the bearing required. Run a traverse to station (3) at the incline and find the dip and strike [1] of the vein, then calculate the closing line from (1) to (3) and solve a triangle for the horizontal dis-

[1] See definitions at end of this chapter.

tance from (3) to (4), i.e., a to s, this latter being the point where a vertical plane through the tunnel cuts the outcrop or strike of the vein, on the surface, and from (1) to (4). If the point (4) is found on the ground and the difference in elevation between it and station (1) determined, the required length is calculated as follows: Assume the tunnel is horizontal, then the distances a to (4) and a to (1) are known, (4)–a being a vertical line. A plane passed through a–(4) and perpendicular to the plane of the vein intersects it in the line b–(4), and intersects the horizontal plane through the tunnel in ab. Having a–(4), solve a right triangle for ab, the acute angle being 90°—the dip; then solve the right triangle abc for ac, the acute angle β, being the angle between directions of strike and tunnel; then $\overline{ac} + \overline{a(2)}$ is the distance the tunnel is to be driven. If it is driven to grade, it can easily be figured what difference in length would result. When it is required to drive a tunnel at right angles to the strike of the vein, i.e., to run the shortest tunnel from a given point to intersect a given vein, the above solution becomes much simpler.

325. Surface Surveys. It is always necessary, or at least desirable, to verify and tie in the monuments that mark the boundaries of the mining property, and to establish the true meridian very carefully, at a point convenient for connection with the underground surveys. If expensive plants are to be erected, it may be advisable to make a topographical survey with transit and stadia for the purpose of locating shafts, buildings, dumps, streams, etc., together with the contour lines and any important geological features.

Sometimes the traverses of underground surveys must be duplicated on the surface to find the exact relation of underground work to the surface. This is in case a topographical survey has not been made, or when exactness is required.

326. Court Maps. This name is one given to maps that are to be used as evidence in court in a mining lawsuit. When the maps are used in this way the surveyor is generally called upon

to verify the map in court. Court maps may be classified according to issues involved.

First. When the case is to determine the ownership of ground, as in an adverse suit, the map usually shows the surface ground of the claim in dispute, the position of workings, and any point that may have a bearing on the case, but ordinarily there is no need of showing the underground workings. These maps should be accurately made and drawn to a fairly large scale, and the desired special points should be distinctly brought out so as to be clearly understood by a jury.

Second. When the suit is to determine the ownership of veins, etc., as in the endless variety of apex suits of which the mining laws of the United States are so fruitful a cause. In this case it is necessary to show the boundary lines and all the underground workings that have any bearing on the question at issue. It is often necessary to construct a number of cross-sections to show the continuity of the vein or the reverse. In this class of work, especially in important cases, large sums of money are spent on the surveying and mapping of the properties. No exact rule can be laid down for this kind of work, as the conditions vary so widely, but the surveyor should in all cases study thoroughly the problem in hand so that the maps may be made to show clearly the matters in dispute.

327. Surveying Boreholes. The location of ore bodies is frequently made by drilling. The two most common drills are the churn and the diamond drill. The latter has the advantage of removing a core, but it is very rare that such a bore hole is straight. The trend of such holes have been successfully traced by the use of a solution of gelatin contained in a small vessel with a compass needle and a plumb-bob and of a shape which allows its alignment with any part of the hole in which it may be placed.

"Another instrument takes a photographic record of the position of the compass needle and plumb-bob, after the lapse of such an interval of time as is necessary to place the instru-

Connection. A passageway which is driven from one accessible part of a mine to another.

Dip. The angle of the plane of the vein makes with the horizontal.

Drift. An underground passage driven along the vein or parallel to the strike.

Heading. Any preliminary passageway driven to explore the mine or to facilitate its future workings.

Incline. Applied to a passage having a more or less fixed inclination from the horizontal; a slope.

Level. One of a number of horizontal passages along, or parallel to, the deposit, and placed at more or less fixed intervals, generally 100 ft., for the systematic working of the mine.

Manhole. A small passage from a *level* into stopes or to the next level above.

Mill-hole. A passage left from *stope* to *level* for dropping down ore or rock.

Outcrop. That portion of the vein intersecting the surface.

Pitch. The direction in azimuth and altitude of the longest axis of an ore body.

Shaft. A hole sunk more or less vertically downward.

Stope. The workings above or below the *levels* where the mass of the ore-body is broken.

Strike. The direction taken by the intersection of the vein with a horizontal plane.

Stulls. Cross-timbers between the walls of the excavation.

Sump. An opening at bottom of shaft or at any *level* for the collection of water.

Tunnel. A horizontal passage from the surface; properly speaking it should be open at both ends, but it is not always so used in mining.

Upraise. A shaft excavated upwards from below.

Vein (also called *lode*, *ledge*, *lead*). A mineral body of a flattened shape.

Wall. The rock on each side of the mineral body. The upper wall is called the "hanging wall," and the lower the "foot wall."

Winze. A shaft sunk from a *level.*

EXERCISES

1. A borehole cuts a 6-foot core from a vein whose strike is N. 65°. E. and whose dip is 50° to the northwest. The borehole bears N. 30° E. and dips 75°. Required the true thickness of the vein.

2. A horizontal tunnel cuts a vein at 1000 feet from the portal. The vein strikes N. 45° W. and dips 70° to the southwest. The tunnel is driven N. 30° E. What will be the distance (horizontal) at which a tunnel, on a 2% grade, driven from the same point and N. 5° 10′ W., will cut the same vein?

3. The following traverse is the result of the survey of a shaft and tunnel:

Station.	H. I.	V. A.	Course.	M. D.	F. S.
1	2.9	−72°	N. 23° E.	176.0	2
2	3.4	−68	N. 28 E.	104.6	3
3	3.8	− 3	N. 38 W.	99.2	4
4	3.2	− 2 55′	S. 72 W.	98.0	5

Station 1 is at the collar of the shaft; station 5 at the portal of the tunnel. Required the course 5 to 1, the horizontal and vertical distances 5 to 1. Where necessary a top telescope mounted 0.3 feet above the main telescope was used.

4. From *A*, *B* bears N. 25° 10′ E. 1500 feet; from *A*, *C* bears S. 65° E. 1200 feet. At *A* a vertical borehole cuts a vein at 290.1 feet; at *B* an inclined borehole bears N. 87° 15′ E., dips 77° and cuts the vein at 515 feet; at *C* a vertical hole cuts the vein at 375 feet. Elevation of *A*, 9801.4 feet; *B*, 9641.0 feet; *C*, 9910 feet. Required, the strike and dip of the vein.

5. From a monument at the mouth of a tunnel a line is run in the tunnel, azimuth 38° 44′, slope distance 424 feet, vertical angle +2° 20′; thence azimuth 65° 32′, slope distance 288.5 feet, vertical angle +2° 28′ to breast. From the same monument a line is run on the surface, azimuth 97° 43′, slope distance 317.5 feet, vertical angle −3° 32′; thence azimuth 38° 42′, slope distance 647 feet, vertical angle +13° 13′ to the centre of a vertical shaft. How deep must the shaft be to meet a connecting drift run on a grade of +2.1% from the breast of tunnel, and what is the slope length and azimuth of this drift?

6. The strike of a certain vein at point of outcrop is N. 44° E. and the dip is 73° 40′, pitch S.E. From this point of outcrop a surface line is run, N. 85° 35′ E., slope distance 235 feet, vertical angle −11° 34′; thence S. 3° 44′ E., slope distance 205.5, vertical angle −13° 34′ to a point from which the tunnel is to be driven in the direction N. 70° W., and with a grade of +3.5% until it intersects the vein. (*a*) What would be the slope length of such a tunnel? (*b*) What would be the slope length and bearing of the shortest possible tunnel run on a +1.3% grade to intersect the vein?

7. A vein has a strike of S. 66° W. and its dip is 54°. What is the azimuth of an incline on the vein having a slope of 43°?

8. From the bottom of vertical shaft No. 1 a horizontal traverse was run in the mine to the bottom of vertical shaft No. 2, as follows: Assumed azimuth 0°, distance 243 feet; thence azimuth 340°, distance 120 feet; thence southeasterly a distance of 470 feet along a vein which shows a strike of 60° (azimuth) and a dip of 35°; thence azimuth 52°, distance 25 feet to the centre of shaft No. 2. From a point vertically above the last point a line is run on the surface with true

azimuth 116° 20′, distance 411 feet (horizontal) to a point *A* from which the centre of shaft No. 1 is sighted at azimuth 71° 30′. (*a*) How much deeper will shaft No. 2 have to be sunk to reach the vein? (*b*) What is the true strike of the vein?

REFERENCES IN MINING SURVEYING

1. Mineral Land Surveying. A text on Mining Surveying, by James Underhill. Mining Reporter, Dec. 7, 1905.
2. Mining at Great Depths. B. H. Borough. London Engineer, Nov. 3, 1899.
3. Outline Scheme for Mine Location. School of Mines Quar., April, 1896.
4. A Few Suggestions on How to Survey a Mine. Mining World, Oct. 31, 1908.
5. End and Side Lines. R. W. Raymond. Proc. Am. Inst. Mining Eng., 1889.
6. Surveys in Inclined Shafts. J. Barrell. Mines and Mining, April, 1900.
7. Method of Plumbing Shafts. Illustrated. Eng. News, 1893, I, p. 302.
8. Plumbing Shafts. Geo. B. Hadesty. Colliery Eng., July, 1897, and Aug., 1896.
9. Shaft Plumbing. Eng. and Min. Jour., Sept. 14, 1907.
10. The Divergence of Long Plumb-lines. F. W. McNair, Eng. and Min. Jour., Vol. LXXVIII, p. 578.
11. The Real Error of Mining Survey. J. Barrell. Mines and Mining, Feb., 1900.
12. The Am. Law Relating to Minerals. C. H. Shawel. School of Mines Quar., Nov., 1905.
13. Weak Points in the Federal Law in Relation to Mineral Lands. Geo. W. Ritter. Eng. and Min. Jour., Jan. 26, 1907.
14. Mine Plans—Preparation and Use. J. D. Kendall. Mining Reporter, Aug. 20, 1903.
15. Assessment Work on Mining Claims. W. E. Colby. Mining and Sci. Pr., Dec. 12, 1908.
16. Surveying Mining Claims. Chas. Tappen. Jour. Asso. Eng. Soc., July, 1897.
17. General Practice in Surveying Claims. Colliery Eng., Nov., 1889 March, 1897.

CHAPTER XII

CITY SURVEYING

329. Land Surveying Methods Inadequate for City Work. The methods described in the chapter on Land Surveying are inadequate for the needs of city surveying. This is due to two principal reasons: First, land in cities may be more than a million times as valuable as on a farm; and, second, while farm buildings are generally placed far within the farm boundaries, the owner of a city lot is generally compelled to cover his entire lot with his building, in fact, the entire block may be so built up.

An error of a foot or so in locating the farm boundary could cause no particular hardship, while a fraction of such an error in locating the boundaries of a city lot might involve a money loss greater than the entire value of the farm. City surveying differs from land surveying also because of the greater variety and difficulty of its problems. Some of the most important of the surveying problems will be treated in the present chapter. For detail information regarding the engineering problems connected with sewer, water supply, and pavement construction the student should consult the standard texts on those subjects.

330. What a City Resurvey may Include. When a new building is to be erected on a lot in a downtown district, there are many matters which the city surveyor must investigate besides that of the correct location of the lot boundaries. This fact is

emphasized by the following list of questions[1] which may be answered in the surveyor's survey and map:[2]

1. To what "datum" will the surveyor's plat refer, at a standard with which all levels are to be compared?

2. What is elevation of sidewalk grade at the building line, as compared with said "datum"?

3. What is elevation of sidewalk at the curb as compared with said "datum"?

4. If the level of sidewalk varies along the building lines or street curbs, state what the pitch is, from point to point, referred to said "datum."

5. Give location, size, depth of invert compared with "datum," rate of fall and direction, and all other information available, regarding the sewers in both streets.

6. Same information for water pipes as mentioned above for sewers, gas pipes, and electric conduits.

7. Give any information in your possession, or in that of the city or county authorities, regarding the nature of the ground enclosed by this lot. There may be plats on file showing whether springs, brooks, or water courses of any sort anciently ran over this territory.

8. Show dimensions and location of every wall, pier, or footing coming against the boundary line of this lot, or lapping over onto the neighbor's lots. It is of great importance that the statements regarding these walls and piers be accurate and complete.

9. State the height and quality of each of the buildings now on the lot.

10. Do the gas, water, and sewer pipes come through the curb walls anywhere, and if so, of what sizes and where are they?

11. State the water pressure per square inch at street level in smallest main, noting location of said main.

12. Give established widths of sidewalks on both streets.

13. Does any basement run out under the street, beyond curb wall line; if so, where, how deep, how wide, how roofed over, and how is earth retained?

14. Give exact angles, each and every one of them, at the corners of this lot. This is of the utmost possible importance.

15. Give the width of streets and alley from building to building.

16. Is there any dispute between surveyors regarding any important measurement or angle of this lot?

17. Mark all lot lines where practicable, on a building or street curb, or sidewalk, that will not be disturbed while tearing down and rebuilding on this lot.

---

[1] This list is required of Chicago city surveyors.

[2] The tracing and four blue prints are usually given the client or the client's architect.

18. Note the location and character of masonry and footings on adjoining lots, which run nearly parallel with the lines of this lot at those places; only those near at hand are considered.

19. What curb walls are already built, and what condition are they in now?

20. Give depth of foundations of party walls, and of foundations of present buildings below "datum."

21. Give street-gutter grades.

22. Give heights and thicknesses of party walls and height of adjoining walls.

It must be obvious that the accurate answering of the above questions requires a most thoroughgoing investigation and survey. In the larger cities the collection of such data is usually in the hands of a comparatively few old and well-established surveying firms. Indeed, this work involves such intimate knowledge of local records, both public and private, as to make it undesirable for inexperienced surveyors to attempt it.

Instruments Used

Because of the necessity of greater accuracy, the most improved engineer's transit level and steel tape should be used in making all city surveys.

331. City Engineer's Transit and Level. The omission of the compass needle makes possible the use of the U-shaped standards, thereby giving greater stability to the instrument. The verniers usually read to 10, 20, or 30 seconds. The 10-second instrument would be of special value in the triangulation of a large city. Fig. 228, p. 593, shows a form of this instrument. Another type of instrument, shown in Fig. 176, is now frequently used. This form is especially suited for turning right angles. The upper telescope will transit in the standards, while the lower telescope is provided with both a small spirit level for ordinary levelling, and also an accurate striding level for precise levelling. The city engineer's transit should be provided with stadia wires and a vertical circle so it can be used in topographic surveys. Experience has shown that with a sufficiently sensitive level vial a transit is capable of running very accurate

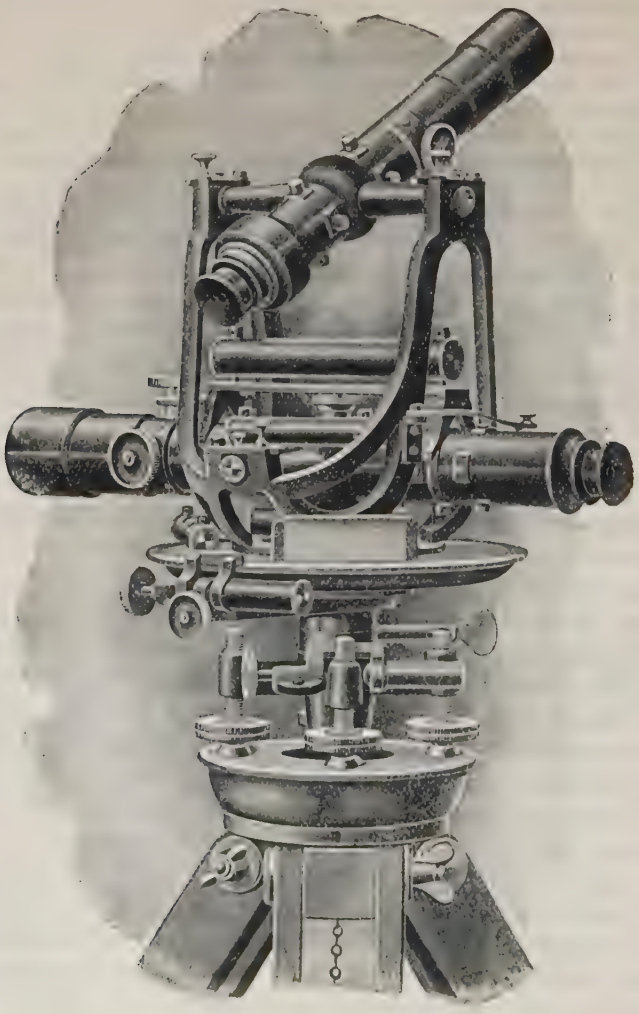


FIG. 176.—Improved City Engineer's Transit

levels; in fact, as accurate as is needed except for the primary bench levels. The self-reading rod (see Fig. 23) is most frequently used in the ordinary profile and cross-section work, but a target should be provided for unusually long shots.

In the larger cities it is desirable and usually possible to have one or more precise levels and transits which can be kept in reserve for the highest grade of work only, the lighter instruments being used for the less accurate routine work.

332. The Steel Tape. The 100-foot steel tape, graduated to hundredths of a foot (Fig. 2) is nearly always used in city work. Such a tape should be compared with the standard city tape before being used and its correction noted. A field thermometer, spring scales, plumb-bobs, marking pins, etc., are also indispensable.

A great improvement over the old form of steel tape is the newly devised nickel-steel alloy, "invar tape," which can now be purchased at reasonable prices.[1] These tapes have a remarkably low coefficient of expansion, only $\frac{1}{30}$ of ordinary steel tapes, or even less. With tapes of so constant a length all temperature corrections may be neglected except on base-line work. For example, if a distance of 2000 feet were measured with this tape under a temperature of 20° different from that of the standard and the correction were neglected, the resulting error due to temperature would be only 0.008 foot. See Art. 421.

City Land Surveys

333. The Requirements of Original City Surveys. City land surveys naturally divide into original surveys and resurveys. By original surveys are meant the land surveys made to locate the lot and block monuments when the city was originally platted.

The public is interested in these surveys, first, to the end

[1] They may be obtained of J. Agar Baugh, 92 Hatton Gardens, London, E.C., England.

that the land be laid out in a way to best serve both present and future public needs (see Arts. 349–351); and, second, to see that the locations are accurately made and permanently monumented. Legislatures in nearly every State have passed laws regulating the manner of surveying and platting land. The laws of Michigan are fairly representative of these laws and are given in full in Appendix H. They are worthy careful study.

The map of an original survey should conform closely to the following requirements in order that it may be entitled for record in the register of deeds office:[1]

1. The length of every side of every lot and block.

2. An exact description of all the exterior boundaries of the entire tract, referred to United States land subdivisions if possible.

3. The exact angle of all intersecting lines and especially all block corners.

4. The exact position of all monuments set, together with a careful description of same.

5. The number of each block and lot.

6. The area of all parcels larger than one acre.

7. The names of all streets, bodies of water, and recognized monuments.

8. If a curve forms the boundary of any lot or block there should appear on its long chord, its length and bearing (or angle from tangent), the radius of the curve, the deflection angle, and chord distance used in laying the curve, as well as the total angle.

9. A simple, but complete title, including the date and name of the surveyor.

10. The scale of map and an arrow for the meridian. In most States the law requires certificates from the surveyor, from the owner, and from the notary. The surveyor's certificate shows:

(*a*) The description of the exterior boundaries.

(*b*) By whose direction the survey was made.

(*c*) That certain described monuments had been set.

[1] Read also Appendix G.

(*d*) That the map is a correct representation of the field measurements.

(*e*) That he had fully complied with the law governing the manner of platting lands.

(*f*) Description of monuments.

The owner's certificate simply recites the fact that he caused the land described in the surveyor's certificate to be platted, as shown on the map, and dedicates the streets to the public use. For the form of such a certificate, as well as that of the notary public, see Appendix H.

334. Subdivisional Work, Field and Office. The first step toward making a city plat should be to secure an accurate description of the boundaries of the land in question. This, not infrequently, requires a careful search in the register of deed's office for still earlier descriptions of the same tract, in order to throw light on disputed or deficient courses.

This investigation and interpretation of deeds for the use of the land surveyor, dealing with the harmony or conflict of the descriptions, is an entirely different work from that of the investigator of titles, which deals with the legal completeness of the conveyance. In the older parts of a town the deed of the present proprietor frequently does not give information sufficient to fix the correct location. The key may lie in some boundary in an early deed referring to a still earlier conveyance of adjacent property; or the earlier deeds may give clearly defined locations, while the later ones say "more or less," at every point.

In some cases the deeds are in such a condition that it is impossible to tell what they mean until it is known what the possession is. Skill in this work can come only after considerable experience; local practices must largely determine what is necessary.

The second step should be a thorough search and survey in the field for all old monuments which define the limits of the land. Especially important are the U. S. land monuments, for, if possible, the plat should be "tied on" to government corners.

This search should not overlook the effect of adverse possession (see Art. 150) in locating boundary lines. A "closed" survey with transit and tape, of the exterior boundaries, completes this step. The third step includes a test of the closure error in such resurvey, by computing the latitudes and departures. If not within reasonable limits, the mistake should be located by remeasurements. If the error is within the limits set for such work,[1] the same is carefully balanced (see Art. 193), the *lengths* as well as the *bearings* of all courses being corrected. Such balanced data are now mathematically consistent. If the city authorities have not already prescribed the proper plan for the subdivision, a plan must be prepared which will conform to State laws (Art. 350). The fourth step consists in setting the block and lot corners according to the plan. The minimum party for this work should include a transitman, two tapemen, and an axeman. For the best work, the block monuments should be set first, after which the lot stakes are set, generally along the lines parallel to the lot frontage.

A preliminary measurement shows where the stake should be driven. After the stake is driven a tack may be placed in the stake to mark the exact frontage. It is necessary to measure at least all angles of all blocks, while in irregular lots additional lot angles may be needed. In addition, the length of all sides of all blocks and lots must be known. The field notes are best kept in the form of a sketch.

The fifth step consists in preparing a map in accordance with the State laws and including all the certificates required. Finally, the sixth and last step is completed by recording the plat after its certificates have been duly executed, witnessed, and the plat approved by city or State authorities.

335. Monuments. How many monuments shall be located, and where shall they be placed? What material shall be used and how set? Answering the first question, it is plain that no

[1] 1 in 5000 to 1 in 15,000 or more.

more work should be attempted than can be done well. Better one point and an azimuth than points everywhere and no two agreeing, either in distance or direction with the relation described by the plat. But so much should be done well that the labor of locating any point in the subdivision from existing monuments shall not be excessive. The points chosen for placing monuments should be such as will continue to be accessible and will not be ambiguous. The centre lines of intersecting streets are sometimes marked, giving one monument to each intersection. This, however, is objectionable because of the liability of the centre line to disturbance due to sewer, gas, water, or pavement construction. Others choose the side lines, giving four monuments to each intersection of streets, but a still better practice is to put the monuments on offset lines in the sidewalk area.

Stones, or concrete monuments made in place, are more often chosen than any other material; iron bars and gas pipe, etc., are sometimes used, driven in place with a sledge; cedar posts, say, 4×4 inches, are quite durable, and hard-burned pottery is sometimes used. Whatever material is chosen, the foundation, which should be flat—not pointed—should reach below frost; and the centre of gravity be kept as low as possible, so that there shall be no tendency to settle out of place when the ground is soft in the spring. When the tops are much above the surface of the ground, there is a liability that they may be displaced by traffic. Probably the surveyor does not see any traffic, or the prospect of it, when he is doing his work, but the traffic must come before the work of the monument can be spared. It is better to bury the stone wholly and indicate where to dig for it by bearings than to run the risk of losing the whole work through indiscretion in placing the monument. In situations where every rainstorm produces a slight fill, it is safe to place the top considerably higher than would otherwise be reasonable. The stones to be set are so placed in the excavation, with the heavy end down, that when the top is in the proper position and before any earth is refilled there is no tendency to fall in any direction; then while the earth

is being refilled and thoroughly tamped about the stone, the top is kept in place. It is better that the mark denoting the point for which the stone stands should be cut upon before it is placed in the ground. When this is done, if the mark is worn off by traffic or knocked off by accident, the centre of that portion of the stone which remains is a very close approximation to the original point. Fig. 177 shows the manner of setting the stone by first referencing the exact point by two intersecting lines from opposite stakes, *AB* and *CD*.

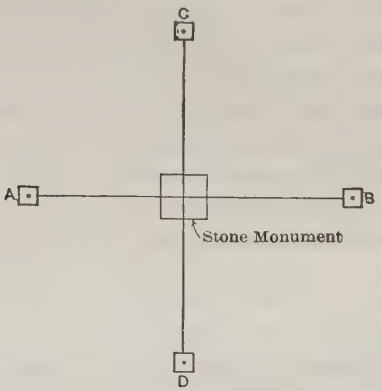


FIG. 177.—Setting a Monument.

336. Defective Plats. The following are important defects commonly found in the very old plats.

Very few of such plats show what stones have been set by the surveyor, or indeed indicate that he has any knowledge that such monuments may ever be useful. If the custom were once established of noting upon the record the location and description of monuments, any monument found during a resurvey, but not shown on the record, would be discredited. As matters now stand it must be proved incorrect to be discredited—a thing not always easy, for a system of quadrilateral blocks whose angles are not recorded and whose street lines are not necessarily straight is far from being rigid.

Many plats require measurements to be made along lines which are easily measured while the land is vacant, but which will become inaccessible as soon as the property is built up. The obstacles to be overcome before the result can be reached by the method described on the record will each add to the uncertainty of the result.

There are many ways in which plats are made which are all justly subject to criticism. Two examples will suffice. Irregularly shaped blocks are sometimes treated as in the annexed sketch, Fig. 178. The outline is subdivided mechanically, and proportional distances are given on interior lines which are not consistent with any trigonometrical relation of the exterior lines,

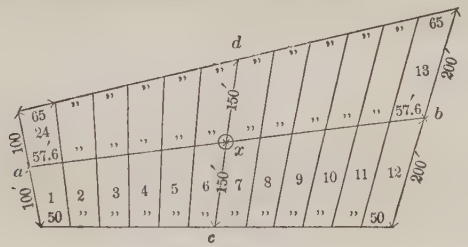


FIG. 178.

much less with that which does exist but is not recorded. The point x has nine distinct locations directly from the plat. On the theory that ab an cd are straight lines, their intersection gives one; ab straight, the distances ax and bx give each one; cd straight, the distances cx and dx give two. Combine the distances ax and cx, bx and cx, etc., and get four more. But this is not all, for the point x stands related to each of the ten other points along the line ab, and each of these has also nine locations which accord with the plat, and our point x may be located from either of them or any combination of them when they have been located by any of the methods described.

Besides the difficulty of determining how interior points should be located, this fan-like subdivision wastes ground in

each lot which results in wedge-shaped remnants about the buildings. These remnants would be valuable if thrown together into the corners, thus keeping the remaining lots rectangular at the front. The attempt to reach a rectangular front sometimes fails through inattention to very simple matters, as in Fig. 179. Here no angles are recorded. The rear corners of the lots are located along the line *ab* by distances from *a* or *b*; but the record depths do not fall upon a straight line. The line *ab* should bisect the angle between the block lines or be parallel to such bisector in order that with a constant distance along *ab* common to the series of lots on each side of that line, their angles with

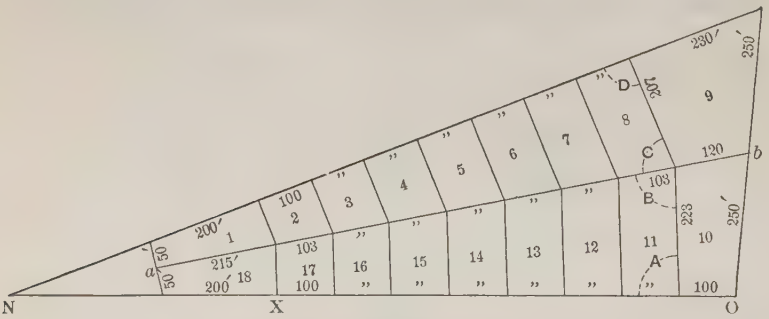


FIG. 179.

their respective fronts may remain constant. In the case given, every lot line has an angle with the block line upon which it fronts different from that of every other lot line, and all dependent on some block angle which is not recorded. If it is not desirable to bisect the block by the line *ab*, its direction may be chosen as desired, the distances along it are fixed by the fronts on one and the angular divergence from that side which is chosen, and the lot fronts on the other side of the block must be correspondingly increased or diminished.

337. City Resurveys. In Monumented Cities. The retracing of city lot lines becomes necessary as soon as building improvements are made, nor do they cease even when the entire

block is covered with houses. The healthy growth of a city often means the replacement of residences with business blocks, and these again with others of still greater pretensions. Every change is likely to call for a careful survey, and the necessity for accuracy of course increases as the character of the improvement and the real estate becomes more valuable.

The method of making such resurveys in a city whose blocks are well monumented is much easier and scientific than the methods applicable in cities with few or no monuments. This can be most clearly stated by use of an example. Suppose in Fig. 179, the original block monuments at N and O are still existent but the lot stakes have all disappeared, and it is required to locate the lot corner at X. In this case X is located by the time-tried rule of proportional measurements. Thus if the block distance NO recorded on the plat was 1264 feet, and the resurvey measurement was 1264.38, the resurvey distance for OX would be derived from the proportion,

$$1264 : 1264.38 = 800 : Y.$$

$$Y = \frac{800 \cdot 1264.38}{1264} = 800.24.$$

This shows that the chain used in the original survey was 0.03 foot longer than the resurvey tape. This method should give the same location, independent of the length of tape used in the resurvey.

338. Resurveys in Non-monumented Cities. By far the larger number of smaller cities and many of our larger cities, at least in part, are included in this class. For, in the days when cities were laid out with needle compasses, it was thought that one monument only was needed to reproduce all lot and block locations, and even this monument finally yielded to the road-maker's plow. In such cases, the proper location for the lot corner must needs rest quite as much on the good judgment

of the surveyor as on any rule of scientific measurement. No surveyor would doubt that his duty was to find the *original* location, right or wrong, but how can two surveyors be expected to agree on a location which must be based upon individual judgment. The courts rule with considerable unanimity that in such cases next in authority to the original monument come fences, buildings, party walls, etc., which were erected at a time when such original monuments marked the line; the assumption being that the occupancy was with respect to such original monuments.[1] In such cases a careful and systematic survey is made of all such marks of lot occupation in the entire block and even the adjacent blocks. The problem is now to make the location which will be more reasonable than any other one that can be suggested. A study of the data secured will generally show what the surplus or deficiency in the block is. A more difficult problem is to know just how to distribute this difference in measurement among the subdivisions in the block. In fact, no theory invented for the treatment of surplus and deficiency has been able to stand the test of the courts against all combinations of circumstances.

The distribution of the whole front in proportion to the recorded distances has been generally upheld by the courts. Care must always be taken not to confuse the present problem with a case where property lines have become fixed by virtue of adverse possession (Art. 150).

In the larger cities, street lines have been measured and remeasured very many times by different surveyors, each of whom leaves his *private* block corner marks or references. In time this generally results in fixing the corner exactly or within very narrow limits. Where such turns out not to be the case, and the marks of possession are inconsistent with each other, the surveyor's duty to his client is simply to show, by a map and a report, the conditions as he finds them and await further instruc-

[1] See Decision by Wisconsin Supreme Court, Vol. XLIV, p. 104.

tions. For him to usurp the powers of the courts is likely to result in unnecessary controversies. Frequently the surveyor may act the part of a peacemaker and thereby effect a compromise, much to his client's advantage and his own reputation. Such an agreement becomes at once binding on the parties thereto,[1] and fixes the controverted lines for the future, quite independent of the actual original positions, should these ever become known.

It is much more important to the owner that no uncertainty shall exist regarding his boundaries than that the actual land shall have the dimensions, exact to an inch, stated on the recorded plat.

339. The Value of an Existing Monument is based (1) on the fact that it corresponds in character and position to a monument described on the recorded plat; (2) on the custom of placing monuments upon the completion of a survey, and on the supposition that the monument in question was set in pursuance of such custom, although no monuments are noted on the plat; (3) on recognition by surveyors and owners of land affected by it; (4) on the knowledge that it was placed by a competent surveyor at a time when data were accessible which are not now in existence. The value of the evidence which establishes or tends to establish the reliability of the monument is primarily a question for the judgment of the surveyor. His decision must be reviewed and defended before courts and juries when there is a difference of opinion.

The monument is valueless, or less valuable in all degrees, when there is evidence that it has been disturbed. It sometimes happens that there is no better way to establish a corner than to straighten up a stone which is leaning, but has not been thrown entirely out of the ground. Inquiry often brings out the fact that a stone, after being completely out of the ground, has been reset either by agreement of owners adjacent, or by the reckless

[1] To prevent misunderstanding, the agreement should be in writing and duly recorded in the register of deeds' office.

individual who did the mischief, and is still pointed out as the stone the surveyor set. As a recognized corner such a stone has some value, i.e., it is to be supposed that it is somewhere in the right neighborhood; but if its position can be verified from other points which have not been disturbed the work should be retraced. If the original survey was made in a careless way or the corner-stones were badly set, they may help a careful man to come to an average line which shall correspond with the recorded plat. Monuments are sometimes moved or destroyed maliciously. It is wise for a surveyor to test discreetly everywhere, but to be especially careful where there has been quarrelling about lines.

There is a principle, recognized to some extent by the courts, that the existing monument is the evidence of the original survey, whether or not it is called for by the recorded plat. The custom that the surveyor making the subdivision and the plat for record shall set corner-stones is so far followed that this is generally true, cases of accident, carelessness, and mischief, and such cases as that mentioned below, being somewhat exceptional, but many times very real. It is sometimes attempted to go a step further and affirm that the recorded plat is the record of the survey. This reverses the order of events in most cases, the survey being made in order to mark upon the ground the chief points of a plan already fixed upon; and as to all the main lines, the plat is not altered, however carelessly the survey may be made. There are subdivisions where no monuments were set and where no certain evidence is in existence of how or where the original survey was made, or whether any survey was made at all, and yet there is a recorded plat. A surveyor being called upon to make a survey of some parcel in such a subdivision, sets stones in order to secure recognition for his theory of the proper location. If he does his work carefully he undoubtedly does the public a service. Can any amount of ignorance of when or why these stones were set ever make them evidence of the original survey? In other cases some monuments may be in existence,

but more would be convenient—points are determined from existing monuments in *accordance with the recorded plat*, and stones are set. Another surveyor may feel a little nervous about manufacturing this sort of evidence of the original survey, or more likely, may think it too much trouble and a damage to the business, for the more doubt the more work for the surveyor, so drives his stake. Then comes the owner, who, desiring to secure a permanent corner, digs a hole about the stake without taking offsets, throws it out, and sets in a stone—an existing monument! This is no fancy sketch, nor are such facts so very rare. The young man who thinks he would like to be a surveyor, but has no eyes nor ears for facts like these, had better turn his attention to some other business. The resurveying of land is an art, not an exact science.[1]

340. The Preservation of Lines after the monuments have disappeared is accomplished by means of notes and marks on buildings, sidewalks, curbing, pavements, fences, etc. Notes on buildings describe not only the character of the building, but the particular part noted, so that another man years afterward using the same note would have no doubt of the identity of the point. In a growing town the work of keeping up the notes goes on without ceasing—buildings are remodelled or rebuilt, streets reconstructed, destroying old marks. The old becomes the new so constantly that the surveyor who would preserve the information which he already has must be constantly employed at the work of renewal. There is no place either in the street or out of it where the surveyor can place his mark and say to all comers, "Touch not." It follows that whenever it is necessary to use any mark, about the permanence of which there can be a shadow of a doubt, the permanence of the mark must be shown by some practicable test; it is careless to assume it.

341. The Cause of Disagreement between Surveyors arises from differences of information or of judgment, and in a

[1] Original land surveys at the present time can and should be made accurate within reasonable limits of error.

less degree from differences of skill. These are all just as human elements as the lawyer deals with in his work. Testimony is affected by the interests of those who speak, and the judgment varies with the temperament of the individual. Perhaps one of the most difficult lines for a surveyor to draw is that which separates his confidence in his own skill in retracing a survey, confessedly inaccurate, from his reliance on testimony evidently biassed as to the position or disturbance of monuments. Errors in execution may be kept within such limits that work which shows differences in closing of 1 in 5000 should be retraced, and the average observed differences in one surveying party's work will not exceed 1 in 20,000. Two sets of men working to reach the same standard may err in opposite directions, so that differences between two surveyors may reasonably be expected to be somewhat larger than either would tolerate in his own work.

342. City Standards.—Much disagreement in surveys would be avoided if every city had a *standard of length*, say, 50 or 100 feet long, with which all surveyor's tapes could be compared and made to correspond. In the larger cities it is the custom to maintain such a standard near the city hall, sometimes in a stone curbing or other place secure from disturbance. Sometimes the standard consists of a steel rod, supported on rollers on the side of a building[1] and protected from the sun. It is desirable that the length of such a city standard should correspond to that of the National Bureau of Standard, Washington.[2]

[1] See illustrated paper by Chas. W. Sherman, The 100 Foot Standard of Length of the Boston Water Works, in Jour. Assoc. of Eng. Soc., Vol. XVIII, April 1897.

[2] This bureau will determine, for a small fee, the true length of any tape sent to them.

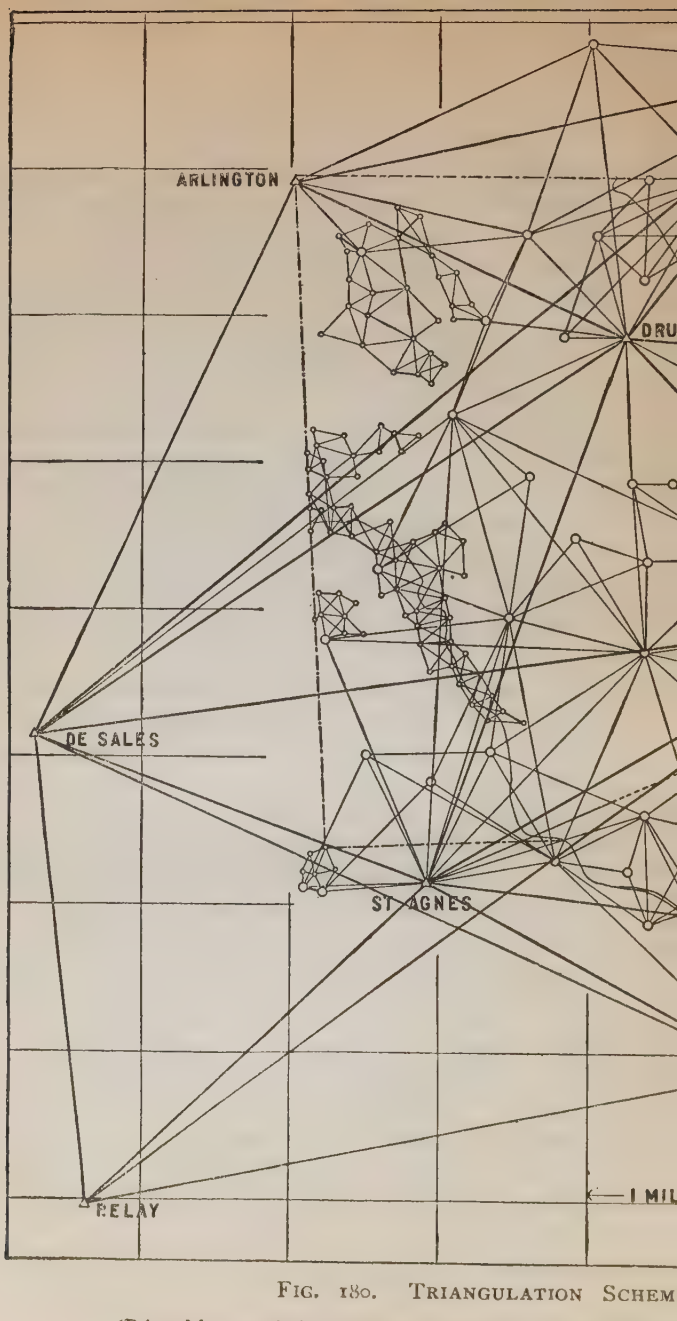
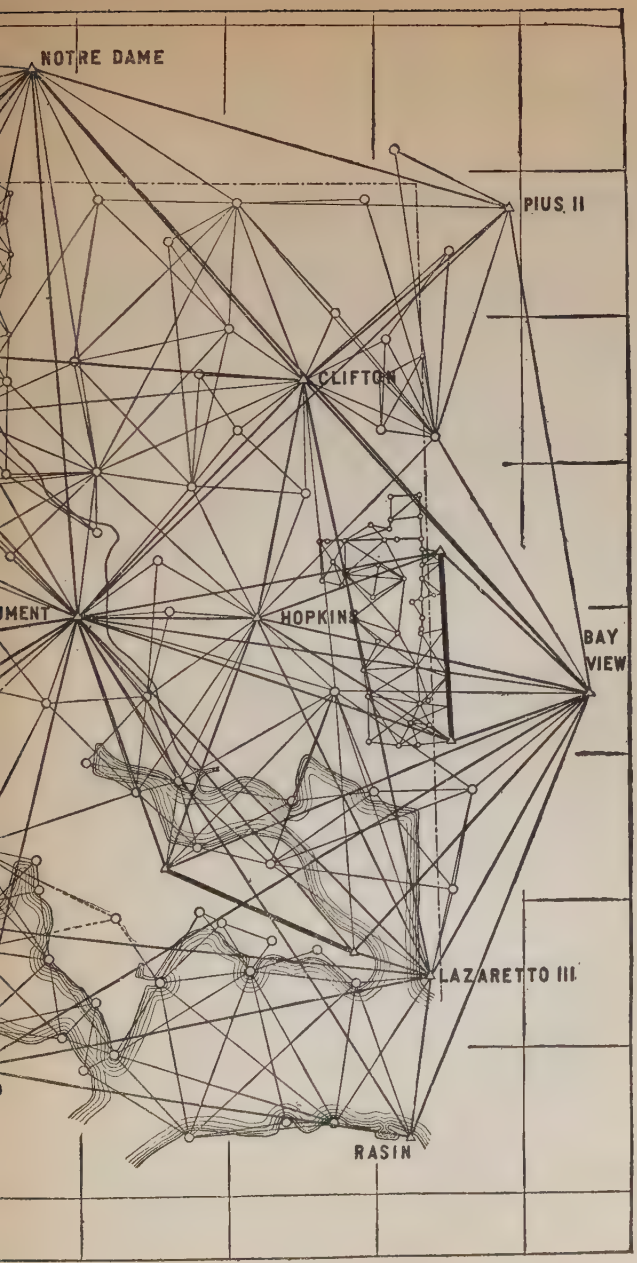


FIG. 180. TRIANGULATION SCHEM

(Printed by permission of Major Joseph W. Shirley, Chief

From Breed a



VEY OF THE CITY OF BALTIMORE.

he Topographical Survey Commission, Baltimore, Md.)

" Surveying."

[*To face page* 463.]

City, Cadastral, and Topographic Surveys

343. General Description. An accurate topographic map of its entire area is one of the most valuable possessions a city can have, for the reason that it makes possible the most economic location and construction of its sewers, water supply, and street improvements. The true importance of such surveys has only tardily been recognized by American cities, but in Europe nearly every city bases its extensions upon such surveys.[1] Without such maps a city is forced to design its improvement piecemeal, with the result that they later have to be changed, abandoned, or operated at a loss. Great economies result from combining the topographic work with a complete property or cadastral survey.

Because of the limited area involved, the effect of curvature of the earth is disregarded in such city surveys, and instead, the map is referred to rectangular axes, one being the true meridian. A convenient origin is selected and all points in the city are located by its distance E. or W. and N. or S. of such origin. The city of Baltimore, for example, referred all co-ordinates to the Washington Monument as an origin (see Fig. 180). Such a survey should have sufficient and accurate control, both vertical and horizontal.

344. Triangulation System. The horizontal control is obtained by a triangulation which starts from one or more accurately measured base-lines and expands until accurately determined points are fixed in all parts of the city.[2]

For the purpose of such work a repeating theodolite is pre-

[1] See Jour. Assoc. Eng. Soc., 1893, p. 1, for a report on the excellent survey of St. Louis.

[2] On the St. Louis Survey, 87 triangulation stations were located, or about two stations to the square mile. On the Baltimore survey, a total of 116 stations were located, or two per square mile, besides 70 steel-tape traverse stations per square mile.

ferred to the direction instrument because of the fact that the former requires less walking around the instrument. Perhaps the greater number of the triangulation stations are forced to be located on roof tops, and if the observer were forced to change his position while reading the angle, the lack of stability would result in too large errors. For a full description of instruments, programme of observation, measurement of the base and all office reductions, see Chapter XIV. The grade of work required might vary from the highest accuracy (primary) to tertiary triangulation.

345. Steel-Tape Traverses. Following the completion of the triangulation a large number of accurate traverses are sometimes run by the transit and tape between known triangulation points. The errors of closure of these traverses are adjusted on the assumption that the triangulation work is free from error. Finally, when the street lines are connected with the co-ordinate system, the co-ordinates of all block and lot corners become known.

346. Precise Level Bench-Marks. To prevent large accumulative errors in the vertical control of the map, a system of precise level circuits should be run, starting with some central bench mark or directrix, and gradually extended outward until the circuits cover the city. The special instruments and all field work thus included is quite fully explained in Arts. 480 to 494. Permanent bench-marks are located at places where they are likely to be undisturbed by frost or traffic. In the St. Louis survey, 240 miles of precise levels were run and B. M.'s set out at an average distance apart of 380 meters. In the Baltimore city survey 282 miles of precise levels were run which determine 600 B. M.'s located at an average interval of 1200 feet along the route.[1] Many hundreds of miles of both wye levels and transit levels were initiated from these precise B. M.'s for topographic control.[2]

[1] The cost of the precise levelling on the St. Louis survey, including field and office, was $45.38 per mile, that of the Baltimore survey cost $23.56 per mile. The average error of both surveys was .003 ft. per mile.

[2] See Chapter XVII. for detail description of precise levelling.

347. The Datum Plane. The plane of reference to which all levels are referred is called the datum. This usually is chosen with reference to the topography. For example, cities situated on the ocean or other bodies of water would find it convenient to use some established stage of such body of water (mean, high, or low stage) as such a plane of reference. This datum would have special significance in cities whose sewers empty into the body of water. Again sea-level datum may be used.

348. City Topographic Methods. The following three methods were tried on the Baltimore survey and in the following order: (1) By transit and stadia method, the notes being worked up and platted in the office, after which the sheets were taken into the field and the contours drawn; (2) by plane table and stadia, the map being prepared on the plane table in the field; (3) by transit and stadia, but with the notes worked up and platted in the field on an 18×24 inch drawing board, mounted on a tripod. The second method "was not found well adapted for working on so large a scale (1 inch = 200 feet), and where so many details were to be located."

The third method was used in over three-quarters of all the area. The field party comprised one topographer, one recorder, two rodmen, one axeman, one computer, and one draftsman. The locations included "all streets, alleys, drives, and roads; all fences, walls, and hedges; all public buildings, churches, schools, factories, depots, warehouses, and residences; all railroads and street railroads; all streams, creeks, ravines, lakes, and ponds; all property and block corner-stones, etc.[1] Previous to going into the field each field party was furnished a sheet on which had been platted all triangulation and primary traverse locations with all lengths and azimuths of such lines and also the elevations of all precise level bench-marks. As rapidly

[1] For a full description of this survey see Eng. News, 1894, Vol. II, p. 83, and 1895, Vol. I, p. 33, and Vol. II, p. 32; also 1896, Vol. II, p. 307, 312, 364, 400.

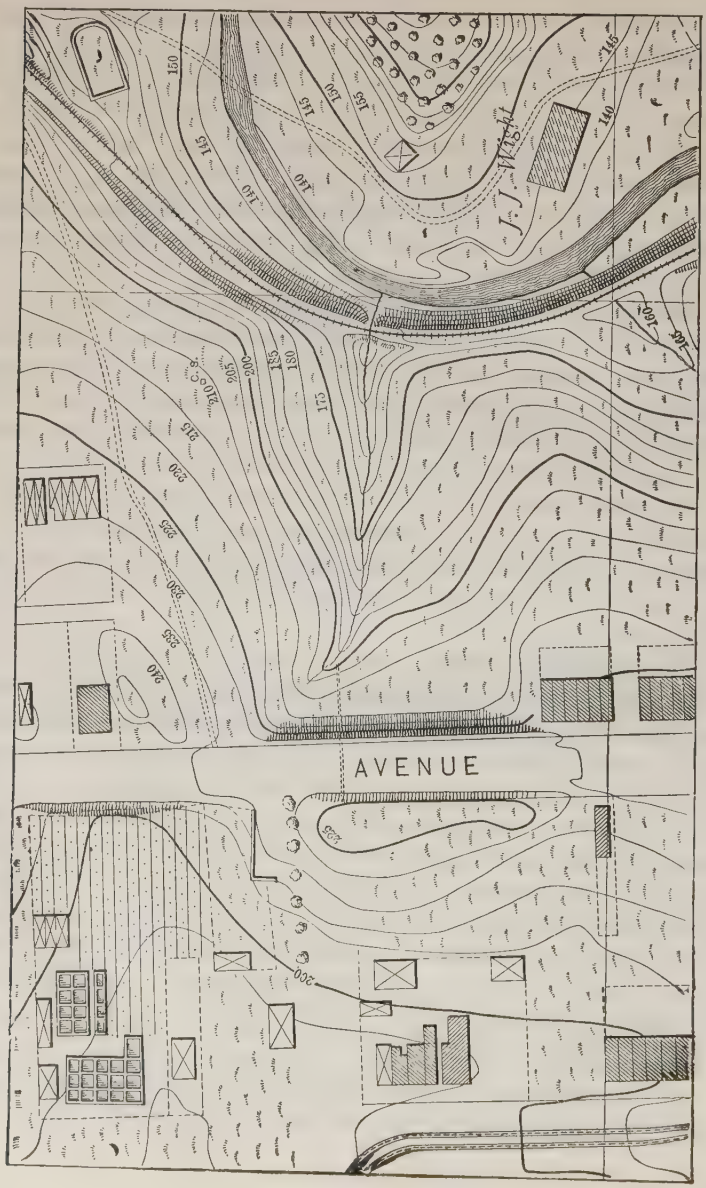


FIG. 181.—Topographic Sheet, City of Baltimore.

as the topographer read azimuths, distances, and vertical angles, the draftsman platted the same, while the recorder worked out the elevations. When all necessary locations were completed the contours were drawn on the field board, and the party moved on to the next station. In all, 2320 stadia stations were occupied and 134,000 sights taken, or an average of 10 per acre. Fig. 181 is a reproduction of part of one of the finished topographic maps. In the original the contours are drawn in brown.

The City Plan

349. The Location of the Site. The location of the city site, instead of being selected on its merits, is too often a matter of accident. It is a reflection on any people that they will allow a mere cross-roads, a waterfall, or a sawmill to locate their cities for all time. This should be especially true of a new country. No city should be located where an adequate supply of good water is not obtainable. Again, city sites should be selected with some reference to the topography and ease of transportation. A swamp,[1] a universal hillside, or a mountain top should not be chosen. Streets with sufficient grade to insure good drainage are preferable to perfectly level ones. Excessive street grades are expensive to maintain and prohibitive of all except slight traffic. The location of a city on an unfavorable site results in an unnecessary tax on the health and wealth of its future citizens.

350. Street Plan. No ideal plan can be laid down as a model for all cities, but instead, each city requires a special design for the size and arrangement of its streets. This design should depend largely upon the topography, and the probable future needs of the city for transportation, both in direction and amount.[2]

[1] Chicago is located in what was originally a swamp, and as a result the extra cost of the foundations for its buildings has been untold millions of dollars.

[2] The plan of New York unfortunately provided for a maximum traffic in an east and west direction, instead of north and south. This resulted in so few north and south avenues as to add immeasurably to the difficulties of the transportation problem.

Special provision should be made for locating and orienting the streets of residence districts so as to give pleasing land or water views as well as plenty of sunlight.

Washington, D. C., is usually pointed to as a model plan (Fig. 182), but an attempt to use the same plan in another city might not be desirable. The unusually large amount of street area (43 per cent of the total area) would prove too great a burden in many cities, but its feature of radial streets is one which could be adopted, at least in part by nearly all large cities.

The practice, common in Europe, of dedicating the land adjacent to all bodies of water for street or other public purpose and of requiring the owners of land to make proper provision for transportation, has not been generally followed in the United States. In recent years legislation has been adopted which compels land owners to submit all plats to the city authorities for approval.[1]

351. Location of Streets. The location of streets is the foremost consideration in city planning. It is the street which makes the city possible and it is the street which permits the growth of the city.

City street plans are of three general types: First, the checkerboard or rectangular system, which gives the maximum number of blocks and lots; second, the first system, modified by occasional diagonal streets for the heaviest traffic (Fig. 182), is a good example; and, third, a ring or concentric arrangement of city streets. Examples of the last are found mostly in Europe[2] (Fig. 183). Outline Map of Paris.

Unfortunately, the first form is very generally used in United States. However, much of the present efforts toward replanning our cities is with the view of adopting the second system, while many European cities are spending many tens of millions of dollars

[1] See example of good and bad planning of water front in the Charities and the Commons, now called "The Survey," Feb. 1, 1908, p. 1509.

[2] This is in part due to the peculiar way in which these cities were forced to grow during the middle centuries, viz., surrounded by a wall for defence.

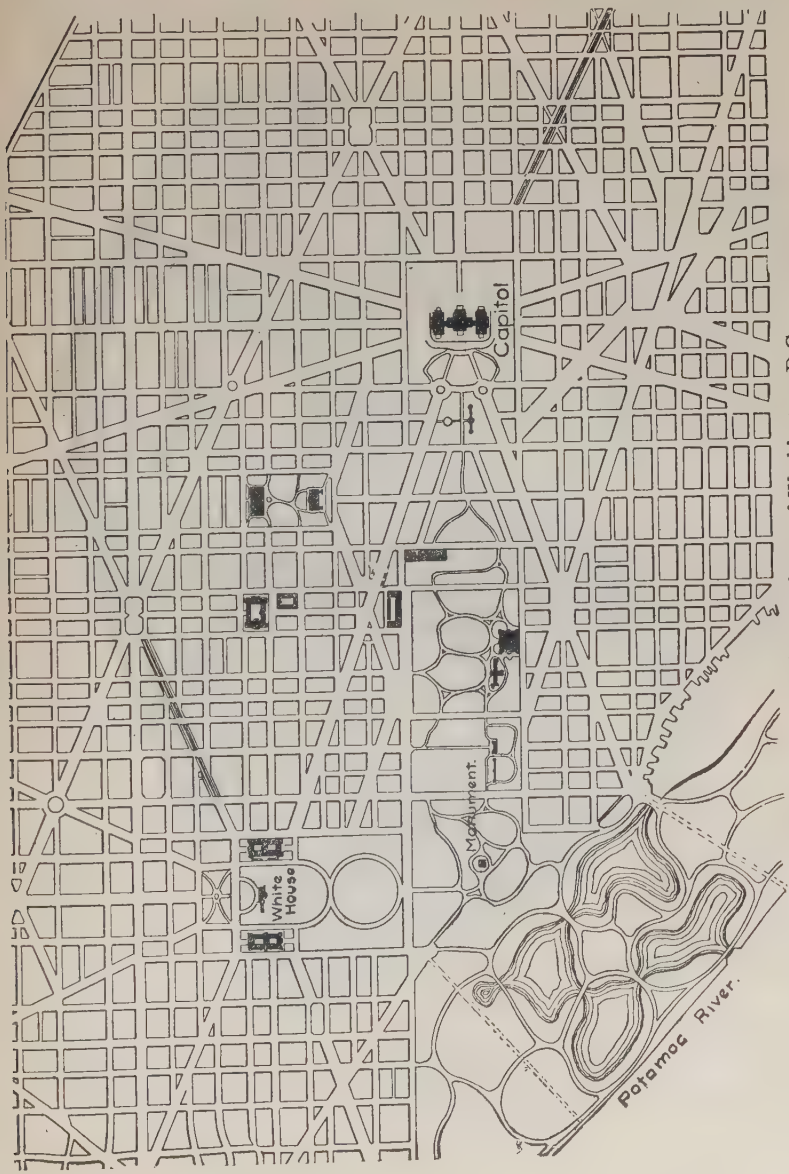


FIG. 182.—Plan of Central part of Washington, D.C.

OUTLINE MAP
OF
PARIS
METRES
1 200 400 600 800 1000
EXPLANATION
Extension of the Western and Orleans Railways
Metropolitan R.R. routes

FIG. 183.

in securing additional diagonal streets or in widening present ones.

The general adoption of the gridiron system of streets is responsible for most faulty street plans. The two main objections to such systems are first, that it compels traffic to go on two sides of a right-angled triangle, instead of using the hypothenuse or diagonal, and second, the excessive cost of street improvements and excessive damage to abutting lands. Fig. 182 shows how the general use of diagonal streets may be made to assist in the transportation problems of a city. Fig 184 shows clearly how the substitution of diagonal streets along natural drainage channels in place of the rigid rectangular system is sometimes a real necessity. It is obvious that the construction of pavements and sewers through such diagonal streets will be at a great saving of cost over the rectangular system, besides the great saving in the maintenance and use of the pavements.

The rectangular system can be adopted on fairly even ground, but in every case it should be supplemented by wide avenues which lead by radial paths from one important civic centre to another. Past mistakes in a city plan are difficult to remedy, but the application of the above principles to the new development in suburban districts should be easy, and under the direct control of the city.

352. Size of City Blocks and Lots. The size of blocks and lots will vary to meet different conditions. The length of city lots is usually between 100 and 180 feet, and their width between 20 and 30 feet. The extreme length of 180 feet would be suitable for the highest class residence district, and provides room for a large lawn. A width of 25 feet seems to be preferred by most authorities, as this is a convenient width for business. If large lots are provided, their subdivision is certain to mix up the records.

A long, narrow block is the most common except sometimes in the business centre. An alley, varying in width from 10 to 30 feet, is usually run lengthwise through the block, especially in a

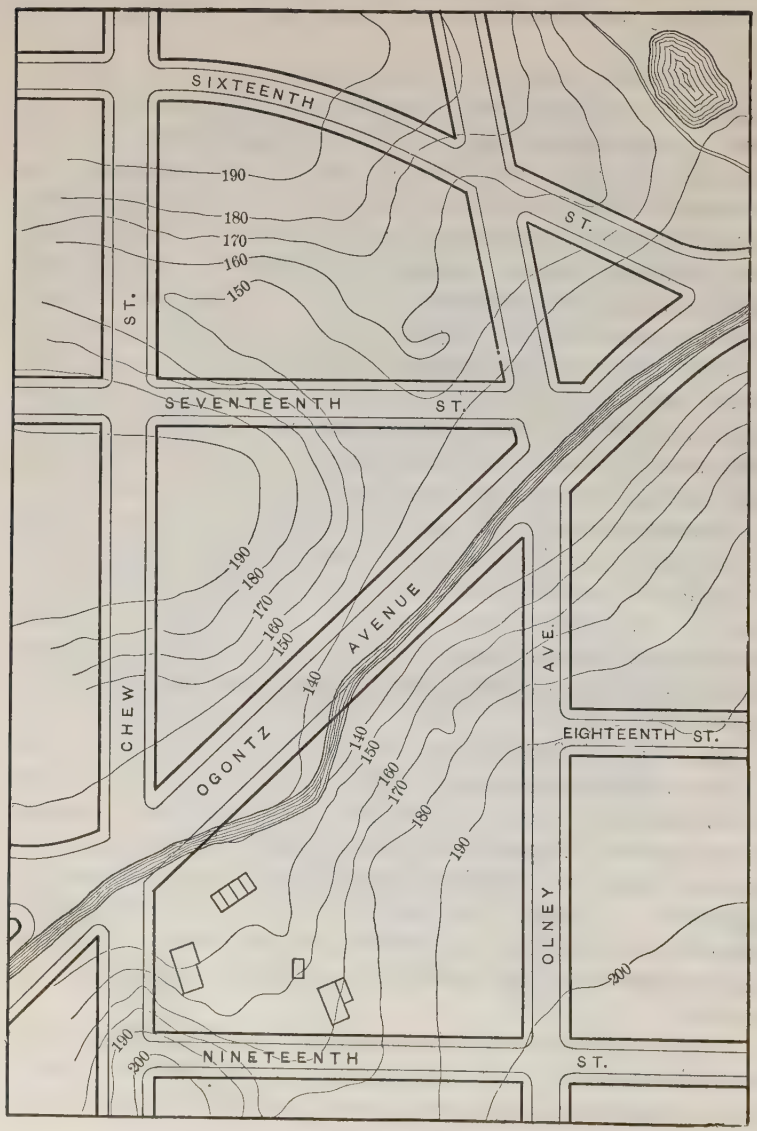


FIG. 184.—Utilization of Diagonal Streets for Drainage. (From City Plan, No. 178, Philadelphia.)

business district. The following are the dimensions of typical blocks in the following cities: Boston, 220×400 feet and 100×550 feet; N w York, 200×900 and 200×400 feet; Philadelphia, 400×500 and 500×800 feet; Washington, 400×600 and 300×800 feet; Chicago, 300×350 and 300×500 feet.

The present type of block when built up seems to result in the unsanitary and inconvenient conditions shown on the left of

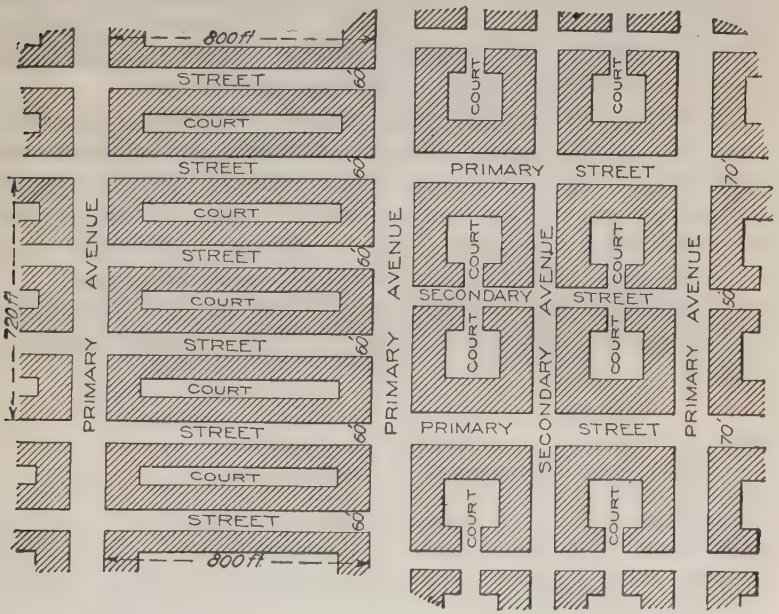


FIG. 185.—Street Plan proposed by Mr. J. F. Harder.

Fig. 185. The right-hand side of this same figure shows a proposed modification [1] of much merit. The objectionable closed court is here replaced by an open court. The two plans give substantially the same area for building and for street purposes.

[1] Proposed by Mr. J. F. Harder in Municipal Affairs, Vol. II, pp. 41–44.

353. Width of Streets. The typical city street in a downtown district resembles a tunnel, the light and air being effectually cut off by the excessively high buildings. Wider streets would greatly assist in bettering sanitary conditions. And yet all streets do not need to be of the same width. Instead, they should vary in width from, say, 210 feet, the width of Ocean Parkway, Brooklyn, with its three roadways, to 40 feet on some unimportant short side street. Such a plan provides adequately for the city needs without undue waste of land. In a residence district a width of from 50 to 80 feet is common. Main thoroughfares, however, should have a width of not less than 100 feet.[1]

Events have sometimes shown that the widening of a narrow street has had the direct effect of increasing the adjacent real estate values more than the cost of enlargement. In platting land it is worthy to note that land is sometimes more valuable to the lot owner when it goes to make a *sufficiently* wide street than when it is added to the lot, thus making a *too narrow* street.

The following table, taken from the census, gives the proportion of the area devoted to streets in a few extreme American cities:

MINIMUM.		MAXIMUM.	
Worcester, Mass.	5.43%	Duluth, Minn.	86.7%
Framingham, Mass.	7.55	Dallas, Texas	78.3
Philadelphia, Pa.	8.42	Denver, Colo.	73.9
Boston, Mass.	8.76	Indianapolis, Ind.	56.4
Lowell, Mass.	8.92	Washington, D. C.	43.5
Fall River, Mass.	9.17	Davenport, Ia.	42.1

354. Street Grades. One of the most important services which the city engineer can render is the design of a good system of street grades. If this design is made before the adjoining property is improved the problem is much simplified. The considerations which control grades are the following: (1) the drain-

[1] Macon, Ga., has the widest streets of any city in United States, they being alternately 180 and 120 feet.

age; (2) the accommodation of the traffic; (3) the cost of cut and fill; (4) the effect upon abutting lands; (5) the appearance of the street. Frequently too much weight has been given to (3) and too little to (1) and (2).

As a basis for a good design an accurate topographic map or profiles of the centre and side lines of the street are indispensable. The grade or vertical alignment is then referred to bench-marks (see Art. 346) in the same manner that the horizontal alignment of the street is controlled. Where the topography allows, opposite curbs should have the same elevation, and street intersections

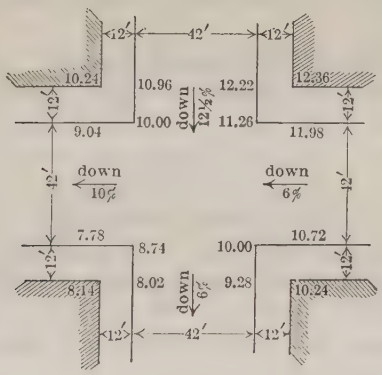


FIG. 186.—Street Grades at Street Intersections.

should be level. When the grades will not allow this, the following rule has been proposed: [1]

"In the business section all the street grades of 3 per cent or less should be continued unbroken over the intersection; and streets having a steeper grade than 3 per cent should have an intersection of 3 per cent between curb lines. The grade of the curb between the other curb line and the property line should in no case be greater than 8 per cent. The grade at the corner

[1] Proposed by Hering and Rosewater. See Eng. News, Vol. XXV, pp. 148–149, or Eng. Record, Vol. XXII, p. 53.

of the property should be determined by adding to each of the grades of the curb opposite the corner, the rise of the sidewalk and taking the mean." Fig. 186 is a street intersection adjusted by the above rule. There are many difficult problems arising out of street grades and the beginner should familiarize himself with the literature [1] on the subject before attempting a grade design.

Street grades should be fixed by the grade of the curbs or the property lines and not by the centre line of the street. The latter will change with each paving material, while improvements to the abutting property are always made with reference to the curbs.[2]

355. Minimum and Maximum Grades. An effort should be made to reduce the grades to about 5 per cent and 3 or 4 per cent in downtown district.[3] This, however, is frequently impossible. New York city has grades on residence streets of 10.15 and even 18 per cent. Pittsburg has grades of 17 per cent, Duluth 12.2 per cent, and Kansas City 16.5 per cent. On very steep grades zig-zag roads are sometimes constructed in order to reduce the grade. A minimum grade from 0.5 to 1 per cent is necessary for drainage. If this cannot be secured it is common to design a summit at centre of block and run the street water toward the block corners.

356. Vertical Curve at Grade Intersections. For the benefit of both appearance and traffic, a vertical curve should be used wherever there is any considerable change of grade either at street intersections or within the block. The parabola is the best form of curve and one most easily put in. In Fig. 187 AB and AC represent two grade lines meeting at A. CB is a vertical

[1] See list of references at close of chapter.

[2] For a model city ordinance establishing a grade system see McCullough's Engineering Work in Towns and Small Cities, Technical Book Agency, Chicago, Ill.

[3] The city of Seattle, Wash., has cut down grades of over 100 feet. One grading contract included about 2,500,000 cubic yards.

parabola tangent to the grade line at C and B. The curve is located by computing the vertical ordinates at the points 1, 2, 3, 4, etc., as follows: DE is the rise in half the length of the curve, i.e., from B to A, while EC is the fall in the second half. If n represents the number of equidistant points necessary to be established on the curve (including the second tangent point C), then from the law of the curve we have the ordinate at the first point,

$$Y = \frac{DE + EC}{n^2},$$

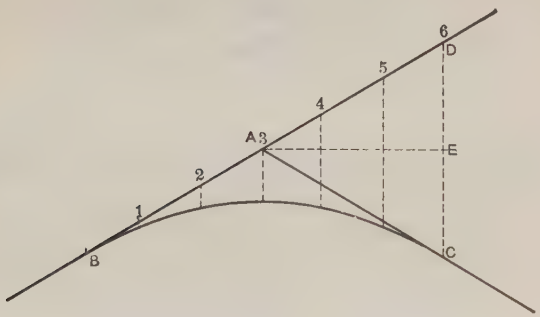


FIG. 187.—Vertical Curve at Grade Intersection.

while at any other point, $Y_n = n^2 Y$. Thus at 2, $Y_2 = 4Y$, at 3, $Y_3 = 9Y$.

The elevations of the curve are obtained by subtracting such computed ordinates from the elevations of the corresponding points on the straight grade. From 10 to 15 feet of curve should be provided for each per cent of change of grade.

357. Curved Streets. "The curved line is the line of beauty" in street planning as in many other situations, and its moderate use even in flat topography contributes also to the charm of variety. The houses show far better on a curved than on a straight line, while the view from the windows both front and side is much more pleasing. In hilly or rolling topography, the

use of curved streets is frequently mandatory for economic reasons, especially in a residence section.

The following brief discussion of the manner of laying out circular curves is thought sufficiently full for ordinary city survey-

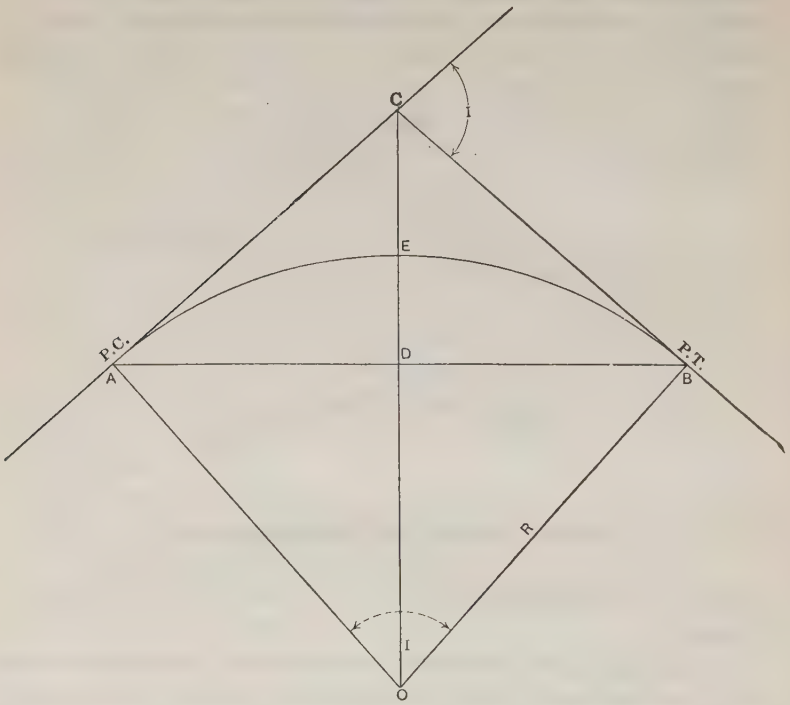


FIG. 188.—Principles of a Circular Curve.

ing. A complete discussion of such a subject may be found in any of several excellent manuals on railroad curves.[1]

358. Principles of a Circular Curve. The straight lines *AC* and *CB*, Fig. 188, are called the tangents, while the curves

[1] Field Engineering, by W. H. Searls, published by John Wiley & Sons, New York, is a standard text.

are designated by the lengths of their radii.[1] The angle between one tangent produced as AC, and the second tangent as CB, is called the intersection angle I.

In Fig. 188, which represents a circular curve, we note the following relations:

$AO = BO$ = radius $= R$;
AEB = length of arc $= L_c$;
AB = long chord $= C$;
$CA = CB$ = tangent distance $= T$;
EC = external distance $= E$;
DE = middle ordinate $= M$;
I = intersection or central angle.

From the figure it is evident that

$$T = R \tan I/2; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$C = 2R \sin I/2; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$M = R \text{ vers } I/2; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$E = R \text{ exsec } I/2; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$L_c = R \times$ circular measure of I.

359. Laying Out Circular Curves. The first step is to locate the point of intersection (P. I.) of the curve. If this point is accessible, the intersection angle is then carefully measured with the transit. If the P. I. is inaccessible, the intersection angle can be computed from a short connecting traverse between the tangents (one course may suffice). From a plat of the premises

[1] In railroad surveying the curves are known by the number of degrees which an arc of 100 feet subtends at the centre, thus if 100 feet subtend 2° at the centre it is called a 2° curve.

often a topographic map, the proper R is determined. The tangent distance then becomes known by use of the formula, $T = R \tan \frac{1}{2}I$. The P. C. and P. T. should be set first as a check, after which the curve is run in by turning off angles from the tangent (called deflection angles) determined as follows: If the total length of the curve be divided into n equal parts, the angle at the centre, corresponding to each of these even arcs, would be $\frac{I}{n}$, and the deflection angle for one chord would be $\frac{I}{2n}$, which in Fig. 189 is angle VAb. It is evident that the deflection angle to point

$$c = 2 \times \frac{I}{2n} = \frac{I}{n},$$

$$d = 3 \times \frac{I}{2n} = \frac{3}{2}\frac{I}{n},$$

$$e = 4 \times \frac{I}{2n} = \frac{2I}{n},$$

etc.

Therefore, after the first deflection angle VAb is turned (see Fig. 189), each succeeding deflection is formed by adding $\frac{I}{2n}$ as an increment to the preceding deflection. The check on the work would be that the last deflection angle should equal $\frac{I}{2}$, and when this last point is set it should coincide with the P. T., as already set. The chords Ab, bc, cd, etc., can be readily computed from the formula, chord length $c = 2R \sin\left(\frac{I}{2n}\right)$.

The transit is now set over the P. C., and with the A vernier set on 0°, the line of sight is directed by the lower motion and clamped on the tangent AV. The angle $VAb = \left(\frac{I}{2n}\right)$ is now turned off in the proper direction, right or left, by the upper motion and a stake sighted in by the transit and set at the computed chord distance (Ab) from A. A tack may be driven in

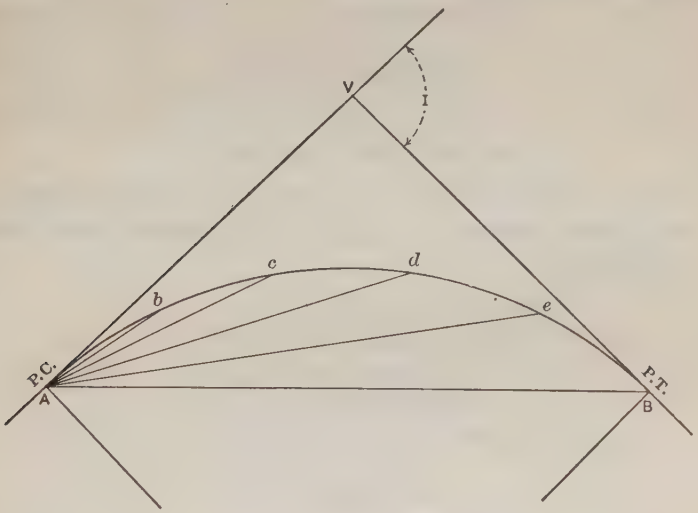


FIG. 189.—Lay-out of Circular Curves.

the stake by which to mark the exact point. Point C is located by adding $\frac{I}{2n}$ to the vernier reading and placing c on line and at the computed chord distance from b. This process is continued until the P. T. is reached. If preferred, the work could start at the P. T. and finish at the P. C.

Before going into the field it is very desirable to compute the deflection angles and the chord distances to be used in running the curves, wherever this is possible.

360. Computation of Deflection Angles and Chords for a Circular Curve. *First. Given.* Curve to the left $R=240$, $I=43^\circ\ 32'\ 30''$:

P. C. = 10 + 51.40 width of street 80 feet.

$T = R \tan 21^\circ\ 46'\ 15'' = 240 \times 0.37928 = 91.03.$

From Table XII:

$43^\circ = 0.7504916$

$32' = 0.0093084$

$30'' = 0.0001454$

$0.7599454 \times 240 = 182.39 = L_c.$

P. C. = 10 + 51.40

1 + 82.39

P. T. = 12 + 33.79

$l_c = 45.60 \pm.$

I. DEFLECTION ANGLES

If the curve be divided into four equal parts corresponding to an equal central angle of 10° 53′ 7″.5, the deflection angle from the P. C. will be:

P. C. =	10 + 51.40		
	10 + 97.00 =	5°	26′ 34″
	11 + 42.60 =	10	58 8
	11 + 88.20 =	16	19 42
P. T. =	12 + 33.79 =	21	46 15

II. CHORDS

$\sin 5^\circ\ 26'\ 34'' = 0.09484$

$2R = 480$

758720

37936

45.52 + centre chord

$.0948 \times 2 \times 40 =$ 7.59 ±

53.11 right chord

37.93 left chord

361. To Set Lot Stakes at Fixed Arc Distances. *Second.* It is sometimes necessary to set lot stakes on a curve at certain fixed arc distances apart. The method of computing the proper deflection angles and chord distance can be best shown by an example. Suppose, in the example above, it is required to compute the deflection angles and chords necessary to set lot stakes at every 50 feet on the street lines. The computations for the right side are as follows:

$$R_r = 240 + 40 = 280 \text{ feet.}$$

The corresponding tangent distance is

$$T_r = 280 \tan 21^\circ\ 46'\ 15'' = 106.20.$$

From Table XII we have the length of this arc or

$$L_c = 280 \times 0.7599454 = 212.78$$

P. C. = 10 + 51.40
2 + 12.78

P. T. = 12 + 64.18

I. DEFLECTION ANGLES

Deflection angle for 50 feet arc on *right* side of street

$$= \frac{50}{212.78} \times 21^\circ\ 46'\ 15'' = 21.7708$$

50

log 1088.54 = 3.036844
log 212.78 = 2.327931

0.708913 = 5°.1158
60

6'.9480
60

56''.88

Defl. angle for 50 ft. $= 5^\circ\ 6'\ 57''$

Defl. angle for 12.78 ft. $= \frac{12.78}{50} \times$ defl. for 50 ft.

$= 0.2556 \times$ defl. for 50 ft.

$\log 0.2556 = 9.407561$

$\log$ defl. 50 ft. $= 0.708913$

$0.116474 \equiv 1^\circ.3076 = 1^\circ\ 18'\ 27'' =$ defl. for 12.78 ft.

P. C.	10 + 51.40			
	11 + 1.40 =	5°	6′	57″
	11 + 51.40 =	10	13	54
	12 + 1.40 =	15	20	51
	12 + 51.40 =	20	27	48
P. T.	12 + 64.18 =	21	46	15 check $= I/2$

II. CHORDS

From Eq. (2), page 479, $C = 2R \sin I/2$, we have

for 50 ft. arc, chord $= C = 560 \sin 5^\circ\ 6'\ 57''$

$= 560 \times 0.089169 = 49.93$

For a 12.78 ft. arc, $C = 560 \times \sin 1^\circ\ 18'\ 27''$

$= 560 \times 0.022765 = 12.75$

The computations for the left-side curve are similar.

362. Form of Field Notes. The following alignment notes of a curved street refer to the centre line as is customary. However, the parallel side lines must also be run out by the use of chords and deflection angles. The notes begin at the bottom for convenience of making a sketch. In the second column are recorded the distances measured on the actual arc along the centre line. The following three columns, headed "Chords," contain the chord measurements from station to station on the right side, centre line, and left side of the street. The sixth

column laid off by the transit at the P. C. is used in running all three curves. The last column contains descriptions of the curve.

FIELD NOTES OF PROBLEM 1. ART. 360

Station.	Arc Distance.	Chords.			Deflection Angles.	Remarks.
		Right.	Centre.	Left.		
12+33.79	45.60	53.11	45.52	37.93	21° 46′ 15″	P. T. Street 80 ft. wide.
11+88.20	45.60	53.11	45.52	37.93	16 19 42	Curve { $R=240$ ft.
11+42.60	45.60	53.11	45.52	37.93	10 58 8	$T=91.03$ ft.
10+97.00	45.60	53.11	45.52	37.93	5 26 34	$I=43°\ 32'\ 30''$
						To the left
10+51.40	.....	.....	.....	.....	.....	P. C.

It will be seen that the above notes locate points on all the three curves in *sets of three*, each set being on a single radial line.

PROBLEM 2. FIELD NOTES OF RIGHT-SIDE CURVE. ART. 361.

Station.	Arc Distance.	Chord Distance.	Deflection Angles.	Remarks.
12+64.18	12.78	12.75	21° 46′ 15″	P. T. Right side of street.
12+51.40	50	49.93	20 27 48	Curve { $R=280$
12+ 1.40	50	49.93	15 20 51	$T=106.20$
11+51.40	50	49.93	10 13 54	$I=43°\ 32'\ 30''$
11+ 1.40	50	49.93	5 6 57	$L_c=212.78$
10+51.40	.....	.....	.....	P. C. Curves to the left.

363. When the whole Curve is Not Visible from One End. In cities, buildings are very likely to prevent running the entire curve from a single transit station. When this is the case, the curve is run, as shown in Art. 359, until the obstacle is reached, the last point being very carefully set, as at *d*, Fig. 190. The transit is then set up over such forward point *d* and the vernier

set to read the old deflection angle VAd, on the *proper* side of 0°. The instrument is next sighted and clamped on the first transit station A, by the *lower* motion. It will be seen that, if the alidade be now turned by the *upper* motion until the vernier reads 0°, the line of sight must be on the tangent to the curve.

The telescope is now reversed and the remainder of the curve from d to B is run, using the same deflection angles and chord

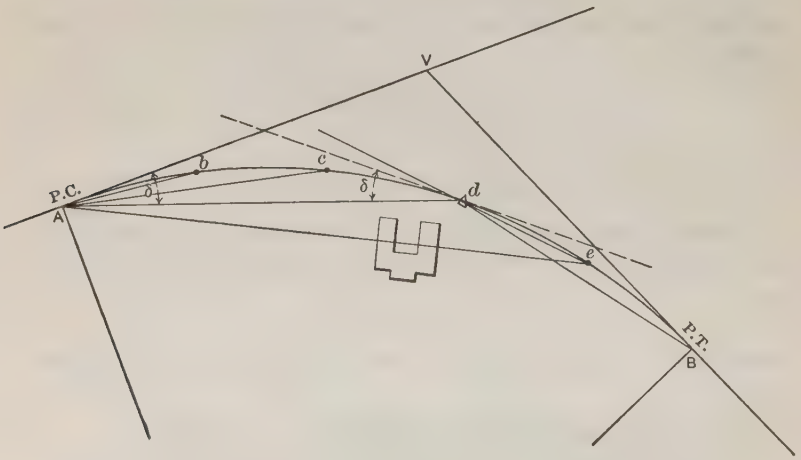


FIG. 190.—Whole Curve not Visible from P.C.

distance as if dB were a separate curve. In hilly land long curves may require several such turning-points.

Instead of the above, some engineers prefer the following method: After the transit has been set at the forward point d, the vernier is set at 0° and clamped on A. The deflection angle VAe is now laid off, the telescope transited and the point e set on line at the proper chord distance. The next point B would be set by laying off the deflection angle VAB.

364. Street Cross-Section. All streets should have a crown in order to insure the drainage of surface water which would

injure the pavement or impede transportation.[1] The amount of crown depends upon the kind of paving, the longitudinal and traverse profile, etc. For example, in New York the standard crown for asphalt streets is $\frac{1}{100}$ of the distance between curbs. (about $\frac{1}{4}$ inch per foot of half width), while for brick or granite, the crown is $\frac{1}{75}$ of the half width, or $\frac{3}{8}$ inch per foot. In Chicago the standard crown for asphalt, brick, or granite block varies from 2 per cent to 5 per cent of the half width, while for macadam the crown varies between 4 and 8 per cent of half width. The above rules are modified when the two sides of the street are of unequal heights. Byrne's "Highway Construction" gives the following crown for different pavements:

Kind of Surface.	Ratio of Rise to Width of Carriageway.
Earth	1–40
Gravel	1–50
Broken stone	1–60
Stone block, brick, and asphalt	1–80
Wood	1–100

The form of the road surface is sometimes two planes from gutter to centre, but more often the surface has the curved form of either the circle or the parabola.[2] The curved cross-section gives a flatter surface in the centre of the street and does not wear concave on the sides as quickly as the plane surface.

The first step in paving a street should be the setting of the curbs.[3] This is followed by setting the grade stakes for the sub-

[1] See Chapter X of Baker's "Roads and Pavements" for an excellent explanation of the means of securing street drainage.

[2] For the usual limits of a city street these two curves practically coincide, but the parabola is easier to compute. Thus to compute the ordinate of any point X (Fig. 187) from the tangent $A'C$, we have from the equation of a parabola,

$$XX' = AB \times \frac{l^2}{\frac{W}{2}}.$$

[3] Concrete curb and gutter are cheapest and retain their position better than stone curbs.

grade, later the top grade of the foundation, and finally that of the finished pavement. If the pavement does not exceed 40 feet in width, a template can be prepared to the desired cross-section, and used with its ends resting on the gutter and its centre grade

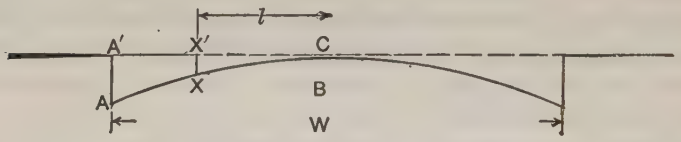


FIG. 191.—Cross-section on Level Ground.

stake. In the case of very wide pavements, additional grade stakes need to be driven, at half- and quarter-distance points.

As stated in Art. 354, wherever possible, an effort is made to design opposite curbs of equal height (Fig. 191). This can be

FIG. 192.—Cross-section of Street, on Side Hill.

done even on streets with moderate side slope by placing one sidewalk higher than the other, thereby requiring a different grade for the parking on the two sides. Sometimes a small

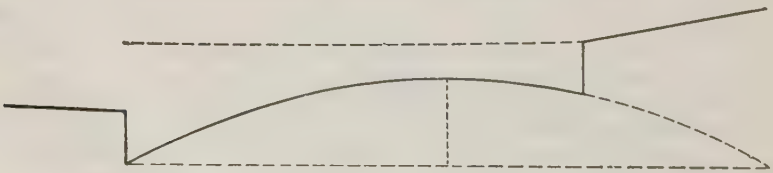


FIG. 193.—Cross-section of Street, on Side Hill.

difference of level is overcome by raising the curb and increasing the depth of the gutter on the *higher* side and lowering the curb on the *lower side*, but keeping the crown of the pavement in the

centre of the pavement. Such an arrangement is shown in Fig. 192. More frequently in such cases the crown is moved toward the high side of the street, as shown in Fig. 193.

The details of such work and the many exceptional cases properly belong to a treatise on pavement.[1]

365. Surveys for Sewers. Because of the fact that the flow of sewage must be by gravity, the location of the sewers must be carefully studied in order to secure their efficient working. The topographic map has been referred to (Art. 343), as furnishing a valuable basis for a sewer design. If this is not available, sufficient levels must be run to secure accurate profiles of all streets concerned. It is usual to divide the city up into sewer districts, the limits of which depend upon the topography. The grades of the main trunk sewers are determined by the elevation of the outfall and the lowest district that must be sewered. When the profiles are platted the sewer grade lines are drawn on same in red, after which the necessary excavation can be computed. The line of the sewer should be laid out with transit and tape, the line stakes being driven level with the ground and at least one foot beyond the side of the excavation, at intervals of from 25 or 50 feet. If construction is not to follow promptly, these stakes should be carefully referenced to building and street lines.

366. The Sewer Field Notes. The main points noted in the transit notes are (1) the location of the sewer with reference to the centre line of the street; (2) the pluses of all changes in alignment; (3) the deflection angles or azimuths at each change in direction; (4) the intersection of both lines of all crossing streets; (5) the intersection with all stream or railroad crossings or anything which could affect the construction work.

The level notes should show the levels at 25- or 50-foot intervals. In addition, where there is much side slope, elevations should be taken of curb lines and also of the bottoms of any cel-

[1] See Baker's "Roads and Pavements," Chapter X, Street Design. See also Trans. Am. Soc. C.E., Vol. XLII, pp. 1–27.

lars, especially on the lower side. It is also useful to determine the elevation of the ground water from nearby wells. The following are the notes used on sewer construction, although there is little uniformity in the form used in different cities.

FIELD NOTES OF SEWER GRADES, MILTON STREET, PARK TO BROOKS. June 14, 1910.

B.S. on B.M. and Stations.	F.S.	Elevation of Stake.	Elevation of Sewer Invert.	Below Grade Stake.	Grade Line Below Guide Line.	Off-set of Guide Line.
B.M. 0.54	**9.92 H.I.**	9.38				
0.....	6.27	3.65	−0.10	3.75	4.25	
1.....	5.79	4.13	+ .04	4.09	3.91	
2.....	5.26	4.66	.18	4.48	3.52	
3.....	4.94	4.98	.32	4.66	3.34	8 ft.
4.....	4.77	5.15	.46	4.69	3.31	
5.....	4.46	5.46	.60	4.86	3.14	
6.....	4.06	5.86	.74	5.12	2.88	
T. P.....	0.31	9.61			Centre of Book	
10.46	**20.07 H.I.**					
7.....	10.89	9.18	.88	8.30	6.70	15 ft.

The description and elevation of the B.M's. would appear in the right-hand page, only part of which is shown. To save space the usual H.I. column is here omitted and the H.I. set in full-face type and put in the F.S. column. The B.S. also is placed in the station column for a similar reason. No confusion need result however.

367. Maps and Profiles. The profiles are prepared on the engraved profile paper, and show any borings, the pluses of all intersecting streets, manhole locations, etc. The map will show the location of the sewers and their size as well as the location of manholes, lampholes, flush tanks, house connections, etc. (see Fig. 194).

The alignment of the sewer is shown between manholes. The figures within the street limits are the distances from the manholes to the house connections. The figures opposite the end of the house connections locate their *outer* ends. The house numbers are also put on the lots for convenience of future reference.

FIG. 194.—Sewer Map, University Heights, Madison, Wis.

368. Indexing the City Plans and Records. The best practice demands that original field books and all similar records shall be kept in a fire-proof vault. All plans and profiles should be indexed and filed according to some good system. This requires that the numbering of the plans should be designed so as to show by the number, the character of the drawing, i.e., whether the original, the tracing, or a print. The card catalogue system, because of its flexibility, is recommended for keeping track of the drawings.

The best way of filing plans is to keep them flat wherever possible. This is made possible by requiring standard sizes of drawings. Plans so filed have the index number in the proper corner. Some city offices require that a drawing shall be placed in its proper order as well as in its proper drawer. Rolled drawings should have their mark on both ends. For detail information regarding such a system, see an article in *Engineering Record*, Feb. 8, 1908, entitled, "A System of Filing Engineer's Notes and Records."

REFERENCES

I. The City Lay-out

1. Street Extensions in Queens Borough, New York City. A. H. Chandler, Trans. Asso. of Civ. Eng., Cornell Uinversity. See also Eng. News, March 14, 1907, for methods used in chaining.
2. Marking Lines of Streets. Bromal. Am. Soc. C. E., July, 1894, p. 111. See also Eng. News, 1894, Vol. I, p. 477, and Vol. II, pp. 55 and 370; 1895, Vol. II, p. 26.
3. Laying Out the 23d and 24th Wards, New York City. Eng. News, Feb. 12 and 18; March 5, 12, 26; April 9, all 1881.
4. The Curved Streets of Washington. Eng. News, Nov. 28, 1895, p. 368; Aug. 29, 1891, p. 192; Nov. 21, 1895, p. 344.
5. Laying Out Curved Streets. Ernest McCullough. Eng. News, May 11, 1905.
6. The Survey of Lowell, Mass. G. A. Nelson. Jour Asso. Eng. Soc., Oct., 1894.

II. City Resurveys

7. The History of Surveying in Chicago. 1905 Annual Report of the Illinois Society of Engineers. S. S. Greeley.
8. (*a*) City Surveying in Its Relation to Judicial Functions and Actual Practice. (*b*) Practice of City Surveyors in City of New York. Proceedings Brooklyn Engineers' Club, 1906.
9. Survey of Roanoke, Va. Adjustment of Errors Found. Eng. News, Vol. I, p. 163, 1889; March 16, 1899.
10. Elimination of Error. W. Bouton. Jour. Asso. Eng. Soc., Vol. V, p. 151.
11. Establishment of Old Lot Lines, etc. S. W. Hoag. Eng. News, May 9 and 16, 1885.
12. The Co-ordinate Survey of Boston. Frank O. Whitney. Jour. Asso. Eng. Soc., Vol. XX, p. 365, 1898.
13. The Staking Out Curves on Rough Ground. Robert Laird. Eng. Contracting. Nov. 25, 1908.
14. Common Sources of Error in Field Work. A. M. Shaw. Eng. News, April 19, 1906.
15. Hints upon Transit Surveys and the Checking of Errors. E. Sherman Gould. Eng. News, Jan. 4, 1900.
16. Law of Resurveys. Decisions of Wis. Supreme Court, Vol. XXVII, Wis. Reports, p. 446; Vol. XIX, Wis. Reports, p. 429; Vol. XCVIII, Wis. Reports, p. 87; Vol. LXXVIII, Wis. Reports, p. 27.
17. Constitutionality of the Boston Resurvey Law. Jour. Asso. Eng. Soc., Vol. XX, p. 386

III. City Planning and Replanning

18. City Planning and Replanning. Editorial Eng. News, Oct. 31, 1907.
19. Planning the City Site. L. W. Haupt. Eng. Mag., Jan., 1895.
20. Principles for Laying Out Cities. J. Stubben. Trans. Am. Soc. C. E. Vol. XXIX, p. 718.
21. The City Plan. A series of eighteen excellent articles well illustrated. Edited by Charles Mulford Robinson. Charities and the Commons, Feb. 1, 1908.
22. The Shaping of Towns. Eng. News, July 7 and 28, 1877.
23. The Streets and Street Plan of Paris. Eng. News, June 2, 1904.
24. The Planning and Control of Town Extensions in Germany. Eng. News, Dec. 20, 1906.
25. Recent Reports on City Plans. Munic. Jour. and Eng., April 3, 1907.
26. Town and Street Planning. Surveyor, Aug. 14, 1908, p. 196.
27. The Evolution of the Street, House, and Garden. Nov. and Dec., 1903, and Jan., Feb., and March, 1904. Chas. Mulford Robinson.

28. City Improvement. Jour. Asso. Eng. Soc., Jan., 1908. The Evolution of City Streets. W. Fish. Street Railway Review, Aug. 15, 1900.

IV. Street Grades and Cross-sections

29. City Ordinance Establishing Grades. McCullough's Eng. Work in Towns and Small Cities.
30. Street Grades. N. P. Lewis. Eng. Record, Oct. 16, 1909.
31. Some Difficult Street Intersections. Eng. Record, Oct. 14, 1903.
32. How to Compute the Cross-section of a Curved Street Paving. Eng Contracting, June 12, 1907.
33. Street Grades and Cross-sections in Asphalt and Cement. Robert P. Woods. Trans. Am. Soc. C. E., Vol. XLII, pp. 1–27.
34. Method of Adjusting Street Grades, etc. F. A. Calkins. Eng. News, Vol. XVII, p. 134; also Vol. XXXV, p. 150.
35. Grades and Grade Systems of Cities. B. Schreiner. Jour. Asso. Eng. Soc., Vol. II, p. 130.
36. Street Grades and Intersections. W. B. Fuller. Jour. Asso. Eng. Soc., Nov., 1894.
37. Method of Fixing Grades. Hering and Rosewater. Eng. News, Sept. 8, 1892.
38. Establishing Street Grades. C. P. Chase, Jour. West. Soc. of Eng., Feb., 1899.
39. Cross-section of European Streets. R. Grimshaw. Municipal Eng., March, 1897.
40. The Seattle Regrade. L. P. Zimmerman. Eng. News, Nov. 12, 1908.
41. Methods of Keeping Data Relating to Street Lines and Grades. Eng. News, July 8, 1909.

V. Cadastral and City Topographical Surveys

42 The St. Louis Topographical Survey. B. H. Colby. Jour. Asso. Eng. Soc., Vol. XII, pp. 1–40.
43. The Baltimore Topographic Survey. Eng. News, 1894, Vol. II, p. 83; 1895, Vol. II, p. 32; Vol. I, p. 33; 1896, Vol. II, pp. 307, 312, 364, 366, 400.
44. The Roanoke Survey. Eng. News, 1892, Vol. I, pp. 30 and 32.
45. The Co-ordinate Survey of Greater Boston. Jour. Asso. Eng. Soc., June, 1898, p. 365.
46. The Cadastral Survey of Lowell, Mass. Jour. Asso. Eng. Soc., Oct., 1894, p. 579.
47. The Topographic Survey of Staten Island. Eng. Record, July, 1907.

CHAPTER XIII

THE MEASUREMENT OF VOLUMES

369. Proposition. *The volume of any doubly-truncated prism or cylinder, bounded by plane ends, is equal to the area of a right section into the length of the element through the centres of gravity of the bases, or it is equal to the area of either base into the altitude of the element joining the centres of gravity of the bases, measured perpendicular to that base.*

Let *ABCD*, Fig. 195, be a cylinder, cut by the planes *OC* and *OB*, the unsymmetrical right section *EF* being shown in

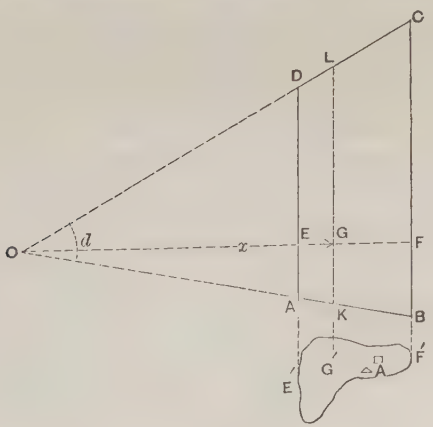


FIG. 195.

plan in *E'F'*. Whatever position the cutting planes may have, if they are not parallel they will intersect in a line. This line of intersection may be taken perpendicular to the paper, and

the body would then appear as shown in the figure, the line of intersection of the cutting planes being projected at O.

Let A = area of the right section;
ΔA = any very small portion of this area;
x = distance of any element from O;
then ax = height of any element at a distance x from O.

An elementary volume would then be $ax\Delta A$, and the total volume of the solid would be $\Sigma ax\Delta A$.

Again, the total volume is equal to the mean or average height of all the elementary volumes multiplied by the area of the right section.

The mean height of the elementary volumes is, therefore, $\frac{\Sigma ax\Delta A}{A} = \frac{a\Sigma x\Delta A}{A}$. But $\frac{\Sigma x\Delta A}{A}$ is the distance from O to the centre of gravity, G, of the right section,[1] and a times this distance is the height of the element LK through this point. Therefore, the mean height is the height through the centre of gravity of the base, and this into the area of the right section is the volume of the truncated prism or cylinder. The truth of the alternative proposition can now readily be shown.

Corollary. When the cylinder or prism has a symmetrical cross-section, the centre of gravity of the base is at the centre of the figure, and the length of the line joining these centres is the mean of any number of symmetrically chosen exterior elements. For instance, if the right section of the prism be a regular polygon, the height of the centre element is the mean of the length of all the edges. This also holds true for parallelograms, and hence for rectangles. Here the centres of gravity of the bases lie at the intersections of the diagonals; and since these bisect each other, the length of the line joining the inter-

[1] This is shown in mechanics, and the student may have to take it for granted temporarily.

sections is the mean of the lengths of the four edges. The same is true of triangular cross-sections.

370. Grading over Extended Surfaces. Lay out the area in equal rectangles of such a size that the surfaces of the several rectangles may be considered planes. For common rolling ground these rectangles should not be over 50 feet on a side. Let Fig. 196 represent such an area. Drive pegs at the corners, and find the elevation of the ground at each intersection by means of a level, reading to the nearest tenth of a foot, and referring the elevations to some datum plane below the surface after it is graded. When the grading is completed, relocate the inter-

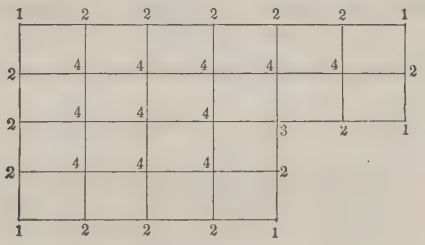


FIG. 196.

sections from witness points that were placed outside the limits of grading, and again find the elevations at these points. The several differences are the depths of excavation (or fill) at the corresponding corners. The contents of any partial volume is the mean of the four corner heights into the area of its cross-section. But since the rectangular areas were made equal, and since each corner height will be used as many times as there are rectangles joining at that corner, we have, in cubic yards,

$$V = \frac{A}{4 \times 27}[\Sigma h_1 + 2\Sigma h_2 + 3\Sigma h_3 + 4\Sigma h_4]. \quad . \quad . \quad . \quad (1)$$

The subscripts denote the number of adjoining rectangles the area of each of which is A.

From this equation we may frame a

RULE.—Take each corner height as many times as there are partial areas adjoining it, add them all together, and multiply by one-fourth of the area of a single rectangle. This gives the volume in cubic feet. To obtain it in cubic yards, divide by twenty-seven.

If the ground be laid out in rectangles, 30 feet by 36 feet, then $\frac{A}{4\times 27}=\frac{1080}{108}=10$; and if the elevations be taken to the nearest tenth of a foot, then the sum of the multiplied corner heights, with the decimal point omitted, is at once the amount of earthwork in cubic yards. This is a common way of doing this work. In borrow-pits, for which this method is peculiarly fitted, the elementary areas would usually be smaller.

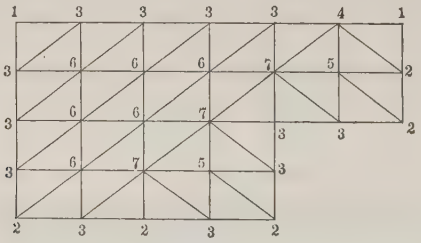


FIG. 197.

In general, on rolling ground, a plane cannot be passed through the four corner heights. We may, however, pass a plane through any three points, and so with four given points on a surface either diagonal may be drawn, which with the bounding lines makes two surfaces. If the ground is quite irregular, or if the rectangles are taken pretty large, the surveyor may note on the ground which diagonal would most nearly fit the surface. Let these be sketched in as shown in Fig. 197. Each rectangular area then becomes two triangles, and when computed as triangular prisms each corner height at the end of a diagonal is used twice, while the two other

corner heights are used but once. That is, twice as much weight is given to the corner heights on the diagonals as to the others. In Fig. 197 the same area as that in Fig. 196 is shown, with the diagonals drawn which best fit the surface of the ground. The numbers at the corners indicate how many times each height is to be used. It will be seen that each height is used as many times as there are triangles meeting at that corner. To derive the formula for this case, take a single rectangle, as in Fig. 198, with the diagonal joining corners 2 and 4. Let A be the area of the rectangle. Then from the corollary, p. 496, we have for the volume of the rectangular prism, in cubic yards,

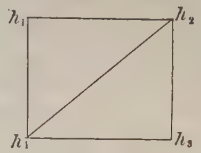


FIG. 198.

$$V = \frac{A}{2 \times 27}\left(\frac{h_1 + h_2 + h_4}{3} + \frac{h_2 + h_3 + h_4}{3}\right)$$

$$= \frac{A}{6 \times 27}(h_1 + 2h_2 + h_3 + 2h_4). \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

For an assemblage of such rectangular prisms as shown in Fig. 197, the diagonals being drawn, we have, in cubic yards,

$$V = \frac{A}{6 \times 27}[\Sigma h_1 + 2\Sigma h_2 + 3\Sigma h_3 + 4\Sigma h_4 + 5\Sigma h_5$$

$$+ 6\Sigma h_6 + 7\Sigma h_7 + 8\Sigma h_8]; \quad . \quad . \quad . \quad . \quad (3)$$

where A is the area of one rectangle, and the subscripts denote the number of triangles meeting at a corner.

As a check on the numbering of the corners, Fig. 197, add them all together and divide by six. The result should be the number of rectangles in the figure. In this case, if the rectangles be taken 36 feet by 45 feet, or, better, 40 feet by 40.5 feet, then the sum of the multiplied heights with the decimal point omitted

is the number of cubic yards of earthwork, the corner heights having been taken out to tenths of a foot.

The method by diagonals is more accurate than that by rectangles simply, the dimensions being the same; or, for equal degrees of exactness larger rectangles may be used with diagonals than without them, and hence the work materially reduced. In any case some degree of approximation is necessary.

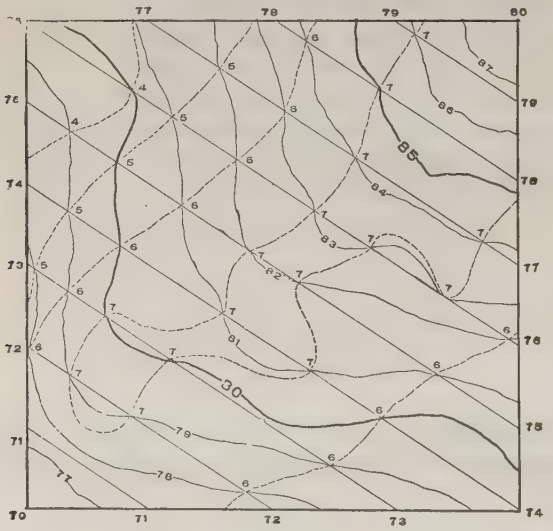


Fig. 199.

371. Approximate Estimates by Means of Contours. (*A*) Whenever an extended surface of irregular outline is to be graded down, or filled up to a given *plane* (not a warped or curved surface), a near approximation to the amount of cut or fill may be made from the contour lines. In Fig. 199 the full curved lines are contours, showing the original surface of the ground. Every fifth one is numbered, and these were the contours shown on the original plat. Intermediate contours one foot apart have been interpolated for the purpose of making this estimate. The

figures around the outside of the bounding lines give the elevations of those points after it is graded down. The straight lines join points of equal elevation after grading; and since this surface is to be a plane these lines are surface or contour lines after grading. Wherever these two sets of contour lines intersect, the difference of their elevations is the depth of cut or fill at that point. If now we join the points of equal cut or fill (in this case it is all in cut), we obtain a new set of curves, shown in the figure by dotted lines, which may be used for estimating the amount of earthwork. The dotted boundaries are the horizontal projections of the traces on the natural surface of planes parallel to the final graded surface which are uniformly spaced one foot apart vertically. These projected areas are measured by the planimeter and called A_1, A_2, A_3, etc. Each area is bounded by the dotted line and the bounding lines of the figure, since on these bounding lines all the projections of all the traces unite, the slope here being vertical. For any two adjoining layers we have, by the prismoidal formula [1] as well as by Simpson's one-third rule,

$$V_{1-3}=\frac{h}{3}(A_1+4A_2+A_3), \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where h is the common vertical distance between the projected areas.

For the next two layers we would have, similarly,

$$V_{3-5}=\frac{h}{3}(A_3+4A_4+A_5); \quad . \quad . \quad . \quad . \quad . \quad (2)$$

or for any even number of layers we would have, in cubic yards,

$$V=\frac{h}{3\times 27}(A_1+4A_2+2A_3+4A_4+2A_5+ \ . \ . \ . \ . \ A_n), \quad . \quad (3)$$

[1] For the demonstration of the prismoidal formula see Art. 372.

where n is an odd number, h and A being in feet and square feet respectively.

(*B*) Whenever the final surface is not to be a plane, but warped, undulating, or built to regular outlines like a fortification, a reservoir embankment, or terraced grounds, a different method should be employed.

In the former method the areas bounded by the dotted lines were areas cut out by planes parallel to the final plane surface, passed one foot apart *vertically*. But since the map shows only the *horizontal projections* of these planes, these projections, multiplied by the vertical distance between them, would give the true volumes.

When the final surface is not to be a plane, proceed as follows: First make a careful contour map of the ground. Then lay down on this map a system of contour lines, corresponding in elevation to the first set of contours, but in a different colored ink, which will accurately represent the final surface desired. This second set of contours would be a series of straight lines if a regular surface, composed of plane faces, was to be constructed, but would be curving lines if the ground were to be brought to a final curving or undulating surface.

The closed figures bounded by the two sets of intersecting contours of the same elevation are *horizontal* areas of cut or fill, separated by the common vertical distance between contours. The volumes here defined are oblique solids bounded by horizontal planes at top and bottom, and are a species of prismoid. The volume of one of these prismoids is found by applying the prismoidal formula to it, finding the end areas by means of a planimeter, and taking the length as the vertical distance between contours. If the contours be drawn close enough together, then each alternate contour area may be used as a middle area, and the length of the prismoid taken at twice the vertical distance between contours; or the volume may be computed by either of the formulas (12), (13), (14), or (15) of Appendix C, where the h's would here become the end areas and l the vertical distance between contours.

Example: Let it be required to build a square reservoir on a hillside, which shall be partly in excavation and partly in embankment, the ground being such as shown by the full contour lines in Fig. 200.[1]

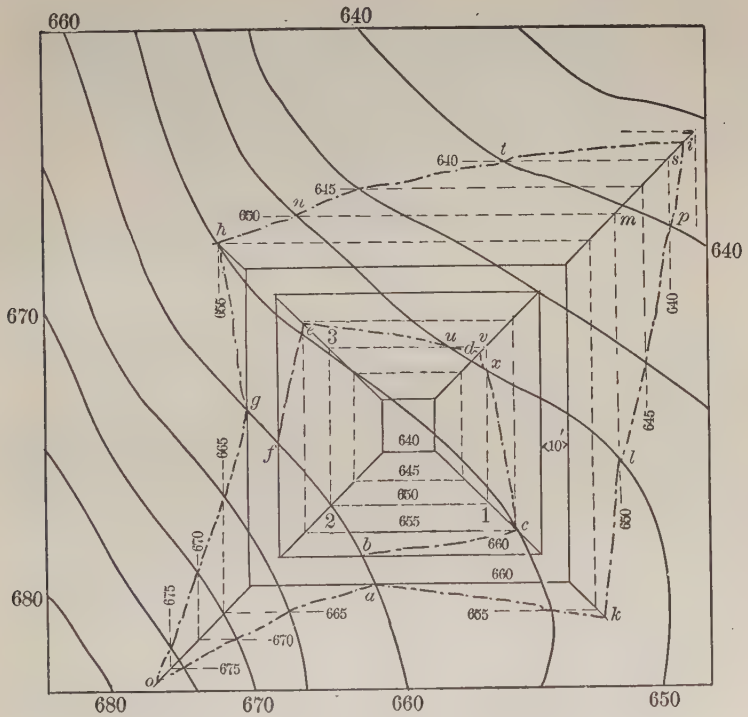


FIG. 200.

The contours, for the sake of simplicity and brevity, are spaced five feet apart. The top of the wall, shown by the full lines making the square, is 10 feet wide and at an elevation of

[1] This figure is taken from a paper describing the method by Professor William G. Raymond.

660 feet. The reservoir is 20 feet deep, with side slopes, both inside and outside, of two to one, making the bottom elevation 640 feet, and 20 feet square, the top being 100 feet square on the inside. The dotted lines are contours of the finished slopes, both inside and out, at elevations shown in the figure. The areas in fill all fall within the broken line marked *abcdefghik*, and the cut areas all fall within the broken line marked *abcdefgo*. These broken lines are grade lines. The horizontal sectional areas in fill and cut are readily traced by following the closed figures formed by contours of equal elevation, thus:

At 640-foot level sectional area in fill is *pst.*
" 650 " " " " " *lmnuvxl.*
" 650 " " " " cut is 123*ux.*

The other areas are as easily traced. In the figure the lines have all been drawn in black. In practice they should be drawn in different colors to avoid confusion.

This second method should be used in all cases where the graded area is considerable and the final relief form is not a plane. If the contours be carefully determined and be taken near enough together, the method will give as accurate results as may be obtained in any other way. The volume may be computed by Eq. (3) of this article, where the areas are the horizontal sectional areas bounded by contours of equal elevation, and h is the vertical distance between contours.

When these methods are used for final estimates, the contours should be carefully determined, and spaced not more than 2 feet apart on steep slopes and 1 foot apart on low slopes.

372. The Prismoid is a solid having parallel end areas, and may be composed of any combination of prisms, cylinders, wedges, pyramids, or cones or frustums of the same, whose bases and apices lie in the end areas. It may otherwise be defined as a volume generated by a right-line generatrix moving on the bounding lines of two closed figures of any shapes which

lie in parallel planes as directrices, the generatrix not necessarily moving parallel to a plane director. Such a solid would usually be bounded by a warped surface, but it can always be subdivided into one or more of the simple solids named above.

Inasmuch as cylinders and cones are but special forms of prisms and pyramids, and warped surface solids may be divided into elementary forms of them, and since frustums may also be subdivided into the elementary forms, it is sufficient to say that all prismoids may be decomposed into prisms, wedges, and pyramids. If a formula can be found which is equally applicable to all of these forms, then it will apply to any combination of them. Such a formula is called

373. The Prismoidal Formula.

Let A = area of the base of a prism, wedge, or pyramid;
$A_1 A_m$, A_2 = the end and middle areas of a prismoid, or of any of its elementary solids;
h = altitude of the prismoid or elementary solid.

Then we have,
For Prisms,

$$V = hA = \frac{h}{6}(A_1 + 4A_m + A_2). \quad . \quad . \quad . \quad . \quad . \quad (1)$$

For Wedges,

$$V = \frac{hA}{2} = \frac{h}{6}(A_1 + 4A_m + A_2). \quad . \quad . \quad . \quad . \quad . \quad (2)$$

For Pyramids,

$$V = \frac{hA}{3} = \frac{h}{6}(A_1 + 4A_m + A_2). \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Whence for any combination of these, having all the common altitude h, we have

$$V = \frac{h}{6}(A_1 + 4A_m + A_2), \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

which is the prismoidal fo mula.

It will be noted that this is a rigid formula for all prismoids. The only approximation involved in its use is in the assumption that the given solid may be generated by a right line moving over the boundaries of the end areas.

This formula is used for computing earthwork in cuts and fills for railroads, streets, highways, canals, ditches, trenches, levees, etc. In all such cases, the shape of the figure above the natural surface in the case of a fill, or below the natural surface in the case of a cut, is previously fixed upon, and to complete the closed figure of the several cross-section areas only the outline of the natural surface of the ground at the section remains to be found. These sections should be located so near together that the intervening solid may fairly be assumed to be a prismoid. They are usually spaced 100 feet apart, and then intermediate sections taken if the irregularities seem to require it.

The *area* of the middle section is never the mean of the two end areas if the prismoid contains any pyramids or cones among its elementary forms. When the three sections are similar in form, the *dimensions* of the middle area are always the means of the corresponding end dimensions. This fact often enables the dimensions, and hence the area of the middle section, to be computed from the end areas. Where this cannot be done, the middle section must be measured on the ground, or else each alternate section, where they are equally spaced, is taken as a middle section, and the length of the prismoid taken as twice the distance between cross-sections. For a continuous line of earthwork, we would then have, in cubic yards,

$$V=\frac{l}{3\times 27}(A_1+4A_2+2A_3+4A_4+2A_5+4A_6\ .\ .\ +A_n), \quad . \quad (1)$$

where l is the distance between sections in feet. This is the same as Eq. (3), p. 501. Here the assumption is made that

the volume lying between alternate sections conforms sufficiently near to the prismoidal forms.

374. Areas of Cross-Sections. In most cases, in practice at least, three sides of a cross-section are fixed by the conditions of the problem. These are the side slopes in both cuts and fills, the bottom in cuts and the top in embankments, or fills. It then remains simply to find where the side slopes will cut the natural surface, and also the form of the surface line on the given section. Inasmuch as stakes are usually set at the points where the side slopes cut the surface, whether in cut or fill, such stakes are called slope-stakes, and they are set at the time the cross-section is taken. The side slopes are defined as so much horizontal to one vertical. Thus a slope of $1\frac{1}{2}$ to 1 means that the horizontal component of a given portion of a slope line is $1\frac{1}{2}$ times its vertical component, the horizontal component always being named first. The *slope ratio* is the ratio of the horizontal to the vertical component, and is therefore always the same as the first number in the slope definition. Thus for a slope of $1\frac{1}{2}$ to 1 the slope ratio is $1\frac{1}{2}$.

375. The Centre and Side Heights. The centre heights are found as follows: Place in one column of the notebook the surface elevation of the ground at the centre stake, as given in the level book. Then take off from the profile the elevation of the points of *change of grade* only, and compute the elevation of grade at each station, from the known distance and grade. Place these elevations of grade in a column alongside the first. Then take the differences and put in a third column as the centre heights. These centre heights, together with the width of base and side slopes in cuts and in fills, are the necessary data for fixing the position of the slope stakes. When these are set for any section as many points on the surface line joining them may be taken as desired. In ordinary rolling ground usually no intermediate points are taken, the centre point being already determined. In this case three points in the surface line are known, both as to their distance out from the centre line and

as to their height above the grade line. Such sections are called "three-level sections," the surface lines being assumed straight from the slope stakes to the centre stake.

376. The Area of a Three-level Section.

Let d and d' be the distances out, and
h and h' the heights above grade of right and left slope stakes, respectively;
D the sum of d and d', c the centre height, r the slope ratio, w the width of bed.

Then the area $ABCDE$ is equal to the sum of the four triangles AEw, BCw, wCD, and wED. Or,

$$A = \frac{(d+d')c + (h+h')\frac{w}{2}}{2}. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This area is also equal to the sum of the triangles FCD and FED, minus the triangle AFB. Or,

$$A = \left(c + \frac{w}{2r}\right)\frac{2}{D} - \frac{w^2}{4r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Eq. (2) can also be obtained directly from Eq. (1) by substituting for h and h' in (1) their values in terms of d and w, $h = \frac{d - \frac{w}{2}}{r}$, and then putting $D = d + d'$. Eq. (2) has but two variables, c and D, and is the most convenient one to use.

377. Cross-Sectioning. It will be seen from Fig. 201 that there are three elevations to be determined above (or below) grade, and two distances out to be determined. A regular line of levels is carried, checking on all pre-established benches. At

each position of instrument from which slope stakes are to be set, the " height of instrument " is taken out, and the difference between this and the " elevation of grade " figured for the several sections, the " elevation of grade " having been taken from the profile, and already entered up for all stations and " grade points." By holding the rod at the section on line and taking a reading, and subtracting this from the " height of instrument," we obtain " elevation of profile " at that section; subtracting this from the " elevation of grade " for that section in fills, and *vice versa* in cuts, we obtain the amount of fill or cut, which can be roughly checked from the profile itself, if desired. The Railroad Field

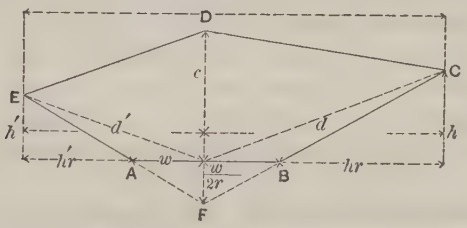


FIG. 201.

Books usually give forms for keeping these notes. If the ground were level transversely, the distance to the slope stakes would be

$$d = cr + \frac{w}{2}.$$

But this is not usually the case, and hence the distance out must be found by trial. If the ground slopes $\left\{\begin{matrix}\text{down}\\ \text{up}\end{matrix}\right\}$ from the centre line in a $\left\{\begin{matrix}\text{fill}\\ \text{cut}\end{matrix}\right\}$ the distance out will evidently be more than that given by the above equation, and *vice versa*. The rodman estimates this distance, and holds his rod at a certain measured distance out, d_1. The observer reads the rod, and

deducts the reading from the height of instrument above grade (or adds it to the depth of instrument below grade), and this gives the height of that point, h_1, above or below grade. Its distance out, then, *should* be $d = h_1 r + \frac{w}{2}$. If this be more than the actual distance out, d_1, the rod is set farther out; if less, it is moved in. The whole operation is a very simple one in practice, and the rodman soon becomes very expert in estimating nearly the proper position the first time.

In heavy work—that is, for large cuts or fills, and for irregular ground—it may be necessary to take the elevation and distance out of other points on the section in order to better determine its area. These are taken by simply reading on the rod at the critical points in the outline, and measuring the distances out from the centre. The points can then be plotted on cross-section paper and joined by straight or by free-hand curved lines. In the latter case the area should be determined by planimeter.

378. Three-level Sections, the Upper Surface Consisting of Two Warped Surfaces. If the three longitudinal lines joining the centre and side heights on two adjacent three-level sections be used as directrices, and two generatrices, one on each side the centre, be moved parallel to the end areas as plane directors, two warped surfaces are generated, every cross-section of which parallel to the end areas is a three-level section. These same surfaces could be generated by two longitudinal generatrices, moving over the surface end-area lines as directrices. The surface would therefore be a prismoid, and its exact volume would be given by the prismoidal formula. *The middle area* in this case is readily found, since the centre and side heights are the means of the corresponding end dimensions.

The prismoidal formula, giving volumes in cubic yards,

$$V = \frac{l}{6 \times 27}(A_1 + 4A_m + A_2), \quad . \quad . \quad . \quad . \quad . \quad (1)$$

could therefore be written,

$$V=\frac{l}{12\times 27}\left[\left(c_1+\frac{w}{2r}\right)D_1+\left(c_2+\frac{w}{2r}\right)D_2 +4\left(c_m+\frac{w}{2r}\right)D_m\right]-\frac{lw^2}{4\times 27r}. \quad (2)$$

This equation is derived directly from Eq. (1) above, and Eq. (2), p. 508. The quantity $\frac{w}{2r}$ is the distance from the grade-plane to the intersection of the side slopes, and is a constant for any given piece of road. It would have different values, however, in cuts and fills on the same line.

For brevity, let

$$\frac{w}{2r}=c_0; \qquad \text{and} \qquad \frac{lw^2}{4\times 27r}=\frac{lwc_0}{54}=K.$$

Here K is the volume of the prism of earth, 100 feet long, included between the roadbed and side slopes. It is first included in the computation and then deducted. It is also a constant for a given piece of road.

Eq. (2) now becomes

$$V=\frac{l}{12\times 27}[(c_1+c_0)D_1+(c_2+c_0)D_2+4(c_m+c_0)D_m]-K, \quad (3)$$

where c_m and D_m are the means of c_1c_2 and D_1D_2, respectively.

This equation involves but two kinds of variables, c and D, and is well adapted to arithmetical, tabular, or graphical computation. Thus if $l=100$; $w=18$; and $r=1\frac{1}{2}$; then $c_0=6$; and $K=200$; and Eq. (3) becomes

$$V=\tfrac{100}{324}[(c_1+6)D_1+(c_2+6)D_2+4(c_m+6)D_m]-200. \quad . \quad (4)$$

The computation may be tabulated as follows:

Sta.	Width, $D=d+d'$.	Height, $C=c+c_0$.	Partial Volume.	Volume of Prismoid.
11	71.9	25.3	562	
M	69.6	23.4	$503\times4=2012$	
12	67.4	21.5	447	
			3021 − 247	2774
M	63.3	19.2	$374\times4=1496$	
12+56	59.2	17.0	311	
			.56 (2254 − 247)	1124

Entering the table (No. XI) for a width of 71 and a height of 25, we find 548, to which add 7 for the 3 tenths of height, and 7 more for the 9 tenths in width, both mentally, thus giving 562 cu. yds. for this partial volume. Similarly for the width 67.4, and height 21.5, obtaining 447 cu. yds. The corresponding result for the middle area is 503, which is to be multiplied by 4, thus giving 2012 cu. yds. The sum of these is 3021 cu. yds., from which is to be subtracted the constant volume K, which in this case is 247 cu. yds., leaving 2774 cu. yds. as the volume of the prismoid.

The next prismoid is but 56 feet long, but it is taken out just the same as though it were full, and then 56 hundredths of the resulting volume taken. The data for the 12th station is used in getting this result without writing it again on the page.

EXAMPLE 2. *Five-level Ground having four Warped Surfaces.*—Find the volume of a prismoid of which the following are the field notes, the width of bed being 20 feet, and the slopes 1½ to 1:

11.	$\frac{28.9}{+12.6}$	$\frac{15.0}{+12.0}$	$\frac{0}{+18.6}$	$\frac{20.0}{+21.0}$	$\frac{43.0}{+22.0}$
12.	$\frac{27.1}{+11.4}$	$\frac{12.5}{+12.0}$	$\frac{0}{+14.8}$	$\frac{18.5}{+19.6}$	$\frac{40.3}{+20.2}$

This is the same problem as the preceding, with intermediate heights added.

To compute this from the table, it is separated into three prismoids, as shown in Fig. 202.

Let $ABDGCFE$ be the cross-section. This may be separated into the triangle

ABC, and the two quadrilaterals $BCGD$ and $ACFE$. The area of the triangle is $\frac{1}{2}cw$. That of the right quadrilateral is, from Art. 199, p. 245.

$$\tfrac{1}{2}\left[c\left(d_k-\frac{w}{2}\right)+k(d_h-0)+h\left(\frac{w}{2}-d_k\right)\right]=\tfrac{1}{2}\left[(c-h)\left(d_k-\frac{w}{2}\right)+kd_h\right].$$

Similarly the area of the left quadrilateral is $\tfrac{1}{2}\left[(c-h')\left(d'_k-\frac{w}{2}\right)+k'd'_h\right]$.

The total area of the section then is

$$A=\tfrac{1}{2}\left[(c-h')\left(d'_k-\frac{w}{2}\right)+k'd'_h+cw+kd_h+(c-h)\left(d_k-\frac{w}{2}\right)\right]. \quad . \quad (1)$$

If the interior side elevations be taken over the edges of the base, then $d'_k-\frac{w}{2}$ and $d_k-\frac{w}{2}$ both become zero, and the first and last terms disappear. Or if the centre and extreme side heights are the same, these terms go out. Experience shows that these terms can usually be neglected without material error.

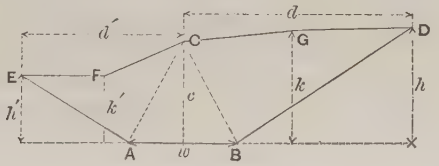


FIG. 202.

If they are retained, each partial volume will be composed of five terms, while if they are neglected there will be but three. The signs of these terms also must be carefully attended to. When the interior side readings are taken over the edges of the base, therefore, this equation becomes

$$A=\tfrac{1}{2}(k'd'_h+cw+kd_h). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The tables are well adapted to compute the prismoidal volume for five-level sections by either of these formulæ. Thus, if the adjacent section also has five points determined in its surface, its area may be represented by an equation similar to one of these, and from these end-area data mean values may be found for the corresponding middle-area points, and the volumes taken out as before. In this case the prism included between the roadbed and side slopes, whose volume is K,

is not included, and hence its volume is not to be deducted from the result. The computation by Table XI of Eq. (1) would be as follows:

Sta.	h'.	d'_h.	k'.	d'_k.	c.	d_k.	k.	d_h.	h.	Partial Volumes	Total Volume
11	12.6	28.9	12.0	15.0	18.6	20.0	21.0	43.0	22.0	$+9+108+114+279-10 = 500$	
M	12.0	28.0	12.0	13.8	16.7	19.2	20.3	41.6	21.1	$4(+6+104+102+260-12) = 1840$	
12	11.4	27.1	12.0	12.5	14.8	18.5	19.6	40.3	20.2	$+3+100+90+242-13 = 422$	2762

The use of the table is the same as before. First take out from the table the volume corresponding to $(c-h')\left(d'_k-\frac{w}{2}\right)$, which when evaluated for section 11 is $(18.6-12.6)(15.0-10)=6.0\times5.0$. This is positive, and the volume corresponding to a depth of 6.0 feet and a width of 5.0 feet is 9 cubic yards. Proceed to evaluate the remaining terms of Eq. (1) in a similar manner, the last term coming out negative. The dimensions of the mid section are the means of the corresponding end dimensions, as before. If one end area is a three-level section and the next a five-level section, the included prismoid is computed as a five-level prismoid, the vanishing points in the three-level section corresponding to the interior side elevations on the five-level section being indicated in the field. Partial stations, or prismoids, are first computed as though they were 100 feet long (for which the table is constructed), and then multiplied by their length and divided by 100 as before.

If Eq. (2) may be used, the work is shortened very much. The columns in h', d'_k, d_k, and h, may be omitted, and there will also be but three terms in each partial product. Thus, if sections 11 and 12 had been taken with the interior elevations, each 10 feet from the centre line, we might have had something as follows:

11.	$\frac{28.9}{+12.6}$	$\frac{10.0}{+15.4}$	$\frac{0}{+18.6}$	$\frac{10.0}{+19.8}$	$\frac{43.0}{+22.0}$
12.	$\frac{27.1}{+11.4}$	$\frac{10.0}{+12.5}$	$\frac{0}{+14.8}$	$\frac{10.0}{+17.4}$	$\frac{40.3}{+20.2}$

The computation then, by Eq. (2), would have been:

Sta.	d'_h.	k'.	c.	k.	d_h.	Partial Volumes.	Total Volume.
11	28.9	15.4	18.6	19.8	43.0	$137+114+263 = 514$	
M	28.0	14.0	16.7	18.6	41.6	$4(121+102+239) = 1848$	
12	27.1	12.5	14.8	17.4	40.3	$104+90+215 = 409$	2771

By this method the computation of a five-level section is little more trouble than that of a three-level section, and yet the intermediate points taken at a distance of $\frac{w}{2}$ from the centre, are apt to increase the accuracy considerably on ordinary rolling ground.

380. Three-level Sections, the Surface divided into four Planes by Diagonals. If the surface included between two three-level sections be assumed to be made up of four planes formed by joining the centre height at one end with a side height at the other end section on each side the centre line (Fig. 203), these lines being called diagonals, an exact computation of the volume is readily made without computing the mid area. Two diagonals are possible on each side the centre line, but the one is drawn which is observed to most nearly fit the surface. They are noted in the field when the cross-sections are taken.

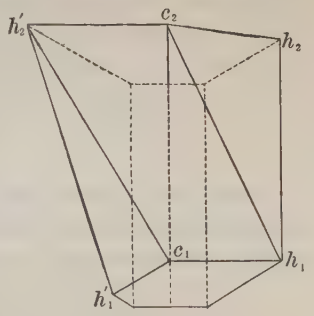


FIG. 203.

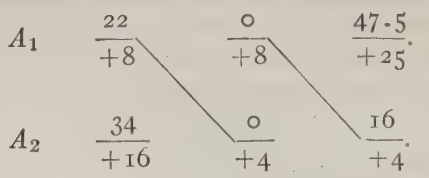
The total volume of such a prismoid in cubic [1] yards is

$$V=\frac{l}{6\times 27}\Big[(d_1+d_1')c_1+(d_2+d_2')c_2+DC+D'C' + \frac{w}{2}(h_1+h_2+H+h_1'+h_2'+H')\Big], \quad (1)$$

where c_1, h_1, and h_1' are the centre and side heights at one section and d_1 and d_1' the distances out, c_2, h_2', h_2, d_2, and d_2' being

[1] This volume is made up of six pyramids, three on each side of the centre plane, these having their apices in the side heights to which the diagonals are drawn, and their bases in the opposite cross-section, in the central vertical plane, and in the roadbed respectively. The student should derive the formula.

the corresponding values for the other end section. C and C' are the centre heights, H and H' the side heights, and D and D' the distances out on the right and left diagonals. Although this formula seems long, the computations by it are very simple. Thus let the volume be found from the following field notes for a base of 20 feet and side slopes $1\frac{1}{2}$ to 1.



The upper figures indicate the distances out and those below the lines the heights, the plus sign being used for cuts. The computation in tabular form is as follows:

Sta.	d.	h.	c.	h'.	d'.	$d+d'$.	$(d+d')c$.	DC.	$D'C'$.
1	22	8	8	25	47·5	69·5	556	····	····
2	34	16	4	4	16	50.0	200	88	128
			$h_1+h_2=24$				88		
			$H+H'=12$				128		
			$\frac{w}{2}\Sigma h's=65\times 10$				$=650$		

6) 162200

27) 27033

1001 cu. yards.

The great advantage of the method consists in the data all being at hand in the field notes.

Hudson's Tables [1] give volumes for this kind of prismoid.

[1] "Tables for Computing the Cubic Contents of Excavations and Embankments." By John R. Hudson, C.E. John Wiley & Sons, New York, 1884.

They furnish a very ready method of computing volumes when this system is used.

381. Comparison of Methods by Diagonals and by Warped Surfaces. Although the surveyor has a choice of two sets of diagonals when this method is used, the real surface would usually correspond much nearer the mean of the two pairs of plane surfaces than to either one of them. That is, the natural surface is curved and not angular, and therefore it is probable that two warped surfaces joining two three-level sections would generally fit the ground better than four planes, notwithstanding the choice that is allowed in the fitting of the planes. More especially must this be granted when the truth of the following proposition is established.

PROPOSITION: *The volume included between two three-level sections having their corresponding surface lines joined by warped surfaces, is exactly a mean between the two volumes formed between the same end sections by the two sets of planes resulting from the two sets of diagonals which may be drawn.*

If the two sets of diagonals be drawn on each side the centre line and a cross-section be taken parallel to the end areas, the traces of the four surface planes on each side the centre line on the cutting plane will form a parallelogram, the diagonal of which is the trace of the warped surface on this cutting plane. Since this cutting plane is any plane parallel to the end areas, and since the warped surface line bisects the figure formed by the two sets of planes formed by the diagonals, it follows that the warped surface bisects the volume formed by the two sets of planes. The proposition will therefore be established if it be shown that the trace of the warped surface is the diagonal of the parallelogram formed by the traces of the four planes formed by the two sets of diagonals. Fig. 204 shows an extreme case where the centre height is higher than the side height at one end and lower at the other. Only the left half of the prismoid is shown in the figure. The cutting plane cuts the centre and side lines and the two diagonals in *efgh* on the plane, and in

$e'f'g'h'$ on the vertical projection. For the diagonal c_1d_2 the surface lines cut out are $e'f'$ and $f'h'$. For the diagonal c_2d_1 they are $e'g'$ and $g'h'$. For the warped surface the line cut out is $e'h'$, this being an element of that surface. It remains to show that $e'f'h'g'$ is a parallelogram.

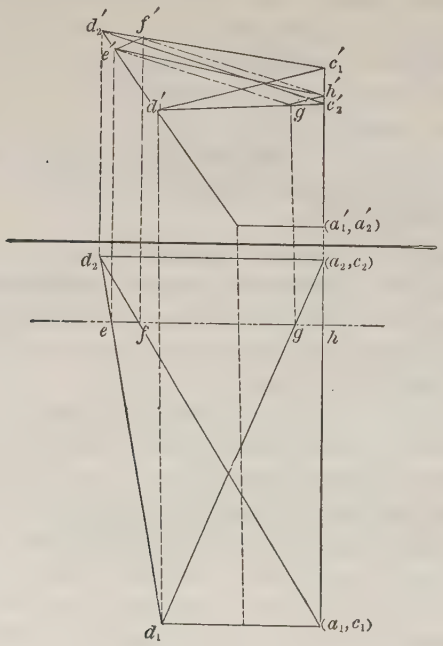


Fig. 204.

Since the cutting plane is parallel to the end planes all the lines cut are divided proportionally. That is, if the cutting plane is one n^{th} of l from c_2, then it cuts off one n^{th} of all the lines cut, measured from that end plane. But if the lines are divided proportionally, the projections of those lines are divided proportionally, and hence the points e', f', h', g' divide the sides of the quadrilateral d_2', c_1', c_2', d_1' proportionally. But it is a

proposition in geometry that if the four sides of a quadrilateral, or two opposite sides and the diagonals, be divided proportionally and the corresponding points of subdivision joined, the resulting figure is a parallelogram. Therefore $e'f'h'g'$ is a parallelogram, and $e'h'$ is one of its diagonals and hence bisects it. Whence the surface generated by this line moving along c_1c_2 and d_1d_2 parallel to the end areas bisects the volume formed by the four planes resulting from the use of both diagonals on one side the centre line. Q.E.D.

It is probable, therefore, that the warped surface would usually fit the ground better than either of the sets of planes formed by the diagonals. Furthermore, the errors caused by the use of the warped surface (Table XI) are compensating errors, thus preventing any marked accumulation of errors in a series of prismoids.[1] There are extreme cases, however, such as that given in the example, Fig. 203, which are best computed by the method by diagonals.

382. Preliminary Estimate from the Profile. If the cross-sections be assumed level transversely then for given width of bed and side slopes, a table of end areas may be prepared in terms of the centre heights. From such a table the end areas may be rapidly taken out and *plotted as ordinates from the grade line.* The ends of these ordinates may then be joined by a free-

[1] The two methods here discussed are the only ones that have any claims to accuracy. The method by "mean-end areas," wherein the volume is assumed to be the mean of the end areas into the length, always gives too great a volume (except when a greater centre height is found in connection with a less total width, which seldom occurs), the excess being *one-half of the volume of the pyramids involved in the elementary forms of the prismoid.* This is a large error even in level sections, and very much greater on sloping ground, and yet it is the basis of most of the tables used in computing earthwork, and in some States it is legalized by statute. Thus in the example computed by Henck's method on p. 518 the volume by mean-end areas is 1193 cubic yards; by the prismoidal formula it is 1168 cubic yards, while by the method by diagonals it was only 1001 cubic yards. This was an extreme case, however, and was selected to show the adaptation of the method by diagonals to such a form.

hand curve, and the area of this curve found by the planimeter. The ordinates may be plotted to such a scale that each unit of the area, as one square inch, shall represent a convenient number of cubic yards, as 1000. The record of the planimeter then in square inches and thousandths gives at once the cubic yards on the entire length of line worked over by simply omitting the decimal point. Evidently the scale to which the ordinates are to be drawn to give such a result is not only a function of the width of bed and side slopes, but also of the longitudinal scale to which the profile line is plotted. The area of a level section is

$$A = wc + rc^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where w, c, and r are the width of base, centre height, and slope ratio respectively.

Now if h = the horizontal scale of the profile, that is the number of feet to the inch, and if one square inch of area is to represent 1000 cubic yards, the length of the ordinate must be

$$y = \frac{hA}{1000 \times 27} = \frac{h(wc + rc^2)}{27000}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If values be given to h, w, and r, which are constants for any given case, then the value of y becomes a function of c only, and a table can be easily prepared for the case in hand. Since y is a function of the second power of c, the second difference will be a constant, and the table can be prepared by means of first and second differences. Thus if c takes a small increment, as 1 foot, then the first difference is

$$\Delta' y = \frac{h}{27000}(w + 2rc + r). \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

But this first difference is also a function of c, and hence when c takes an increment this first difference changes by an amount equal to

$$\Delta'' y = \frac{h}{27000} \cdot 2r, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

which is constant. An initial first difference being given for a certain value of c, a column of first differences can be obtained by simply adding the $\Delta''y$ continuously to the preceding sum. With this column of first differences the corresponding column of values of y may be found by adding the first differences continuously to the initial value of y for that column.[1]

TABULAR VALUES OF y IN EQUATION (2) FOR $w=20$, $r=1\frac{1}{2}$, AND $h=400$.

c	0.′0	0.′1	0.′2	0.′3	0.′4	0.′5	0.′6	0.′7	0.′8	0.′9
	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.
0	0.00	0.03	0.06	0.09	0.12	0.15	0.19	0.22	0.25	0.28
1	.32	.35	.39	.42	.46	.49	.53	.57	.61	.64
2	.68	.72	.76	.80	.84	.88	.92	.96	1.00	1.05
3	1.09	1.13	1.17	1.22	1.26	1.31	1.35	1.40	1.45	1.49
4	1.54	1.59	1.63	1.69	1.73	1.78	1.83	1.88	1.93	1.99
5	2.04	2.09	2.14	2.19	2.24	2.30	2.36	2.41	2.47	3.52
6	2.58	2.63	2.69	2.75	2.80	2.87	2.92	2.98	3.04	3.10
7	3.16	3.22	3.28	3.35	3.41	3.47	3.54	3.60	3.66	3.73
8	3.79	3.86	3.92	3.99	4.05	4.13	4.19	4.26	4.33	4.40
9	4.47	4.54	4.60	4.68	4.75	4.82	4.89	4.97	5.04	5.11
10	5.18	5.26	5.33	5.40	5.48	5.56	5.64	5.72	5.79	5.87
11	5.95	6.03	6.10	6.18	6.26	6.35	6.43	6.51	6.59	6.67
12	6.76	6.84	6.92	7.00	7.09	7.18	7.26	7.35	7.43	7.52
13	7.61	7.70	7.78	7.86	7.96	8.05	8.14	8.23	8.32	8.41
14	8.50	8.60	8.68	8.77	8.87	8.97	9.06	9.16	9.25	9.35
15	9.44	9.54	9.63	9.73	9.83	9.94	10.03	10.13	10.23	10.33
16	10 43	10.53	10.62	10.73	10.83	10.94	11.04	11.15	11.25	11.35
17	11.46	11.56	11.66	11.77	11.88	12.00	12.10	12.21	12.31	12.42
18	12.53	12.64	12.75	12.86	12.97	13.09	13.20	13.32	13.42	13.54
19	13.65	13.77	13.87	13.99	14 10	14.23	14.34	14.47	14.58	14.70
20	14 81	14.93	15.04	15.16	15.29	15.42	15.53	15.66	15.78	15.90

The preceding table was constructed in this manner, for $w=20$ feet, $r=1\frac{1}{2}$; and $h=400$ feet to the inch.

383. Borrow Pits are excavations from which earth has been "borrowed" to make an embankment. It is generally preferable to measure the earth in cut rather than in fill, hence when the earth is taken from borrow pits and its volume is to

[1] For a further exposition of this subject, see Appendix C.

be computed in cut, the pits must be carefully staked out and elevations taken before and after excavating. The methods given in Art. 370 are well suited to this purpose, or they may be computed as prismoids by the aid of Table XI, if preferred. To use the table it is only necessary to enter it with such heights and widths as give twice the elementary areas (triangles or quadrilaterals) into which the end sections are divided, and then multiply the final result by the length and divide by 100. The table is entered for both end-area dimensions and also the mid-area dimensions, four times this latter result being taken the same as before.

384. Shrinkage of Earthwork. Excavated earth first increases in volume, when removed from a cut and dumped on a fill, but it gradually settles, or shrinks, until it finally comes to occupy a less volume than it formerly did in the cut. Both the amounts, initial increase and final shrinkage, depend on the nature of the soil, its condition when removed, and the manner of depositing it in place. There can therefore be no general rules given which will always apply. *For ordinary clay and sandy loam, dumped loosely, as from wheelbarrows, the first increase is about one-twelfth, and then the settlement about one-sixth of this increased volume, leaving a final volume of about nine-tenths of the original volume in cut.*

Embankments made with carts or wagons will shrink from the first volume in fill from five to ten per cent, while wheel-scraper work will shrink from one to five per cent, depending on the condition of the material when moved and the weather conditions during the progress of the work. One may judge of the probable shrinkage by estimating the proportion of voids which will probably be ultimately filled by settlement.

For rock the permanent increase in volume is from 60 to 80 per cent, the greater increase corresponding to a smaller average size of fragment.

385. Excavations under Water. It is often necessary to determine the volume of earth, sand, mud, or rock removed

from the beds of rivers, harbors, canals, etc. If this be done by soundings alone, it is likely to work injustice to the contractor, as he would receive no pay for depths excavated below the required limit; and besides, foreign material is apt to flow in and partially replace what is removed, so that the material actually excavated is not adequately shown by soundings within the required limits. It is common, therefore, to pay for the material actually removed, an inspector being usually furnished by the employer to see that no useless work is done beyond the proper bounds. The material is then measured in the dumping scows or barges. The unit of measure is the cubic yard, the same as in earthwork. There are two general methods of gauging scows, or boats: One is to actually measure the inside dimensions of each load, which is often done in the case of rock, and the other is to measure the displacement of the boat, which is the more common method with dredged material. When the barge is gauged by measuring its displacement, the water in the hold must always be pumped down to a given level, or else it must be gauged both before and after loading and the depth of water in the hold observed at each gauging. A displacement diagram (or table) is prepared for each barge, from its actual external dimensions, in terms of its mean draught. There should always be four gaugings taken to determine the draught, at four symmetrically located points on the sides, these being one-fourth the length of the barge from the ends. Fixed gauge scales, reading to feet and tenths, may be painted on the side of the barge, or if it is flat-bottomed, a gauging rod, with a hook on its lower end at the zero of the scale, may be used and readings taken at these four points. Any distortion of the barge under its load, or any unsymmetrical loading, will then be allowed for, the mean of the four gauge readings being the true mean draught of the boat.

To prepare a displacement diagram, the areas of the surfaces of displacement must be found for a series of depths uniformly spaced. This series may begin with the depth for no load, the hold being dry. They should then be found for each

five-tenths of a foot up to the maximum draught. If the boat has plane vertical sides and sloped ends these areas are rectangles, and are readily computed. If the boat is modelled to curved lines, the water lines can be obtained from the original drawings of the boat, or else they must be obtained by actual measurement. In either case they can be plotted on paper, and their areas determined by a planimeter. These areas are analogous to the cross-sections in the case of railroad earthwork, and the prismoidal formula may be applied for computing the displacement. Thus:

Let A_0, A_1, A_2, A_3, etc., be the areas of the displaced water surfaces, taken at uniform vertical distances h apart. Then for an even number of intervals we have in cubic yards

$$V=\frac{h}{3\times 27}(A_0+4A_1+2A_2+4A_3+\ldots A_n). \quad . \quad . \quad (1)$$

If the total range in draught be divided into six equal portions, each equal to h, then Weddel's Rule [1] would give a nearer approximation. With the same notation as the above we would then have, in cubic yards,

$$V=\frac{3h}{10}[A_0+A_2+A_4+A_6+5(A_1+A_3+A_5)+A_3]. \quad . \quad (2)$$

These rules are also applicable to the gauging of reservoirs, mill ponds, or of any irregular volume or cavity.

After the displaced volume of water is found, the corresponding volume of earth or rock is found by applying a proper constant coefficient. This coefficient is always less than unity, and is the reciprocal of the specific gravity of the material. This must be found by experiment. In the case of soft mud it is nearly

[1] For the derivation of this rule, see Appendix C.

unity, while with sand and rock it is much more. When rock is purchased by the cubic yard, solid rock is not implied, but the given quality of cut or roughly quarried rock, piled as closely as possible. When rock is excavated, solid rock is meant. A *measured volume* of any material put into a *gauged scow* will give the proper coefficient for that material. Thus if the measured volume V' give a displacement of V, then $\frac{V'}{V} = C$ is the coefficient to apply to the displacement to give the volume of that material.

NOTE.—The computation of "haul" does not properly come within the province of this work. A very excellent article on this subject, showing the use of the "mass curve" in earthwork, by Professor Walter L. Webb, will be found in the Proc. Eng. Club, Phil., Vol. XIV (1897), p. 249, and also in the Railroad Gazette for December 17, 1897, p. 885. Also a paper by Professor C. Frank Allen in Railroad Gazette of May 24, 1895, p. 325.

BOOK III

GEODETIC SURVEYING

CHAPTER XIV

TRIANGULATION AND BASE LINE

386. The Objects of a Geodetic Survey. The main object of a geodetic survey is to furnish the necessary control for all other surveys. This control includes the determination of relative positions of a comparatively few widely separated points on the earth's surface and the directions and lengths of the lines joining them. Without such accurate locations the errors of ordinary surveys would accumulate until they vitiated the needed accuracy of the survey and map. Indeed, the primary triangulation of the U. S. Coast and Geodetic Survey, by its corrective function, stands in the same relation to other surveys as the U. S. Supreme Court stands to all lesser courts.[1]

The above work involves (*a*) the measurement of base lines; (*b*) the measurement of the angles of the triangulation system; and (*c*) the astronomic determinations of latitude, longitude, and azimuth. All of these operations will be briefly discussed in Book III.

[1] In fact, during the past few years this primary triangulation has been the means of settling boundary line disputes between more than ten States.

The secondary object of a geodetic survey and one incidental to the above operations is the determination of the size and shape of the earth or that part of it covered by the survey. This is best accomplished by frequent *astronomic* determinations of latitude, longitude, and azimuth at triangulation stations.

387. The Size and Figure of the Earth. Geodesy is a natural development of land surveying, but because of the great areas [1] covered by the triangulation, account must be taken of the *sphericity of the earth.* The first measurements ever made to determine the radius of the earth were made by the famous school of Alexandria about 200 B.C. It is remarkable that the *principle* there employed is the same that has been used ever since. Stripped of all details the method consisted in measuring a long north and south line and of determining the latitude of its two ends by measuring the altitude of the sun or other star. The difference between the observed latitudes gives the curvature of this arc in degrees. The length of the earth's radius is easily computed from these data. No further attempts to solve the size of the earth were made until a thousand years [2] later. Eight centuries later the introduction of the principle of triangulation gave a great impetus to geodetic investigations.

Modern geodesy may be said to begin with the discovery of the law of gravitation by Newton late in the seventeenth century. "He proved that the earth, as a revolving and not wholly rigid body subject to its own attraction, must take the form of a slightly flattened sphere." [3] Modern precise measurements prove that any section of the earth parallel to the equator is with considerable

[1] The U. S. C. & G. S. transcontinental triangulation extends from Cape May on the Atlantic to the Golden Gate on the Pacific, or through 45° of longitude. See Bulletin of University of Wisconsin, No. 38, by Mr. John F. Hayford.

[2] By the Arabs in the Mesopotamia about 825.

[3] These theoretical conclusions were fully confirmed by the expeditions sent to the equatorial region of Peru (1735–41), and to the polar region of Lapland (1736–37). Their work proved Newton's theory, since they found the length of one degree to be greater, or in other words,, the arc flatter, near the pole than near the equator.

exactness a circle, and every section passing through both poles is an ellipse. The dimensions and form of this ellipsoid of revolution are usually given by its equatorial and polar radii or diameters. The two most notable computations of these dimensions were made by Bessel in 1841 and by Clarke in 1866, the latter being now in most common use. These dimensions are given in the following Table I:

TABLE I.

	Bessel, Meters.	Clarke, Meters.	Bessel English Statute Miles.	Clarke, English Statute Miles.
a. Equatorial radius	6,377,397	6,378,206	3962.72	3963.23
b. Semipolar axis	6,356,079	6,356,584	3949.48	3949.79
Difference $a-b$	21,318	21,622	13.25	13.44
Compression $\frac{a-b}{a}$	$\frac{1}{299.12}$	$\frac{1}{295}$		

388. Minor Irregularities in the Shape of the Earth. By the shape of the earth is meant the surface of the sea, which includes three-quarters of its entire surface. But it must not be understood that this shape, as defined above, is entirely free from such irregularities as small *humps* and hollows.[1] The ocean surface at any given point is at right angles to the direction of the plumb line. On the continents, the horizontal planes defined by the level vials are again perpendicular to the direction of the plumb line. Minor deviations of the plumb line (and the corresponding irregularities in the shape of the earth) are made manifest by geodetic surveys in the following way: If the standard geodetic latitude, longitude, and meridian of one station of a triangulation system be known the geodetic latitude and longitude of every

[1] See "Precise Surveying," Chapters VI and VII, by Mansfield Merriman, John Wiley & Sons; also "Figure of the Earth and Isostasy," by J. F. Hayford, U. S. C. and G. S., 1909 and 1910.

other station of such system can be computed (Art. 461). If now the astronomic latitude and longitude of all or part of these same stations be observed, the differences between corresponding geodetic and astronomic latitudes and longitudes furnish at once a determination of the local shape of the earth. For the geodetic latitudes and longitudes are all referred to the Clarke ellipsoid of revolution, while the astronomic latitudes and longitudes are referred to the actual irregular figure of the earth. The differences therefore between the regular and the irregular figures constitute the humps and hollows.

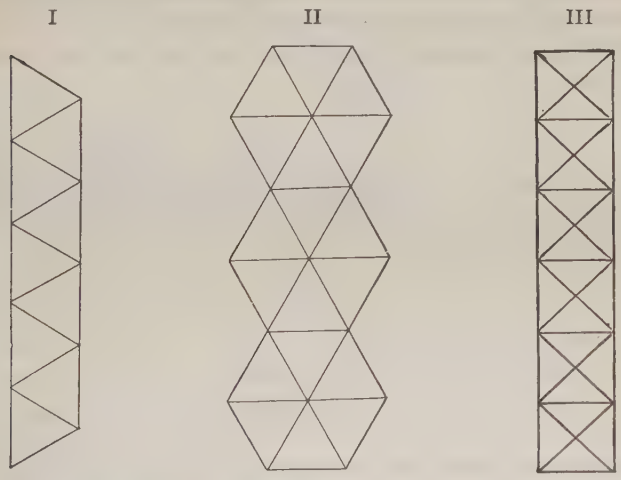


FIG. 205.

389. Triangulation Figures. If it is desired to connect two distant points by a system of triangulation at the least expense, system I, shown in Fig. 205, may be used. This system is, however, rarely used because of its lack of accuracy.

If such distant points are to be joined, or such double system of stations established, with the greatest attainable accuracy, then system III should be used. This system is also best

adapted to secondary work, where it is desired to simplify the work of reduction. Each quadrilateral may be independently reduced.

If the greatest area is to be covered for a given degree of accuracy or cost, then system II is the one to use.

System I consists of a single row of simple triangles, system II of a double row of simple triangles or of simple triangles arranged as hexagons, and system III of a single row of quadrilaterals. A quadrilateral in triangulation is an arrangement of four stations with all the connecting lines observed. This gives six lines connecting as many pairs of stations, over which pointings have been taken from both ends of the line.

For the same maximum length of lines we have the following comparison of the three systems:[1]

System.	Composition.	Distance Covered.	No. of Stations.	Total Length of Sides.	Area Covered.	No. of Conditions.
I	Equilateral triangles....	5	11	19	4.5	$n-2=9$
II	Hexagons.....	5.2	17	34	9	$\frac{7n-14}{5}=21$
III	Quadrilaterals (squares).	4.95	14	29	3.5	$2n-4=24$

Thus, for the same distance covered, the number of stations to be occupied and the total length of lines to be cleared out are about one-half more for systems II and III than for system I. The area covered by system II is twice that by system I, but the number of conditions is much greater in system III than in either of the others. Since almost all the error in triangulation comes from erroneous angle measurements, the results will be more accurate according to our ability to reduce the

[1] Taken from the U. S. C. and G. Survey Report for 1876.

observed values of the angles to their true values. The "conditions" mentioned in the above table are rigid geometrical conditions, which must be fulfilled (as that the sum of the angles of a triangle shall equal 180°), and the more of these geometrical conditions we have, the more nearly are we able to determine what the true values of the angles are. The work will increase in accuracy, therefore, as the number of these conditions increases, and this is why system III gives more accurate results than systems I and II. This will be made clear when the subject of the adjustment of the observations is considered. See also Art. 394.

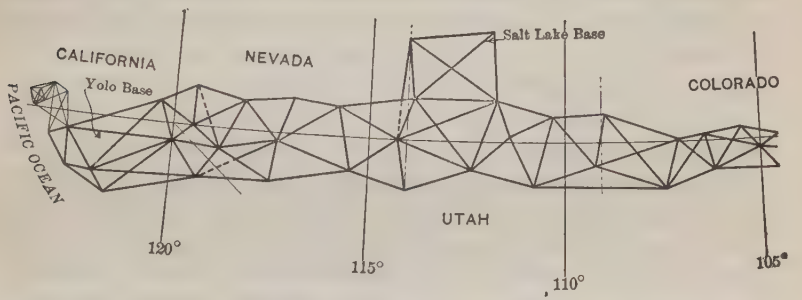


FIG. 206.—Part of the U. S. C. and G. S. Transcontinental Triangulation of U.S.

Because of the variations in topography it will rarely be found practical to lay out the system of perfect figures shown above. Instead extra stations will need to be introduced and the size of the figures changed. This is well illustrated by Fig. 206.[1]

390. Classification of Triangulation. For the various grades of work, different degrees of accuracy are required involving the use of different instruments and different programmes of observation and computation. These grades of triangulation are distinguished as Primary, Secondary, and Tertiary.

[1] Part of trans-continental triangulation of the U. S. C. and G. S.

391. Primary Triangulation. As its name would imply, primary triangulation is of the highest grade of accuracy obtainable at reasonable cost. It employs the longest sides practicable and its angles are measured by the most accurate instruments and methods. Formerly, it was thought necessary to go to excessive trouble and expense to secure very long sides, 100 miles long being common with many sides much in excess of even this. The minimum length of such work on the U. S. Coast and Geodetic Survey is now 6 kilometres (3.8 miles). The specifications for this work state that "there is little, if any, advantage, in so far as accuracy is concerned, in making the lines much longer than this." Instead of the excessively long line and attending large expense, it is better practice to use that length which will make the total cost of the work of reconnoissance, station building, and angle measurement a minimum per mile of progress.

The office computations for this grade of work are also very refined and laborious. For example, all angles must be reduced to what they would have been if made at sea level (due to the fact that the normals of the earth's surface do not intersect), and astronomical latitudes must be corrected for elevation above the sea. In general, primary triangulation is used where it is unsupported over long distances and so constitutes the main framework for all other surveys.

392. Tertiary Triangulation. This, the lowest grade of triangulation, is used for the immediate control of topographic and hydrographic surveys or for engineering purposes in general. Even here the triangulation system should not be allowed to degenerate into single triangles except in the rare cases when it is extremely difficult to avoid this. The lower limit in the length of the sides is fixed only by the difficulty of closing the triangles with the required accuracy, and by the fact that short lines are apt to be accompanied by poor geometrical conditions.

393. Secondary Triangulation. Secondary triangulation is intermediate between primary and tertiary triangulation and is used to connect the two, or it may be used where the total distance

covered is not great, or where it is supported at frequent intervals by the primary triangulation.

Where the angles are measured by small and inferior instruments, or in case only part of the angles are actually measured, or all measured but by an inferior programme, it is usual to designate the work as *reconnoissance*, and not in any of the above classes.

394. Strength of Figures. By strength of figures is included the relation of the *shape* of the figure to the *accuracy* of the computed lengths of sides.

It has recently been shown that much misinformation exists on this subject. The effect of a given figure upon such accuracy is shown by the computed probable errors. Now the square of the probable error in the logarithm of a triangle side is given by

$$\tfrac{4}{3}(d)^2\frac{Nd-Nc}{N_d}\Sigma\delta_A{}^2+\delta_A\delta_B+\delta_B{}^2.\ ^1 \quad . \quad . \quad . \quad (1)$$

In the formula d is the probable error of an observed direction (angle) ordinarily computed from the adjustment of the triangulation. It expresses the accuracy of the observing only and is not dependent upon the geometrical conditions in the figure. The two terms following this, viz., $\frac{N_d-N_c}{N_d}$ and $\Sigma\delta_A{}^2+\delta_A\delta_B+\delta_B{}^2$ are independent of the accuracy of the observing and are entirely fixed by the figure chosen. These two terms therefore express the strength of the figure.

N_d is the number of directions observed in the figure and N_c is the number of geometrical conditions to be satisfied. The formula shows that the smaller the value of this fraction, $\frac{N_d-N_c}{N_d}$, the greater the accuracy of the lengths. In other words, the greater the number of rigorous conditions in proportion to the number of unknowns (the directions), the greater the accuracy.

[1] See Wright and Hayford's "Adjustment of Observations," pp. 237–8, where the equation is given in slightly different form.

In the quadrilateral, for example, $N_d = 10$ and $N_c = 4$, and therefore $\frac{N_d - N_c}{N_d} = .60$, while for a six-sided central-point figure, $N_d = 22$, $N_c = 7$, and hence $\frac{N_d - N_c}{N_d} = 0.68$. This shows that so far as effect of rigorous conditions is concerned, the former figure is a stronger figure than the second.

The expressions δ_A and δ_B are the logarithmic differences corresponding to a change of one second in the *distance* angles of each triangle concerned. Thus in Fig. 213 in computing the length of line *CD* from the line *AB* as a known side, the computation may utilize the triangles *ABD*, *BDC*. In every case the computation of the sides involves but two of the three angles of each triangle. These angles used in the length computation are called the distance angles, thus in *ABD* angle *D* and *A*, and in triangle *BDC* angles *C* and *B* are distance angles.

The remaining angle of each triangle can be used most conveniently to carry forward the azimuth (see Art. 461), and are therefore called the azimuth angles. The distance angles are always opposite the *known* and *required* sides of the triangle.

It should be noted from the formula (1) that the smaller the $\Sigma\delta_A^2 + \delta_A\delta_E + \delta_E^2$ the greater the resulting accuracy in the length. Therefore it is important to select such figures as will keep this summation small. The differences in the table of logarithmic sines are very small for angles near 90° and very large for angles near 0° or 180°. Hence to keep this $\Sigma\delta_A^2 + \delta_A\delta_B + \delta_B^2$ small, it is desirable to avoid distance angles near 0° and 180°. It is immaterial as far as errors of lengths are concerned whether the azimuth angles are small or not. The force of this may be seen from the following analysis of Figs. 207 and 208. In both these figures the known and required sides are drawn heavy, *AB* being known and *CD* being required. In Fig. 207 no matter which pair of triangles is used in the computation of *CD* no distance angle less than 44° nor greater than 125° is used. The resulting

value of $\Sigma\delta_A^2+\delta_A\delta_B+\delta_B^2$ is here found to be not greater than 4,[1] corresponding to a high degree of accuracy in the length.

The azimuth angles concerned in this figure are very small, being from 8° to 13°, but this fact has no effect upon the accuracy of the length CD, as they are not concerned in the computation of its length. On the other hand, in Fig. 208 the distance angles used in computing the required side CD from the known side AB range from 9° to 125° and the use of two very small angles is unavoidable. The resulting value of $\Sigma\delta_A^2+\delta_A\delta_B+\delta_B^2$ is 270 or 290, depending upon the route chosen for the computation, corresponding to a very low degree of accuracy in the computed length.

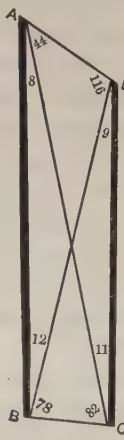


FIG. 207.

The current idea then that in order to secure a satisfactory degree of accuracy in the computed length of sides, *all* angles in a triangulation must be kept between 30° and 150° is erroneous. The fact is that it is important to keep the *distance angles* between these limits and preferably near 90°, but in so far as accuracy of length is concerned the size of the *azimuth angles* is immaterial.

In practice the selection of strong figures involves a complicated set of judgments. On the U. S. Coast and Geodetic

FIG. 208.

Survey this judgment is guided by a table of values of $(\delta_A^2+\delta_A\delta_B+\delta_B^2)$ used in connection with each particular figure under discussion. One general principle which has been found valuable in

[1] The δ_A and δ_B are expressed in units of the sixth place of logarithms.

forming a correct judgment is that a figure much narrower than its length in the direction of the triangulation belt, such as Fig. 208, is a weak figure, while the broad figures, such as Fig. 207, are strong in so far as concerns the accuracy of their lengths.

This principle, however, must be applied with caution, as there are other important elements involved.[1]

395. Base Line Sites. The line actually measured is called the base line. The number of such base lines required, as well as their lengths, depends upon the average lengths of the sides of the triangulation and upon the accuracy desired in the latter. The site of the base lines should be chosen before, or at the same time, as the triangulation stations. The frequency of bases in primary triangulation should be such as to insure no larger discrepancy between adjacent bases when checked through the triangulation than one part in 25,000. This ratio reduces to one part in 10,000 in secondary and one part in 5000 in tertiary triangulation. Past experience of the U. S. Coast and Geodetic Survey shows that the above requirements correspond to a chain of fifteen to thirty-five triangles in primary work and to a chain of from ten to thirty-five triangles in the case of secondary and tertiary triangulation.

Because of the greater ease of measuring base lines at the present time, due to the use of the invar steel tape (Arts. 421–423), it is now economical to measure base lines more frequently than was formerly the case.[2]

396. Connection of Base Line with the Triangulation. Several steps or figures are usually necessary in order to transfer from the relatively short base line to the sides of the main triangulation. Even with well-conditioned figures, a very large propor-

[1] See Wright and Hayford's "Adjustment of Observations," published by Macmillan, New York, for a full discussion of this subject.

[2] On the U. S. C. and G. S. triangulation completed in 1900 there were nine base lines between latitudes 26° and 42½°, or an average interval of less than 120 miles. The older transcontinental triangulation work covering 45° of longitude is checked by only ten bases, the average interval being 270 miles.

tion of the high accuracy of the base is necessarily lost and dissipated in the so-called base net.

Some of the most common methods of expanding from a base are shown in Figs. 209 to 212 inclusive. The substitution of

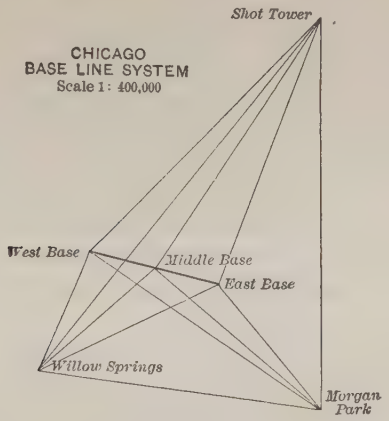


FIG. 209.

tapes in large part for bars in base work makes it possible to utilize much rougher ground for base sites than was formerly possible. Thus the grades may equal 10 per cent while ravines of less than 50 metres do not form obstacles to base measurements.

FIG. 210.

As a result of this it is now both possible and desirable to locate the base with reference to making the geometric conditions in the base figures, such as to be most favorable to accuracy. Each

base net should not be longer than two ordinary figures of the main chain between bases.

The location of the base should be such as to enable one side of the main system to be computed with the greatest accu-

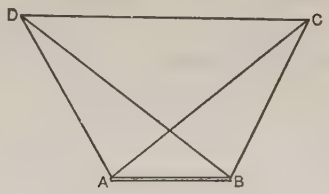


FIG. 211.—Base Net.

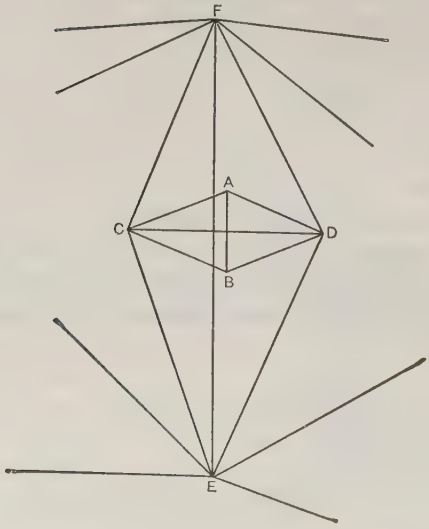


FIG. 212.—Base Net.

racy and with the least number of auxiliary stations for a given length of base. In flat open country the base should be chosen to suit the location of the triangulation stations in the main system; but in rough country some of the main stations must

often be chosen to suit the location of the base line. In Fig. 209 the location of the base line is almost an ideal one, being taken directly across one of the main lines of the system. By referring to Fig. 210 it will be seen that the line Willow Springs—Shot Tower is one of the fundamental lines of the main system, and the base is located directly across it. Here the ground is a flat prairie, and the base was chosen to suit the stations of the main system. The station at the middle of the base is inserted in order to furnish a check on the measurements of the two portions as well as to increase the strength of the system by increasing the number of equations of conditions.

Fig. 211 is a common form of base net.

397. The Reconnoissance. The success of the triangulation depends in large measure upon the intelligent investigations made in the reconnoissance. These investigations should include the selection of the strongest and most feasible figures for the triangulation, the location of the base lines, public roads, sources of supplies for men and animals, and in general the collection of all information affecting the economy of the operations to follow. Such information cannot be too full and all should be carefully recorded.

The most favorable ground for a line of triangles is a valley of proper width, with bald knobs or peaks on either side. Stations can then be selected giving well-conditioned triangles, with little or no clearing out of lines, and with low stations. In a wooded country the lines must be cleared out or else very tall stations must be used. In general, both expedients are resorted to. Stations are built so as to avoid the greater portion of the obstructions, and then the balance is cleared out.

So much depends on the proper selection of the stations in a system of triangulation, as to time, cost, and final accuracy, that the largest experience and the maturest judgment should be made available for this part of the work. The form of the triangles; the amount of cutting necessary to clear out the lines and the probable resulting damage to private interests; the

height and cost of stations, and the accessibility of the same; the avoidance of all sources of atmospheric disturbance on the connecting lines, as of factories, lime- or brick-kilns, and the like, which might either obstruct the line by smoke or introduce unusual refraction from heat; the freedom from disturbance of the stations themselves during the progress of the work, and the subsequent preservation of the marking stones—these are some of the many subjects to be considered in determining the location of stations.

It is the business of the reconnoissance party not only to locate the stations, but to determine the heights of the same. A station that has been located is temporarily marked by a flag fastened upon a pole, and this made to project from the top of a tall tree in the neighborhood. In selecting a new station it is customary to first select from the map the general locality where a station is needed, and then examine the region for the highest ground available. When this is found, the tallest trees are climbed and the horizon scanned with the aid of a pair of field glasses to see if the other stations are visible. If no tree or building is available for this purpose ladders may be spliced together and raised by ropes until the desired height is obtained.

398. Instrumental Outfit. The reconnoissance party requires a convenient means of measuring angles and of determining directions and elevations. For measuring angles a pocket sextant would serve very well, provided the stations are distinct or provided distinct range points in line with the stations may be selected by the aid of field glasses. A prismatic pocket compass will often be found very convenient in finding back stations which have been located and whose bearings are known. An aneroid barometer is desirable for determining approximate relative elevations. For methods of using it in such work, see Chapter VI. If to the above-named instruments we add field glasses, drawing instruments, steel tape, two heliotropes for testing the intervisibility of stations, best maps of the country

available, and creepers for climbing trees, the instrumental outfit is fairly complete.

399. The Direction of Invisible Stations. It often happens that one station cannot be seen from another on account of forest growth, which may be cleared out. In such a case the station may be located and the line cleared from one station or from both, the direction of the line having been determined. This direction may always be computed if two other points can be found from each of which both stations and the other auxiliary point are visible. Thus in Fig. 213 let AB be the line to be cleared out, and let C and D be two points from which all the stations may be sighted. Measure the two angles at each station and call the distance CD unity. Solve the triangle BCD for the side BC, and the triangle ADC for the side AC. We now have in the triangle ABC two sides and the included angle to find the other angles. When these are found the course may be aligned from either A or B. It will often happen that either C or D or both can be taken at regular stations. Of course a target must be left at either C or D to be used in laying out the line from A or B. The above is a modification of the problem (5), p. 110. A use of this expedient will often greatly facilitate the work.

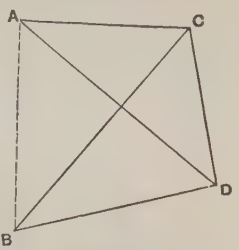


FIG. 213.

400. The Heights of Stations depend on the relative heights of the ground at the stations and of the intervening region. If the surface is level, then the heights of stations depend only on their distance apart. In any case the distance apart is so important a function of the necessary height that it is well to know what the heights would have to be for level, open country.

The following table[1] gives the height of one station when the other is at the ground level, for open, level country:

[1] Taken from Report of U. S. Coast and Geodetic Survey for 1882.

Table II.

DIFFERENCE IN FEET BETWEEN THE APPARENT AND TRUE LEVEL AT DISTANCES VARYING FROM 1 TO 66 MILES

Distance. Miles.	Difference in Feet for—			Distance. Miles.	Difference in Feet for—		
	Curvature.	Refraction.	Curvature and Refraction.		Curvature.	Refraction.	Curvature and Refraction.
1	0.7	0.1	0.6	34	771.3	108.0	663.3
2	2.7	0.4	2.3	35	817.4	114.4	703.0
3	6.0	0.8	5.2	36	864.8	121.1	743.7
4	10.7	1.5	9.2	37	913.5	127.9	785.6
5	16.7	2.3	14.4	38	963.5	134.9	828.6
6	24.0	3.4	20.6	39	1014.9	142.1	872.8
7	32.7	4.6	28.1	40	1067.6	149.5	918.1
8	42.7	6.0	36.7	41	1121.7	157.0	964.7
9	54.0	7.6	46.4	42	1177.0	164.8	1012.2
10	66.7	9.3	57.4	43	1233.7	172.7	1061.0
11	80.7	11.3	69.4	44	1291.8	180.8	1111.0
12	96.1	13.4	82.7	45	1351.2	189.2	1162.0
13	112.8	15.8	97.0	46	1411.9	197.7	1214.2
14	130.8	18.3	112.5	47	1474.0	206.3	1267.7
15	150.1	21.0	129.1	48	1537.3	215.2	1322.1
16	170.8	23.9	146.9	49	1602.0	224.3	1377.7
17	192.8	27.0	165.8	50	1668.1	233.5	1434.6
18	216.2	30.3	185.9	51	1735.5	243.0	1492.5
19	240.9	33.7	207.2	52	1804.2	252.6	1551.6
20	266.9	37.4	229.5	53	1874.3	262.4	1611.9
21	294.3	41.2	253.1	54	1945.7	272.4	1673.3
22	322.9	45.2	277.7	55	2018.4	282.6	1735.8
23	353.0	49.4	303.6	56	2092.5	292.9	1799.6
24	384.3	53.8	330.5	57	2167.9	303.5	1864.4
25	417.0	58.4	358.6	58	2244.6	314.2	1930.4
26	451.1	63.1	388.0	59	2322.7	325.2	1997.5
27	486.4	68.1	418.3	60	2402.1	336.3	2065.8
28	523.1	73.2	449.9	61	2482.8	347.6	2135.2
29	561.2	78.6	482.6	62	2564.9	359.1	2205.8
30	600.5	84.1	516.4	63	2648.3	370.8	2277.5
31	641.2	89.8	551.4	64	2733.0	382.6	2350.4
32	683.3	95.7	587.6	65	2819.1	394.7	2424.4
33	726.6	101.7	624.9	66	2906.5	406.9	2499.6

$$\text{Curvature} = \frac{\text{square of distance}}{\text{mean diameter of earth}};$$

Log curvature = log square of distance in feet − 7.6209807;

$\text{Refraction} = \frac{K^2}{R}m$, where K represents the distance in feet, R the mean radius of the earth ($\log R = 7.3199507$), and m the coefficient of refraction,[1] assumed at .070, its mean value, sea coast and interior.

$$\text{Curvature and refraction} = (1 - 2m)\frac{K^2}{2R}.$$

Or, calling h the height in feet, and K the distance in statute miles, at which a line from the height h touches the horizon, taking into account terrestrial refraction, assumed to be of the same value as in the above table (.070), we have

$$K = \frac{\sqrt{h}}{.7575}, \qquad h = \frac{K^2}{1.7426}.$$

401. Intervisibility of Stations. The following examples will serve to illustrate the use of the preceding table:

I. *Elevation of Instrument Required to Overcome Curvature and Refraction.* Let us suppose that a line, A to B, was 18 miles in length over a plain, and that the instrument could be elevated at either station, by means of a portable tripod, to a height of 20 or 30 or 50 feet. If we determine upon 36.7 feet at A, the tangent would strike the curve at the distance represented by that height in the table, viz., 8 miles, leaving the curvature (decreased by the ordinary refraction) of 10 miles to be overcome. Opposite to 10 miles we find 57.4 feet, and a signal at that height erected at B would, under favorable refraction, be just visible from the top of the tripod at A, or be on the same apparent level. If we now add 8 feet to tripod and 8 feet to signal pole, the visual ray would certainly pass 6 feet above the tangent point, and 20 feet of the pole would be visible from A.

[1] See discussion on refraction, Arts. 477–479.

II. *Elevations Required at Given Distances.* If it is desired to ascertain whether two points in the reconnoissance, estimated to be 44 miles apart, would be visible one from the other, both elevations must be at least 278 feet above mean tide, or one 230 feet and the other 331 feet, etc. This supposes that the intervening country is low, and that the ground at the tangent point is not above the mean surface of the sphere. If the height of the ground at this point should be 200 feet above mean tide, then the natural elevations should be 478 or 430 and 531 feet, etc., in height, and the line is barely possible. To insure success, the theodolite must be elevated at both stations to avoid high signals.

Since the height of station increases as the square of the distance, it is evident that the minimum aggregate station height is obtained by making them of equal height. Or, if the natural ground is higher at one station than the other, then the higher station should be put on the lower ground—that is, when the intervening country is level. If, however, the obstruction is due to an intervening elevation, the higher station should be the one nearer the obstruction.

III. *To determine whether the line of sight between two stations would pass above or below the summit of an intervening hill, and how much in either case.*

h_1 = height of lower station. d_1 = distance h_1 to h_2.
h_3 = height of higher station. d_2 = distance h_2 to h_3.
h_2 = height of intervening hill.

EXAMPLE

h_1 = 625 feet..... 625 feet strikes horizon at.... 33 miles
h_3 = 2065 feet..... 65 − 33 = 32 miles..... 588 feet of elevation
h_2 = 1200 feet..... 32 − 15 = 17 miles..... 166 feet of elevation
d_1 = 50 miles..... 2065 − 588 = ... 1477 feet
d_2 = 15 miles..... $\frac{65}{15} = 4.33$ and $\frac{1477}{4.33} =$ ..... 341.0 feet

and h, the height of the line at the hill, = 1477 + 166 − 341.0 = 1302 feet.

Hence, the line passes 1302 − 1200 or 102 feet above the hill at h_2.

This problem of intervisibility may also be determined by the following formula:[1]

$$X = h_1 + (h_3 - h_1)\frac{d_1}{d_1 + d_2} - 0.5803 d_1 d_2,$$

in which a mean value of the coefficient of refraction, 0.065, is used.

Sometimes, however, a very high or abnormal degree of refraction is utilized to make a connection on long lines. Thus on the primary triangulation of the Great Lakes three lines, respectively—100, 93, and 92 miles in length, were observed across Lake Superior, which could not have been done except that the the refraction was found sometimes to exceed twice its average amount. The line from station Vulcan, on Keweenaw Point, to station Tip-Top in Canada, was 100 miles in length. The ground at station Vulcan was 726 feet above the lake, and the observing station was elevated 75 feet higher, making 801 feet above the surface of the lake. The station at Tip-Top was 1523 feet above the lake, the observing tripod being only 3 feet high. From the above table we find that the line of sight from Vulcan would become tangent to the surface of the lake at a distance of 37.4 miles, and that from Tip-Top at a distance of 51.5 miles, thus leaving a gap of about eleven miles between the points of tangency, for ordinary values of the refraction. If this interval were equally divided between the two stations and these raised to the requisite height, we would find from the table that Tip-Top would have to be elevated some 340 feet and Vulcan some 260 feet. Since this was not done, we must conclude that an occasional excessive value of the refraction was sufficient to bend these rays of light by about these amounts in addition to the ordinary curvature from this source. In other words, the actual refraction when one of these stations was visible from the other must have been more than double its mean amount.

[1] Gillespie's "Surveying," Vol. II, p. 22.

The following is a synopsis of the heights of the stations built for the observation of horizontal angles in the primary triangulation of the Great Lakes:

Total number of stations [1]							243
Combined height of stations							14,100 feet
Average height of stations							58 "
Average height of stations from Chicago to Buffalo							81.3 "
Number of stations less than 10 feet high							22
" " from	10	feet to	24	feet in height			18
" " "	25	"	49	"			50
" " "	50	"	74	"			71
" " "	75	"	99	"			47
" " "	100	"	109	"			18
" " "	110	"	119	"			15
" " "	120	"	124	"			2

The heights above given are the heights at which the instrument was located above the ground. The targets were usually elevated from 5 to 30 feet higher.

The excessive heights of the stations from Chicago to Buffalo are due to the country being very heavily timbered, and the surface only gently rolling. In the vicinity of Lake Superior they averaged only about 35 feet high, while from Buffalo to the eastern end of Lake Ontario they averaged 51 feet in height.

The present practice of the U. S. Coast and Geodetic Survey is to make the length of the triangle sides the most economical for the particular region. This has reduced the length of the sides and the height of the towers without reducing the accuracy of the triangulation.

402. Construction of Stations.[2] If it is found necessary to build tall stations, two entirely separate structures must be erected, one for carrying the instrument and one for sustain-

[1] Only stations built expressly for the work are here included. Sometimes buildings or towers were utilized in addition to these.

[2] For the most approved detail methods of constructing and erecting stations, see report by Mr. J. F. Hayford, in Appendix 4, p. 829, Report U. S. Coast and Geodetic Survey for 1903. Figs. 214, 215 are taken from this Bulletin.

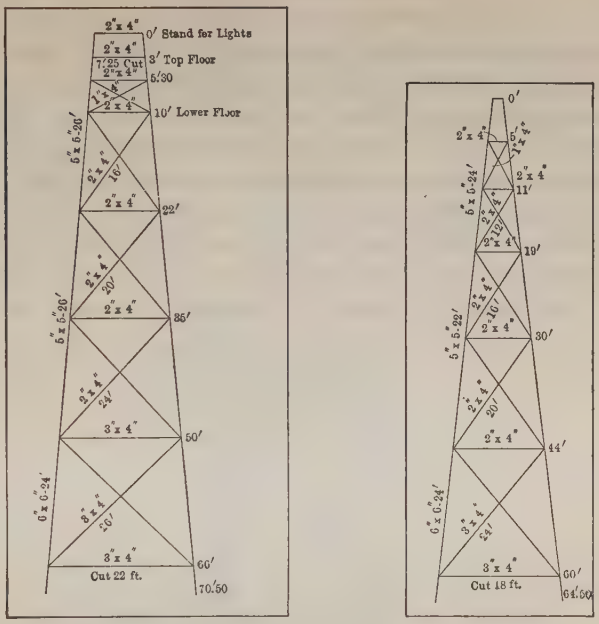


FIG. 214.—Side View of Triangulation Tower and Tripod.

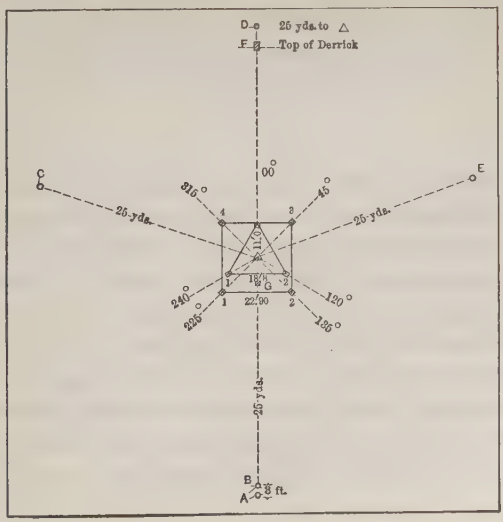


FIG. 215.—Plan of Tower, Tripod and Guy Wires.

ing the platform on which the observer stands. These should have no rigid connection with each other. These structures are shown in plan and elevation in Figs. 214 and 215. The inner station is a tripod on which the instrument rests; this is surrounded by a quadrangular structure, shown separately in elevation to prevent confusion. Both structures are built entirely of wood, the outer one being usually carried up higher than the tripod (not shown in the drawing), and the target fixed to its apex. This upper framework serves also to support an awning

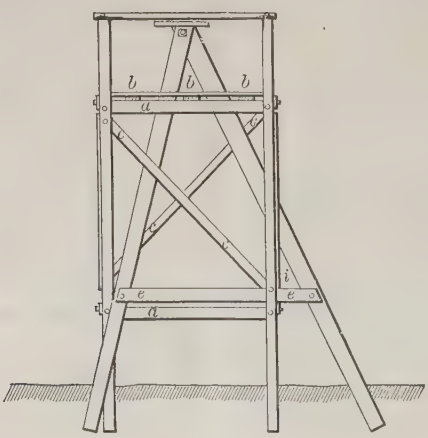


FIG. 216.

to shade the instrument from the sun. For lower stations a simpler construction will serve, but the observer's platform must in all cases be separate from the instrument tripod. The wire guys and wooden braces shown in Fig. 215 were not used on the U. S. Lake Survey stations.

For stations less than about 15 feet in height the design shown in Figs. 216 and 217 may be used. Here the outer platform on which the observer stands is entirely separate from the tripod which supports the instrument. For ground stations a post firmly planted serves very well, or a tree cut off to the proper

height. The common instrument tripod will seldom be found satisfactory for good work. Sometimes extra heavy and stable tripods of the ordinary pattern have given excellent results, when the tripod legs were supported by 2×4 inch stakes firmly driven into the ground. Precautions also should be taken to have the tripod legs securely clamped to the tripod head.

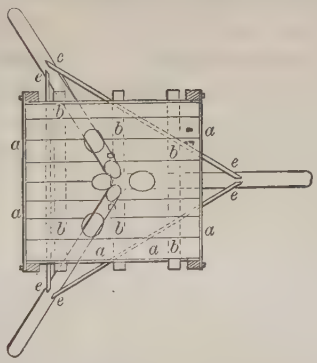


FIG. 217.

403. Targets. The requisites of a good target are that it shall be clearly visible against all backgrounds, readily bisected, rigid, capable of being accurately centred over the station, and so constructed that the centre of the visible portion, whether in sun or in shade, shall coincide with its vertical axis.

It is not easy always to fulfil these conditions satisfactorily. To make it visible against light or dark backgrounds, it is well to paint it in alternating black and white belts. For ready bisection it should be as narrow as possible for distinctness. This is accomplished by making the width subtend an angle of from two to four seconds or arc. Since the arc of one second is three-tenths of an inch for 1 mile radius, an angle of four seconds would give a target one-tenth of a foot in diameter for 1-mile distances, or 1 foot in diameter for 10 mile distances. Something depends on the magnifying power of the telescope used.

The design shown in Fig. 218 will satisfactorily satisfy the conditions as to rigidity and convenience of centering. Of course it should stand vertically over the station so that a reading could be taken on any part of its height. The last condition is not so easily satisfied. If a cylinder or cone be used the illuminated portion only will appear when the sun is shining, and a bisection on this portion may be several inches to one side of the true axis.

The target is then said to present a phase, and corrections for this are sometimes introduced. It is much better, however, to use a target which has no phase. If the target is to be read mostly from one general direction, a surface, as a board, may be used; but if the target is to be viewed from various points of the compass, then from those stations which lie nearly in the plane of the target it would not be visible, from its width being so greatly foreshortened.

In this case two planes could be set at right angles, one above the other. One or both would then be visible from all points, and since their axes are coincident, either one could be used. The objection to this would be that the upper disk would cast its shadow at times on the lower one, leaving one side in sun and the other in shade, thus giving rise to the very evil it is sought to eliminate. A very satisfactory solution of this problem was made on the Mississippi River Survey by means of the following device (Fig. 219): Four galvanized-iron wires, about three-sixteenths inch in diameter, are bent into a circle of, say, 4 inches in diameter, and soldered. To these four circles are attached four vertical wires about one-fourth inch in diameter and 4 feet long, as shown in the accompanying figure. All joints to be securely soldered, the size of the wire increasing with the size of the target. The target is now divided into a number of zones by stretching black and white canvas alternately and in opposite ways between the opposing uprights, making diametral sections. If there are more than two zones, those marked by the same color should have the canvas crossing in different ways, so that if one plane is nearly parallel to any line

of sight the other plane of this color will be nearly at right angles to it. This target has no phase, is visible against any background, and readily mounted. A wooden block may be inserted at bottom, with a hole in the axis of the target. This may then be set over a nail marking the station. The target is held at top by wire guys leading off to stakes in the ground. Such a

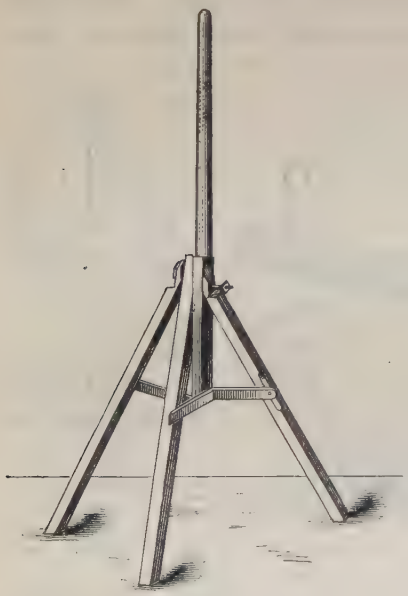


FIG. 218.

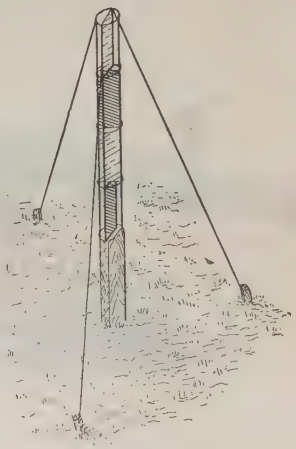


FIG. 219.

a target could be mounted on top of the pole shown in Fig. 218, if it should be found necessary to elevate it.

404. Heliotropes. When the distance between stations is such that, owing to the distance, the state of the atmosphere, or the small size of the objective used, a target would appear indistinct, or perhaps not be visible at all, the reflected rays of the sun may be made to serve in place of a target. This limiting distance is usually about 20 miles. Any device for accom-

plishing this purpose may be called a heliotrope. In Figs. 220 and 221 are two forms of such an instrument. That shown in Fig. 220 is a telescope mounted with a vertical and horizontal motion. This is turned upon the station occupied by the observer, and is then left undisturbed. On the telescope are mounted a mirror and two disks with circular openings. The mirror has two motions so that it can be put into any position. Its centre is coincident with the axis of the disks in all positions. The

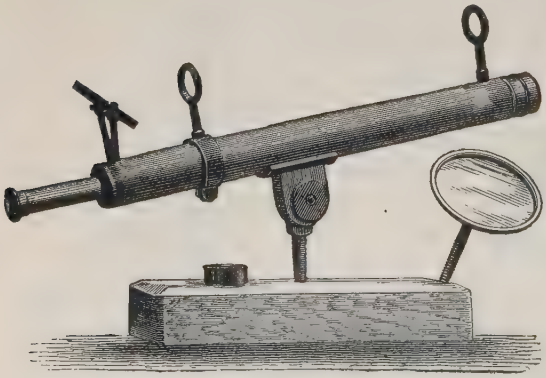


FIG. 220.

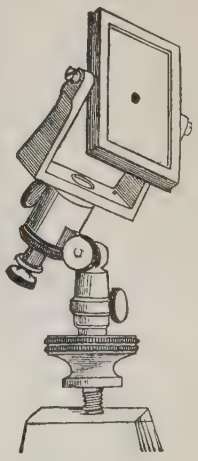


FIG. 221.

mirror may be turned so as to throw a beam of light symmetrically through the forward disk, in which position the reflected rays are parallel to the axis of the telescope, and hence fall upon the distant point.

The heliotrope shown in Fig. 221 is to be used in conjunction with a single disk, which may be a plain board mounted on a plank with the mirror. The silvering is removed from a small circle at the centre of the mirror. The disk has a small hole through it as high above its base as the clear space on the mirror is above the plank. The operator points the apparatus by

sighting, through the clear spot on the mirror and the opening in the disk, to the distant station. If the plank be fastened in this position the attendant now has only to move the mirror so as to keep the cone of reflected rays symmetrically covering the opening in the disk, and the light will be thrown to the distant station.

Since the cone of incident rays subtends an angle of about thirty-two minutes, the cone of reflected rays subtends the same angle. The base of this cone has a breadth of about 50 feet to the mile distance, or at a distance of 20 miles the station sending the reflection is visible over an area in a vertical plane 1000 feet in diameter. This reflected cone of rays is greatly increased by imperfections in the surface of the mirror and by the variable terrestrial refraction in the air passed through. The alignment of the heliotrope need not be, therefore, very accurate. This alignment may vary as much as fifteen minutes of arc on either side of the true line. This is nearly 0.01 of a foot in a distance of 2 feet. If the bearing, or direction, of the distant station is once determined, it may be marked on the station by some means within this limit, and a very rude contrivance used for sending the reflected ray, or flash, as it is called. Thus, a mirror and a disk with the requisite movements may be mounted on the ends of a board or pole from 5 to 20 feet long, and when this is properly aligned it serves as well as any other more expensive apparatus. The hole in the disk should usually subtend an angle at the observer's station of something less than one second of arc, which is a width of three-tenths of an inch to the mile distance. On the best work with large instruments it should subtend an angle of less than one-half a second, the minimum effective opening depending almost wholly on the condition of the atmosphere.[1]

[1] Reflected sunlight has been seen a distance of 60 miles, through an opening one inch in diameter, which then subtended an angle of but one-eighteenth of one second of arc at the instrument. This would require a very clear atmosphere.

More uniform results can be secured if the various images are of nearly uniform brightness. To secure this it is necessary to vary the size of the mirror with the length of the line to be observed. For ordinary atmospheric conditions, and for distances greater than about 10 miles the formula

$$x = 0.046d$$

may be used, where x is the side of the square mirror in inches and d the distance in miles, or

$$y = \tfrac{3}{4}s,$$

where y is in millimetres and s in kilometres. In general, the mirror should be as small as possible and still give a clear and distinct light. Better work can be done with small and sharp signals than upon large and very bright ones.

Whatever form of heliotrope is used, an attendant is required to operate the apparatus. Evidently it can be used only on clear days, whereas cloudy weather is much better adapted to this kind of work, since the atmosphere then transmits so much clearer and steadier an image.

The heliotrope can be used as a means of communication between distant stations by some fixed code of flashing signals, and it has been so used very often with great advantage to the work. The attendant on the heliotrope, usually called a flasher, can thus know when the observer is reading his signals, when he is through at that station, and, in general, can receive his instructions from his chief direct from the distant station.

405. Geodetic Night Signals. Mr. C. O. Boutelle, of the U. S. Coast and Geodetic Survey, made a series of experiments in 1879 at Sugar Loaf Mountain, Maryland, for the purpose of testing the efficiency of certain night signals and the comparative values of day and night work. His report is given in Appendix No. 8 of the Report of the U. S. Coast and Geodetic

Survey for 1880. It seems that either the common Argand or the " Electric " coal-oil lamp, assisted by a parabolic reflector or by a large lens, gives a light visible for over 40 miles. His conclusions are:

1. That night observations are a little more accurate than those by day, but the difference is slight.

2. That the cost of the apparatus is less than that of good heliotropes.

3. That the apparatus can be manipulated by the same class of men as those ordinarily employed as heliotropers.

4. That the average time of observing in clear weather may be more than doubled by observing at night, and thus the time of occupation of a station proportionately shortened; " clear-cloudy " weather, when heliotropes cannot show, can be utilized at night.

Experiments made in 1902 by Mr. Wm. Bowie with acetylene lamps [1] gave even better results than with the oil lamp. Experience has shown that satisfactory results can be secured on lines not exceeding 20 to 35 miles in length. Not only are the signals steadier at night, but the period of good seeing is very much greater than during the day. Partly as a result of this, the work done by Mr. Bowie shows that the observation of angles was completed at five stations in a single day and at twenty-seven stations in two days each. This is very much shorter time than has been taken previously.

406. Station Marks. If the triangulation is to serve for the fixing of points for future reference, then these points must be marked in some permanent manner. In this case the station should be so chosen that the surface for a few feet around the station shall always remain undisturbed. To insure against disturbance from frost or otherwise, the real mark is usually set several feet underground. Many different means are employed to mark these

[1] Report of J. F. Hayford, U. S. Coast and Geodetic Survey Report, 1903, pp. 824–27. The lamp used was a modified bicycle lamp.

points. The underground mark is to serve only when the superficial marks have been disturbed, there being always left a mark of some kind projecting above ground. On the U. S. Lake Survey, "the geodetic point is the centre of a $\frac{1}{4}$-inch hole drilled in the top of a stone 2 feet by 6 inches by 6 inches, sunk $2\frac{1}{2}$ feet below the surface of the ground. When the occupation of the station is finished, a second stone post, rising 8 inches above the ground, is placed over the first stone. Three stone reference posts, 3 feet long, rising about a foot above the ground, are set within a few hundred feet of the station, where they are the least likely to be disturbed. A sketch of the topography within a radius of 400 metres about the station is made, and the distances and azimuths of the reference marks are accurately determined."

When the station is located in natural rock a copper bolt may be set to mark the geodetic point, with one or more arrows cut in the rock pointing to the station.

On the Mississippi River survey, stations had to be set on ground subject to overflow. These were to serve both for geodetic points and for bench marks, both their geographical position and their elevation being accurately determined. Both the rank growth and the sedimentary deposits from the annual overflows would soon obliterate any mark which was but slightly raised above the surface. After much study given to the subject, the following method of marking such points was adopted: A flat stone 18 inches square and 4 inches thick, dressed on the upper side, has a hole drilled in the centre, into which a copper bolt is leaded, the end projecting a quarter of an inch above the face of the stone. The stone is marked thus, $\frac{U \cdot S}{B \cdot M'}$, and is placed 3 feet under ground. On this stone, and centred over the copper bolt, a cast-iron pipe 4 inches in diameter and 5 feet long is placed, and the dirt tamped in around it. The pipe is large enough to admit a levelling-rod. The top is closed with a cap, which is fastened to the pipe by means of a bolt. The elevations of both the top of the pipe and of the stone are determined.

A careful description of each station and all the reference marks together with a sketch of their position should be turned in for record. It would be useful if information regarding the best route to reach the station is also included.

Measurement of the Base Line

407. Preparation of the Line. The considerations which govern the location of the base line are discussed in Art. 395. The preparation of the base will depend upon whether bars or tapes are to be used in measurement. At the present time steel tapes are in general use. The first step consists in placing substantial stone monuments for the purpose of marking the ends of the base. If the subsoil be unstable these monuments should be set on a wide concrete base, the exact point is marked by fine cross lines cut on a copper bolt sunk into the stone. To insure against disturbance an underground mark should be set in addition to the surface mark. Sufficient intermediate stone should also be set to divide the base into convenient sections of about a mile.

If the tape is to be used, 4×4 inch posts should next be firmly set along the entire base at distances of the shortest tape length. Careful levels are then run over the posts, and intermediate supports are set at such intervals as may be throught proper, and usually so as not to break grades between posts. Wire nails may be driven on such grade to support the tape. Care should be taken to set the tape posts with considerable accuracy so as to avoid many "set ups" or "set backs" when the line is measured. Finally, brass or zinc strips, firmly nailed to the tops of the posts, serve to record and hold the tape contacts.

If some form of base bars is to be used, the base line only needs subdivision into kilometre or perhaps half-mile sections.

408. Base Apparatus. The essential features sought for in a base-measuring apparatus are:

1. The terminal points must remain at an unvarying distance

FIG. 222.—The New Duplex Base Apparatus of the U. S. C. and G. Survey.

apart, during the measurement, or else the variations in length must admit of easy and accurate determination.

2. The unit used in the measurement must be compared with some known standard in order that its absolute length may be accurately determined.

3. Base apparatus must be so constructed as to admit of ease and rapidity in handling, stability in its supports, and great accuracy and certainty in making exact contact.

Base apparatus may be classed as follows:

1. Contact apparatus, including compensating bimetallic bars and monometallic bars.

2. Optical apparatus, including compensating bimetallic bars and monometallic bars.

3. Tapes and wires.

409. Contact Apparatus. In this form of apparatus the ends of the bars are brought into successive contacts, since the length of the bar is the distance between its extreme ends. Contact bars usually are provided with a slide carrying the bar terminal and working against a spring. The length of such a bar is the distance between its terminal points when a line on the slide coincided with a line on the bar. The best example of this type of apparatus is the U. S. Coast and Geodetic Survey " Duplex Bars." See Figs. 222 and 223.

410. Optical Apparatus. In this type of apparatus the bar length is fixed by a fine line cut on its neutral axis. Only one bar is required, but two or more microscopes properly mounted on secure standards are needed. These microscopes are pointed and read on two end marks, after which the bar is moved forward, the forward microscope meanwhile holding the measure. The rear end is brought under this same microscope while at the same instant a pointing and reading is taken to the forward end line by another microscope, and the process repeated. A good example of this type of apparatus is the U. S. Coast and Geodetic Survey " Iced Bar." [1]

[1] For description see U. S. Coast and Geodetic Survey Reports for 1892 and 1897.

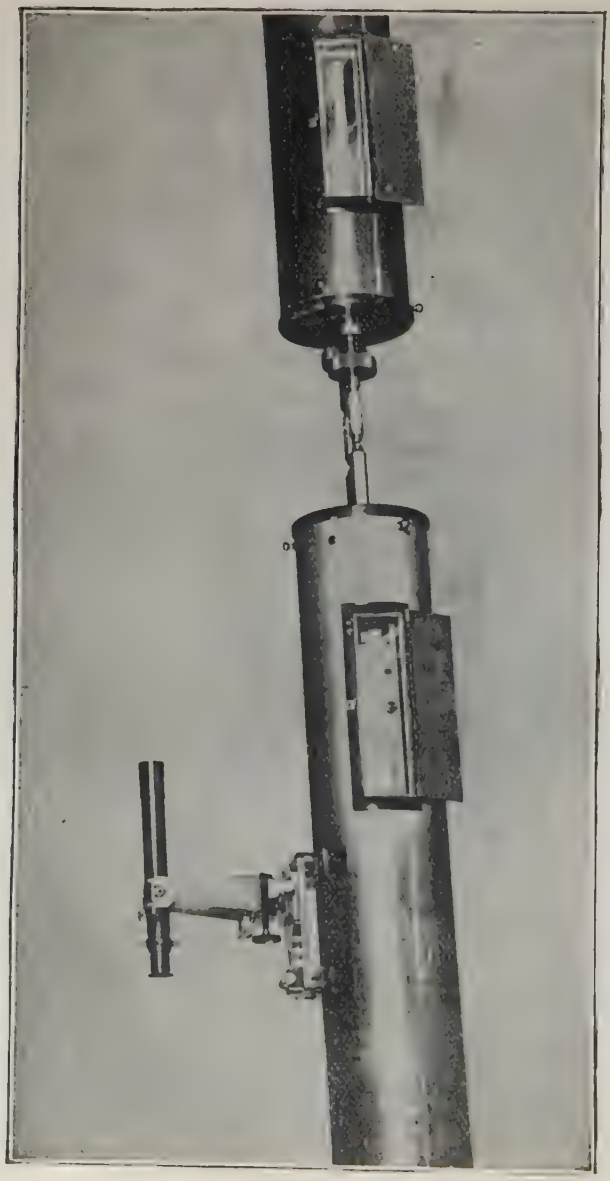


FIG. 223.—The End Contacts of the New U. S. C. and G. Duplex Base Apparatus.

411. Compensating Apparatus. In this form of apparatus the attempt is made to keep the two ends of the apparatus at the same distance apart, independent of any change of temperature. Because of mechanical difficulties in the construction none of the attempts to produce a compensating apparatus have been successful. The most satisfactory form of this apparatus is one designed by Schott. See U. S. Coast and Geodetic Survey Report for 1882, p. 107.

412. The Iced-Bar Apparatus.[1] This consists of a steel bar, 5.02 metres long, 8 millimeters thick, and 32 millimetres deep, so supported in a steel trough filled with melting ice as to allow vertical and horizontal alignment. All uncertainty regarding temperature is thus removed. The bar length is indicated by microscopic lines engraved upon the surface of platinum-iridium plugs set near the ends of the bar and on its neutral axis. The trough is supported by a carriage movable upon a track. A sector for obtaining the vertical alignment is attached to the trough. The successive positions of the ends of the bar are fixed by powerful microscopes firmly attached to stout posts solidly planted in the ground. This apparatus is too slow for measuring a base line. Its chief use has been to calibrate the apparatus used in base-line measurements.

413. The " Duplex " Bars. This base apparatus, invented by Wm. Eimbeck, is shown in Figs. 222 and 223.[2] It consists of two measuring tubes, 5 metres long, each containing two hollow cylindrical bars, one of steel and one of brass. As the expansion of brass is about one and one-half times that of steel, there will be only one temperature at which both bars are of the same length. This temperature is called the normal or duplex temperature. The coefficient of expansion of both bars and the

[1] For a full description of this apparatus and manner of use see Report of U. S. Coast and Geodetic Survey for 1892. The apparatus was designed by Mr. R. S. Woodward.

[2] For a full description of this interesting mechanism see Appendix II, U. S. Coast and Geodetic Survey Report for 1897.

exact length and temperature of the bars when of the *same* length constitutes their calibration. This is accomplished by measuring a known distance with the apparatus at two very different temperatures. (The known distance usually is 100 metres as determined by the iced-bar apparatus, Art. 412.) With the absolute (as well as the relative) lengths of both bars, and their coefficients of expansion thus determined, the length of each bar at any temperature is readily obtained.

414. Field Methods with Duplex Bars. Fig. 222 shows a single measuring tube, mounted on two wooden tripods. Two such tubes are used in the measurement. In service these are aligned by means of the telescope at the left end, and its vertical angle is determined by means of the sector mounted on the side at the centre of the tube. The measuring bars project at both ends. These bars are brought into end contact by means of thumbscrews, as shown in Fig. 223. These thumbscrews move the bars bodily in the tube against the action of spiral springs. The end contacts are made steel to steel and brass to brass. The rear tube is then carried forward and its rear end brought into contact with the forward end of the stationary tube, and contact made again by moving the bars in the forward tube. These contact ends are agate surfaces, one being a vertical plane and the other which meets it a horizontal knife-edge. The work proceeds in this way at an average rate of some fifty or sixty tubes (250 or 300 metres) an hour, a maximum speed of eighty tubes having been attained. The bars were first used under an awning frame, which was dragged forward on sled-runners by a team. Good results are now obtained, however, by working in the open. The relative position of the bars in the tube can be changed by rotating 180° the inner or "reversing" tube, which carries the bars, the outer or "truss" tube remaining fixed.

Since the brass bar will in general be longer or shorter than the steel bar (longer for temperatures above and shorter for temperatures below the normal), it is clear that by always bringing steel to steel and brass to brass in making the end contacts,

one bar will continually gain upon the other. There is an arrangement, however, for moving the brass bar forward or back with reference to the steel whenever this deviation reaches too large an amount, as 5 centimetres, for instance. There is a vernier attached to the steel bar at each end, which reads upon a scale upon the brass bar, by means of which this occasional adjustment can be determined and its amount recorded in the notes. With these corrections applied there will result two continuous measurements of the base, one by the steel bar and one by the brass bar. Three mercurial thermometers are placed in each tube, and all are read for part of the contacts; these may or may not be given weight in computing the length of the base. *The relative lengths of the entire base as measured by the steel-bar record and by the brass-bar record give the key to the average temperature of both bars for the entire base, provided the two bars be assumed to always have the same temperature.* Since these bars are in reality small tubes, of a relative thickness of metal to compensate for relative specific heats and conductivities, and are both enclosed in a double tubular covering, it has been found by trial that they may be assumed to always have the same temperature.

A single measurement of the base line with this apparatus (including the thermometers) will give three determinations of the base length, viz.:

1. The duplex length with temperature deduced from the difference in the steel and brass measures.

2. The steel bars with the temperature deduced by the mercurial thermometers.

3. The brass bars with the temperature indicated by the mercurial thermometer.

While these measurements are not absolutely independent of each other, they afford a valuable check upon the reliability of the measurements.

EXAMPLE

Both bars are found to have a length of 5 metres at a temperature of 16.6° C. The coefficient of expansion of the steel bars as determined was 0.0000117 and of the brass bar 0.0000175 per degree centigrade. The length of a base line in terms of the steel bar was found to be 8000.551 metres. In terms of brass bar, 8000.319 metres. What is the correct length of the base line?

Coefficient steel,	0.0000117
" brass,	0.0000175
Difference,	0.0000058
	8000
	.0464000 difference in base per degree C.

Σ dif. in length of base = $0^m.232$.

$0^m.232 \div 0^m.0464 = 5$. Hence the mean temperature of the apparatus as used in the field was 16°.6 + 5° = 21°.6 C.

415. Secondary-Base Apparatus. This is an apparatus used by the U. S. Coast and Geodetic Survey, consisting of two separate cylinders. Each cylinder contains a cylindrical steel rod 5 metres long and 9 millimetres in diameter, with usual contact slides and agate terminals (Art. 413). Each steel rod rests in a built-up wooden bar, 7.5 centimetres wide, 14 centimetres thick, and 4.9 metres long. Each rod carries two thermometers inserted near each end on opposite sides of the bar and in metallic contact with it. Because the temperature of the bars lags behind the temperature of the air and of the thermometer bulbs, the best results can be obtained only by dividing the measurement equally between periods of rising and falling temperatures.

416. Steel-Tape Apparatus. *The Steel Tape* furnishes the most convenient, rapid, and economical means for measuring any distance for any desired degree of accuracy up to about 1 in 500,000, and if the most favorable times are chosen, an accuracy of 1 in 1,000,000 may be attained. The conditions of use depend on the accuracy required. If we suppose the absolute length, coefficient of expansion, and modulus of elasticity have been accurately determined, any distance can then be measured

in absolute units within an accuracy of 1 in 1,000,000, by taking due precautions as to temperature and mechanical conditions.[1] For base-line measurement the length should be from 200 to 500 feet,[2] and its cross-section from 0.002 to 0.003 of a square inch. For an accuracy of 1 in 5000 the tape may be used in all kinds of weather, held and stretched by hand, the horizontal position and amount of pull estimated by the chainmen. The temperature may be estimated, or read from a thermometer carried along for the purpose. On uneven ground, the end marks are given by plumb-line.

For an accuracy of 1 in 50,000 the mean temperature of the tape should be known to the nearest degree Fahrenheit, the slope should be determined by stretching over stakes, or on ground whose slope is determined, and the pull should be measured by spring balances. The work could then be done in almost any kind of cloudy weather. For an accuracy of 1 in 500,000, extreme precautions must be taken. The mean temperature must be determined to about one-fifth of a degree F., the slope must be accurately determined by passing the tape over points whose elevations above a given datum are known, the pull must be known to within a few ounces, and all friction must be eliminated. The largest source of error is apt to be the temperature. On clear days, the temperature of the air varies rapidly for varying heights above the ground, and, besides, the temperature of the tape would neither be that of the air surrounding it, nor of the bulb of a mercurial thermometer. In fact, there is no way of determining by mercurial thermometer, even within a few degrees, the mean temperature of a steel tape lying in the sun, either

[1] So far as is known steel tapes were first used for measuring base lines in New Zealand in 1871, by Mr. Edwin Fairburn. The tape was 66 feet long, and the lengths were marked on lead. See *The Surveyor* (Sydney, N. S. W.) for Sept. 21, 1900.

[2] On the U. S. Coast and Geodetic Survey tapes of 50 and 100 metre lengths have been found most economical in use. The longer the tape the larger the number of the party required.

on or at varying heights above the ground. The work must then be done *at night or in cloudy weather, and when air and ground are at about the same temperature.*

There should also be only a slight wind, both on account of its mechanical action on the tape, and from the temperature variations resulting therefrom.

417. Method of Mounting and Stretching the Tape. Friction is sufficiently eliminated if the tape be supported by nails driven on line at intervals of from 25 to 75 feet. The longer the distance between supports the greater the tension needed. These nails may be set on grade or not, as desired; but if not on grade, then each point of support must have its elevation determined. A low point should not intervene between two higher ones, else the pull on the tape will lift it from such support. See also Art. 407 on preparation of line.

Several methods for stretching the tape are in common use. On the U. S. Coast and Geodetic Survey a spring balance attached to a lever (see Fig. 224) is used which is forced into the ground. The height of the balance is regulated by a wheel nut so as to keep the tape level with the top of zinc or copper contact strips.

The use of a weight running over a pulley with bicycle bearings has also given entire satisfaction. The form shown in Fig. 225 has been used for over ten years in the summer school of the University of Wisconsin.[1]

418. Thermometers. Mercurial thermometers, graduated to half degrees and read to fifths of a degree, should be used. At least two thermometers should be used, care being taken to distribute them proportionately along the tape, and to hold them at the same distance from the ground as the tape.

419. Balances. As a spring balance is adjusted to show correct weight when held vertical, a small correction must be introduced to give correct tensions when the balance is held

[1] For a full description see Transactions Am. Soc. C.E., Vol. XLVIII, p. 130, by L. S. Smith, Asso. M. Am. Soc. C.E.

FIG. 224.—Tape-Stretching Device used by U. S. C. and G. S.

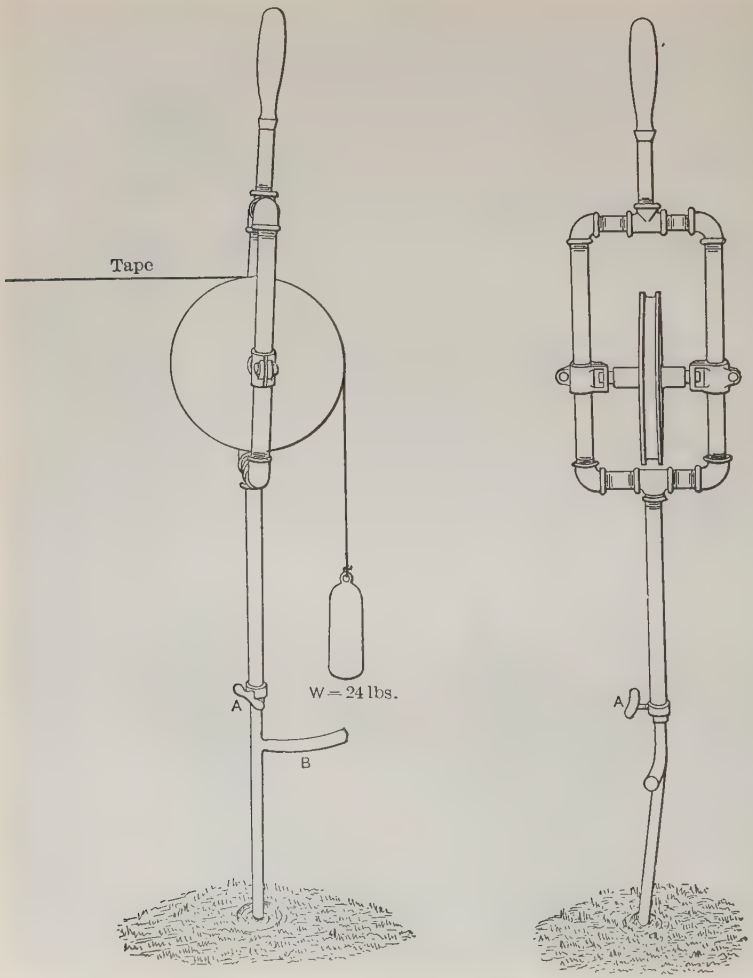


FIG. 225.—Tape-Stretching Device used by University of Wisconsin.

horizontally. An index error may also develop as a result of wear and other causes.

Let e be the index correction to reduce to actual tension in the tape. Then e is the reading of the index when the balance is vertical and the hook end down. Let E be the reading of the index when the balance is suspended by its hook, hook end up. Let W be the correct weight of the balance. Then if T denotes the reading of the index when the balance is held horizontal and T' the corresponding true tension, we have

$$T = T' + e + \tfrac{1}{2}(W - E - e) = T' + \tfrac{1}{2}(W - E + e).$$

420. M. Jäderin's Method. Professor Edward Jäderin, of Stockholm, has brought the measurement of distances by wires and steel tapes to great perfection. He uses a tape 25 metres in length, and stretches it over tripods set in line, as shown in Fig. 226. On the top of the tripod head is a fixed graduation.

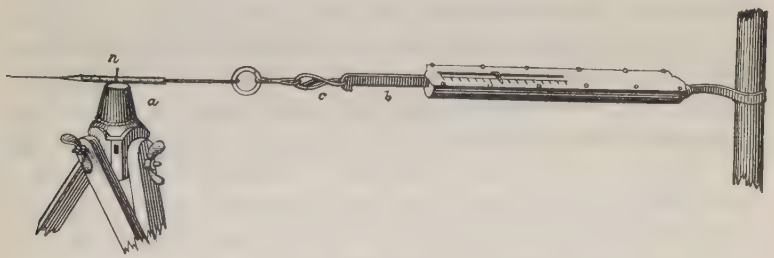


FIG. 226.

At the rear end of the tape there is a single graduation, but at the forward end a scale 10 centimetres in length is attached to the tape, this being graduated to millimetres on a bevelled edge. The middle of this scale is 25 metres from the graduation at the other end of the tape. The tripods are set as near as may be to an interval of 25 metres, but it is evident that the reading may be taken on them if this interval is not more than 5 centimetres more or less than 25 metres. The reading is taken to tenths of

millimetres, the tenths being estimated. The tape is stretched by two spring balances, a very stiff spring being used at the rear end and a very sensitive one at the forward end. The rear balance simply tells the operator here when the tension is approximately right, the measure of this tension being taken on the forward balance, which is shown in the figure.

If a single steel wire or tape be used, Mr. Jäderin also finds that the work must be done in cloudy and calm weather, or at night, if the best results are to be obtained. But he finds that if two wires be used, one of steel and the other of brass, he can continue the work during the entire day, even in sunshine and wind, and obtain an accuracy of about one in 1,000,000 in his results.[1] The wires are stretched in succession over the same tripods, by the same apparatus, one wire resting on the ground while the other is stretched. More accurate results could doubtless be obtained if both wires are kept off the ground constantly, the wire not in use being held by two assistants, or if stakes and wire hooks are used, both wires might be stretched at once in the same hooks. The two wires form a metallic thermometer, the difference between the readings of the same distance by the two wires determining the temperature of both wires, when their relative lengths at a certain temperature and their coefficients of expansion are known. This method is similar in principle to that of the duplex-bar apparatus, where steel and brass bars are used, shown in Fig. 222. In such cases the true length of line is found by Eq. (5), p. 580.

If the appliances above outlined be used with a single tape or wire, and the work be done on calm and densely cloudy days, or at night, or with two wires used even in clear weather,

[1] See Geodätische Längenmessung mit Stahlbänden und Metalldrähten von Edv. Jäderin, Stockholm (1885, 57 pp.). Also, Exposé élémentaire de la nouvelle Methode de M. Edouard Jäderin pour la mesure des droites géodésiques au moyen de Bandes d'Acier et de Fils métalliques, par P. E. Bergstrand, Ingénieur au Bureau central d'Arpentage, à Stockholm (1885, 48 pp.). See also U. S. Coast and Geodetic Survey Report for 1893, p. 125, for a complete translation of this report

it is not difficult to make the successive measurements agree to an accuracy of 1 in 500,000. There still remains, however, the errors in the absolute length, in the coefficient of expansion, in the modulus of elasticity, in the measure of the pull, and in the alignment, none of which would appear in the discrepancies between the successive measurements.

421. Nickel-Steel Tapes. The name "invar" has been given to a alloy of nickel and steel which possesses the valuable property of having a very low coefficient of expansion.[1] By varying the proportions of the materials, almost any coefficient of expansion may be obtained, some specimens even showing a negative coefficient. Tapes with a coefficient of about one-thirtieth of steel have commonly been used. These tapes are rather soft, easily bent, and not nearly as elastic as steel. Without tension the tapes appear crooked and full of small bends, but most of this irregularity disappears upon applying a tension of 20 to 30 pounds. Experiments[2] made in the laboratory indicated that the tapes were entirely satisfactory for base-line work, a conclusion which was fully confirmed by their use in the field later. The graduation marks were ruled on silver sleeves riveted to the tape near its ends. A reel 16 inches in diameter was found suitable for the tapes, but a smaller reel might cause permanent change in their lengths.

422. Comparative Errors of Invar Tape Measures.[3] Any error due to the failure of the thermometers to indicate the temperature of the tape is probably less with invar tapes than

[1] These tapes can be obtained at reasonable prices of J. H. Agar Baugh, 92 Hatton Garden, London, E.C. Certificates of true length of tape as tested by the National Physical Laboratory of Great Britain also furnished.

[2] Experiments by A. W. French, Appendix 4, Report U. S. Coast and Geodetic Survey, 1907, p. 107, show: First, the yield-point is about 70 per cent of the tensile strength ; second, the tensile strength is between 450 and 500 pounds actual or 100,000 pounds per square inch; third, loads up to 60 pounds (at least) do not cause permanent set.

[3] Conclusion based on experience of the U. S. Coast and Geodetic Survey Report, 1907.

for steel, as an error of 2°.8 C. is necessary to produce as large an error with invar tapes as an error of 0°.1 C. will produce with steel tapes. This effect is probably not more than three or four times as much in daylight as at night, hence the temperature errors of invar tapes used in daytime are from one-eighth to one-tenth those of steel tapes used at night.

Increased accuracy can also be obtained from invar tapes, because the errors of observation in daylight are less than at night. Greater speed is due to the fact that all members of the party can see to work and travel without bothering with the use of lights. The invar tapes are handled and used in the same way as steel tapes.

The errors in determining grade and sea level reductions are small and in large part compensating in all tape measurements. The errors of tension are not as easily analyzed, but probably do not compensate as much as could be desired. Experience shows that the small errors made in transferring tape lengths are compensating.

423. Comparison of Costs. The measurement in 1900 of nine bases by the U. S. Coast and Geodetic Survey, using both duplex bars and steel tapes (at night) show that the latter were just as accurate and very much more economical. The measurement of six bases in 1906, using steel tapes at night and invar tapes in the daytime, prove that the latter work was both very much more accurate and more economical. The unit costs have been about as follows: Invar tape measurement, $150 per double mile; steel tape measurement, $160 per double mile; duplex bars, $480 per double mile.

424. The True Length of the Tape. By the true or absolute length is meant the length of the tape or bar in terms of the international metre bar, this being a copy of the "Mètre des Archives," or French standard. The most convenient way of securing such information is to send the tape to the National Bureau of Standards, Washington, D. C., where a comparison will be made, for a small fee, with an accuracy of 1 in 100,000. This accuracy can be

increased to 1 in 1,000,000 or more, at times convenient to this bureau.[1]

If an absolute standard is not available, then the length may be found by measuring a known distance, as a previously measured base line, and computing the temperature at which the tape is standard. Or the tape may be compared with a shorter standard, as a yard or metre bar, by means of a comparator furnished with micrometer microscopes.

425. The Coefficient of Expansion of Steel Tapes may be taken anywhere from 0.0000055 to 0.0000070 for 1° F.[2] If the tape is used at nearly its standard temperature, then the

[1] The Bureau of Standards reports the length of the 50-metre invar tape belonging to the University of Wisconsin, when supported at the ends and middle and under a tension of 15 kilograms and at a temperature of 20° C., as 50 m. − 0.254mm. + (t − 20° C.) 0.026 mm. The probable error of this length is 1 in 2,500,000.

[2] The author made a series of observations on a steel tape 300 feet long, the readings being taken at short intervals for four days and three nights. The tape was enclosed in a wooden box, and supported by hooks every 16 feet. The observations were taken on fine graduations made by a diamond point, there being a single graduation at one end, but some fifty graduations a millimetre apart at the other end. The readings were made by means of micrometer microscopes mounted on solid posts at the two ends. The range of temperature was about 50° F., and the resulting coefficient of expansion for 1° F. was 0.00000699±3 in the last place. Professor T. C. Mendenhall found from six or eight experiments on steel bands used for tapes a mean coefficient of 0.0000059. Steel standards of length have coefficients ranging from 0.0000048 to 0.0000066.

Mr. Edward Jäderin, Stockholm, has obtained a mean value of 0.0000055, from a number of very careful determinations, both from remeasuring a primary base line, and from readings in a water bath. Several steel wires were tested, and their coefficients all came very near the mean as given above.

For brass wires he found a mean coefficient of 0.0000096 F. The 15-foot standard brass bar of the U. S. Lake Survey has a coefficient of 0.0000100, while tabular values are found as high as 0.0000107 F.

There is some evidence that *cold-drawn wires have a less coefficient of expansion than rolled bars and tapes.*

In the past coefficients of expansion have seldom been found with great accuracy, the coefficients of the "Mètre des Archives," the French standard, having had an erroneous value assigned to it for ninety years.

coefficient of expansion plays so small a part that its exact value is unimportant. If it is used at a temperature of 70° F. from its standard temperature, and if the error in the coefficient used be 20 per cent, the resulting error in the work would be 1 in 10,000. This is probably the extreme error that would ever be made from not knowing the coefficient of expansion, some tabular value being used. If nothing is known of the coefficient of expansion, probably 0.0000065 would be the best value to use. It is evident, however, that for the most accurate work the coefficient of expansion of the tape used must be carefully determined. The coefficient of expansion of the invar tapes used by the U. S. Coast and Geodetic Survey in measuring six bases in 1906 was 0.00000041 per degree centigrade (0.00000023 F.). This expansion equals 0.02 mm. per degree C. per tape length of 50 metres.

426. The Modulus of Elasticity. This is readily found by applying to the tape varying weights, or pulls, and observing the stretch. The correction for sag will have to be applied for each weight used, in case the tape is suspended from hooks, which should be done to eliminate all friction.

Let P_1 be the maximum load in pounds;

P_0 " minimum load in pounds;

a " increased length of tape in inches due to the increased pull;

L " length in inches for pull P_0, or the graduated length of tape;

S " cross-section in square inches;

E " modulus of elasticity;

d " distance between supports in inches;

w " weight of one inch of tape in pounds;

s " shortening effect of the sag for the length L;

v " sag in inches midway between supports.

Then we have from the definition of modulus of elasticity,

$$E = \frac{P_1 - P_0}{S} \div \frac{a}{L} = \frac{(P_1 - P_0)L}{Sa}. \quad . \quad . \quad . \quad . \quad (1)$$

But for the pull P_1, the shortening from sag is much less than for the pull P_0. We must therefore find the effect of the sag in terms of the pull.

427. Effect of the Sag. Where the sag is small, as it always is in this work, the curve, although a catenary, may be considered a parabola without an appreciable error.

If we pass a section through the tape midway between supports, and equate the moments of the external forces on one side of this section, we obtain, taking centre of moments at the support

$$Pv = \frac{wd}{2} \cdot \frac{d}{4} = \frac{wd^2}{8},$$

or

$$v = \frac{wd^2}{8P}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If the length of a parabolic curve be given by an infinite series, and if all terms after the second be omitted, which they may when $\frac{v}{d}$ is small, then we may write

$$\text{Length of curve} = d\left(1 + \frac{8}{3}\frac{v^2}{d^2}\right). \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If we now substitute for v its value given in Eq. (1), we have

$$\text{Length of curve} = d\left\{1 + \frac{1}{24}\left(\frac{wd}{P}\right)^2\right\}.$$

If we call the excess in length of curve over the linear distance between supports the *effect of the sag*, we have

$$c = \frac{d}{24}\left(\frac{wd}{P}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

for one interval between supports. If there are n such intervals in one tape length, then $nd=L$, and the effect of the sag in the entire tape length is

$$C_s=\frac{L}{24}\left(\frac{wd}{P}\right)^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If S_1 and S_0 be the effects of the sag for the pulls P_1 and P_0 ($S_1<S_0$ for $P_1>P_0$), then the total movement at the free end due to the pull being increased from P_0 to P_1 would be $a+(S_0-S_1)$. If this total movement be called M, then we would have

$$E=\frac{(P_1-P_0)L}{S(M-S_0+S_1)}=\frac{P_1-P_0}{S\left(\frac{M}{L}-\frac{(wd)^2}{24}\left(\frac{P_1^2-P_0^2}{P_1^2P_0^2}\right)\right)}. \quad . \quad (5)$$

EXAMPLE

Let $P_1=60$ pounds;
$P_0=10$ pounds;
$w=0.00055$ pound per inch of tape;
$d=300$ inches$=25$ feet;
$S=0.002$ square inch;
$M=3.2$ inches;
$L=3600$ inches$=300$ feet.

To find E.

From Eq. (5) we have

$$E=\frac{50}{0.002\left\{\frac{3.2}{3600}-\frac{0.027}{24}\left(\frac{3500}{360000}\right)\right\}}=28{,}500{,}000.$$

From the same data, we find from Eq. (4) the effect of the sag to be 0.040 inch for the 10-pound pull, and 0.001 inch for the 60-pound pull.

Evidently, if the tape is stretched by the same weight when its absolute length is found, and when used in measuring, the stretch, or elongation from pull, would not enter in the computation, and so the modulus of elasticity would be no function of the problem.

Again, the stretch per pound of pull may be observed for the given tape, and then neither E nor S, the cross-section, would enter in the computation.

428. Temperature Correction. If mercurial thermometers are used, their field readings must first be corrected for the errors of their scale reading, each thermometer having, of course, a separate set of corrections. Then the *mean* of the corrected readings may be taken for all the whole tape lengths in the line measured, and the correction for the entire line obtained at once. Thus

Let L = length of line;

T_0 = temperature at which the length of the tape is given for the standard pull P_0, this usually being the temperature at which its true length is its graduated length for that standard pull;

T_m = the *mean corrected* temperature of the entire line;

α = coefficient of expansion for 1°;

C_t = correction for temperature.

Then

$$C_t = +\alpha (T_m - T_0) L. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The temperature correction for a part of a tape length is computed separately.

If the value of α for the tape used is not known, it may be taken at 0.0000065.

If a metallic thermometer is used, as a brass and a steel wire, or a brass and a steel bar as in the U. S. Coast and Geodetic Survey apparatus, shown on p. 560, then we have the following:

429. Temperature Correction when a Metallic Thermometer is Used.

Let l = length of wire or tape used, as 300 feet;

l_s = absolute length of the steel wire at the standard temperature of, say, 32° F.;

l_b = same for brass wire;

L = total length of line for whole tape lengths ($= nl$ approximately);

n = number of lengths of the standard measured;

r_s = *mean value* of all the scale readings on steel wire for the entire line $\left(=\frac{\Sigma r'_s}{n}\right)$;

r_b = same for scale readings on brass wire;

α_s = coefficient of expansion for the steel wire;

α_b = coefficient of expansion for the brass wire;

t_0 = *mean temperature* for the entire line.

Then we have

$$\left.\begin{aligned} L &= n(l_s + r_s)(1 + (t_0 - 32^\circ)\alpha_s) \\ &= n(l_b + r_b)(1 + (t_0 - 32^\circ)\alpha_b) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Since the temperature correction is relatively a very small quantity, we may put $l_s + r_s = l_b + r_b = l$, the length of the tape to which the temperature correction is applied.

We then have from (2),

$$(t_0 - 32^\circ) = \frac{(l_s + r_s) - (l_b + r_b)}{l(\alpha_b - \alpha_s)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Substituting this value of the temperature in (2), we obtain

$$L = n\left[l_s + r_s + \frac{\alpha_s}{\alpha_b - \alpha_s}((l_s + r_s) - (l_b + r_b))\right] . \quad . \quad . \quad . \quad (4)$$

If we put $l_s + r_s = S_s$ and $l_b + r_b = S_b$, we have

$$\left.\begin{aligned} L &= n\left[S_s + (S_s - S_b)\frac{\alpha_s}{\alpha_b - \alpha_s}\right] \\ &= n\left[S_b + (S_s - S_b)\frac{\alpha_b}{\alpha_b - \alpha_s}\right] \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

From either of the Eqs. (5) we may compute the length of the line as corrected for temperature. If, however, it is desired to

find the temperature correction separately, in order to combine it with the other corrections, we have

$$C_{st}=n(S_s-S_b)\frac{\alpha_s}{\alpha_b-\alpha_s}, \quad . \quad . \quad . \quad . \quad . \quad (6)$$

for the temperature correction to be applied to the measured length by the steel wire, or

$$C_{bt}=n(S_s-S_b)\frac{\alpha_b}{\alpha_b-\alpha_s}, \quad . \quad . \quad . \quad . \quad . \quad (7)$$

as the temperature correction to be applied to the measured length by the brass wire.

These formulas all apply only to the *entire tape lengths*. Any fractional length would have to be computed separately, or else a diminished weight given to their scale readings in obtaining the mean values, r_s and r_b.

430. Correction for Alignment, both horizontal and vertical. The relative elevations of the points of support are found by a levelling instrument, and the horizontal alignment done by a transit or by eye. An alignment by eye will be found sufficiently exact if points be established on line by transit every 500 or 1000 feet. The suspending nails and hooks afford considerable latitude for lateral adjustment when the tape is stretched taut; hence the horizontal deviation will be practically zero unless the stakes are very badly set, and the relative elevations of any two successive supports should be determined to less than 0.05 foot. If no care is taken to have more than two suspension points on grade, then each section of the tape will have a separate correction. Usually a single grade may as well extend over several sections, in which case the portion on a uniform grade may be reduced as a single section.

Let l_1, l_2, l_3, etc., be the successive lengths of uniform grades, and h_1, h_2, h_3, etc., the differences of elevation between the

extremities of these uniform grades; then for a single grade we would have the correction

$$C = l - \sqrt{l^2 - h^2},$$

or

$$l^2 - 2Cl + C^2 = l^2 - h^2.$$

But since C is a very small quantity as compared with l, we may drop the C^2, whence we have $C = \frac{h^2}{2l}$ for a single grade.

The exact value of C, in ascending powers of h, is

$$C = \frac{h^2}{2l} + \frac{h^4}{8l^3} + \frac{h^6}{16l^5} + \text{etc.} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

For the entire line, if all but the first term be neglected, the correction is

$$C_g = -\frac{1}{2}\left(\frac{h_1^2}{l_1} + \frac{h_2^2}{l_2} + \frac{h_3^2}{l_3} + \ldots \frac{h_n^2}{l_n}\right). \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If the l's are all equal, as when no two successive suspension points fall in the same grade, then we have

$$C_g = -\frac{1}{2l}(h_1^2 + h_2^2 + h_3^2 + \ldots h_n^2) = -\frac{\Sigma h^2}{2l}. \quad . \quad . \quad (3)$$

Since the relative elevations are determined, and not the angles of the grades, these formulas are more readily applied than one involving the grade angles.

The error made in rejecting the second power of C in the above equations is given in the table on the following page, where l and h are taken in the same unit of length.[1]

[1] From Jäderin's Geodätische Längenmessung.

If the grades are given in vertical angles, as they always are with the ordinary base apparatus, then we have for the correction to each section whose length is l, and whose grade is θ above or below the horizon,

$$C_g = -l(1-\cos\theta) = -2l\sin^2\frac{\theta}{2}.$$

If θ be expressed in minutes of arc, and if the grade angle is less than about six degrees, or if the slope is less than one in ten, we may write

$$C_g = -2l\sin^2\frac{\theta}{2} = -\frac{1}{2}l\theta^2\sin^2 1' = -\frac{\sin^2 1'}{2}\theta^2 l$$
$$= -0.00000004231\,\theta^2 l;$$

or by logarithms,

$$\log C_g = \text{const log } 2.626422 + 2\log\theta + \log l.$$

431. Correction for Sag. From Eq. (4), p. 578, we have

$$C_s = -\frac{L}{24}\left(\frac{wd}{P}\right)^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where C_s is the shortening due to sag in an inelastic tape.

If the standard length be given with the pull P_0, and the distance between supports d_0, while in the field the pull P and distance d between supports be used, then the correction for sag is

$$C_s = \frac{Lw^2}{24}\left(\frac{d_0^2}{P_0^2} - \frac{d^2}{P^2}\right) = \frac{L}{24}\left(\frac{w}{P_0 P}\right)^2 (d_0^2P^2 - d^2P_0^2), \quad . \quad (5)$$

where L, d, and C_s are taken in the same unit of length, and w is the weight of a unit's length of tape in the same units used for P.

Table III.

THE ABSOLUTE ERRORS IN THE FORMULA $C_g = \frac{h^2}{2l}$.

Length of Uniform Grade.	Absolute Error in the Same Units Used for t and h.				
	0.00005	0.00015	0.00025	0.00035	0.00045
l.	h = Rise or Fall in Length l.				
1	0.14	0.19	.....	.....	.....
2	.24	.31	.....	.....	.....
3	.32	.42	.....	.....	.....
4	.40	.53	.....	.....	.....
5	.47	.62	.....	.....	.....
6	.54	.71	0.81	.....	.....
7	.61	.80	.91	.....	.....
8	.67	.88	1.00	.....	.....
9	.73	.97	1.10	1.19	.....
10	.79	1.05	1.19	1.29	.....
11	.85	1.12	1.28	1.39	.....
12	.91	1.20	1.36	1.48	.....
13	.97	1.27	1.45	1.57	1.67
14	1.02	1.35	1.53	1.66	1.77
15	1.08	1.42	1.61	1.75	1.86
16	1.13	1.49	1.69	1.84	1.96
17	1.18	1.56	1.77	1.92	2.05
18	1.24	1.62	1.85	2.01	2.14
19	1.29	1.69	1.92	2.09	2.23
20	1.34	1.76	2.00	2.17	2.31
21	1.39	1.82	2.07	2.25	2.40
22	1.44	1.89	2.15	2.33	2.48
23	1.48	1.95	2.22	2.41	2.57
24	1.53	2.02	2.29	2.49	2.65
25	1.58	2.08	2.36	2.57	2.73
26	1.63	2.14	2.43	2.65	2.82
27	1.67	2.20	2.50	2.72	2.90
28	1k72	2.26	2.57	2.80	2.98
29	1.77	2.32	2.64	2.87	3.06
30	1.81	2.38	2.71	2.95	3.14

432. Correction for Pull. From Eq. (1), p. 576, we may write at once

$$C_p = +\frac{(P-P_0)L}{SE}.$$

Here P is taken in pounds, L and C_p in inches, and S in square inches, since E is usually given in inch-pound units. If E has not been determined by experiment, it may be taken at 28,000,000. The cross-section S is best found by weighing the tape and computing its volume, counting 3.6 cubic inches to the pound. Knowing the length, the cross-section can then be found. If the stretch has been observed for different weights, and the value of E computed, the value of S is of course of no consequence, provided the same value be used for both observations.

433. To Reduce a Broken Base to a Straight Line. It is sometimes necessary or convenient to introduce one or more angles into a base line. These would never deviate much from 180°. Let the difference between the angle and 180° be θ, and let the two measured sides be a and b, to find the side c. If θ be expressed in minutes of arc and if it is not more than about 3°, the following approximate formula will prove sufficiently exact:

$$\text{Side } c = a+b-\frac{\sin^2 1'}{2}\cdot\frac{ab\theta^2}{a+b}$$

$$= a+b-0.00000004231\frac{ab\theta^2}{a+b}.$$

If θ is greater than from 3° to 5°, the triangle would have to be computed by the ordinary sine formula.

434. To Reduce the Length of the Base to Sea Level. In geodetic work, all distances are reduced to what they would be if the same lines were projected upon a sea-level surface by radii passing through the extremities of the lines. It is not necessary, however, to reduce all the lines of a triangulation system in this

manner, since if the length of the base line is so reduced the computed lengths of all the other lines of the system will be their lengths at sea level. The angles that are measured are the *horizontal* angles, and are not affected by the differences of elevation of the various stations. It is necessary, therefore, to know the approximate elevation of the base above sea level.

Let r = mean radius of earth;
a = elevation above sea level;
B = length of measured base;
b = length of base at sea level.

Then $r+a:r::B:b$,

or $$b = B\frac{r}{r+a}.$$

The correction to the measured length is always negative, and is

$$C = b - B = -B\left(1 - \frac{r}{r+a}\right) = -B\left(\frac{a}{r+a}\right).$$

Since a is very small as compared to r, we may write

$$C = -B\frac{a}{r}.$$

The mean radius[1] in feet is

$$\text{Mean } r = \frac{20926062 + 20855121}{2} = 20{,}890{,}592 \text{ feet},$$

$$\log r \text{ (in feet)} = 7.3199507.$$

435. Summary of Corrections. For the significance of the notation used in the following equations, see the preceding articles

[1] Rigidly, we should use the length of the *normal* for the given latitude, but the mean radius as above found is sufficient for most cases.

where they are derived. The corrections are all for the entire line measured, or rather for that portion of it composed of entire tape lengths, and are to be applied with the signs given to the measured length.

1. *Correction for Temperature.*

For a single standard with mercurial temperatures,

$$C_t = +\alpha(T_m - T_0)L. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

For metallic thermometer readings, as found from steel and brass standards, for instance, the correction to be applied to the length as found by the steel wire, or standard, is

$$C_{st} = n(S_s - S_b)\frac{\alpha_s}{\alpha_b - \alpha_s}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

2. *Correction for Grade.* This is always negative.

In terms of the difference of elevation of grade, points at a common distance, l, apart,

$$C_g = -\frac{\Sigma h^2}{2l}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In terms of the grade angles, expressed in minutes of arc

$$C_g = -0.000000042\,31\,\Sigma\theta^2 l. \quad . \quad . \quad . \quad . \quad (4)$$

3. *Correction for Sag.*

For the standard length given for a pull P_0, and a distance between supports d_0, while P and d are used in the field work,

$$C_s = \frac{L}{24}\left(\frac{w}{P_0 P}\right)^2 (d_0^2 P^2 - d^2 P_0^2). \quad . \quad . \quad . \quad . \quad (5)$$

For the standard length given for no pull and no sag, the shortening due to sag alone in an inelastic tape is

$$C_s = -\frac{L}{24}\left(\frac{wd}{P}\right)^2. \quad \ldots\ldots\ldots \quad (6)$$

4. *Correction for Pull.*

$$C_p = +\frac{P-P_0}{SE}L; \quad \ldots\ldots\ldots \quad (7)$$

or

$$C_p = (P-P_0)en. \quad \ldots\ldots\ldots \quad (8)$$

5. *Correction for Broken Base.*

If a and b are the two measured sides which make an angle of $180°-\theta$, the correction to be added to $a+b$ to get the distance between their extremities, θ being less than 5°, and expressed in minutes of arc, is

$$C_b = -0.0000000423\,1\,\frac{ab\theta^2}{a+b}.$$

6. *Correction to Sea Level.*

$$C_l = -L\frac{a}{r},$$

where L is the length of the measured base at an altitude a above sea level.

$$\log r \text{ (in feet)} = 7.3199507.$$

436. To Compute any Portion of a Straight Base which Cannot be directly Measured. It sometimes is convenient to take a base line across a stream or other obstruction to direct measurement. In such a case a station may be chosen as O in Fig. 227, and the horizontal angles $AOB=P$, $BOC=Q$, and $COD=R$ measured. If the parts AB and CD lie in the same

straight line, and $AB=a$ and $CD=b$ are known, then $BC=x$ may be found *by measuring only the angles at O.*

Thus in the triangles ABO and ACO (Fig. 227) we have

$$\frac{CO}{BO}=\frac{x+a}{a}\frac{\sin P}{\sin (P+Q)},$$

also from the triangles BDO and CDO we have

$$\frac{CO}{BO}=\frac{b}{x+b}\frac{\sin (Q+R)}{\sin R}.$$

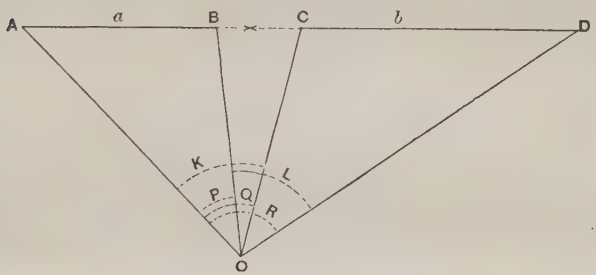


FIG. 227.

Let $K=P+Q$ and $L=Q+R$, then by equating the above values of $\frac{CO}{BO}$ we have

$$(x+a)(x+b)=\frac{ab(\sin K \sin L)}{\sin P \sin R},$$

whence

$$x=-\frac{a+b}{2}\pm\sqrt{\frac{ab(\sin K \sin L)}{\sin P \sin R}+\left(\frac{a-b}{2}\right)^2}.$$

Evidently only the positive result is to be taken.

The points A, O, and D should be chosen so as to give good intersections at A and D.

437. Accuracy Attainable by Steel-Tape and Metallic-Wire Measurements. The following results have been attained by using the methods herein described:

1. In Sweden, Mr. Edw. Jäderin measured a primary base line two kilometres in length three times, by means of steel and brass wires 25 metres long, in ordinary summer weather, mostly clear, with a probable error of a single determination of 1 in 600,000, and a probable error of the mean result of 1 in 1,000,000, as compared with the true length of the line as obtained by a regular primary base apparatus.

2. Professor R. S. Woodward, when assistant on the U. S. Coast and Geodetic Survey, in 1892, made five measurements of a base line 3807 metres long, in four sections, using two steel tapes, making two measurements with each at night, and one measurement in the daytime in clear sunlight. These results gave a probable error in the mean of all of the results of $\pm \frac{1}{2{,}000{,}000}$ part, not including the error in the length of the tape itself, and a probable error of $\frac{1}{1{,}280{,}000}$ part when all sources of error are taken into account.[1]

3. The remarkable measurement of nine base lines in a single season, 1900, by a single field party of the U. S. Coast and Geodetic Survey, marks a new era in base-line work.[2] With the object of eliminating the constant errors of the apparatus, five different sets of apparatus were used in the measurement of each base. While no expense was made to increase the accuracy beyond $\frac{1}{500{,}000}$, an average accuracy of 1,200,000 was secured, as will be seen from the following table:

[1] See a paper on "The Use of Long Steel Tapes for Measuring Base Lines," Trans. Am. Soc. C. E., Vol. XXX (1893), p. 81.

[2] See Appendix 3, Report U. S. Coast and Geodetic Survey for 1901, by Mr. A. L. Baldwin, for a full discussion of the specifications prepared by Mr. John F. Hayford.

TABLE IV

ACCURACY OF STEEL TAPE BASE MEASUREMENTS

Name of Base.	Probable Error.	Length.	Probable Error as Proportional Part.	Speed in Miles per Hour.
	mm.	m.		
Shelton	± 9.0	7,884.72	1/876000	1.04
Page	± 5.6	8,250.99	1/1473000	.91
Anthony	± 4.8	6,034.70	1/1257000	1.06
El Reno	± 9.7	12,886.68	1/1329000	1.20
Bowie	± 5.0	8,196.04	1/1639000	1.20
Stephenville	± 6.0	6,255.08	1/1042000	1.33
Lampasas	± 4.1	5,961.13	1/1454000	1.04
Alice	±10.1	6,971.69	1/ 690000	1.30
Seguin	± 5.1	6,794.61	1/1333000	1.08

These bases have a total length of 43.2 miles, an average length of 4.8 miles and control over 1000 miles of the triangulation along the 98° meridian.

4. The results of another remarkable season of base-line measurement, using steel and "invar" tapes, during 1906,[1] are summarized in the following table:

TABLE V

ACCURACY OF INVAR TAPE BASE MEASUREMENTS

Base.	Probable Error.	Length.	Probable Error as Proportional Part.	Speed with Steel Tapes in Miles per Hour.
	mm.	m.		
Point Isabel	±3.0	7384.92	1/2450000	1.05
Willamette	±4.1	14019.38	1/3430000	1.34
Tacoma	±4.0	12055.57	1/3020000	1.40
Stephen	±4.2	9221.83	1/2170000	1.34
Brown Valley	±2.8	8223.57	1/2910000	1.34
Royalton	±3.2	9637.55	1/2980000	1.49
	Average, 10,090		1/2760000	1.32

[1] See Appendix 4, U. S. Coast and Geodetic Survey, 1907, by Mr. O. B. French, for an interesting description of this work. See also Appendix 1 U. S. Coast and Geodetic Survey, Report 1910, for cost data.

The measurements with invar tape averaged 7 per cent faster than with steel tapes and 70 per cent more accurate. The average discrepancy between measurements of the same distance by steel and "invar" tapes was one part in 526,000.

Measurement of the Angles

438. Instruments for Measuring Angles. The instruments used in triangulation are designed especially for the accurate measurement of horizontal angles. This demands very accurate centering and fitting at the axis, and strict uniformity of graduation. It was formerly supposed that the larger the circle the more accurate the work which could be done. It is now known that there is no advantage in having the horizontal limb more than 10 or 12 inches in diameter.

There are two general methods of reading fractional parts of the angle, smaller than the smallest graduated space on the limb. One is by verniers, the other by micrometer microscopes. Verniers may be successfully used to read angles to the nearest 10 or 20 seconds of arc, but if a nearer approximation is desired microscopes should be employed.

The names of the instruments corresponding to these two widely different methods are respectively the *repeating* and *direction* theodolites. It will be noted that both forms insure great stability by means of three widely spaced foot screws and long vertical axes. This stability is increased by the U-shaped standards.

439. The Repeating Instrument. Excellent forms of the 8-inch repeating instrument are shown in Figs. 228 and 231. The construction of this instrument is the same as that of the engineer's transit, viz., the alidade is attached to an upper plate carrying the verniers and the inner *solid* axis, while the graduated (lower) limb is fitted with a *hollow* axis as nearly concentric with the solid axis as it is possible to make it.

The construction allows of the movement of the solid axis carrying the alidade in the hollow axis, and also the movement

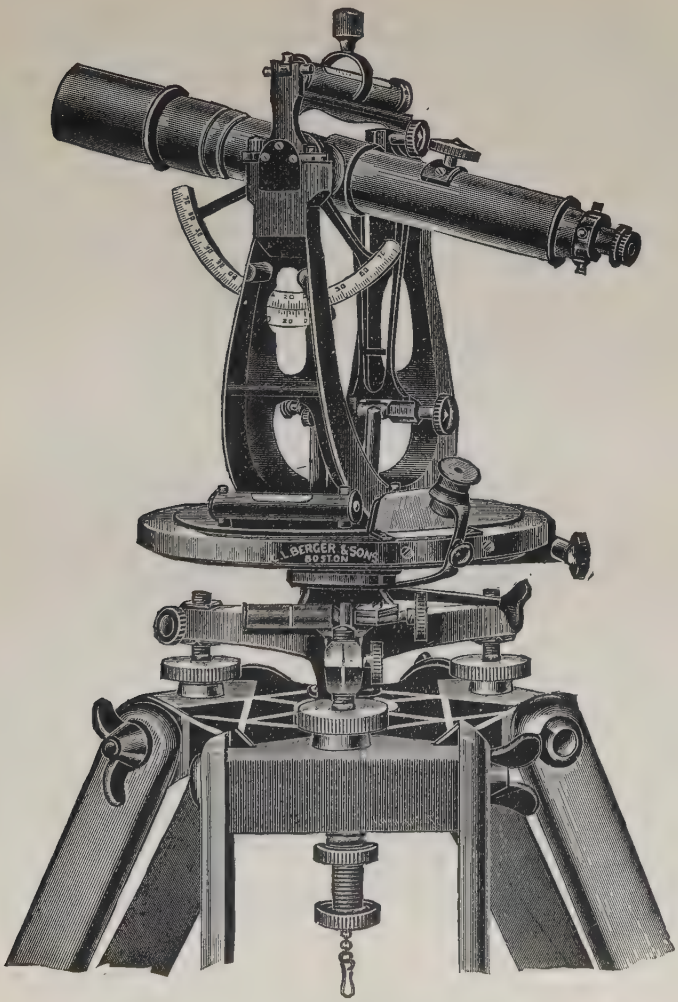


FIG. 228.—8-inch Repeater Transit.

of the hollow axis carrying the graduated circle in its own socket. Each motion is provided with a clamp and slow-motion screw.

The two (or more) verniers usually read to 10 seconds directly with an eye-piece of correspondingly high power. Theoretically, the above arrangement should lessen the effect of errors of graduation, of pointing, and of reading. In practice, however, it is found that certain mechanical defects, such as slipping of the clamps, eccentric centres of the two axes, etc., counterbalance a large part of the advantages. This slipping of the clamps makes a particular instrument read always too much or always too little, a defect that can be overcome in part by the use of a proper programme, see Art. 446. This type of instrument is seldom used in primary triangulation.

440. Direction Instrument. A modern form of this non-repeating instrument designed by the U. S. Coast and Geodetic Survey is shown in Fig. 229. A study of the programme of observation (Art. 447) will show the significance of its name. Such an instrument is used to measure the direction of the stations from some initial station. The angles between the lines of the triangulation are the computed differences between the directions. This instrument has but one movement and one axis. The graduated limb is simply a flat disc. Means of clamping the circle and of changing the position of its zero point are provided. Two or more microscopes are provided for reading the circle. Such an instrument has the circle divided into 5-or 10-minute divisions, the fractional reading being read by a micrometer to single seconds. This instrument is used by the U. S. Coast and Geodetic Survey on its primary triangulation. Its manner of use is further explained in Art. 445.

Fig. 230 shows a smaller and more portable form of a similar type of instrument.

441. Altazimuth Instrument. Altazimuth is the name given to an instrument (Fig. 231) designed to measure accurately vertical as well as horizontal angles. Both horizontal and vertical limbs are read by means of micrometer microscopes. Such an

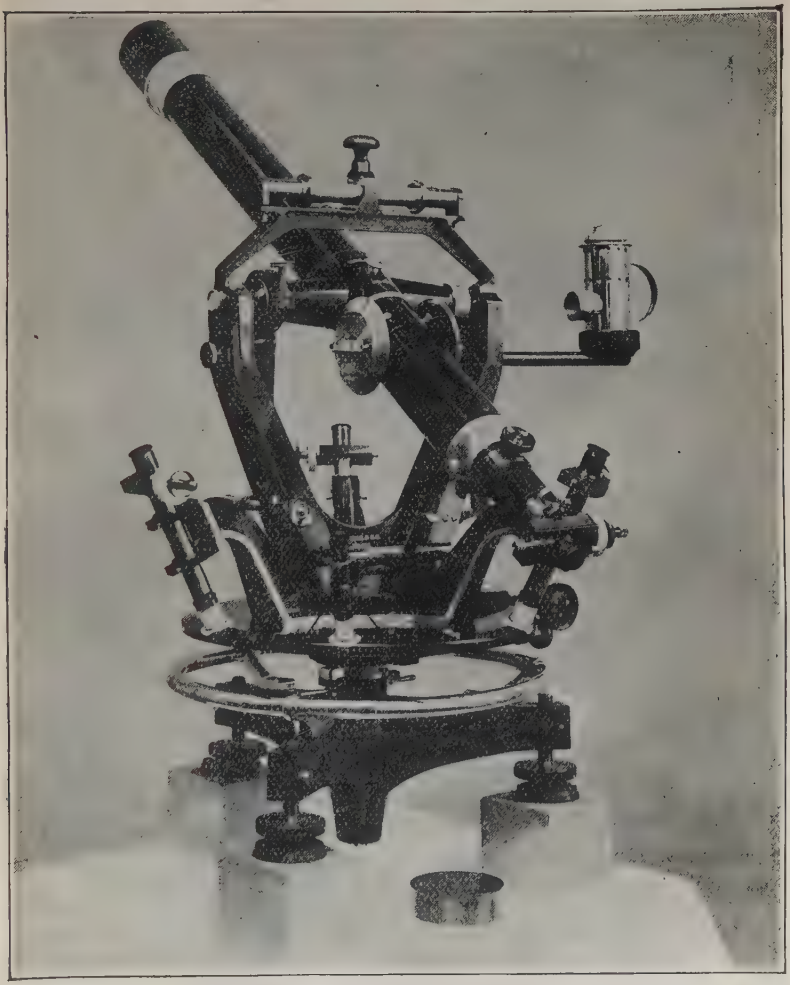


FIG. 229.—Direction Theodolite (U. S. Coast and Geodetic Survey). Circle 12″ divided into 5′ Divisions read by Three Microscopes to Single Seconds.

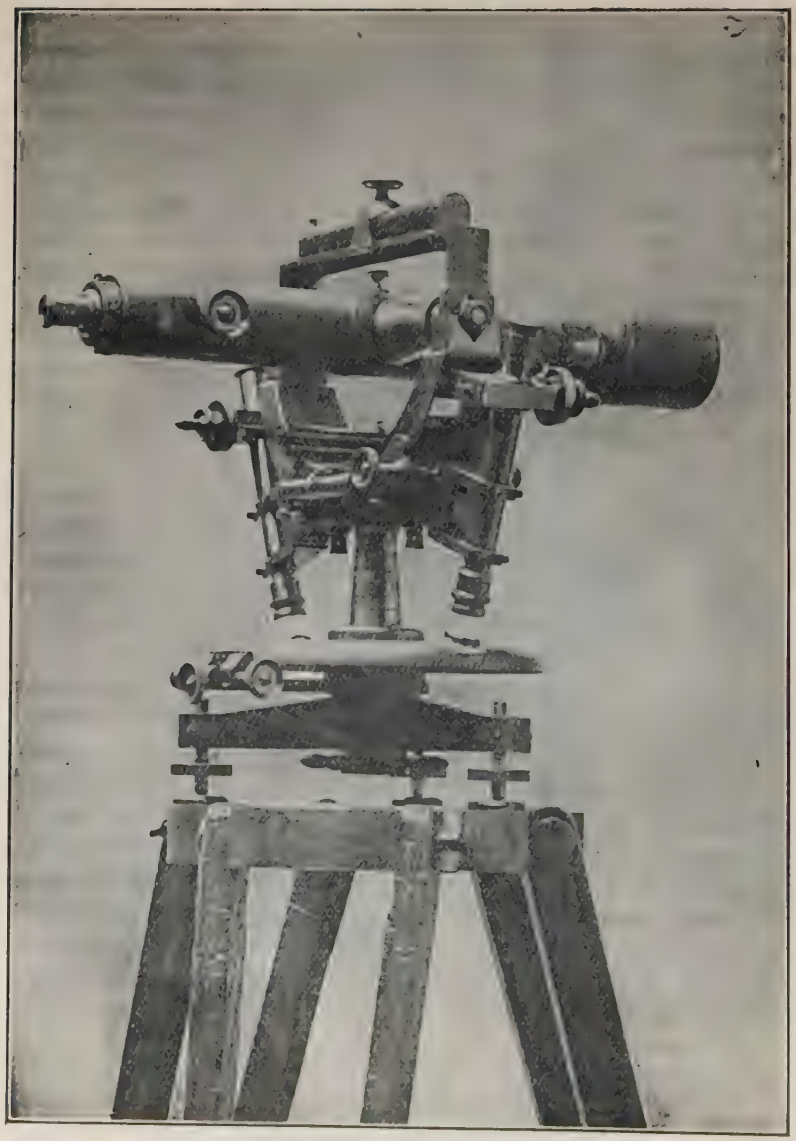


FIG. 230.—8-inch Direction Instrument Circle Divided into 10′ Divisions and read by Two Micrometer Microscopes to Single Seconds.

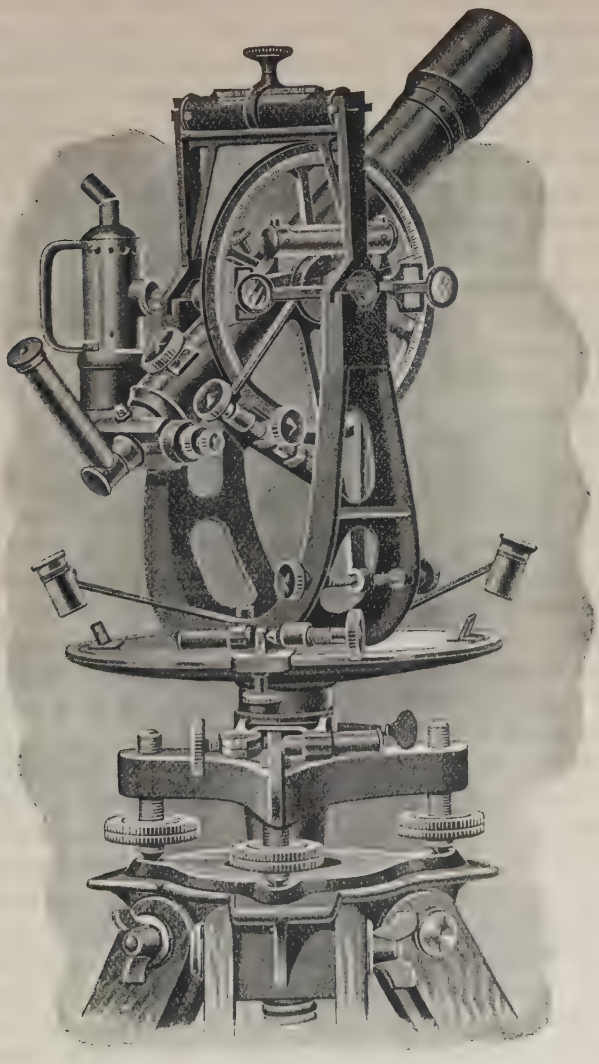


FIG. 231.—Altazimuth Transit (8-inch).

instrument is designed especially for astronomical observations for latitude and azimuth, but may also be used as a meridian or transit instrument for observing time as well as for measuring horizontal and vertical angles in triangulation. It is in fact the universal geodetic instrument, just as the complete engineer's transit is the universal instrument in ordinary surveying. In almost all cases where micrometers are used in reading the angles the limbs are graduated to five or ten minutes and the readings made to single seconds.

442. Adjustments of the Theodolite. The adjustments are very similar to those of a common engineer's transit, and should be made in the following order:

1. *Plate Level Adjustment.* The axes of all fixed levels are made perpendicular to the vertical axis of the instrument and therefore parallel to the plate. For explanation of method with figures, see Art. 72.

2. *The Adjustment for Wind*, or the lack of parallelism between the level vial and the line joining the centres of the points of supports, is tested by tipping the level and noting whether the bubble changes its position or not. If it runs toward the left, for example, when tipped toward the observer, then the left-hand end must be moved toward the observer, or the right-hand end moved away from the observer by means of the lateral adjusting screws until the bubble remains in the centre.

3. *Striding Level Adjustment.* This consists in making the axis of the level parallel to the points of support. To test this adjustment, note the reading of the bubble (most conveniently when in the middle) when resting on the horizontal axis of the telescope. Reverse the level on its supports and correct one-half of the run of the bubble by the vertical adjusting screws.

4. *Horizontal Axis Adjustment.* This consists in making the horizontal axis of the telescope truly horizontal. The striding level having been adjusted the vertical axis is made truly vertical, i.e., the foot screws are moved until the bubble remains unchanged when the transit is moved in azimuth.

If the striding level is still in adjustment any movement of the bubble from the centre must be due to the inclination of the horizontal axis, and should be corrected by bringing the bubble to the centre by raising or lowering that pivot provided with adjusting screws.

5. *Collimation Adjustment.* This adjustment consists in making the line of sight perpendicular to the horizontal axis of the telescope. It is tested by pointing the telescope on some well-defined point, preferably, a mile away. The telescope is then carefully lifted out of its wyes, turned 180° about its long axis, and replaced in the wyes again.[1] If the same point is still bisected, the collimation is perfect; if not bisected, correct half the error by means of the screws on the reticule. If the telescope is inverting, the cross wires must be moved in a direction opposite to that apparently required.

443. The Filar Micrometer[2] is used for the accurate measurement of small distances or angles, when the required exactness is greater than can be obtained by means of a vernier scale. It is usually combined with a microscope, the micrometer threads and scale lying in the plane of the image produced by the objective. This image is always larger than the object itself in microscopes, and therefore a given movement of the wires in the micrometer corresponds to a very much less distance on the object sighted at, according to the magnifying power of the objective.

The frame holding the movable wires has a screw with a very fine thread working in it, called the micrometer screw. This screw has a graduated cylindrical head, or disc, attached to it, there usually being sixty divisions in the circumference when used in angular measurements. The number of whole revolutions are recorded by noting how many teeth of a comb scale are passed over, this scale being nearly in the plane of the

[1] Some instruments do not allow the telescope to be lifted out. In this case the collimation would be adjusted as in the common engineer's transit (Art. 74).

[2] From *filum*, thread; *micros*, small, and *metros*, measure. The thread is in this case a spider's web, or scratches on glass.

wires and therefore in the focus of the eye-piece. The fractional parts of a revolution are read on the graduated screw-head outside. These micrometer attachments are shown on the two microscopes in Fig. 230 and on the three in Fig. 229. Fig. 232 is a sectional view of a filar micrometer. The graduated head *h* is attached to the milled head *m*, forming a nut into which the micrometer srcew *a* works. The screw is rigidly attached to the frame *b*, to which are fastened the movable wires *f'*. The comb scale *s* and fixed wire *f* are attached to the frame *c*, which is adjusted to a zero reading of the graduated head by the capstan serew *d*. The lost motion on both of these frames is taken up

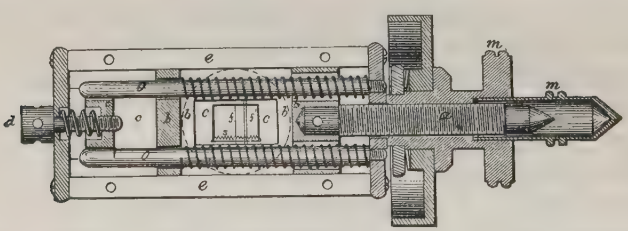


FIG. 232.—Filar Micrometer.

by springs. The complete revolutions of the screw are counted on the comb scale, and the fractional part of a revolution on the graduated head. The reading is made by bringing the double wires symmetrically over a graduation, the space between the wires being a little more than the width of the graduation, when the exact number of revolutions and sixtieths are read on the comb scale and on the head. If the limb is graduated to ten minutes and each revolution corresponds to one minute, then if the reading is taken on the *nearest* graduation, the number of revolutions need never exceed five. If, however, the reading be always taken to the last ten-minute mark counted on the limb, then ten revolutions may have to be read on the screw. The movement of the threads is as they appear to be, there being no inversion of image between the wires and eye. The movement on limb is, however, opposite from the apparent motion.

444. Correction for Run. If the limb is graduated to ten minutes, and a single revolution of the screw corresponds to the space of one minute, then just ten revolutions of the screw should move the wires from one graduation to the next. If this is not exactly true, then the value of a ten-minute space should be measured a number of times, by running the wires back and forth, the mean result taken, and from this the value of *one revolution* of the screw determined. This value is called the "run of the screw," and a correction is applied to the readings, which are always made in degrees, minutes, and seconds, counting one revolution a minute and one division on the head a second of arc. These corrections are no longer necessary, because of the adoption of an ingenious programme of observation.[1]

This programme is given in Art. 447. It will be easily seen that the sixteen readings on the initial station are nearly uniformly distributed over the five-minute interval covered by the micrometer. This insures that the sum of the corrections for run (if applied) will be nearly zero on this signal as well as on every other signal. The laborious corrections for run are therefore unnecessary. If the run of any single micrometer is found to be greater than four seconds the micrometer should be adjusted. The run is also adjusted if the error in the mean of the three micrometers is greater than one second.

445. Reading the Circle. In reading the circle, the observer first notes directly the last degree mark passed over, then the number of full five- or ten-minute divisions on the circle between the micrometer zero and the last degree mark; he then counts the odd number of minutes on the comb scale between the last five- or ten-minute mark on the circle and the zero of the micrometer; and, finally, to secure the number of seconds, he runs the micrometer wires backward until the last five- or ten-minute mark on the scale is reached, when the reading of the micrometer

[1] This was devised by Mr. J. F. Hayford, inspector of geodetic work of the U. S. Coast and Geodetic Survey.

gives the number of seconds. The average of the micrometers determines the pointing.

For reading on graduated straight lines the double threads give better results than either the single thread or the intersecting threads. The space between the threads should be a little greater than the width of the image of the graduation line, so that a narrow strip of the limb's illuminated upper surface may appear on either side of the graduation and inside the wires. The setting is then made so as to make these illuminated lines of equal width. It is conceded that such an arrangement will give more exact readings than any other that has been used.

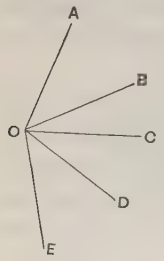


FIG. 233.

The magnifying power of the microscope is from thirty-five to sixty-five diameters.

446. Programme with Repeaters. To measure the angle AOB (Fig. 233), the verniers are clamped at zero, then the telescope directed to the left-hand station, A, and the lower motion clamped. The upper motion is then unclamped and the telescope directed and clamped on B. This is repeated six times by the following programme:

PROGRAMME

Telescope Normal

1. Set on left station, and read both verniers.
2. Unclamp above and set on right station.
3. " below " left "
4. " above " right "
5. " below " left "
6. " above " right "

etc. etc.

After the sixth repetition both verniers are read and recorded. The total angle turned off divided by 6 gives a partial value for

the angle *AOB*. The telescope is then reversed [1] and the explement of the angle *AOB* measured with the same programme. This value of the explement subtracted from 360° gives a value α' for the angle. An inspection of this programme will show that the mean value, $\alpha = \frac{a_1 + \alpha'}{2}$, should, as far as is possible, eliminate error due to lack of instrumental adjustment as well as the clamp errors.[2] The number of sets will depend upon the accuracy sought. On the U. S. Coast and Geodetic Survey, using 10-inch theodolites, the number of sets vary from two to four on secondary triangulation.[3]

In the figure, the angle *EOA*, which closes the horizon, would be measured with the same programme as though it formed a part of the triangulation. If this be done the five angles are adjusted by correcting each by one-fifth the failure of their sum to equal 360°. The practice of measuring summation angles is no longer followed on the U. S. Coast and Geodetic Survey.

The following field notes show one method of using the repeater for measuring horizontal angles.[4] Another method is

[1] In case the instrument used is a theodolite, and its telescope cannot be revolved on its horizontal axis, it should be lifted from the pivot bearings and turned over end for end, leaving the pivots in their former bearings. If this cannot be done conveniently, then the limb should be shifted by $360 \div n$ each time, and this will result in mostly eliminating these same errors of collimation and inclination of horizontal axis. If it be found that the vertical axes are not parallel, then at least four sets of readings should be taken and these should be distributed upon the horizontal limb symmetrically with reference to the plane of greatest inclination between the two vertical axes.

[2] For example, if the clamp error always gives too large readings then α_1 and its explement are both too large. This results in α' too small. $\frac{\alpha_1 + \alpha'}{2}$ should therefore be free from all of the clamp error that can be so eradicated.

[3] The accuracy here sought is that the maximum error of closure shall seldom exceed eight seconds and shall average two or three seconds.

[4] Taken from the records of the University of Wisconsin Summer School of Surveying.

to make half of the repetitions with telescope direct and half with telescope reversed, both in measuring the angle as well as its explement.

REPEATING METHOD

HORIZONTAL ANGLES

Station: Water Tower, Portage, Wis. Date: June 20, 1906.
Observers: Fisher. Recorder: Sherman. Instrument: B. & B. Repeater No. 854.

Objects Observed.	Time, h. m.	D R	Reps.	Angle. ° ′	A ″	B ″	Mean of Verniers.	Arc Passed Over.	Angle, Mean D and R. ° ′ ″	Remarks.
Sand Hill.	9:30	D		0 0	0	10	5	261°		Clear
Hubbard.		D	6	261 22	10	20	15	22′ 10″	43 33 41.6	Mean: 43° 33′ 41″2.
Expl'mn't	9:45	R		0 0	0	10	5	1898°		
		R	6	98 38	0	0	0	37′ 55″	316 26 19.1	

447. Non-Repeating or Direction Instrument Programme. An "initial" station is chosen to which the circle positions described below are referred. This station should be, by preference, the one most apt to be always plainly visible.

With the proper circle reading clamped on the initial station, the pointings are made to the stations in succession (clockwise), and all microscopes read at *each* pointing. Thus if the instrument were at *O* (Fig. 233), the pointings would be made to *A*, *B*, *C*, *D*, *E* with telescope direct. The telescope is now lifted out of the standards and turned 180° about its longitudinal axis. The alidade is then turned 180° in azimuth and the telescopes replaced in the standards.[1] A new pointing is now made on *E* recorded and similarly on *D*, *C*, *B*, *A*. The average of the two pointings on any station (direct and reversed) is the set value of such direction. On primary triangulation one set

[1] It will be noted that this programme keeps the telescope in the same position as though it had been possible to transit it, i.e., the horizontal axis rests in the same trunnions as before reversion.

is taken in each of the following sixteen positions on the initial station:[1]

	°	′	″		°	′	″
No. 1,	0	00	40	No. 9,	128	00	40
No. 2,	15	01	50	No. 10,	143	01	50
No. 3,	30	03	10	No. 11,	158	03	10
No. 4,	45	04	20	No. 12,	173	04	20
No. 5,	64	00	40	No. 13,	192	00	40
No. 6,	79	01	50	No. 14,	207	01	50
No. 7,	94	03	10	No. 15,	222	03	10
No. 8,	109	04	20	No. 16,	237	04	20

On secondary triangulation, one set on each of the following five positions has secured reasonable closures.[2] Less experienced observers using less accurate instruments would of course need to increase the number of sets.

	°	′
No. 1,	0	1
No. 2,	72	3
No. 3,	144	5
No. 4,	216	7
No. 5,	288	9

The following is a field record[3] of the measurement of horizontal directions with an 8-inch instrument.

[1] It has been found that with experienced observers and using a 12-inch theodolite (Fig. 229) one set in each of the sixteen positions insures the maximum error of closing a triangle not greater than three seconds and an average of one second. It will be noted that with three microscopes the readings in each group of four positions is nearly uniformly spaced at intervals of 15° over the entire 360°.

[2] Maximum closure of eight seconds and average closure of two or three seconds has been the experience of the U. S. Coast and Geodetic Survey.

[3] Taken from the records of the University of Wisconsin Summer School of Surveying.

HORIZONTAL DIRECTIONS

Station: Water Tower. State: Wisconsin. County: Columbia. Date, June 24, 1909.
Observer: Bandley. Recorder: McNown. Instrument: Heyde Direction. Pos.: No. 5.

Objects Observed.	Time. h. m.	Tel. *D* or *R*.	Mic.	°	′	″	Mean. ′ ″	Direction.[1] °	′	″	Remarks.
Caledonia . .	4:10	*D*	1	288	9	0		288	8	9.2	Clear and windy
			2	108	9	0	9 0				
Sand Hill . .			1	314	59	21		314	58	33.8	
			2	134	59	27	59 24				
Hubbard . . .			1	358	33	11		358	32	23.5	
			2	178	33	20	33 15.5				
Lewiston . . .			1	17	38	37		17	37	47	
			2	197	38	50	38 43.5				
Lewiston . . .	4:15	*R*	1	197	36	58					
			2	17	36	43	36 50.5				
Hubbard . . .			1	178	31	37					
			2	358	31	26	31 31.5				
Sand Hill . .			1	134	57	49					
			2	314	57	38	57 43.5				
Caledonia . .	4:20		1	108	7	22					
			2	288	7	15	7 18.5				

448. Time of Observing. In order to eliminate the effect of lateral refraction it was formerly thought necessary to observe angles on several days. Recent specifications[2] of the Coast Survey allow work at any hour, and do not require work on more than one day at any one station. (See Art. 405.) These specifications have not resulted in lower accuracy, but instead in much lower cost of the field work.

449. Reduction to Centre. It not infrequently happens that the instrument cannot be set over the geodetic point, as when a tower or steeple is used for such point. In such a case the observations must be reduced to what they would have been if the instrument or signal had been placed directly over the station mark. Thus in Fig. 234 let C be the true station and C' the eccentric point occupied by the instrument. The line AB is a

[1] By direction is here meant the mean of direct and reversed reading.

[2] Prepared by Mr. John F. Hayford. See Report U.S. Coast and Geodetic Survey for 1902, p. 819.

side of the system whose length has been found. From the measured angles at A and B the approximate value of the angle C is found and the lengths of the sides a and b computed. At C' the angle $AC'B$ is measured with the same exactness as though it were the angle C itself and the angle $CC'B=\alpha$ is measured by a single observation. The distance $CC'=r$ is also found. Since the exterior angle at the intersection D, as ADB, is equal to the sum of the opposite interior angles, we have

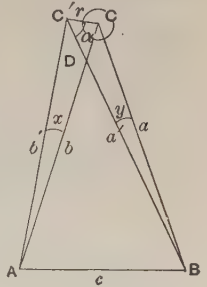


FIG. 234.

$$C+y=C'+x, \quad \text{or} \quad C=C'+(x-y). \quad . \quad (1)$$

In the triangle $AC'C$ we have the sides b and r and the angle $AC'C$ known, whence

$$\left.\begin{aligned} \sin x &= \frac{r \sin (C'+\alpha)}{b}; \\ \text{similarly,} \qquad \sin y &= \frac{r \sin \alpha}{a}. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Since x and y are very small angles, their sines are proportional to their arcs, and we may write $\sin x = x \sin 1''$, where x is expressed in seconds; similarly, $\sin y = y \sin 1''$, and Eqs. (2) become

$$\left.\begin{aligned} x &= \frac{r \sin (C'+\alpha)}{b \sin 1''}; \\ y &= \frac{r \sin \alpha}{a \sin 1''}. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Substituting these values in (1) we have

$$C = C' + \frac{r}{\sin 1''}\left(\frac{\sin [C'+\alpha]}{b} - \frac{\sin \alpha}{a}\right), \quad . \quad . \quad . \quad (4)$$

where the correction to C' is given in seconds of arc. The signs of the trigonometrical functions of the angle α must be carefully

attended to, as it is measured continuously from B around to the left to 360°.

In case the *directions* of the sides have been observed instead of the angles, the following method [1] may be used to correct the directions.

Referring to Fig. 235. Let C be the centre mark of the

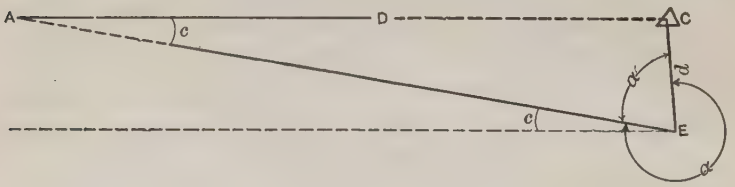


Fig. 235.

station, E the eccentric station, d the eccentricity, D the distance of the line AC, α the angle between C and the object observed, A, and c the angular correction to reduce to centre. Then we have in the triangle ACE,

$$\frac{\sin c}{d} = \frac{\sin \alpha}{D},$$

or $$c \text{ (in seconds)} = \frac{d \sin \alpha}{D \sin 1''} \text{ (since } c \text{ is a small angle).}$$

The following example [2] shows a convenient form for making this computation:

Example. Station, Insane Asylum, eccentric station:

d = 14 feet 4.87 inches = 172.87 inches,	log = 2.23772
Reduction inches to metres,	log = 8.40483
sin 1″ a.c.,	log = 5.31442
$\log \frac{d}{\sin 1''}$ =	5.95697

[1] Taken from Gillespie's "Surveying."

[2] Taken from Vol. II, Gillespie's "Surveying."

TABLE VI

Station.	α	log sin α	log D	log $\frac{\sin \alpha}{D}$	log $\frac{d \sin \alpha}{D \sin 1''}$	Correction. $''$
Insane Asylum centre..	00 00					
Morgan.....	119 18	9.9406	4.1083	5.8323	1.7893	+ 61.6
Minoma.....	172 50	9.0961	4.0252	5.0709	1.0279	+ 10.7
Stand pipe.....	212 38	9.7318	3.9825	5.7493	1.7063	− 50.9
Prs. church.....	237 07	9.9242	3.8598	6.0644	2.0214	−105.1

In case angles were observed the correction to any angle may be obtained by subtracting the correction opposite left-hand station from that opposite the right-hand station, and apply the difference as the correction to this angle. This is shown in the following table:

TABLE VII

Object Observed.	Observed Angles.	Corrections.	Reduced Angles.
Insane asylum.....			
Stand pipe.....	212° 37′ 30″		
Morgan stand pipe...	93° 19′ 8″.6	(− 61″.6 − 50″.9) = − 1′ 52″.5	93° 17′ 16″.1
Minoma stand pipe..	39° 47′ 45″.2	(− 10″.7 − 50″.9) = − 1′ 1″.6	39° 46′ 43″.6
Pres. church.....	295° 42′ 51″.7	(+ 105″.1 + 10″.7) = + 1′ 55″.8	295° 44′ 47″.5

ADJUSTMENT OF THE MEASURED ANGLES

450. Equations of Conditions. When any continuous quantity, as an angle or a line, is measured, the observed value is always affected by certain small errors.[1] Indeed, it would not be possible even to express exactly the value of a continuous quantity in terms of any unit, as degrees or feet and fractional

[1] See Appendix D for a discussion of the sources of error in field work and methods for computing probable error.

parts of the same, even though this value could be exactly determined. If, therefore, the measured values of the three angles of a triangle be added together, the sum will not be exactly 180°. But we know that a rigid *condition* of all triangles is that the sum of the three angles is 180°. An equation which expresses a relation between any number of observed quantities which of geometrical necessity must exist is called an equation of condition, or a condition equation. Thus, in the above case, if A', B', and C' be the mean observed values of the angles, and A, B, and C their true values, we would have for our condition equation

$$A+B+C=180°. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

We would also have

$$A'+a'=A; \quad B'+b'=B; \quad C'+c'=C,$$

where a', b', and c' are small corrections to the measured values A', B', and $C,'$ which are to be determined.

Let us suppose that the length of the side b has been exactly measured,[1] then when the true values of the angles are found we may compute the other two sides. If the sides b and c have both been measured, the length of the side c as computed from b must agree with its measured length, and so we might write the condition equation

$$c=\frac{b \sin (C'+c')}{\sin (B'+b')}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Again, if the side a had been measured and its exact length found, we would obtain the third condition equation,

$$c=\frac{b \sin (A'+a')}{\sin (B'+b')}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

[1] This assumption is made in regard to the measured base lines in a triangulation system, since its exactness is so much greater than can be obtained in the angle measurements.

We now have three independent equations involving three unknown quantities, and can, therefore, find the quantities a', b', and c'. But if only one side had been measured, we should have had but one equation from which to determine three unknown quantities. Evidently there is an infinite number of sets of values of a', b', and c', which would satisfy this equation. If we now impose the condition that the corrections shall be the *most probable* ones, then there is but one set of values that can be taken.

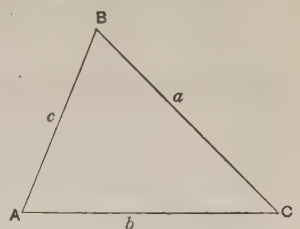


FIG. 236.

Equation (1) is called an *angle equation*, since only angles are involved; while Eqs. (2) and (3) are called *side equations*, since the lengths of the sides are also involved.

451. Local or Station Adjustment. Where the angles at a given station were measured by the repeating method one or more rigorous conditions result, the use of which should lead to still more accurate values for the angles measured thus:

(*a*) The sum of the angles around the horizon should equal 360°, and

(*b*) An angle may be formed from two or more others.[1]

Such a problem is best solved by use of the least square method.[2] Otherwise the following less accurate method may be employed. In Fig. 237 we have

$$A+B=E \quad \text{or} \quad A+B-E=\varepsilon, \quad . \quad . \quad . \quad . \quad (1)$$

and also

$$A+B+C+D=360^\circ \quad \text{or} \quad A+B+C+D-360^\circ=\varepsilon'. \quad (2)$$

[1] This practice is not encouraged on the U. S. Coast and Geodetic Survey, as was formerly the case, for the reason that the additional accuracy was not commensurate with the labor involved.

[2] See Wright and Hayford's "Treatise on the Adjustment of Observations," p. 259.

From (1) we have (if ε is a plus quantity),

$$\left(A-\frac{\varepsilon}{3}\right)+\left(B-\frac{\varepsilon}{3}\right)-\left(E+\frac{\varepsilon}{3}\right)=0, \quad . \quad . \quad . \quad . \quad (3)$$

and if ε' be a plus quantity,

$$\left(A-\frac{\varepsilon'}{4}\right)+\left(B-\frac{\varepsilon'}{4}\right)+\left(C-\frac{\varepsilon'}{4}\right)+\left(D-\frac{\varepsilon'}{4}\right)-360^\circ=0. \quad (4)$$

Values for A and B (A' and B') are obtained from (3) and substituted in (4), which is then solved.[1] Eq. (4) is the only one commonly used.

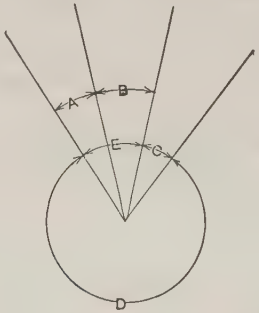


FIG. 237

Having satisfied the station equations we next eliminate the discrepancies disclosed by the geometrical conditions discussed in Art. 455, viz.:

1. That the sum of the angles of each triangle must equal 180° plus the spherical excess.

2. That the length of any side as computed from the known side should be the same by every route. These equations are called angle and side equations, respectively.

452. Spherical Excess. Unlike a plane triangle, the sum of the angles of a spherical triangle must need be in excess of 180°.

[1] This method is only approximate because the corrections obtained are successively obtained.

If r = the radius of the sphere and ε the spherical excess in seconds, then we have

$$\varepsilon = \frac{\text{area of } \Delta}{r^2 \sin 1''}.$$

It is not usual to take account of the spherical excess in computing a single triangle or quadrilateral; but if azimuth is to be carried over a series of triangles it is necessary that all the angles be spherical angles. In this discussion spherical excess will be omitted; but if it is desirable to introduce it, it may be inserted in Eqs. (1), (2), (3), pp. 615–6, the right members then becoming l_1+e_1, l_2+e_2, and l_3+e_3, where e_1 is the residual excess of the angle AOB over that of the angle DOC (being negative in this case), e_2 is the excess of angle BOC over that of the angle AOD, and e_3 is the spherical excess for the entire quadrilateral. The spherical excess may be taken as $1''$ for each 75 square miles of area, and this is to be divided equally among the angles of the figure. The formula for spherical excess E (in seconds) $= \frac{206000A}{r^2}$, where A is area in square miles, and r is radius of the earth in miles.

453. Adjustment of a Triangle. The finding and applying of the most probable corrections to the measured values of the angles of a system of triangulation is called *adjusting the system.* In the case of a single triangle with one known side and three measured angles, we have seen that there is but one equation of condition. If the three angles have been equally well observed, then it is most probable [1] that they are all equally in error, and hence this condition of highest probability gives us the probability equation

$$a' = b' = c', \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

which enables the corrections to be determined.

[1] That is, this relation is more probable than is any other *single* relation that can be assigned, but of course it is not more probable than *all* other cases combined. See also Appendix D on Errors.

Thus, let $A'+B'+C'-180°=a'+b'+c'=V$,

then from (4) we have

$$a'=b'=c'=\frac{V}{3}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where V is the error of closure of the triangle.

If the angles have not been equally well observed, as evidenced by their computed probable errors, or by use of instruments of different grades, the most probable corrections would be those which are proportional to the computed probable errors of the angles. Thus, if the probable errors of the angles A, B, C were 1′, 1″.5, 2″, and c_1, c_2, c_3, are the corrections to be applied to the measured values we would have

$$c_1 : 1 :: c_2 : 1.5 :: c_3 : 2,$$

or (1) $1.5c_1=c_2=0.75c_3$.
(2) $2 \quad c_2=1.5c_3$.
(3) $\quad c_1+c_2+c_3=4.5$.

From these equations the values for the correction to each angle can be easily computed.

Adjustment of a Quadrilateral

454. General Statement. The rigid least square adjustment of a quadrilateral is to be preferred to any substitute method like the following. The main advantage of this substitute is that it can be used by the many who are not familiar with least square methods. It gives substantially the same results as the more rigid method one when applied to a *single* quadrilateral.

455. The Geometrical Conditions. In the quadrilateral in Fig. 238 there are eight observed angles, A_1, B_2, B_3, C_4, etc. The geometrical conditions which must here be fulfilled are:

(*a*) The sum of all the angles of any triangle must be 180° plus the spherical excess and the opposite angles at the intersection of the diagonals must be equal.

(*b*) The computed length of any side, as *DC*, when obtained from any other side, as *AB*, through two independent sets of triangles, as *ABC*, *BDC*, and *ABD*, *ADC*, shall be the same in both cases.

The probability condition is that the set of corrections applied to the several angles shall be more probable than any other one of the infinite number of sets of corrections which would satisfy the other condition.

The condition given in (*a*) gives rise to the angle equations, and that given in (*b*) gives one side equation.

There are evidently eight unknown corrections to be determined.

456. The Angle-Equation Adjustment. In the quadrilateral *ABCD* we have four triangles in which all the angles have been observed, two sets of opposite angles where the other two angles of the corresponding triangles have been observed, and the quadrilateral itself in which all the angles have been observed; making, in all, seven geometric conditions to be fulfilled. Only three of these conditions are independent, however, since where any three independent conditions are fulfilled the remaining four are fulfilled also. Thus, a great variety of conditioned equations could be formed, but we will choose the three which give the simplest equations, viz.: that the opposite central angles shall be equal, and that the sum of all the angles of the quadrilateral shall be 360°. These give rise to the following equations:

If A_1, B_2, B_3, C_4, etc., be the observed angles, and l_1, l_2, and l_3 the residuals in the several combinations, due to erroneous determinations, then we have:

$$180° - (A_1 + B_2) - \{180 - (C_5 + D_6)\} = l_1,$$

$$\text{or} \quad -A_1 - B_2 \qquad + C_5 + D_6 \qquad = l_1. \quad (1)$$

Similarly, $\quad -B_3 - C_4 \qquad\qquad + D_7 + A_8 \qquad = l_2, \quad (2)$

and $\quad A_1 + B_2 + B_3 + C_4 + C_5 + D_6 + D_7 + A_8 - 360 = l_3. \quad (3)$

If the angles have all been equally well observed,—that is, if their mean observed values have equal credence,—then they are said to have equal weight, and any residual arising from any combination of angles should be distributed uniformly among the angles forming such combination.[1] Thus l_1 arises from the

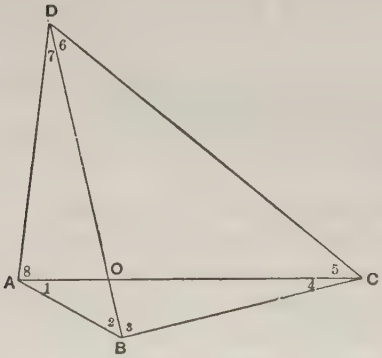


FIG. 238.

angles A_1, B_2, C_5, and D_6. This residual should therefore be divided equally between these four angles. When this is done we have

$$-\left(A_1 + \frac{l_1}{4}\right) - \left(B_2 + \frac{l_1}{4}\right) + C_5 - \frac{l_1}{4} + D_6 - \frac{l_1}{4} = 0. \quad . \quad . \quad (4)$$

[1] The errors in the mean observed values of the angles are supposed to result from the small incidental errors and approximations made in pointing, reading, etc.; in other words, they are supposed to be *errors of observation* and not *instrumental errors*, these latter having been eliminated by the method of making the observations. Since the sources of the errors of observation are the same for small as for large angles, it follows that they should be credited with equal portions of the aggregate error of any combination of angles, regardless of the size of the angles themselves. See Appendix D for further explanation.

Similarly

$$-\left(B_3+\frac{l_2}{4}\right)-\left(C_4+\frac{l_2}{4}\right)+D_7-\frac{l_2}{4}+A_8-\frac{l_2}{4}=0. \quad . \quad . \quad (5)$$

It is evident that if l_3 be now divided uniformly among the eight observed angles, it will not affect the two adjustments already made; neither have the adjustments already made affected the third condition, expressed by Eq. (3), since equal amounts have been added and subtracted. Hence these adjustments may be made in sequence as well as simultaneously, and we shall have for the total corrections for angle equations

$$\begin{aligned} &A_1-\left(\frac{l_3}{8}-\frac{l_1}{4}\right)+B_2-\left(\frac{l_3}{8}-\frac{l_1}{4}\right)+B_3-\left(\frac{l_3}{8}-\frac{l_2}{4}\right)+C_4 \\ &-\left(\frac{l_3}{8}-\frac{l_2}{4}\right)+C_5-\left(\frac{l_3}{8}+\frac{l_1}{4}\right)+D_6-\left(\frac{l_3}{8}+\frac{l_1}{4}\right)+D_7 \\ &-\left(\frac{l_3}{8}+\frac{l_2}{4}\right)+A_8-\left(\frac{l_3}{8}+\frac{l_2}{4}\right)-360^\circ=0. \quad . \quad . \quad . \quad . \quad . \quad (6) \end{aligned}$$

Or, if v_1, v_2, v_3, etc., be the total corrections to the several mean observed angles for angle equations, we have

$$\left.\begin{aligned} v_1=v_2&=-\frac{l_3-2l_1}{8}, \quad & v_5=v_6&=-\frac{l_3+2l_1}{8}; \\ v_3=v_4&=-\frac{l_3-2l_2}{8}, \quad & v_7=v_8&=-\frac{l_3+2l_2}{8}. \end{aligned}\right\} \quad . \quad . \quad . \quad (7)$$

457. The Side-Equation Adjustment. In the quadrilateral shown in the figure, let AB be the known side, and CD the required side, which is to be computed through two independent sets of triangles. Let A_1', B_2', B_3', etc., be the several angles corrected for angle conditions by the corrections found in Eq. (7).

As computed through the first set of triangles, we have

$$DC = \frac{BC \sin B_3'}{\sin D_6'} = \frac{AB \sin A_1' \sin B_3'}{\sin C_4' \sin D_6'} \quad . \quad . \quad . \quad (8)$$

Similarly,

$$DC = \frac{AD \sin A_8'}{\sin C_5'} = \frac{AB \sin B_2' \sin A_8'}{\sin C_5' \sin D_7'} \quad . \quad . \quad . \quad (9)$$

Whence

$$\frac{\sin A_1' \sin B_3'}{\sin C_4' \sin D_6'} = \frac{\sin B_2' \sin A_8'}{\sin C_5' \sin D_7'};$$

or

$$\frac{\sin A_1' \sin B_3' \sin C_5' \sin D_7'}{\sin B_2' \sin C_4' \sin D_6' \sin A_8'} = 1, \quad . \quad . \quad . \quad (10)$$

which is called the side equation.

It is evident that in any case where the angles have all been observed, even after they have been adjusted for the angle conditions, this equation will not hold true, the value of the left member being a little more or less than one. When put into the logarithmic form for computation, therefore, we will have

$$\log \sin A_1' + \log \sin B_3' + \log \sin C_5' + \log \sin D_7' - \log \sin B_2' - \log \sin C_4' - \log \sin D_6' - \log \sin A_8' = l_4, \quad (11)$$

where l_4 is the logarithmic residual due to erroneous observations.

We must now distribute this residual l_4 among the log sines according to the most probable manner of the occurrence of the errors which caused it. For a given small error, as 1″, in any angle, the effect on the log sine is measured by the logarithmic tabular difference for 1″ for that angle. This tabular difference varies for different angles, being large for angles near zero or 180°, and small for angles near 90°.

Let v_1', v_2', v_3', etc., be the corrections[1] to be made to the angles A_1', B_2', B_3', etc., for the side equation (11), and let d_1, d_2, d_3, etc., be the corresponding logarithmic tabular differences for 1″.

Now, the influences on l_4 of the small angular errors were in direct proportion to the tabular differences[2] of the corresponding log sines; therefore the corrections should be in proportion to the corresponding tabular differences.[3] In other words, the corrections are weighted in proportion to their tabular differences.[2] We therefore have the numerical relation:

$$v_1' : d_1 :: v_2' : d_2 :: v_3' : d_3, \text{ etc.},$$

or, paying attention to signs,

$$\frac{v_1'}{d_1} = -\frac{v_2'}{d_2} = \frac{v_3'}{d_3} = -\frac{v_4'}{d_4} = \text{etc.} \quad . \quad . \quad . \quad . \quad (12)$$

But since the log-sine correction is the angular correction multiplied by the tabular difference, and since the sum of these would equal l_4, we would have

$$v_1'd_1 - v_2'd_2 + v_3'd_3 - v_4'd_4 + v_5'd_5 - v_6'd_6 + v_7'd_7 - v_8'd_8 = -l_4. \quad (13)$$

From Eqs. (12) and (13) we are to find the side-equation corrections v_1', v_2', v_3', etc.

[1] By Eq. (11) it will be seen that the corrections to the angles having odd subscripts must be of opposite sign from those having even subscripts.

[2] It should be noted that the tabular differences of angles greater than 90° must be negative.

[3] To illustrate this principle more fully, let us suppose that for a change of 1″ in the angles A_1 and A_2 the corresponding changes in the log sines are 1′ for A_1 and 2 for A_2; then for a given error of 1 in log sin A_1 + log sin $A_2 = l$ there are two chances that it came from A_2 to one chance that it came from A_1, when these angles were equally well observed. If the error is to be divided between the angles A_1 and A_2, therefore, we should make the correction to A_2 twice as great as the correction to A_1, or $v_1' : v_2' :: d_1 : d_2$, whence $\frac{v_1'}{d_1} = \frac{v_2'}{d_2}$. The same reasoning would hold evidently for any number of angles, hence Eq. (12).

Dividing Eq. (13) by Eq. (12), term by term, we have

$$d_1^2+d_2^2+d_3^2+d_4^2+d_5^2+d_6^2+d_7^2+d_8^2$$
$$= -\frac{l_4 d_1}{v_1'} = +\frac{l_4 d_2}{v_2'} = -\frac{l_4 d_3}{v_3'} = +\text{etc.}$$

Whence we have, for the values of these corrections,

$$\frac{v_1'}{d_1} = -\frac{v_2'}{d_2} = \frac{v_3'}{d_3} = -\frac{v_4'}{d_4} = \frac{v_5'}{d_5} = -\frac{v_6'}{d_6} = \frac{v_7'}{d_7}$$
$$= -\frac{v_8'}{d_8} = -\frac{l_4}{\Sigma(d^2)}. \quad . \quad . \quad . \quad . \quad . \quad (14)$$

We have now found a set of corrections, v_1, v_2, v_3, etc. (Eq. (7)), for the angle equations, and a set of corrections, v_1', v_2', v_3', etc. (Eq. (14)), for the side equation; but they were determined independently and not simultaneously, and therefore, when successively applied, each set of corrections will disturb the former adjustment somewhat. Thus, if the corrections in Eq. (7) be first applied, and then those of Eq. (14), using the partially corrected angles in finding l_4 by Eq. (11), we would find Eq. (10) would be satisfied, but l_1, l_2, and l_3, in Eqs. (1), (2), and (3), would now not be zero when the newly adjusted angles were used. Another set of corrections v_1'', v_2'', v_3'', etc., might now be found by Eq. (7) for the adjusted angles A_1'', B_2'', B_3'', etc., and so on by successive approximations, using the corrections of Eqs. (7) and (14) alternately, until both sets of conditions were satisfied within the desired limits. It will usually be found, however, that the adjustment for side equation does not materially disturb that for angle equations. If the angles were all the same size, so that the corrections to the log sines would have equal weight, the first adjustment would remain undisturbed. In this case, the corrections for side equation would all be numerically equal, the odd and even subscripts having opposite signs. If the observed angles range between 30° and 60°, as they would

in a fairly symmetrical quadrilateral, then the errors of this approximation would be quite inappreciable.

458. Rigorous Adjustment for Angle- and Side-Equations. Let the angle-equation adjustments be applied as given by Eq. (7). Then, using these adjusted angles, let the corrections to the angles for side equation be so expressed that they shall not be inconsistent with the angle-equation conditions, whatever their values. This may be done by letting

$$\left.\begin{aligned} v_1' &= x_0 + x_1, & v_5' &= x_0 + x_3; \\ v_2' &= x_0 - x_1, & v_6' &= x_0 - x_3; \\ v_3' &= -x_0 + x_2, & v_7' &= -x_0 + x_4; \\ v_4' &= -x_0 - x_2, & v_8' &= -x_0 - x_4. \end{aligned}\right\} \quad . \quad . \quad . \quad (15)$$

Then, analogous to Eq. (13), we may write

$$\begin{aligned} &d_1(x_0+x_1) - d_2(x_0-x_1) - d_3(x_0-x_2) + d_4(x_0+x_2) \\ &\quad + d_5(x_0+x_3) - d_6(x_0-x_3) - d_7(x_0-x_4) + d_8(x_0+x_4) = -l_4; \end{aligned} \quad (16)$$

or

$$\begin{aligned} &(d_1 - d_2 - d_3 + d_4 + d_5 - d_6 - d_7 + d_8)x_0 + (d_1 + d_2)x_1 \\ &\quad + (d_3 + d_4)x_2 + (d_5 + d_6)x_3 + (d_7 + d_8)x_4 = -l_4, \quad . \quad . \quad . \quad . \end{aligned} \quad (17)$$

wherein l_4 is given by Eq. (11), and the d's are the tabular differences for one second for several log sines as before.

If, for simplicity, we write for the coefficients of x_0, x_1, x_2, x_3, x_4, respectively, C_0, C_1, C_2, C_3, and C_4, then (17) becomes

$$C_0x_0 + C_1x_1 + C_2x_2 + C_3x_3 + C_4x_4 = -l_4. \quad . \quad . \quad (18)$$

It now remains to find the values of x_0, x_1, x_2, x_3, and x_4, such that their combinations which make up the angle corrections as given in Eqs. (15) shall be the most probable.

To make (18) symmetrical with (15),[1] we may put it in the following form:

$$\left(\frac{C_0}{4}x_0+C_1x_1\right)+\left(\frac{C_0}{4}x_0+C_2x_2\right)+\left(\frac{C_0}{4}x_0+C_3x_3\right)$$
$$+\left(\frac{C_0}{4}x_0+C_4x_4\right)=-l_4. \quad \ldots \ldots \quad (19)$$

In the argument preceding the derivation of Eq. (12) it was found that the measured angles required to be corrected by a series of quantities (v_1', v_2', etc.), which quantities were found to be to each other as the tabular differences of the angles themselves; and Eq. (13), which is a summation of log sine corrections, shows that when Eq. (12) is true it is equivalent to saying that the most probable set of angle corrections (v_1', v_2', etc.) is that set which are respectively proportional to their numerical coefficients (d_1, d_2, etc.). This is, in fact, a general law of the theory of probabilities; and hence we say, in Eq. (19), that the most probable corrections (x_0, x_1, etc.) are those which are proportional to their several numerical coefficients, or, we may write at once, as a condition of the greatest probability:

$$x_0:x_1::\frac{C_0}{4}:C_1; \quad x_0:x_2::\frac{C_0}{4}:C_2, \text{ etc.};$$

or,

$$\frac{4x_0}{C_0}=\frac{x_1}{C_1}; \quad \frac{4x_0}{C_0}=\frac{x_2}{C_2}, \text{ etc.}$$

Therefore the condition of the highest probability gives

$$\frac{4x_0}{C_0}=\frac{x_1}{C_1}=\frac{x_2}{C_2}=\frac{x_3}{C_3}=\frac{x_4}{C_4}. \quad \ldots \ldots \quad (20)$$

[1] This is done in order to reduce the *weight* of x_0 to that of each of the other four x components of the v corrections in (15), as x_0 enters eight times in those equations while each of the others enters only twice.

Dividing (18) by (20), term by term, we have

$$\frac{C_0^2}{4}+C_1^2+C_2^2+C_3^2+C_4^2=-\frac{C_0 l_4}{4x_0}=-\frac{C_1 l_4}{x_1}$$
$$=-\frac{C_2 l_4}{x_2}=-\frac{C_3 l_4}{x_3}=-\frac{C_4 l_4}{x_4};$$

whence

$$\frac{4x_0}{C_0}=\frac{x_1}{C_1}=\frac{x_2}{C_2}=\frac{x_3}{C_3}=\frac{x_4}{C_4}=-\frac{l_4}{\frac{C_0^2}{4}+\Sigma(C^2)}.^1 \quad . \quad . \quad . \quad (21)$$

From Eq. (21) the side-equation corrections can be computed, which will not disturb the angle-equation adjustment, and which are the most probable corrections to the several angle values.

The second or rigid method will be found much more satisfactory than the method by approximations. The complete adjustment consists in applying to the mean measured values, the corrections from angle equations given by Eq. (7), and then applying to these corrected angles the corrections found by Eq. (21).

NOTE. The results obtained in the above adjustments are identical with those found by the method of least squares, and the fundamental principle by which they are obtained is also the same as that of least squares, viz.: that the arithmetic mean of properly weighted observations is the most probable result, and is identical with that obtained by making the sum of the squares of the corrections a minimum. For least-square solutions of this problem, see Clarke's "Geodesy," pp. 263–6, and Wright and Hayford's "Adjustment of Observations," pp. 303–8.

[1] Note that $\Sigma(C^2)$ does not include C_0^2.

EXAMPLE

The following is the numerical computation of the quadrilateral shown in the figure. AB is the known side, and CD is to be found. The mean observed values of the angles are given in the second column. The corrections for angle equations are given in the third column, and are the same for all three methods of solution given above. The spherical excess is here applied only to the quadrilateral as a whole, or to l_3 thus distributing it equally among the several angles. This is a common way of doing it, although if the excess is considerable, and the several triangles very unequal in size, as is the case here, it should be applied to the several triangles according to their size, as stated in Art. 452.

In columns 7 and 8, the corrections for side equation are worked out by the two methods given to show the relative results. Thus, from Eq. (14) we find the values of v_1', v_2', etc., for the first approximation. Applying these to the first corrected values in column 4, and again taking out the values of l_1, l_2, and l_3, for angle-equation conditions, we find they are not zero, but very small. It would probably be sufficient to work out a new set of angle corrections by Eq. (7), and then consider the quadrilateral adjusted. In this example, the final values thus found would then differ from the final values by the rigid adjustment by not more than $0''.2$ for any angle.

If we compute CD from AB, assuming the latter to be 25,000 feet in length, we obtain 88,670.9 feet in passing though the triangles ABC and BCD, while if we pass through the triangles ABD and ADC we obtain 88,671.1 feet, a discrepancy of 0.2 foot, and giving a mean value of 88,671.1 feet. The discrepancy of 0.2 foot in the two results by the rigid solution results from not computing the corrections beyond tenths of a second.

If simply a check on the final corrected values is desired, it may be obtained by adding them, when their sum should equal 360°+spherical excess, or by taking out the log sines and seeing if l_4' in Eq. (11) is zero. In this case it is not zero, but 9, resulting from not carrying out the corrections beyond tenths of seconds as mentioned above.

Adjustment of Larger Systems

459. Used only in Primary Triangulation. The simultaneous adjustment of all the angles in an extended system of triangles with one measured base which is taken as exact, is a very complicated problem. The methods of least squares must here be applied, so that a discussion of this problem belongs rather to a treatise on geodesy than to one on surveying. The

TABLE VIII

COMPUTATION OF ANGLE CORRECTIONS FOR A QUADRILATERAL ADJUSTMENT

Angles.	Observed Values.	Corrections for Angle Equations.	First Corrected Values.	Log. Sines.	Tabular Differences.	Corrections for Side Equation. First Approx. First Method.	Corrections for Side Equation. By Second Method, $\pm x_0$, $\pm x_1$, etc.	Final Corrected Angles Rigid Solution (Third Method).	Check. Log. Sines of the Corrected Angles.
A_1.....	30° 12′ 52″.4	−0″.6	51″.8	9.7017725	+36.2	+0″.4	+0″.6	30° 12′ 52″.4	9.7017747
B_3.....	88 55 19 .3	−0 .3	19 .0	9.9999232	+ 0.4	0 .0	+0 .6	88 55 19 .6	9.9999229
C_5.....	39 48 06 .4	−0 .8	05 .6	9.8062686	+25.2	+0 .3	+0 .5	39 48 06 .1	9.8062698
D_7.....	18 49 38 .3	−1 .2	37 .1	9.5088142	+61.8	+0 .6	+0 .4	18 49 37 .5	9.5088166
				0.0167785					9.0167840
B_2.....	47 31 20 .4	−0 .7	19 .7	9.8677851	+19.3	−0 .2	−0 .4	47 31 19 .3	9.8677843
C_4.....	13 20 30 .1	−0 .3	29 .8	9.3631540	+88.8	−0 .9	−0 .8	13 20 29 .0	9.3631467
D_6.....	37 56 06 .8	−0 .8	06 .0	9.7887106	+25.0	−0 .3	−0 .3	37 56 05 .7	9.7887098
A_8.....	83 26 13 .1	−1 .2	11 .9	9.9971443	+ 2.5	0 .0	−0 .6	83 26 11 .3	9.9971441
Sums ...	360 00 06 .8	−5 .9	00 .9	0.0167940	261.2	−0 .1	0 .0	360 00 00 .9	9.0167849

$l_4 = -155$ $\qquad$ $l'_4 = -9$

Spher. ex. = 0.9
$l_3 = +5''.9$
$l_2 = +2\ .0$
$l_1 = +0\ .4$

Hence from Eq. (7) we find v_1, v_2, v_3, etc.

FIRST METHOD.

$-\frac{l_1}{\Sigma(d^2)} = +\frac{155}{14756} = +0.01$, whence, from Eq. (14) we find v_1', v_2', etc., and then from Eqs. (1), (2), and (3), find l_1', l_2', and l_3' to be: $l_1' = -0''.2$; $l_2' = +1''.3$; and $l_3' = -0''.1$.

SECOND METHOD.

$C_0 = 44.2$
$C_1 = 55.5$
$C_2 = 89.2$
$C_3 = 52.2$
$C_4 = 64.3$

$\frac{1}{4}C_0^2 = 488$
$C_1^2 = 3080$
$C_2^2 = 7957$
$C_3^2 = 2725$
$C_4^2 = 4134$

$$\frac{\Delta x_0}{44.2} = -\frac{l_4}{\frac{C_0^2}{4} + \Sigma(C^2)} = \frac{155}{18384} = 0.0083$$

$x_0 = +0.1$
$x_1 = +0.5$
$x_2 = +0.7$
$x_3 = +0.4$
$x_4 = +0.5$

Whence from Eq. (15) find v_1', v_2', etc.

adjustments of a triangle and of a quadrilateral will be found sufficient for all secondary work, or such as is intended to serve only for topographical or geographical purposes. Especially is this true if the stations be so selected that the observed lines will form a series of quadrilaterals. The adjustment of these quadrilaterals by the rigid method given above gives nearly as good results as could be obtained by reducing the work as a single system. For a discussion of the least-square methods of adjustment of an extended system of triangles the student is referred to "Primary Triangulation of the U. S. Lake Survey," being Professional Papers, Corps of Engineers U. S. A., No. 24; Report of the U. S. Coast and Geodetic Survey for 1875; Clarke's "Geodesy"; and especially to Wright's "Adjustment of Observations," p. 259.

The facility and accuracy with which base lines may now be measured by means of long steel tapes will result in actually measuring many more lines than has heretofore been done, and so errors from angular measurements will not be allowed to accumulate to any great extent. Indeed, geodetic methods have already been materially influenced by this new method of accurate measurement.

460. Computing the Sides of the Triangles. After the angles of the system are adjusted, the sides of the triangles are computed by the ordinary sine ratio for plane triangles. If the system consists of simple triangles, then one side is known and the other two sides computed from it. In this case there is no check on the computation except what the computer carries along with him, or what may be obtained from a duplicate computation.

If the system be made up of a series of quadrilaterals, then the line which is common to two successive quadrilaterals is computed through two sets of triangles from the previous known side. Thus if the quadrilateral of Fig. 238 be one of a series the lines in common being AB and CD, then AB is computed in duplicate from the previous quadrilateral, and the mean of

the two results taken. In the triangle *ABD* compute *AD*, and then in the triangle *ADC* compute *DC*; in the triangle *ABC* compute *BC*, and then in the triangle *BCD* compute *DC* again. There are thus obtained two independent values of *DC*, as computed from *AB*. If the adjustment had been exact these values would have agreed exactly, but the adjusted angles were computed only to the nearest second, or tenth of a second; hence the two values of *DC* will agree only to a corresponding exactness. If the system be composed of quadrilaterals and the adjustment be made to the nearest second, then the two values of *DC* would probably differ in the fifth or sixth significant figure. If the adjustment be made to the nearest tenth of a second, and a seven-place logarithmic table be used, then the two values of *DC* should begin to differ in the sixth or seventh place. Of course the adjusted values are not the *true* values of the angles, but simply the *most probable* values.[1] If the angles were not accurately measured the adjusted values may still be considerably in error, but any such errors would not prevent the two values of *CD* from agreeing, since this agreement is one of the conditions which the adjustment is made to satisfy. The disagreement between the two computed values of *CD* comes only from the *inexactness* of the computed corrections to the angles, an angle, like a length, being an incommensurable quantity, and hence some degree of approximation is necessary in its expression. If the true computed values of *CD* differ by more than the amounts above signified, then it is probable that an error has been made in the computation.

Such computations as the above are best arranged as in the table on following page.

Here the angles are written in such an order that when the arithmetical complement (a. c.) of the subtractive log. sine is taken out, the three logs. will be in convenient relative positions for adding. This will become evident on a study of the table.

[1] See Appendix D for discussion of probable error, etc.

In computing the second and fourth triangles it is evidently unnecessary to write the log. distances again in their proper lines, since they already stand conveniently just above. It will be noted that in the above form the diagonals of the quadrilateral were not computed.

TABLE IX

Stations.	Adjusted Angles.	Log. sines.	Log. Distances.	Distances in Feet.	Sides.
D_7	18° 49′ 37″.5	9.5088166	3.8989072	7,923.32	Base AB
A		a. c. (.4911834)			
B_2	47 31 19 .3	9.8677843	4.2578749	18,108.19	AD
C_5	39 48 06 .1	9.8062698			
D		a. c. (.1937302)			
A_8	83 26 11 .3	9.9971441	4.4487492	28,102.77	CD
C_4	13 20 29 .0	9.3631467	3.8989072	7,923.32	Base AB
B		a. c. (.6368533)			
A_1	30 12 52 .4	9.7017747	4.2375352	17,279.67	BC
D_6	37 56 05 .7	9.7887098			
C		a. c. (.2112902)			
B_3	88 55 19 .6	9.9999229	4.4487483	28,102.71	CD

When a series of triangles in a chain are to be computed from a base line and the adjusted angles, whether these form a single row, as in I, p. 531, or a double row, making complete quadrilaterals, as in III of the same page, it is customary to compute all the sides; and then, in case of a series of quadrilaterals, the average length (or log.) of the sides common to two adjacent quadrilaterals is used as a new base for the next one. Thus, if all the sides in the above quadrilateral were to be computed, and if the side CD be computed through the two diagonals instead of through the outer sides, the computations would be arranged as follows:

Table X

Stations.	Adjusted Angles.	Log. sines.	Log. Distances.	Distances in Feet.	Sides.
C	13° 20′ 29″.0	9.3631467	3.8989072	7,923.32	Base *AB*
		a. c. (.6368533)			
B	136 26 38 .9	9.8382579	4.3740184	23,660.20	*AC*
A	30 12 52 .4	9.7017747	4.2375352	17,279.67	*BC*
D	18 49 37 .5	9.5088166	3.8989072	7,923.32	Base *AB*
		a. c. (.4911834)			
A	113 39 03 .7	9.9618982	4.3519888	22,449.09	*BD*
B	47 31 19 .3	9.8677843	4.2578749	18,108.19	*AD*
C	53 08 35 .1	9.9031638	4.3519888	22,449.09	*BD*
		a. c. (.0968362)			
D	37 56 05 .7	9.7887098	4.2375348	17,279.65	*BC*
B	88 55 19 .6	9.9999229	4.4487479	28,102.69	*CD*
D	56 45 43 .2	9.9224146	4.3740184	23,660.20	*AC*
		a. c. (.0775854)			
C	39 48 06 .1	9.8062698	4.2578736	18,108.14	*AD*
A	83 26 11 .3	9.9971441	4.4487479	28,102.69	*CD*

In this form the logs. of the angles and of the sides opposite to them are placed on the same line, the known side and its angle opposite being always written on the first line, and the a. c. of this log. sine taken out as shown. The log. distances of another side of any triangle is then the sum of its corresponding (opposite) log. sine, the a. c., and the log. distance of the known side. These sums are readily taken without copying off the figures, and they are written at once in the log. distance column. The original observed, but uncorrected, mean angles are commonly given in addition to the corrected angles, but these have been omitted here as not essential to explain the form. In the above computation it so happens that the log. side *CD* is the same in each case. Had they differed, as in the previous tabulation, the mean log. would be used as the log. of this known side for the next set of triangles.

461. Computing the Geodetic Positions. After the angles of the system are adjusted, and the sides of the triangles computed, we have the plane angles and linear distances from point to point in the system. It now remains to compute the latitudes and longitudes of the several stations, and the azimuths of the lines.

The following formulas, though not exact, are quite sufficient when the sides of the triangles do not exceed ten or fifteen miles in length:[1]

NOTATION

Let L = latitude of the known point;
L' = latitude of the unknown point = $L + dL$;
M = longitude of the known point;
M' = longitude of the unknown point = $M + dM$;
Z = azimuth of the unknown point from the known, counting from the south point in the direction S.W.N.E.;
Z' = azimuth of the known point from the unknown, or the back azimuth = $Z + dZ + 180°$;
K = length in metres of line joining the two points;
e = eccentricity of the earth's meridian section;
N = length of the normal, or radius of curvature of a section perpendicular to the meridian of the middle latitude, in metres;
R = radius of curvature of the meridian in metres.

Then we have in terms of the length and azimuth of a given line, in seconds of arc, when the distances are given in metres,

$$\left.\begin{aligned} L - L' = -dL &= BK\cos Z + CK^2 \sin^2 Z + Dh^2; \\ M - M' = +dM &= AK \sin Z \sec L'; \\ Z + 180° - Z' = -dZ &= dM \sin \tfrac{1}{2}(L + L'). \end{aligned}\right\} \quad . \quad . \quad (1)$$

[1] For a summarized derivation of these formulas for computing the L M Z's from triangulation data, together with extended tables of factors used, see Report U. S. Coast and Geodetic Survey, 1894, Appendix No. 9.

where $L' = L + dL$;

$M' = M + dM$;

$Z' = Z + dZ + 180°$.

$$A^1 = \frac{1}{N' \text{ arc } 1''}; \quad B = \frac{1}{R' \text{ arc } 1''}; \quad C = \frac{\tan L'}{2R'N' \text{ arc } 1''};$$

$$D = \frac{\frac{3}{2}e^2 \sin L' \cos L' \sin 1''}{(1 - e^2 \sin^2 L')}; \quad L_m = \text{mean latitude} = \frac{L + L'}{2};$$

and h = value of first term in right member $= BK \cos Z$. *Careful attention must be paid to the signs of the Z functions.*

TABLE XI

OF *L M Z* COEFFICIENTS

Latitude.	Log A + 10.	Log B + 10.	Log C + 10.	Log D + 10.
30°	8.5093588	8.5115729	1.16692	2.3298
31	3363	5054	.18416	.3382
32	3134	4368	.20108	.3460
33	2901	3669	.21772	.3532
34	2665	2959	.23409	.3597
35	8.5092425	8.5112239	1.25024	2.3656
36	2182	1510	.26617	.3709
37	1936	0772	.28193	.3756
38	1687	8.5110027	.29753	.3797
39	1437	8.5109275	.31299	.3833
40	8.5091184	8.5108517	1.32833	2.3863
41	0930	7755	.34358	.3888
42	0675	6989	.35875	.3907
43	0419	6220	.37386	.3921
44	8.5090162	5449	.38894	.3930
45	8.5089904	8.5104677	1.40400	2.3933
46	9647	3905	.41906	.3932
47	9390	3134	.43414	.3924
48	9133	2364	.44926	.3912
49	8878	1598	.46443	.3894
50	8.5088623	8.5100835	1.47968	2.3871

Logarithmic values of the coefficients A, B, C, and D are given in the above table for each degree of latitude from 30°

[1] A is to be evaluated for L. Log. sin $1'' = 4.6855749$.

to 50°. By the aid of this table the *LMZ*'s are readily found. These tabular values are computed from the constants of Clarke's spheroid. In this we have

Equatorial semidiameter = 6,378,206 metres
Polar semidiameter = 6,356,584 "

Whence the ratio of the semidiameters is $\frac{294.98}{293.98}$.

Clarke's value of the metre has been taken, which is

1 metre = 39.37000 inches.

The difference of azimuth of the two ends of a line is due to the convergence of the meridians passing through its extremities, this convergence, as seen from the last of Eqs. (1), being equal to the difference of longitude into the sine of the mean latitude.

When the sides of a system of triangulation have been computed, and the azimuths of the lines are desired from the several stations, the successive differences of latitude and longitude are first computed, and from these the azimuths of the lines, using Eqs. (1). If the longitude is unknown, the longitude of the first station may be assumed without affecting the accuracy of the computed relative positions. The last of Eqs. (1) gives the difference betewen the forward and back azimuth of the line joining the two stations. This difference being applied, with the proper sign, gives the azimuth of the first station as seen from the second. But when the azimuth of one line from a station is known, the azimuths of all other lines from that station are found from the adjusted spherical angles at that station, provided the spherical excess had been deducted or allowed for, in the adjustment. If no account has been taken of spherical excess, the error in azimuth accumulates in working eastward or westward, and soon becomes appreciable.

For any other station the azimuth correction is again found for the line joining this station with a station where azimuths have been computed, which when applied gives the azimuth of

this line as taken at the forward station, whence the azimuths of all the lines from this station are known, and so on. The following examples should make the methods of computing clear:

EXAMPLE. Given the L and M of Tomales Bay, and Azimuth Tomales Bay-Bodega to compute L and M of Bodega and the back azimuth Bodega-Tomales Bay. U. S. C. and G. S. Report, 1894.

Z	Tomales Bay to Sonoma.....	244°	08′	30″.9
L	Bodega to Sonoma.....	83°	14	34 .7
Z	Tomales Bay to Bodega.....	160°	53′	56″.2
dZ			2	01 .9
180°		180°		
Z'	Bodega to Tomales Bay...	340	51′	54″.3

L	38°	10′	47″.982	Tomales Bay. ..	M	122°	56′	47″.301
dL	+	7	28 .222	K=14,626.8 m...	dM	+	3	16 .993
L'	38	18′	16″.204	Bodega.....	M'	123°	00′	04″.294

$\frac{1}{2}(L+L')$ 38° 14′ 32″		K	4.1651480	K^2	8.3033		
		$\cos Z$	9.9754055n	$\sin^2 Z$	9.0297	h^2	5.303
		B	8.5109892	C	1.3003	D	2.380
1st term	−448.″273	h	2.6515427n		8.6603		7.683
2d and 3d terms	+ .051				.046		.005
$-dL$	−448 .222						

K	4.1651480		
$\sin Z$	9.5148602		
A	8.5091611	dM	2.29445
$\sec L'$	0.1052810	$\sin \frac{1}{2}(L+L')$	9.79168
	2.2944503		2.08613
dM	+196″.993	$-dZ$	+121″.9

NOTE. Take out A from table for values of L'.

462. The Inverse Problem.[1] The above formulas can also be used for solving the inverse problem, viz.: Given the latitude and longitude of two points, to find their distance and mutual azimuths, i.e., given L, M, L', M' (or L, L', dM) to find K, Z, and Z'. For the direct solution put

$$\left.\begin{aligned} K\cos Z = x = -\frac{1}{B}[dL + c\cdot y^2 + D(dL)^2 + E(dL)y^2 + E\cdot Cy^4] \\ K\sin Z = y = \frac{dM\cos L'}{A}; \end{aligned}\right\} \quad (2)$$

$$\tan Z = \frac{y}{x} \quad \text{and} \quad K = x\sec Z = y\operatorname{cosec} Z.$$

Divide $dM = K\sin Z\cdot A\sec L'$ by the first term for dL, $h = K\cos Z\cdot B$, whence we get

$$\tan Z = \frac{dM\cdot B}{A\sec L' h}.$$

This would give the azimuth at once provided we knew h, since dM is known. We therefore compute the smaller terms for the differences of latitude in order to obtain h by subtracting them from the known value dL. The only addition to the usual form is the term $\log\tan Z = \log(K\sin Z) - \log(K\cos Z)$. The value of K will best be taken from the term h when $\sin Z$ exceeds $\cos Z$ and from the term dM when the reverse is true. When K is large additional corrections must be made. The following example will explain the method:

[1] Taken from Appendix 9, Report U. S. Coast and Geodetic Survey for 1894.

PROBLEM Given the L M of Bodega and Tomales Bay and azimuth Tomales Bay-Bodega to compute the length of the line and the back azimuth Bodega-Tomales Bay.

Z	Tomales Bay to Bodega.....	160°	53′	56″.2
dZ		—	2	01 .9
180°		180°		
Z'	Bodega to Tomales Bay	340	51′	54″.3

L	38°	10′	47″.982	Tomales Bay....	M	122°	56′	47″.301
dL	+	7	28″.222	K = 14,626.8 m...	dM	+	3	16 .993
L'	38°	18′	16″.204	Bodega.....	M'	123°	00′	04″.294

$\frac{1}{2}(L+L')$ 38° 14′ 32″	K	4.1405535n	K^2	7.3600	h^2	5.303
	$\cos Z$		$\sin^2 Z$			
	B	8.5109892	C	1.3003	D	23.80
	h	2.6515427n		8.6603		7.683
1st term	−448.273			0.046		0.005
2d and 3d terms	+ .051					
$-dL$	−448.222					

K	3.6800082n			$K \sin Z$	3.6800082
$\sin Z$					
A	8.5091611	dM	2.29445	$K \cos Z$	4.1405535n
[1] cos L	0.1052810	$\sin Lm^2$	9.79168	$\tan Z$	9.5394547n
Ar. Comp.					
	2.2944503		2.08613	Z	160° 53′ 56″.2
				$\sin Z$	9.5148602
dM	+196.″993	$-dZ$	+121.″9	K	4.1651480

[1] The sec L can be used instead. [2] $Lm = \frac{1}{2}(L+L')$.

REFERENCES ON FIGURE AND SIZE OF THE EARTH

1. On the Position and Shape of the Geoid. Bulletin U. S. Coast and Geodetic Survey, No. 48, 1888. R. S. Woodward.
2. Measurement of Arcs. U. S. Coast and Geodetic Survey Report for 1893, p. 360.

3. Comparison of Local Deflections of Plumb Lines. U. S. Coast and Geodetic Survey Report, 1879, p. 110–23.
4. The Figure of the Earth and Isostasy. Mr. J. F. Hayford. U. S. Coast and Geodetic Survey, 1909, and its supplement, 1910.
5. Is the Ocean Surface Depressed? R. S. Woodward. Science, Vol. VII, No. 177, pp. 570–1. Year, 1886.
6. Mathematical Theories of the Earth. R. S. Woodward. Proc. A. Asso. A. S., Vol. XXXVII, pp. 49–69. See also Bull. Phil. Soc. Washington, Vol. IX, p. 580, 1888.
7. The Russo-Swedish Expedition for Measuring an Arc of the Meridian. Rev. Gen. des Sciences, Dec. 15, 30, 1902.
8. The Curvature of the Geoid in the Simplon Tunnel. Comptes Rendus, Sept. 10, 1906.
9. See also a Bibliography by Prof. James H. Gore, Appendix 8, Report U. S. Coast and Geodetic Survey, for 1902.

REFERENCES ON TRIANGULATION

1. The Invention of Triangulation. S. Wellisch. Zeitschrift d. Oesterreichischen Ingineur u. Arch. Vereins, May 19, 1899.
2. Early History of Instruments and Art of Observing. C. S. Howe. Jour. Asso. Eng. Soc., March, 1897.
3. Old Methods of Observing. U. S. Coast and Geodetic Survey Report, 1893, p. 286.
4. Methods of Eliminating Error of Graduation in Repeating Instruments. U. S. Coast and Geodetic Survey Report, 1893, p. 419.
5. Primary Triangulation, etc., of the U. S. Government Surveys. H. M. Wilson. Eng. News, April 4, 1901.
6. Station Marks. U. S. Coast and Geodetic Survey Report, 1892, p. 330.
7. Table of Foreign Triangulation. Report U. S. Coast and Geodetic Survey, 1893, p. 278.
8. Short Method of Adjusting Triangulation. Inst. C. E., Vol. CI for 1890, pp. 249–56.
9. Triangulation and Traverse Work in New York City. E. H. Holden. Eng. Record, Oct. 5, 1907.
10. Resurvey of St. Mary's River. E. E. Haskell. Report Chief Eng., 1897, U. S. A., Part VI, p. 4073.
11. Triangulation along the 98th Meridan. J. F. Hayford. Report U. S. Coast and Geodetic Survey, 1903, Appendix 4. Also Report, 1905, Appendix 5.
12. Triangulation in Kansas. J. F. Hayford. Report U. S. Coast and Geodetic Survey, 1902. Appendix 3.

BASE LINES

1. The Invar (Nickel Steel) Tapes on the Measurement of Six Primary Base Lines. O. B. French. Proc. Am. Soc. C. E., Oct., 1907.
2. Recent Rapid Base-line Measurements on U. S. Coast and Geodetic Survey. A. L. Baldwin. Proc. Eng. Club of Phila., Oct., 1903; also Report U. S. Coast and Geodetic Survey, 1901.
3. Base-line Measurements on 98th Meridian. J. F. Hayford. Eng. News, Vol. XLVIII, pp. 162–64.
4. Errors for Contact and Transfer. U. S. Coast and Geodetic Survey Report, 1889, p. 485.
5. Schott's New Compensating Base Apparatus. Report U. S. Coast and Geodetic Survey, 1882, pp. 107–138.
6. Tapes on the Holton Base, Indiana. U. S. Coast and Geodetic Survey Report, 1892, p. 413; also Trans. Am. Soc. C. E., 1893. R. S. Woodward.
7. Jäderin's Report on Steel Tapes and Wires. U. S. Coast and Geodetic Survey Report, 1893, pp. 125–64. See also Trans. Am. Soc. C. E., Vol. XLVIII, p. 123.
8. Use of Steel Tapes on U. S. Coast and Geodetic Survey. Eng. News, 1893, Vol. II, p. 91; 1894, Vol. I, p. 95.
9. Tension Apparatus. U. S. Coast and Geodetic Survey. J. F. Hayford. Trans. Am. Soc. C. E., Vol. XLVIII, p. 115. Univ. of Wisconsin Apparatus. L. S. Smith. Same volume, p. 130.
10. Measurement of Base Lines, Old and New Methods. Eng., London, Aug. 31, 1906.
11. Remeasurement of the Mackinac Base Line with Tapes One Kilometre Long. Eng. Record, Aug. 13, 1898.
12. Mass. Institute of Technology Tape Apparatus. Eng. Record, Feb. 27, 1904.
13. Foreign Base Apparatus. Report U. S. Coast and Geodetic Survey, 1893, pp. 254–59. Number of Base Lines, p. 262.
14. The New Duplex Base Apparatus. U. S. Coast and Geodetic Survey Report, 1897, p. 737. Its Use on the Salt Lake Base, p. 753.
15. Testing Steel Tapes at the National Bureau of Standards. H. T. Wade. Eng. News, Aug. 13, 1908.
16. Quick Processes in Modern Geodesy. C. E. Guillaume. Sci. Am. Supplement, June 17, 1905.
17. The Correction to a Spring Balance Held Horizontally. Proc. Inst. C. E., London, Vol. CLXI.
18. Six Primary Base Lines Measured with Steel and " Invar " Tapes. Mr. O. B. French. Report U. S. Coast and Geodetic Survey for 1907, Appendix 4.

CHAPTER XV[1]

FIELD ASTRONOMY

463. Fundamental Concepts. This chapter is intended to present, as succinctly as possible, certain astronomical methods of determining azimuth, latitude, and time, with surveying instruments and to such degree of precision as may be required for ordinary survey purposes, or in connection with a secondary triangulation. For the most part these methods are adapted from Comstock's "Field Astronomy for Engineers," to which reference may be made for the theory upon which they are based. Their application, which alone is contemplated here, requires an understanding of certain technical terms, some of which, e.g., meridian, horizon, altitude, azimuth, etc., may be presumed familiar to the reader, while to others his attention is especially invited as follows:

The Celestial Poles are the parts of the sky toward which the earth's rotation axis points. The north celestial pole is vertically above the north terrestrial pole, etc.

The Celestial Equator is a great circle of the sky midway between the poles. Every part of it stands vertically over some point of the earth's equator and, through the earth's rotation upon its axis, during the course of each day any given point of the celestial equator is made to pass over every point of the terrestrial equator.

An Hour Circle is a great circle of the sky passing through the poles. Every hour circle stands vertically above some meridian

[1] Prepared by Professor George C. Comstock, Director of Washburn Obser vatory, University of Wisconsin.

of the earth and, by means of the earth's rotation, it is in each twenty-four hours made to pass over each and every terrestrial meridian.

The Celestial Meridian. That hour circle that at any moment stands vertically above the geographical meridian of any place upon the earth's surface is called the celestial meridian of that place. It is a great circle of the sky passing through the poles and the zenith. The axis of the earth and the direction of the vertical both lie in its plane.

The Declination of any celestial body, e.g., sun or star, is the angular distance of its centre from the celestial equator, counted as + when north of the equator, − when south of it. Compare with this definition the similar relation of altitudes to the horizon and latitudes to the terrestrial equator.

The Hour Angle of any celestial body is the angle between its hour circle and the celestial meridian; it is reckoned from the meridian, positive toward the west. Compare this definition with the similar relation of azimuth to the meridian or of longitude with respect to its assumed prime meridian.

The Right Ascension of any celestial body is the angle between its hour circle and the hour circle passing through an arbitrarily assumed origin called the vernal equinox. Compare with terrestrial longitudes. The vernal equinox takes the place of the Greenwich meridian save that right ascensions are reckoned from the equinox along the equator toward the *east* until the star's hour circle is reached, while longitudes are commonly measured from the Greenwich meridian toward the west.

The declination combined with either right ascension or hour angle constitutes a pair of co-ordinates which fix the position of a star with respect to celestial reference points, just as altitude and azimuth are commonly used to fix its position relative to terrestrial reference points. Owing to the rotation of the earth upon its axis the altitude and azimuth of a star continually change in a somewhat complicated manner, and the values which they have at any one moment become something else at the next moment.

The hour angle of a star also changes continuously but simply, having a uniform increase equal to the earth's rate of rotation, 15° per hour. Right ascensions and declinations of stars are unaffected by the earth's rotation; they remain practically unchanged from day to day and may therefore be printed in a table for permanent reference, although they are subject to small variations (in the case of the sun, large variations) which must be taken into account when precision is desired.

These several co-ordinates, together with sundry other useful quantities, will be represented hereafter by the following symbols which may profitably be committed to memory:

Right ascension, α (alpha)	Azimuth, A
Hour angle, t	Altitude, h
Declination, δ (delta)	Mean solar time, M
Geographical latitude, ϕ (phi)	Sidereal time, θ (theta)

Mean Solar Time is the kind of time kept by ordinary clocks and watches and is defined as the hour angle of the mean sun, to distinguish it from true solar time, which is the hour angle of the real sun. In consequence of the earth's orbital motion, the real sun appears annually to make a complete circuit about the sky with a varying speed that makes it impossible for a well-made clock to keep pace with it. Astronomers have therefore invented a fictitious body called the mean sun, that moves around the equator with uniform speed, completing the circuit once each year and keeping always as nearly abreast of the real sun as is consistent with uniform motion. The angle between the hour circles of the real sun and mean sun is called the *Equation of Time*, and its value from day to day is given in the almanac. Use it in order to pass from one kind of solar time to the other, e.g., when it is desired to find mean solar time from an observation of the real sun.

Sidereal Time is the hour angle of the vernal equinox. When the equinox is "on the meridian" of any observer, i.e.,

directly above it, the sidereal time is 0 hours; when, three hours later, the hour circle of the equinox makes an angle of 45° with the same meridian the sidereal time is three hours, etc. At this moment last named, every star whose right ascension is 45° (or three hours as right ascensions are commonly reckoned) will be upon the meridian above named and we may establish the following two equivalent propositions:

(*a*) The right ascension of any star is equal to the sidereal time at which it crosses the meridian.

(*b*) The sidereal time at any moment is equal to the right ascension of any star that is then on the meridian.

464. Relation of Sidereal to Mean Solar Time. Sidereal time gains uniformly upon mean solar time at the rate of one day per year and the two therefore agree only once in each year. If we represent by V the particular instant at which this agreement occurs we shall find it to vary considerably in different years (on account of the leap year cycle, etc.), but for many years to come it will be given with approximate accuracy by the formula

$$V = \text{March } 22.229 + \frac{L}{4} - \frac{N}{128}, \quad . \quad . \quad . \quad . \quad (1)$$

where the first term in the value of V is a date reckoned in decimals of a day from noon, Central Standard Time, i.e., time of the meridian 90° west of Greenwich; L is the number of the given year in the leap-year cycle, and N is the number of the year in the twentieth century, e.g., for 1908, a leap year; $L=0$, $N=8$; for 1909, first year after a leap year, $L=1$, $N=9$, etc. At the date V we have $\theta = M$, and at any other date, which we represent by D,

$$\theta - M = a(D - V), \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where D must be expressed in days and decimals of a day, the same as V, and the coefficient a, which is the daily gain of sidereal

upon mean solar time, has a value that may be expressed by any one of the following relations:

$$a = 3^{m}\ 56^{s}.555$$
$$\log a = 0.595783 \text{ (minutes)}$$
$$a = 4\left(1 - \frac{1}{70}\right) \text{ (minutes, approximate).}$$

465. Longitude and Time. Since time is always measured by some hour angle, one side of which is a meridian, it follows that the hour angle and resulting time will depend upon the particular meridian chosen for this purpose and will be greater the farther east this meridian lies, the exact amount of this excess being one hour for each 15° of longitude. In comparing different times, they must first be reduced to a common meridian by applying a longitude correction of this kind, e.g., if it be desired to obtain the declination of the sun at the instant of its transit over one's own meridian, using for this purpose an almanac constructed for the Greenwich meridian, the local time that is to be used in the interpolation from the almanac must first be reduced to Greenwich time by adding the longitude, expressed in hours, minutes, and seconds. Note in this connection that while the civil date begins at midnight, the date used for astronomical purposes begins at noon, twelve hours later than the civil date, and that its hours are numbered 0 to 24 instead of up to 12 only. For illustration, let it be required to find, in the kind of time in which V is expressed, the equivalent of the local mean solar time, July 13, $9^{h}\ 2^{m}\ 30^{s}$ A.M., at a place whose longitude is 100° 10′ west of Greenwich. Changing to astronomical reckoning, we have in place of the above date, July 12, $21^{h}\ 2^{m}\ 30^{s}$. To reduce this local time to Central Standard Time there must be added the difference of longitude between the local and standard meridians, viz.:

$$100^{\circ}\ 10' - 90^{\circ}\ 00' = 10^{\circ}\ 10' = 0^{h}\ 40^{m}\ 40^{s},$$

and this furnishes us the required standard time, July 12, 21^h 43^m 10^s.

466. Conversion of Time. To illustrate the use of Eqs. (1) and (2), let it be required to find the local sidereal time corresponding to the local mean solar time above given, viz., 1910, July 13, 9^h 2^m 30^s A.M. Corresponding to this date we have, in Eq. (1), $L=2$, $N=10$ from which to find the value of V, and using the equivalent time above found for the given date and changing the hours, minutes, seconds into decimals of a day we obtain the required value of D. The remainder of the computation is as shown below:

D	July 12.905	$a(D-V)$	442.573 (minutes)
V	March 22.651	$\theta-M$	7^h 22^m 34^s
$D-V$	112.254 (days)	M	21 2 30
$\log(D-V)$	2.050202	θ	4 25 4 (after subtracting 24^h)
$\log a$	0.595783		
Sum	2.645985		

Where a nautical almanac is available its data and methods may be used to solve the above problem somewhat more conveniently and accurately than is possible by the method here employed. See page 559 of the American Ephemeris for 1910, where the above problem is solved in a different way and the sidereal time found to be 4^h 25^m $3^s.60$.

467. Finding the Stars. The sidereal time in connection with latitude furnishes a key to the aspect of the sky. At any given moment all stars whose right ascensions are equal to the sidereal time will then stand upon the celestial meridian. Stars of greater right ascension will be to the east of the meridian and stars of lesser right ascension to the west. A star whose declination is equal to the latitude of the place of observation will, in crossing the meridian, pass through the zenith, a star of smaller declination will pass south of the zenith (in the northern hemisphere), a star of declination greater than the latitude will pass north of the zenith. A star of declination equal to 90° would

not move at all, but would remain permanently fixed on the meridian at an altitude equal to the latitude of the place of observation. While no bright star of this kind exists, the pole star, Polaris, and a number of fainter stars have declinations so nearly 90° that they continuously circle about the point above defined (the celestial north pole) without ever getting far from it. The observer should learn to know Polaris by sight, but it is not necessary thus to know any other star, since for the purposes of this chapter they need be observed only when on or near the meridian, and may then be found with an engineers' transit as follows: Place the instrument "in the meridian," i.e., with the horizontal axis level and pointing due east and west. Set the telescope to the altitude the star will have when on the meridian and the star will be in the field of view when the sidereal time is equal to its right ascension. The required altitude of the star and the sidereal time of its culmination must be computed in advance from one of the following formulas:

When star's δ is less than ϕ.....	$h=90°-\phi+\delta$	$\theta=\alpha$	(3)
When star's δ is between ϕ and 90°..	$h=90°+\phi-\delta$	$\theta=\alpha$	
When star is below pole.....	$h=-90°+\phi+\delta$	$\theta=\alpha+12^h$	

In the case last named the star is said to be at lower culmination, i.e., in the act of crossing that part of the meridian that lies between the pole and the horizon, and the sidereal time of this culmination is twelve hours different from that of the transit across the upper half of the meridian.

Having set for a star some minutes before it is due on the meridian, it will frequently be convenient without waiting for its arrival to turn the telescope *slowly* in azimuth toward the star, to make sure that it is at hand, and this should be done to find any star that is to be observed a little way off the meridian, as is frequently convenient. But note that Polaris cannot in general be found in this way. Sight over the telescope at it, or compute its altitude by the table (page 671), and having set the telescope to this altitude sweep *slowly* for it in azimuth.

Observations of stars are most conveniently made in the twilight when it is dark enough to see the star and light enough to see the wires of the instrument. They may, however, be equally well made by night if the wires are illumined by a light thrown obliquely down the telescope tube. Use a bull's-eye or electric hand lamp and hold it as nearly on line with the star as is possible without obscuring the latter. A drop of wax or candle grease placed on the centre of the object glass considerably improves this illumination without in any way injuring the performance of the telescope. Bright stars are much easier to observe than faint ones and the brightness of each star given in the almanac, or in the tables of this book, is indicated by a number called its magnitude. The smaller the number, the brighter the star, and in using an engineer's transit or similar instrument, stars whose magnitudes are represented by a number much greater than 4 should be avoided.

468. Tables. Time Stars. It is shown above in Eq. (3) that to find any given star, its co-ordinates α, δ, must be given, and there must also be known, at least approximately, the observer's latitude, ϕ, and the sidereal time, θ. These latter quantities are to be found from observations made at the place in question, but the co-ordinates of all the brighter stars are given in the nautical almanac and may be interpolated, *from the volume for the current year*, for the particular moment at which their values are desired. The American Ephemeris and Nautical Almanac is a volume of some 700 pages annually issued by the Navy Department at Washington as a repository for astronomical data of all kinds, and is an indispensable reference book for the astronomer. Beginning with the volume for 1912 there are given for the use of the surveyor, in addition to the data hitherto furnished, extensive tables for use in determining azimuth and latitude from observations of Polaris, which, however, like most other tables of the Almanac, are good only for the current year and must be renewed annually. For the convenience of the reader who may not have access to the current volume of the Almanac there are

appended to this chapter certain tables of a more permanent character which for most purposes included within the scope of this work will render him independent of the Almanac, in case of need, while prepared to use its advantages when they are at hand. By means of these tables azimuth, latitude, and time may be determined without using the Almanac. The first of these,[1] Table A, contains the right ascensions and declinations of a limited number of southern stars, together with a column showing the mean solar time at which they cross the meridian. All of these quantities change from year to year, and even from day to day, and provision is therefore made for adapting the tabular numbers to any time and place for which their values may be required. The first three columns of the table, containing a reference number, prefixed to the name and magnitude of the star, seem to call for little explanation. The fourth column shows for each star its right ascension on that day of the year 1910 which is printed in column 10 under the heading, Date. Column 5 contains for each star the amount by which its right ascension increases in a hundred years, and for any year other than 1910 the right ascension may be found by taking the variation in direct proportion to the number of years elapsed since 1910, e.g., for Star No. 1 the right ascension on Nov. 30, 1920, will be

$$\alpha = 0^h\ 39^m\ 8^s + \frac{10}{100} 301^s = 0^h\ 39^m\ 38^s.$$

While the right ascension thus found relates only to the date given in column 10, its variation is so slow that without introducing error greater than 1^s, it may be used in connection with any observation made within three months preceding or two months following that date. This is about the limit of accuracy of the numbers themselves, since the progressive increase of the right

[1] See Comstock's "Field Astronomy," Art. 87.

ascensions with the lapse of time is not strictly uniform, as was assumed in the above example, but is subject to an oscillatory change that will make all of the right ascensions about one second less than their computed values in the years 1909, 1927, and one second greater than the computed values in 1918 and 1937, etc. Columns 6 and 7 give the declinations of the stars and their centennial variations in form entirely comparable with the right ascensions, but it should be noted that the variations marked + imply that the star is moving north and therefore is numerically diminishing a negative declination. The corrections are to be added algebraically.

Column 8 shows the local mean solar time at which, under certain standard conditions, the star crosses the meridian on the date given in column 10. These limiting conditions may be removed and the values made universally applicable by adding to the tabular number a correction interpolated from Table B with the longitude as argument. The numbers contained in Table B are accurate to the nearest second although expressed only in integral minutes. They may be interpolated to fractions of a minute as may also the numbers a_0, b_0 of Table P. These corrections are always to be added to Table A. On the average, a star comes to the meridian each night $3^m\ 55^s.91$ *earlier* than on the preceding night and having found the mean solar time of the star's transit on the date given in column 10, we may find it for any other date by applying the proper multiple of this period as a correction. These multiples are very easily formed when the daily gain is assumed equal to 4^m–4^s, which is sufficiently exact for most purposes.

For example, we may find the mean time of transit of Star No. 1 over the meridian 45° west of Greenwich on Dec. 25, 1910, as follows:

M.T.	$8^h\ 4^m.0$	From column 8.
Correction	$+3.5$	From Table B.
Corr'd. M.T.	$8^h\ 7^m.5$	For Nov. 29.
$26(4^m–4^s)$	$1^h\ 42^m.3$.	December 25–November 29=26 days.
Local M. T.	$6^h\ 25^m.2$.	For December 25.

Like the right ascensions, the numbers M.T. of Table A increase slowly with the lapse of time and their values for years subsequent to 1910 should be found by allowing for the centennial variation given in column 9, and expressed in minutes of time in order to adapt it to the unit used in column 8.

469. Tables—Polaris Orientation.[1] All the stars of Table A are near the equator, cross the meridian south of the zenith, and as they move rapidly are well adapted for use in determining time by comparing the tabular α, or M.T., with the time shown by some watch or clock at the instant of the star's transit across the meridian. For the determination of azimuth, the slower-moving stars near the pole, especially Polaris, are more advantageous, and Tables C, D, E, P, page 671, are designed to facilitate observations of this star. It has above been pointed out that if Polaris were exactly at the pole it would serve as a permanent meridian mark which might also be used for determinations of latitude by measuring its altitude above the horizon. While the actual case is not quite so simple as the one thus suggested, since Polaris is in fact more than a degree distant from the pole and ceaselessly changes both its direction and distance from the latter, the star may be made to furnish both azimuth and latitude by applying to the observations certain corrections whose nature is shown in Fig. 240. Here P is the pole, about which the star S daily revolves in a circular orbit, its position at any moment being determined by the hour angle t, which is reckoned from the top of the orbit toward the west. N is the north point of the horizon and A is the foot of the vertical circle passing through the star. The arcs AN and PB, which we represent by a_0 and b_0 respectively, represent the difference of azimuth and altitude between Polaris and the pole and, corresponding to certain standard conditions, the values of a_0 and b_0 are shown in Table P with the hour angle t as argument. This argument runs through its

[1] See Comstock's "Field Astronomy," Arts. 32, 33, for a more complete exposition of the method.

entire set of values every twenty-four hours and therefore depends upon the time of day as well as upon the day and year for which a_0 and b_0 are required, but its value at any moment is readily obtained through the formula,

$$t = M + D + Y, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

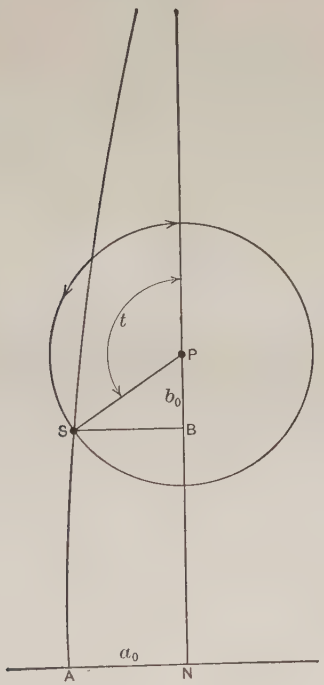


FIG. 240.—Polaris Orientation.

where M denotes the given local mean solar time; D is given in Table D, from which it is to be interpolated for the given date, taken to the nearest tenth of a day and expressed in Central Standard Time; and Y is to be taken from Table C for the given year, without interpolation. To facilitate the interpolation of

D a table of proportional parts is given in the margin of the page; use as the argument of this table the given days, and fraction of a day, of the month.

The tabular values of a_0 and b_0, corresponding to the argument t, thus found, pertain to certain standard conditions of time and place not usually realized in practice, and they must be adapted to the actual conditions by multiplying them by the factors F_1 and F respectively. The latter of these factors is given in Tables C and D,

$$F = F' + F'',$$

and the former must be computed from the relation,

$$F_1 = 0.7612F \sec h, \qquad \log 0.7612 = 9.8815,$$

where h denotes the star's altitude. Since h never differs much more than a degree from the observer's latitude, it is feasible and convenient to prepare such a table as the following, in which are entered all the auxiliary quantities required in Table P for the year 1910 and in latitude 43°. Such a table when once constructed will serve for several weeks or even for an entire year without change, and a blank form of this kind, to be filled in pencil by the reader for use in his own latitude is printed at the foot of Table P.

SAMPLE LOCAL ORIENTATION TABLE

Year.	1910 (July).	h	F_1
Y	$+11^m$	42°	1.031
F	1.007	43	1.048
R	$1'$	44	1.066

Having found t by means of Eq. (4), we may now find the true altitude, and azimuth of Polaris by the formulas,

$$\left.\begin{aligned} h &= \phi + Fb_0, \\ A &= 180° + F_1 a_0 \end{aligned}\right\}. \qquad \ldots\ldots \quad (5)$$

The apparent altitude of Polaris, h', will be slightly greater than the true altitude on account of a number of minor causes, among which the atmospheric refraction holds the chief place. Their combined effect is shown in Table E, under the heading R, and the value of this correction, corresponding to an altitude of 43°, is entered in the little table given above. It is to be used only in determining the latitude from an observed altitude of Polaris.

A determination of any one of the three quantities, azimuth, latitude, or time is very greatly facilitated by a knowledge of the other two, and in general at least an approximate knowledge of all three should be acquired before seeking to make an accurate determination of any one of them. When all three are unknown a beginning of this determination may conveniently be made as follows:

470. Azimuth, Time, Latitude, Polaris Orientation Method.[1] When a precision no greater than a major fraction of a minute of arc is required the quickest and simplest method of determining azimuth, time, and latitude is furnished by the tables, page 671. It is presumed that an engineer's transit, or an equivalent instrument, is available and that the observer is provided with a watch which he should set, as nearly as may be, to local mean solar time, although even a considerable error in this time (half an hour) will be of little consequence. Select in advance some hour and minute at which it will be convenient to make the observations and prior to that time compute from the tables the azimuth that Polaris will have at the chosen minute. The altitude h, required for this purpose, may itself be computed from the tables, if the latitude is approximately known, or it may be derived from a vertical circle reading taken upon Polaris five or ten minutes before the chosen moment of observation. Set the horizontal circle to read the computed azimuth of the star and at the moment for which the computation was made

[1] See Comstock's "Field Astronomy," Arts. 32, 33.

bring the star behind the intersection of the cross wires without changing the reading of the horizontal circle, i.e., use the lower motion. The instrument is now said to be oriented, i.e., its verniers read true azimuths, and, turning on the upper motion, if the line of sight be directed at any object whatever the azimuth of the object will be given by the corresponding reading of the horizontal circle. Mark the position of the instrument and take a reading to some fixed object in order to secure a permanent azimuth line.

If the vertical circle be read before the telescope is turned from Polaris, the resulting altitude h', if free from instrumental error, will furnish the latitude through the formula,

$$\phi = h' - (Fb_0 + R), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where Fb_0 and R have the values explained in the preceding article.

Instrumental error may be eliminated by making a second entirely similar observation in the reversed position of the instrument, but any error in the watch will not be thus eliminated, and unless the watch is known to be correct within a minute or two, in relation to local time, it will be necessary to determine and allow for the amount of its error. This may be done by observing one of the stars of Table A, or a similar star selected from the almanac, as follows: Turn the instrument in azimuth (do not transit the telescope about the horizontal axis) until the horizontal circle reads 0°; the line of sight will then be in the meridian and the star in question may be found by the methods set forth on p. 643. Observe by the watch the time, which we shall designate by T, at which this star in its diurnal motion crosses the line of sight of the telescope, and we shall then have ΔT, the correction required by the watch, given through the relation,

$$\Delta T = \text{M.T.} - T, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where M.T., given by Table A, or computed from the almanac, is the mean time of the star's transit over the meridian. If ΔT

proves to be of sensible amount it should be applied as a correction to the time of observation of Polaris and the azimuth and latitude recomputed with the corrected time. In all strictness any change thus introduced into the azimuth should require a correction to ΔT itself, since a false orientation would result in the observation being made at one side of the meridian instead of upon it; but this error can usually be overlooked unless the error of the watch is of abnormal magnitude.

The value of ΔT found above is the correction required to reduce the watch time to local mean solar time. The correction required to reduce it to local sidereal time may also be obtained through the formula,

$$\Delta' T = \alpha - T, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where α is the star's right ascension and the accent denotes that $\Delta' T$ refers now to sidereal time.

The application of this method is shown in the following record and reduction of a time, latitude, and azimuth determination for which the observer selected 10 P.M. of the given date, July 24, 1910, as a convenient hour for observation. In the preliminary computations the latitude is assumed to be about 43° and the longitude about 90° west of Greenwich.

PRELIMINARY COMPUTATIONS FOR POLARIS ORIENTATION

Time star	No. 20, η Serp.	b_0	$-24'$
M. T., Aug. 24, 1910	$8^h\ 6^m.8$	F	1.007
31 (4^m–4^s)	2 1 .9	Fb_0	−24
M. T., July 24	10 8 .7	ϕ	43 0
δ	− 2° 55′	h	42 36
$90°-\phi$	47 0	a_0	+86.7
h	44 5	F_1	1.041
Azimuth star	Polaris	$F_1 a_0$	+90.3
Assumed M	10^h 0^m	A	181°30.′3
" D	6 29		
" Y	11		
" t	16 40		

FIELD NOTES OF ORIENTATION OBSERVATION

July 24, 1910. At Station K.

Object.	Watch, T.	Horizontal Circle.	Vertical Circle.
Polaris	$10^h\ 0^m\ 0^s$	181° 30′ 20″	42° 41′
η Serp.	$10^h\ 7\ 7$	0 0 0	44 1
Mark	.....	1 7 30	.....

The instrument having been oriented upon Polaris, as set forth above, the record gives immediately as the azimuth of the mark, 1° 7′ 30″. The correction of the watch, referred to local mean solar time, is the computed M.T., minus the observed time,

$$\Delta T = 10^h\ 8^m.7 - 10^h\ 7^m.1 = +1^m.6,$$

and since at the time of this observation Polaris was increasing its azimuth only 7′ per hour, the effect of this error in the watch may be ignored, or the azimuth of the mark may be summarily increased by an amount, x, taken proportional to the error of the watch and the rate of change of the star's azimuth,

$$x : 7' :: 1^m.6 : 60^m. \qquad x = +0'.2.$$

Similarly, we find for the correction of the watch referred to sidereal time, $\alpha - T$,

$$\Delta' T = 18^h\ 16^m\ 42^s - 10^h\ 7^m\ 7^s = +8^h\ 9^m\ 35^s.$$

For the latitude we find from the observed altitude of Polaris,

h	42° 41′
Fb_0	−24
R	+ 1
ϕ	43 4

By the methods above employed all of the quantities, azimuth, latitude, time, should be determined well within one minute of arc.

471. Latitude by Circum-Meridian Altitudes of the Sun. When the position of the meridian is approximately known, e.g., through such a determination as that of the preceding article, or even by a compass bearing, with the magnetic declination taken into account, the following method may be used for a daylight determination of latitude:

When the sun bears 1° east of the meridian observe the altitude of its upper edge. Reverse and relevel the instrument and when the sun bears 1° west of the meridian observe the altitude of its lower edge. Calling the mean of these altitudes h, and the declination of the sun as interpolated from the almanac, δ, the latitude will be given by the relation,

$$\phi = C + \delta - h$$

where C is to be interpolated, with the observed altitude as argument, from the following table:

CIRCUM-MERIDIAN ALTITUDES OF SUN

h	C	h	C
10°	90° 5′.5	50°	90° 0′.4
20	90 2 .4	60	90 0 .2
30	90 1 .3	70	90 0 .1
40	90 0 .7	80	90 0 .1

The tabular number, C, includes the effect of atmospheric refraction, parallax, and the sun's deviation from the meridian. While intended primarily for observations of the sun the table may equally well be used for the reduction of similar observations of a star, provided the numbers C for all values of h less than 70° are diminished by 0′.1. An illustration of its application is shown in the reduction of the following observations:

FIELD NOTES. CIRCUM-MERIDIAN ALTITUDES OF SUN
March 25, 1910

Sun's Azimuth.	Circle.	Vernier *A*.	Vernier *B*.	Altitude Level.
359°	Left	48° 23′ 15″	48° 22′ 30″	8.5 7.6
1	Right	48 52 0	48 49 0	7.0 9.0

The readings of the altitude level show that in the mean the error of level was insignificant, and the other instrumental errors are mainly eliminated by the reversal. We have the following reduction, in which the observer's longitude was assumed to be 90° west of Greenwich, in interpolating the sun's declination:

C	90° 0′.4
δ	+1 40 .6
$C+\delta$	91 41 .0
h	48 36 .7
ϕ	43 4 .3

The instrument employed in the above observation was a good engineer's transit with vertical circle reading to 30″, the sun's azimuths were determined with a compass and the observations were made by projecting the images of sun and wires upon a piece of cardboard held back of the eye-piece. The true latitude is known to have been 43° 4′ 37″ (see Art. 474) and the difference between this number and that found above is fairly typical of what may be expected from the method.

472. Azimuth by Elongation of a Circum-Polar Star.[1] A star is said to be at elongation when, having moved to the east or west as far as its circular path about the pole will permit, it pauses for a moment before starting to return to the meridian. Its azimuth when at elongation, A_E, is given by the very simple formula,

$$\sin A_E = \cos \delta \sec \phi. \quad . \quad . \quad . \quad . \quad . \quad (10)$$

[1] See Comstock's "Field Astronomy," Art. 37.

The declination, δ, must be obtained from the almanac, and if the latitude, ϕ, is fairly well known, e.g., to the nearest minute, observations of such a star at elongation may be made to give a much more precise determination of azimuth than can be obtained through the orientation tables. To this end let the horizontal angle between the star and the mark whose azimuth is desired be measured at least once, and preferably several times, in each position of the instrument, e.g., use the method of repetitions and observe, to the nearest minute, the time at which each pointing to the star is made. If all these pointings be assumed as made exactly at the moment of elongation, the azimuth of the mark, A_M, becomes

$$A_M = A_E + H, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where H denotes the mean value of the measured angle between mark and star, considered positive when the mark is east and negative when it is west of the star. The beginner is recommended to confine his observations of this kind to Polaris and to determine the time of its elongation, E, from the following formula and tables in which Y and D are to be obtained from Tables C and D, p. 671, and M_1 is to be taken from the following special table, with the observer's latitude as argument:

LOCAL MEAN TIMES OF ELONGATIONS OF POLARIS

ϕ	M_1.	
	West Elongation.	East Elongation.
0°	6h 0m	18h 0m
20	5 58	18 2
40	5 56	18 4
60	5 52	18 8

$$E = M_1 - (D + Y). \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

A part of the accuracy contained in the observations is lost, through the assumption above made, that they may be treated as if the star were at elongation at the time of each pointing. It is at elongation for a moment only, and before and after that moment it is nearer to the meridian than the computed azimuth at elongation. It is to take into account this changing position of the star that the times of observation are to be recorded and if we represent each such time by the symbol T and represent by n the total number of pointings made, it may be shown that the star's motion before and after elongation is very approximately taken into account by adding to the declination, as interpolated from the almanac, a correction given in seconds of arc by the formula,

$$\Delta\delta = +\frac{4''}{n}\Sigma\left(\frac{T-E}{10}\right)^2, \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where the symbol Σ represents the sum of the several quadratic terms formed by subtracting from each observed T the computed time of elongation, E, and squaring one-tenth of the resulting difference, expressed in minutes.

We illustrate the above formulas by applying them to an observation of Polaris near eastern elongation made on July 23, 1910, at a place whose latitude is assumed to be 42° 30′. The observation comprised six pointings on both mark and star, the observed times of the latter pointings being as shown below. We proceed to find first the time of elongation, as follows:

D	$6^h\ 26^m$
Y	11
M_1	18 4
$D+Y$	6 37
E	11 27

A preliminary examination having shown that the elongation came near midnight the value of D is interpolated for the date July 23.5. The declination of Polaris as interpolated from the

almanac is $\delta = 88° 49' 21''.4$, to which we have to apply a correction, $\Delta\delta$, computed from the observed times as follows:

Observed T.	$\frac{T-E}{10}$	$\left(\frac{T-E}{10}\right)^2$	
$11^h\ 20^m$	$0^m.7$	0.5	
11 25	0 .2	0.0	
11 31	0 .4	0.2	
11 37	1 .0	1.0	$\Delta\delta = \frac{4}{6}'' \times 5.3 = 3''.5$
11 39	1 .2	1.4	
11 42	1 .5	2.2	
		5.3	

Applying the correction thus determined we find a declination of $88° 49' 24''.9$, which corresponds to a fictitious star that at elongation has an azimuth equal to the mean of the azimuths of Polaris at the several times of observation. With this declination and the mean value of the measured angle Eqs. (10) and (11) furnish the azimuth of the mark as follows:

δ	$88° 49' 24''.9$
$\cos\delta$	8.31241
$\sec\phi$	0.13237
$\sin A_E$	8.44478
A_E	$181° 35' 45''$
H	$-6\ 26\ 19$
A_M	$175\ 9\ 26$

The precision attainable by the above method depends very largely upon the care with which the errors of level are eliminated. On reversing, relevel and bring the azimuth bubble back to the same position in the tube that it occupied in the first half of the series. If this is carefully done an azimuth may be thus determined with an engineer's transit within a few seconds of arc.

If an almanac is not at hand the declination of Polaris may be found within $2''$ or $3''$ by the formula,

$$\delta = 90° - 4208''(F' + F''), \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where F' and F'' are to be taken from the Tables C and D, p. 671.

473. The Two-Star Method—Time and Azimuth.[1] A serious disadvantage of the Polaris elongation method of determining azimuth is the inconvenient hour at which the observations must frequently be made, e.g., 3 A.M. in the month of June. By use of the following method this inconvenience may be avoided at the expense of a little additional computing and results in no way inferior to those found at elongation may be obtained at any convenient hour.

Measure the horizontal angle between Polaris and the mark whose azimuth is required, exactly as in the case of elongation observations, save that the time of each pointing should be observed to the nearest second and that the altitude of the star must also be measured with at least approximate accuracy, i.e., to the nearest minute or two of arc. Similarly measure the horizontal angle between the mark and a southern star, e.g., one of those contained in Table A, noting the times as accurately as possible and measuring the altitude exactly as in the case of Polaris. For both stars eliminate the level error by the procedure indicated in the preceding article. Let H_1 represent the mean of the measured angles between Polaris and the mark and H_2 the mean of the angles between the southern star and the mark. Similarly, let T_1, T_2, h_1, h_2 represent the mean of the observed times and altitudes of the two stars respectively. If the observed times T were true sidereal times, or could be turned into true sidereal times by adding to them the known correction of the watch, $\Delta' T$, we might compute an azimuth of the mark, A, from the observation of either star, since the altitude and sidereal time fully determine the stars' azimuth, and each star should give the same result for A. If the assumed $\Delta' T$ is appreciably wrong, the stars will give different results for the azimuth of the mark and a new value of $\Delta' T$ must be found that will reconcile these discordant azimuths. Let us represent by U the best preliminary estimate that can be made with respect to the cor-

[1] See Comstock's "Field Astronomy," Art. 38.

rection required to eliminate the error of the watch on sidereal time, e.g., such a value of $\Delta' T$ as might be obtained through the orientation method and Eq. (9), and let x be the unknown correction that added to U will give the true $\Delta' T$; we proceed to find x and the true azimuth of the mark, A_M, as follows: From the observation of each star compute an azimuth of the mark, A, and a coefficient, C, showing how much this computed azimuth is vitiated by the error x; using the following formulas that apply equally to each star:

$$\left.\begin{aligned} t &= T + U - \alpha, \\ \sin A_* &= \cos\delta \sec h \sin t, \\ C &= G \cos\delta \sec h \cos t, \\ A &= A_* + H. \end{aligned}\right\} \quad \ldots\ldots \quad (15)$$

The horizontal angle, H, between mark and star is to be considered essentially negative whenever the mark is west of the star. The logarithm of the factor G that occurs in the third equation may be taken from the following table, whose argument is the computed azimuth of the star, A_*. Observe that the sign of G is always minus for a southern star and plus for a northern star.

THE G FACTOR, TWO-STAR METHOD

A_*		$\log G$	A_*	
360°	0°	−1.1761+	180°	180°
359	1	.1762	179	181
358	2	.1764	178	182
357	3	.1767	177	183
356	4	−1.1772+	176	184

The computation made by Eq. (15) gives for the northern star and southern star respectively the equations,

$$\left.\begin{aligned} A_1 &= A_M + C_1 x, \\ A_2 &= A_M + C_2 x, \end{aligned}\right\} \quad \ldots\ldots \quad (16)$$

i.e., the computed azimuth of the mark is equal to its true azimuth plus the effect of the unknown x, and these equations may be solved to determine A_M and x. The coefficient G has been so chosen that if A_1, A_2, be expressed in degrees, minutes, and seconds of arc the solution will give the value of x in seconds of time, and the correction of the watch relative to sidereal time will then be

$$A'T = U + x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Owing to the gain of sidereal upon mean solar time, the value of U for a watch keeping mean solar time continually increases at the rate of about ten seconds per hour, and due heed must be paid to this gain if the observations of the northern and southern stars are not strictly simultaneous, by assigning to the later observations, values of U correspondingly greater than are taken for the earlier ones. It is well to render the observations of the two stars nearly simultaneous by making half the observations of Polaris precede and half follow those of the southern star, or vice versa, as is done in the following example, in which the number of observations is reduced to a minimum, one pointing on each object in each position of the instrument. Increased precision may, of course, be obtained by increasing the number of observations and, with a small instrument, by using the method of repetitions. Heed must be taken in this case not to extend unduly a set of observations, particularly if the star is more than a degree distant from the meridian in azimuth, since in this case a correction analogous to Eq. (13) will be required by the observations. This correction will not usually be needed if the set of pointings to be reduced by Eq. (15) does not extend over a longer time than ten or twelve minutes. The application of these methods is shown in the following record and reduction of a set of observations made with an engineer's transit:

FIELD NOTES—AZIMUTH AND TIME—TWO-STAR METHOD

April 23, 1907. At Station E.

Azimuth of Asylum Light.

Object.	Circle.	Watch, *T*.	Horizontal Circle. Ver. *A*.	Horizontal Circle. Ver. *B*.	Vertical Circle.	Levels. W.	Levels. E.
Mark.....	R	$9^h\ 0^m$	187° 54′ 0″	53′ 45″	.....	W.	E.
* No. 13.....	R	$2\ 16^s.5$	358 14 15	14 15	32° 41′		
Polaris.....	R	5 11	179 5 40	5 30	42 10	25.7	9.3
Polaris.....	L	10 14	359 8 30	8 25	42 3	9.1	25.6
* No. 13.....	L	$14\ 8.5$	181 41 20	40 45	32 39		
Mark.....	L	9 17	7 54 30	54 10	.....		

Comparison of watch with standard time:

C. S. T $9^h\ 23^m\ 40^s$ Watch $9^h\ 25^m\ 37^s.0$

REDUCTION OF OBSERVATIONS—TWO-STAR METHOD

Star.	Polaris.	* No. 13.
δ	88° 48′ 31″	−14° 16′ 41″
α	$1^h\ 24^m\ 46^s$	$11^h\ 14^m\ 42^s.3$
T	9 7 42	9 8 12.5
$T+U$	11 13 42	11 14 12.5
t	9 48 56	− 0 0 29.8
t	147 14 0	− 0 7 27
h	42 6	32 39
log sin t	9.7334	7.3359n
cos δ	8.3179	9.9864
sec h	0.1296	0.0747
cos ϕ	9.9247n	0.0000
G	1.1762	1.1761n
sin A	8.1809	7.3970n
log C	9.5484n	1.2372n
A *	179° 7′ 52″	359° 51′ 25″
H	8 47 5	172 3 33
A	187 54 57	187 47 52

By Eq. (16),

$$187° 54′ 57″ = A_M - 0.35x$$
$$187\ 47\ 52 = A_M - 17.27x$$

and solving these equations we find

U	$+2^h\ 6^m\ 0^s.0$
x	$+25^s.1$
$d'T$	$+2^h\ 6^m\ 25^s.1$
A_M	187° 55′ 6″

The results thus obtained may be presumed correct within a few seconds of arc, e.g., 10″, and this precision may be increased by multiplying the observations.

474. Latitude from Difference of Meridian Altitudes.[1] Eq. (3) may readily be made to furnish the relation,

$$2\phi = (\delta_1 + \delta_2) + (h_1 - h_2), \quad . \quad . \quad . \quad . \quad . \quad (18)$$

where δ and h, are the declination and *meridian altitude* of any star and the subscripts 1 and 2 denote respectively that, of the two stars here considered, the first crosses the meridian north of the zenith, the second south of it. When the northern star is at lower culmination, i.e., in the act of crossing that part of the meridian that lies below the pole, the above equation must be written,

$$2\phi = (180^\circ - \delta_1 + \delta_2) + (h_1 - h_2). \quad . \quad . \quad . \quad . \quad (19)$$

These equations are the basis of precise latitude determinations, which are thus reduced to a measurement of the difference of two altitudes, h_1, h_2, neither of which need itself be known, save very roughly, provided their difference is accurately measured. A special instrument, called a zenith telescope, is the favorite device for this purpose, but any instrument provided with a vertical circle may be used, and it is worthy of special note that the gradienter often found on engineers' transits is excellently well adapted to this purpose. Make a pointing in altitude on one of the stars, read the gradienter head and then *without loosing the altitude clamp* make a pointing on the second star, revolving the instrument in azimuth and turning the gradienter screw until the telescope is brought to the right altitude. If R_1 and R_2 denote the readings of the gradienter head corresponding to these

[1] See Comstock's "Field Astronomy," Arts. 70 to 73.

two pointings and k is the value of a revolution of the gradienter, i.e., the angle through which the telescope is moved by one complete turn of the gradienter, we have, barring the effect of refraction,[1]

$$h_1 - h_2 = (R_1 - R_2)k.$$

An approximate value of the constant k may be found through readings of the horizontal or vertical circle of the instrument, but an accurate value is best obtained through the latitude observations themselves, as is shown in the example below.

Since the altitudes of stars are affected in varying degree by the atmospheric refraction, the observed difference of altitude, however measured, must be freed from this effect in order to make it equal to the $h_1 - h_2$ of the equations. The amount of this refraction correction is directly proportional to the measured difference of altitude, and when this difference is 1° the correction may be taken from the following table whose argument is the mean of the altitudes of the stars. Double the tabular number when the difference of altitude is 2°, etc., and always apply the correction so as to numerically increase the measured difference.

DIFFERENTIAL REFRACTION

Altitude.	Correction for 1°.	Altitude.	Correction for 1°.
10°	32″.	50°	1″.7
20	8 .3	60	1 .3
30	3 .9	70	1 .1
40	2 .4	80	1 .0
50	1 .7	90	1 .0

[1] To secure the full measure of precision attainable with a gradienter its screw must be calibrated. See Comstock's "Field Astronomy," Art. 72, for the method of doing this.

In turning from the northern to the southern star, or vice versa, the instrument must be revolved about the vertical axis and any deviation of this axis from the true vertical should be carefully determined by means of a spirit level, i.e., the altitude level of the instrument. If the level readings show that this axis deviates from the true zenith toward the south by an amount b, then b must be added to the latitude furnished by Eqs. (18) or (19). Subtract b when the axis tips toward the north.

A useful application of Eqs. (18) and (19) may be made to observations of Polaris taken at any hour whatever and combined with observations made upon a southern star when its azimuth is 180° greater than that of Polaris,[2] e.g., point on Polaris, then turn 180° in azimuth and observe the southern star when it reaches the position thus defined. For such an observation we must write in Eq. (18), in place of the declination of each star, a fictitious declination defined by the equation,

$$\cot d = \cot \delta \cos t, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

and find in terms of it,

$$2\phi = (d_1 + d_2) + (h_1 - h_2). \quad . \quad . \quad . \quad . \quad . \quad (21)$$

In order to compute the hour angles of the stars, required in Eq. (20), it will be necessary to note the times of observation, T, and to obtain at least an approximate value of the correction of the watch $\Delta' T$, referred to sidereal time; we shall then have,

$$t = T + \Delta' T - \alpha.$$

[2] Or, more generally, when its distance from the meridian measured along a great circle is numerically equal to the corresponding distance of Polaris from the meridian.

We proceed to apply these equations to the following observations which are a part of the set given on p. 654, and there partially reduced.

FIELD NOTES—GRADIENTER LATITUDE

July 24, 1910. At Station K.

Object.	Watch, T.	Horizontal Circle.	Vertical Circle.	Gradienter.	Levels.
					N. S.
Polaris	$10^h\ 0^m$	181° 30′ 20″	42° 41′	10.108	7.3 7.5
η Serp.	10 11 26	1 30 20	44 0	13.947	7.8 7.3
					+0.15

The value of a revolution of the gradienter is provisionally assumed to be $k = 20'.5$ and the value of a level division to be 12″.

REDUCTION OF GRADIENTER LATITUDE

Star	Polaris	η Serp.	$R_1 - R_2$	3.839
T	$10^h\ 0^m\ 0^s$	$10^h\ 11^m\ 26^s$	$(R_1 - R_2)k$	− 1° 18′ 42″
ΔT	+ 8 9 34	+ 8 9 36	$d_1 + d_2$	87 27 59
θ	18 9 34	18 21 2	Refraction	−3
α	1 26 57	18 16 41	Level	+2
t.	16 42 37	0 4 21	2ϕ	86 9 16
δ.	88 49 21	− 2 55 23	ϕ	43 4 38
cot δ	8.3129	1.2919n		
cos t	9.5202n	9.9999		
d	90° 23′ 25″	− 2° 55′ 26″		

The level correction is here positive because the greatest level reading, 7.8, being at the north end of the tube, shows that the vertical axis was tipped to the south. Its value in terms of a level division, as shown in the record, is the mean of the diagonal

differences of the four printed level readings, and this quantity multiplied by the value of a level division is the correction to be applied in obtaining 2ϕ.

By observing only stars on the meridian, the computation contained in the first two columns above is rendered unnecessary and the reduction correspondingly abridged, but in many cases it is difficult to find suitable northern stars to observe on the meridian and Polaris is always available at the expense of the little extra computation shown above. Theoretically, the latitude here found requires a correction depending upon the azimuth in which the stars were observed, but this may usually be ignored. Its value in the present case is less than $0''.1$.

A more serious source of error lies in the adopted coefficient, k, the value of a revolution of the gradienter, and to control this value a second southern star was observed with Polaris and the resulting double latitude found to be 86° 8′ 56″. The difference of the gradienter readings was in this case 12.385 revolutions and the altitude of Polaris being greater than that of the southern star, this resulted in a term additive to the sum of the declinations in order to obtain 2ϕ. Assuming that the difference in the resulting latitudes is due to an error of unknown magnitude in the assumed value of k, we have the following equations from which to determine x, the required correction to this value:

$$2\phi = 86° 9' 16'' - 3.839x,$$
$$2\phi = 86\ 8\ 56 + 12.385x.$$

From these we find,

$$x = +1''.23,$$
$$k = 20' 31''.23,$$
$$\phi = 43° 4' 36''.$$

Compare this result with that found on p. 654 from the vertical circle reading.

When the gradienter has been duly calibrated the precision attainable by the above method depends chiefly upon the character of the level employed. Not much can be expected from the ordinary plate levels of a transit, but if the striding level is temporarily fastened at right angles to the horizontal axis, the probable error of a single determination of latitude ought not to exceed 2″ to 4″.

475. Longitude. Longitude is determined by comparing the observed error of a chronometer or watch, referred to the local meridian, with its error referred to some standard meridian. The latter error must be obtained by comparison of the watch with standard time, transmitted either by telegraph or other equivalent signals, or by actually transporting a timepiece from one meridian to the other. The latter alternative should be adopted only when the others are impracticable, and great care must then be exercised to disturb the timepiece as little as possible in transit and to allow for its gain or loss in the intervening time.

The Two-Star record at p. 663 contains a comparison of the watch with Central Standard Time which was made by noting at a convenient moment the time simultaneously shown by the watch and a standard clock. We employ this comparison and the results of the reduction made in Art. 473 as follows:

$$\begin{aligned} \text{C.S.T.} - \text{Watch} &= -1^{m}\ 57^{s}.0 \\ \text{Local T.} - \text{Watch} &= +0\ \ 25\ .1 \\ \text{C.S.T.} - \text{Local T.} &= -2\ \ 22\ .1 = -35'\ 32'' \end{aligned}$$

Applying this last number to the longitude of the meridian corresponding to Central Standard Time, 90° west of Greenwich, we obtain as the observer's longitude, 89° 24′ 28″, W.

TABLE A—TIME STARS

1 No.	2 Name.	3 Mag.	4 R.A. 1910.	5 Var.	6 Dec. 1910.	7 Var.	8 M. T.	9 Var.	10 Date.
			h. m. s.	*s.*	° ′	′	*h. m.*	*m.*	
1	β Ceti	2	0 39 8	+301	−18 29	+33	8 4.0	+2.0	Nov. 29
2	τ Ceti	3-5	1 39 56	279	−16 24	+32	8 1.6	1.7	Dec. 15
3	γ Ceti	3-5	2 38 42	310	+ 2 52	+25	8 1.2	2.2	Dec. 30
4	δ Eridani	3-5	3 38 57	287	−10 4	+21	8 1.3	1.8	Jan. 14
5	ν Eridani	4	4 31 50	300	− 3 32	+13	8 2.9	2.0	Jan. 27
6	Rigel	0	5 10 13	288	− 8 18	+ 7	8 1.9	1.8	Feb. 6
7	κ Orionis	2	5 43 30	284	− 9 42	+ 2	8 3.6	1.7	Feb. 14
8	Sirius	2	6 41 11	264	−16 36	− 8	8 2.2	1.4	Mar. 1
9	Procyon	0	7 34 36	314	+ 5 27	−15	8 0.5	2.2	Mar. 15
10	30 Monoc.	4	8 21 11	300	− 3 37	−19	8 3.7	2.0	Mar. 26
11	α Hydrae	2	9 23 11	295	− 8 16	−26	8 2.6	2.0	April 11
12	μ Hydrae	4	10 21 45	290	−16 23	−31	8 2.0	1.9	April 25
13	δ Crateris	4	11 14 52	300	−14 18	−33	8 0.0	2.0	May 10
14	δ Corvi	3	12 25 14	310	−16 1	−33	8 3.3	2.2	May 27
15	Spica	1	13 20 29	316	−10 42	−32	8 3.3	2.3	June 10
16	ϵ Virginis	4	14 11 19	314	− 5 34	−29	8 2.9	2.2	June 23
17	β Librae	2.5	15 12 12	322	− 9 3	−22	8 0.7	2.4	July 9
18	Antares	1	16 23 56	367	−26 14	−14	8 1.5	3.1	July 27
19	θ Ophiuchi	3	17 16 32	368	−24 55	− 6	8 2.8	3.1	Aug. 9
20	η Serpentis	3	18 16 42	310	− 2 55	+ 1	8 3.8	2.2	Aug. 24
21	π Sagitarii	3	19 4 28	357	−21 10	+ 9	8 0.4	2.9	Sept. 6
22	θ Aquilae	3-5	20 6 42	310	− 1 5	+18	8 3.5	2.1	Sept. 21
23	ν Aquarii	4-5	21 4 45	327	−11 44	+24	8 2.4	2.4	Oct. 9
24	α Aquarii	3	22 1 13	308	− 0 45	+29	8 3.6	2.1	Oct. 20
25	δ Aquarii	3-5	22 49 56	319	−16 18	+32	8 1.1	2.3	Nov. 2
26	ω Aquarii	4-5	23 38 7	+311	−15 2	+33	8 2.0	+2.2	Nov. 14

TABLE B—CORRECTIONS TO M.T.

Longitude.	Leap Year.		After Leap Year.		
	Jan., Feb.	March, Dec.	I	II	III
	m.	*m.*	*m.*	*m.*	*m.*
183° 0′ East	8	4	5	6	7
91° 30′ East	7	3	4	5	6
Greenwich	6	2	3	4	5
91° 30′ West	5	1	2	3	4
183° 0′ West	4	0	1	2	3

POLARIS ORIENTATION

TABLE C—YEAR NUMBERS

Year.	F'	Y
		m.
1910	1.002	+11
11	0.997	10
12	.992	*9,13
13	.987	11
14	.982	10
1915	0.977	8
16	.972	*6,10
17	.968	9
18	.964	7
19	.960	6
1920	0.956	*4,8
21	.952	7
22	.948	6
23	.945	5
24	.941	*2,6
1925	0.936	5
26	.932	4
27	.928	3
28	.923	*0,4
29	.918	3
1930	0.913	+1

* Use only in January and February.

TABLE D—DAY NUMBERS

Month.	D	F''	PP for D.	
	h. m.			*m.*
Jan.	16 59	−0.001	1	4
Feb.	19. 2	− .001	2	8
Mar.	20 53	.000	3	12
April	22 56	+ .001	4	16
May	0 54	+ .003	5	20
June	2 56	+ .005	6	24
July	4 54	+ .005	7	28
Aug.	6 55	+ .005	8	32
Sept.	8 57	+ .003	9	35
Oct.	10 55	+ .001	10	39
Nov.	12 57	− .001	20	79
Dec.	14 56	− .003	30	118
Jan.	16 59	− .005		

TABLE P—POLARIS ORIENTATION

t	a_0	b_0	t
h.	′	′	*h.*
0	− 0+	+70+	24
1	24	68	23
2	46	61	22
3	65	50	21
4	80	35	20
5	89	+18+	19
6	−92+	0	18
7	89	−18−	17
8	80	35	16
9	65	50	15
10	46	61	14
11	24	68	13
12	− 0+	−70−	12

TABLE E

h	R
0	′
10°	5.
20	2.2
30	1.4
50	0.5
70	0.1

Year.	19 .
Y	..
F	..
R	..

h	F_1
..	..
..	..
..	..

$t = M + D + Y.$
$h = \phi + Fb_0.$
$A = 180° + F_1 a_0.$
$\phi = h' - Fb_0 - R.$

REFERENCES IN ASTRONOMY

1. The Completion of the Longitude Girdle of the Globe. Edwin Smith Eng. News, Oct. 6, 1904.
2. Time, Longitude, Latitude, and Azimuth. U. S. Coast and Geodetic Survey Report, 1880, Appendix 14, and Report, 1898, pp. 261–400.
3. The Determination of Latitude. Applied Science, March, 1909.
4. Methods for Determining Latitude, Longitude, and Time on Reconnaissance. Eng. News, March 1, 1900. W. S. Post.
5. Time, Azimuth, and Latitude. G. Wisner. Trans. Am. Soc. C. E., Vol. XXII (1883), p. 267.
6. Differences of Longitude by Chronometers. Comptes Rendus, April 25, 1904.
7. Geodetic Positions. Formulæ and Tables for Computing. U. S. Coast and Geodetic Survey Report, 1894, Appendix No. 9.
8. U. S. Coast and Geodetic Survey Methods. Hayford's "Geodetic Astronomy," John Wiley & Sons, New York. 1898.
9. Manual of Spherical and Practical Astronomy. W. Chauvenet.
10. Wright and Hayford's "Adjustment of Observations." D. Van Nostrand & Co., New York. 1904.

CHAPTER XVI

TRIGONOMETRIC LEVELLING

476. General Description. Trigonometric levelling consists in determining differences of elevation by means of carefully measured vertical angles and horizontal distances. Such work usually accompanies triangulation and because of the fact that the best time of observing vertical angles is during the middle of the day adds but little to the cost of the former work. However, even with the highest grade of instrument the results obtained are much inferior to precise spirit levelling. The most accurate results are obtained in mountainous country, the least accurate in a flat region. This is due to the varying amount of refraction. (See Art. 478.)

477. Instruments and Methods. For the best work the vertical circle should be provided with micrometer microscopes, or else so constructed as to allow the "repeating" method. An instrument of the latter type is shown in Fig. 241. With such an instrument the double zenith distance of the point is measured and repeated before reading the circle by a programme very similar to that used in measuring horizontal angles with a repeating instrument. Thus, to find the zenith distance of a signal, point the telescope and bisect the signal with the horizontal thread, then turn the alidade 180° in azimuth, transit the telescope, and bisect the signal again. At both bisections read the vertical circle and the level parallel to vertical circle. Then one-half of the difference of circle reading, corrected for level, will give a single determination of the zenith angle desired free from

any index error of the vernier. This angle can be repeated, i.e., added on as many times as desired and the mean angle computed as with horizontal angles. The other form of instrument pro-

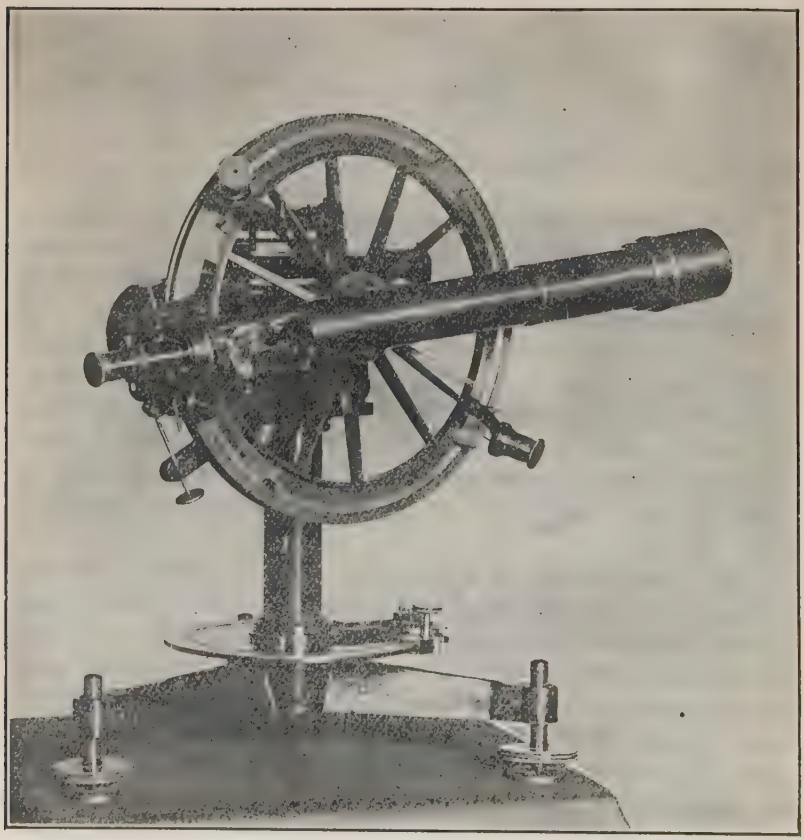


Fig. 241.—Instrument for Measuring Double Zenith Distances by the "Repeating" Programme.

vides a micrometer microscope in the eye piece of the objective. The micrometer is used to measure the small differences of

vertical angle between two or several stations. On long distances the vertical angles to the stations usually differ by amounts easily in the range of the micrometer. For a description of a micrometer, see Art. 443.

478. Refraction. If rays of light passed through the atmosphere in straight lines, then in trigonometrical levelling we should have to correct only for the curvature of a level surface at the locality. It is found, however, that rays of light near the surface of the earth usually are curved downwards—that is, their paths

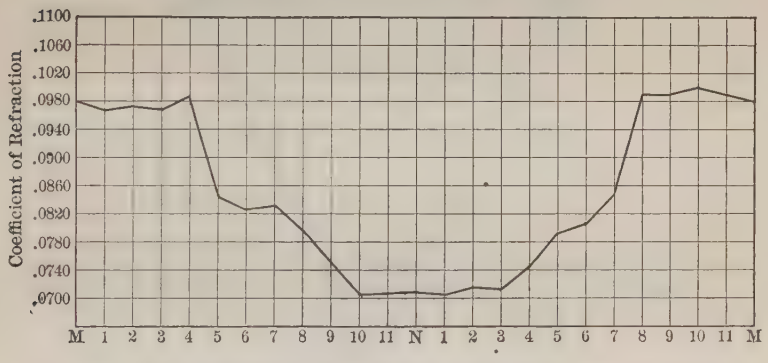


FIG. 242.—The Coefficient of Refraction at Different Hours of the Day as Measured by the U. S. C. and G. S.

are convex upwards. This curve is quite variable, sometimes being actually convex downwards in some localities. It has its greatest curvature about daybreak, diminishes rapidly till 8 A.M., and is nearly constant from 10 A.M. till 4 P.M., when it begins to increase again. The curve may be considered a circle having a variable radius, the mean value of which is about seven times the radius of the earth. The main cause of the inaccuracies in trigonometric levelling is due to atmospheric refraction. Because of the uncertainties in the refraction, work should be attempted only at the time of minimum and slowly changing refraction, viz., the middle of the day. See Fig. 242.

479. Refraction, How Measured. In Fig. 243 let A, B denote the positions of the instrument at two stations with elevations h_1 and h_2 above the sea level s.

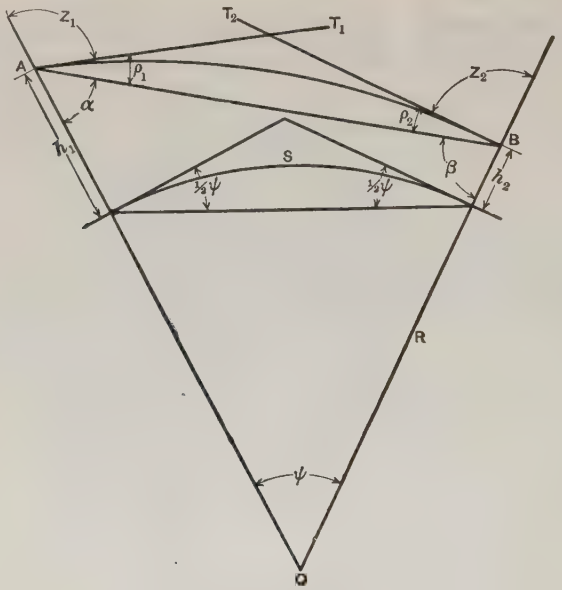


FIG. 243.

O = centre of earth;
R = radius of curvature of earth's surface for line s;
ψ = angle at earth's centre subtended by AB;
Z_1 and Z_2 = observed zenith distances at A and B upon B and A respectively;
ρ_1 and ρ_2 = corresponding refraction angles;
AB = the curved line of sight (nearly a circle);
AT and BT = tangents to curve AB at A and B.

$$\alpha = 180° - (Z_1 + \rho_1). \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\beta = 180° - (Z_2 + \rho_2). \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Now since the curve AB is nearly circular we can write

$$\rho_1 = m_1 \psi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\rho_2 = m_2 \psi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Where m_1 and m_2 are called coefficients of refraction. In other words, the coefficient of refraction at either end of a line joining two stations is the ratio of the refraction angle at that end to the angle at the centre of the earth subtended by the two stations.

480. Determination of Coefficient of Refraction. 1. From the geometric relations of Fig. 243 we have

$$Z_1 + \rho_1 + Z_2 + \rho_2 = 180° + \psi. \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Substituting for ρ_1 and ρ_2 their value, given above,

$$m_1 + m_2 = 1 - \frac{1}{\psi}(Z_1 + Z_2 - 180°) \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

As ψ is a small angle we may substitute $\frac{s}{R \sin 1''}$, whence

$$m_1 + m_2 = 1 - \frac{R \sin 1''}{s}(Z_1 + Z_2 - 180°). \quad . \quad . \quad . \quad (7)$$

If the observations at A and B are simultaneous and the two stations differ but little in elevation we may assume $m_1 = m_2 = m$ and thus obtain the mean refraction coefficient for this line,

$$m = 0.5 - \frac{R \sin 1''}{2s}(Z_1 + Z_2 - 180°) \quad . \quad . \quad . \quad (8)$$

2. When the elevations of two stations are known from precise levelling we can determine the value of m at either end of the line joining the two stations.

Let Z_1' and Z_2' be the true zenith distances at A and B

corresponding to the observed zenith distance Z_1, Z_2. Then we have from Fig. 243,

$$Z_1' - Z_2' = (Z_1 + \rho_1) - (Z_2 + \rho_2) = (Z_1 - Z_2) + \frac{(m_1 - m_2)s}{R \sin 1''}. \quad (9)$$

The last member of (9) will be found in Eq. (19), which determines the difference in height $h_1 - h_2$. Since $h_1 - h_2$ are known we can transpose Eq. (19), substituting $Z_1' - Z_2'$ for its equivalent as given in (9). We then have

$$\tfrac{1}{2}(Z_1' - Z_2') = \tan^{-1}\left\{\frac{h_1 - h_2}{s\left(1 + \frac{h_1 - h_2}{2R} + \frac{s^2}{12R^2}\right)}\right\}. \quad . \quad . \quad . \quad (10)$$

But since $\frac{h_1 + h_2}{2R}$ and $\frac{s^2}{12R^2}$ are always very small we may write

$$\tfrac{1}{2}(Z_1' - Z_2') = \tan^{-1}\left\{\frac{h_1 - h_2}{s}\left(1 - \frac{h_1 + h_2}{2R} - \frac{s^2}{12R^2}\right)\right\}. \quad (11)$$

Also from Fig. 243 we see that

$$\tfrac{1}{2}(Z_1' + Z_2') = 90 + \tfrac{1}{2}\psi = 90 + \frac{s}{2R \sin 1''}. \quad . \quad . \quad (12)$$

From (11) and (12) we can compute the true zenith distance at either end of the line. If now the zenith distances be observed, the coefficient of refraction is easily computed thus:

$$Z_1' - Z_1 = \rho_1 = m_1\psi = \frac{m_1 s}{R \sin 1''}, \quad . \quad . \quad . \quad . \quad (13)$$

$$Z_2' - Z_2 = \rho_2 = m_2\psi = \frac{m_2 s}{R \sin 1''}, \quad . \quad . \quad . \quad (14)$$

whence

$$m_1 = \frac{(Z_1' - Z_1) R \sin 1''}{s}, \quad . \quad . \quad . \quad . \quad . \quad (15)$$

$$m_2 = \frac{(Z_2' - Z_2) R \sin 1''}{s}.^1 \quad . \quad . \quad . \quad . \quad . \quad (16)$$

The following are a few values for the minimum refraction in the localities named:

Locality.	Observed by	Mean m.
Maine	U. S. Coast and Geodetic Survey	0.0710
North Georgia	Do.	0.0715
Central California	Do.	0.0742
New York State	New York State Survey	0.0730

The refraction has been found to be greatest over water or along the coast, and the least in the interior. Near the coast the U. S. Coast and Geodetic Survey uses the mean value 0.078 and 0.065 for the interior.

481. To Find the Difference of Height. The following three cases will be considered:

1. When the zenith distances are observed at both ends of each line but not simultaneously.

2. When the zenith distances are observed at both ends and also simultaneously.

3. When observations of zenith distance are made only at one end of the line.

Case 1. From the triangle AOB, Fig. 243, we have,[2]

$$\frac{OA - OB}{OA + OB} = \frac{\tan \frac{1}{2}(\beta - \alpha)}{\tan \frac{1}{2}(\beta + \alpha)}. \quad . \quad . \quad . \quad . \quad . \quad (17)$$

[1] For a full discussion of this method, see Appendix 12, U. S. Coast and Geodetic Survey Report for 1883.

[2] From Gillespie's "Higher Surveying," p. 233.

But $OA = R + h_1$, $OB = R + h_2$, $\alpha = 180 - Z_1 - \rho$, $\beta = 180 - Z_2 - \rho_2$, and $\alpha + \beta = 180 - \psi$. Substituting these values in (17) we have

$$\frac{h_1 - h_2}{h_1 + h_2 + 2R} = \tan \tfrac{1}{2}\psi \tan \tfrac{1}{2}[(Z_1 - Z_2) + (\rho_1 - \rho_2)]. \quad . \quad (18)$$

Substituting for ρ_1 and ρ_2 their values $m_1\psi$ and $m_2\psi$, developing the term $\tan \frac{1}{2}\psi$ by Maclaurin's series and reducing, we obtain

$$h_1 - h_2 = s \tan \tfrac{1}{2}\left[(Z_1 - Z_2) + \frac{m_1 - m_2}{R \sin 1''}\right]\left(1 + \frac{h_1 + h_2}{2R} + \frac{s^2}{12R} + \text{etc.}^1\right) \quad (19)$$

Case 2. When the zenith angles are measured simultaneously, it is usual to regard $m_1 = m_2$. Eq. (19), then becomes,

$$h_1 - h_2 = s \tan \tfrac{1}{2}(Z_1 - Z_2)\left(1 + \frac{h_1 + h_2}{2R} + \frac{s^2}{12R} + \text{etc.}\right). \quad . \quad . \quad . \quad (20)$$

The factor $\frac{h_1 + h_2}{2R}$ rarely exceeds 2 metres while the term $\frac{s^2}{12R}$ is negligible as compared to the error in assuming $m_1 = m_2$.

The factor $\left(1 + \frac{h_1 + h_2}{2R} + \frac{s^2}{12R^2} + \text{etc.}\right)$ can be tabulated for the arguments $h_1 + h_2$ and s, so that its effect on a certain difference of elevation is quickly applied.

The following formula is also sometimes used:

$$h_1 - h_2 = \frac{s \sin \frac{1}{2}(Z_1 - Z_2)}{\cos \frac{1}{2}(Z_1 - Z_2 + \psi)}. \quad . \quad . \quad . \quad . \quad (21)$$

---

[1] For ordinary distances the term $\frac{s^2}{12R}$ may be neglected.

log $R = 7.32068$ in feet, and log sin $1'' = 4.6855749$.

Case 3. In the common case a line sighted from one end only, we have, since

$$\alpha = 180° - Z_1 - \rho_1,$$
$$\beta = \quad Z_1 + \rho - \psi,$$

by substituting these values in Eq. (17) and reducing, we have

$$h_1 - h_2 = s \cot\left\{ Z_1 - (1-2m)\frac{s}{2R \sin 1''} \right\} \left(1 + \frac{h_1+h_2}{2R} + \frac{s^2}{12R^2} \right) \quad (22)$$

$$= s \cot Z_1 + \frac{1-2m}{2R} s^2 + \frac{1-m}{R} s^2 \cot^2 Z_1. \quad . \quad . \quad . \quad . \quad . \quad (23)$$

When only approximate elevations are required, if the lines are not more than ten miles in length, the following formula may be used instead of (23),

$$h_1 - h_2 = s \cot Z_1 + ks^2, \quad . \quad . \quad . \quad . \quad . \quad (24)$$

where $\log k = 2.3128(-10)$ in feet,

$= 2.8288(-10)$ in metres,

obtained by using $m = 0.07$ and the average value of R for latitude 45°.

When the line is sighted over from both ends we obtain two values of the difference in height, whose weighted mean gives the required result.

The following table gives computed values of the combined mean corrections for curvature and refraction for short distances, either for horizontal or inclined sights. Both the distance d and the correction c_n are in feet, except for the last column, where the distance is given in miles. For a more extended table for long distances, see page 544.

CORRECTION FOR EARTH'S CURVATURE AND REFRACTION

d	c_n	d	c_n	d	c_n	d	c_n	d	c_n	Miles.	c_n
Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.		
300	.002	1300	.035	2300	.108	3300	.223	4300	.379	1	.571
400	.003	1400	.040	2400	.118	3400	.237	4400	.397	2	2.285
500	.005	1500	.046	2500	.128	3500	.251	4500	.415	3	5.142
600	.007	1600	.052	2600	.139	3600	.266	4600	.434	4	9.141
700	.010	1700	.059	2700	.149	3700	.281	4700	.453	5	14.282
800	.013	1800	.066	2800	.161	3800	.296	4800	.472	6	20.567
900	.017	1900	.074	2900	.172	3900	.312	4900	.492	7	27.994
1000	.020	2000	.082	3000	.184	4000	.328	5000	.512	8	36.563
1100	.025	2100	.090	3100	.197	4100	.345	5100	.533	9	46.275
1200	.030	2200	.099	3200	.210	4200	.362	5200	.554	10	57.135

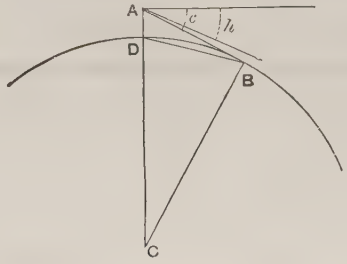


FIG. 244.

482. Levelling by the Sea Horizon. In Fig. 244 let A be the point of observation and B the point on the sea-level surface where the tangent from A falls. Then we have

$$H = AD^1 = AB \tan ABD$$

$$= R \tan C \tan \frac{C}{2}. \quad . \quad . \quad . \quad . \quad . \quad (25)$$

Since the angle C is always very small, we may let the arc equal its tangent, whence

$$H = \frac{R}{2} \tan^2 C. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

[1] Let the student prove this relation.

If the observed angle of depression be $h = C - mC$, then

$$C = \frac{h}{1 - m},$$

and

$$H = \frac{R}{2} \tan^2 \left(\frac{h}{1 - m} \right); \quad \ldots \ldots \quad (27)$$

or

$$H = \frac{R}{2} \left(\frac{h}{1 - m} \right)^2 \tan^2 1'', \quad \ldots \ldots \quad (28)$$

where h is expressed in seconds of arc.

$$\log \frac{R}{2} \tan^2 1'' = 6.39032 \text{ for distances in feet.}$$

REFERENCES ON TRIGONOMETRIC LEVELLING

1. Refraction, Mt. Diablo-Martinez, California. U. S. Coast and Geodetic Survey Report, 1876, pp. 355–67; Report, 1883, pp. 289–321.
2. Abnormal Refraction near the Ground. Report U. S. Coast and Geodetic Survey, 1879, p. 212, and 1893, p. 330.
3. Accuracy of Trigonometric Leveling. (*a*) Report N. Y. State Survey, 1882, p. 000. (*b*) Report of Great Triang. Survey of India, Vol. II. (*c*) Report U. S. Coast and Geodetic Survey, 1876, Appendices Nos. 16 and 17.
4. General Discussion of (*a*) Wright's "Adjustment of Observations," pp. 382–94.
5. (*b*) Gillespie's Higher Surveying, pp. 221–72.
6. Adjustment of a Net of Trigonometric Levels. Wright's Adjustment of Observations, p. 388.
7. See also Jordan's "Handbuch der Vermessungstunde" or Helmert's "Hoeher Geodäsie."
8. Tables Comparing Barometric with Trigonometric Leveling. Eng. News, 1881, pp. 133–35.

CHAPTER XVII

PRECISE SPIRIT LEVELLING

483. General Description. Precise spirit levelling is the operation of determining differences of elevation of any two points on the earth's surface with the utmost accuracy. Work of such refinement calls for specially designed instruments and specially devised methods of observation and reduction of notes.[1]

From such differences of level it is possible to compute the absolute elevations of all points above some standard datum surface. In nearly every country the mean level of the sea is taken as the surface of reference. Precise levelling furnishes the means of transferring levels to the interior of the continent almost free from accumulative error.[2] (See Art. 494).

Accurate elevations are needed for reducing the length of base lines to mean sea level; for the control of topographic surveys; for elevations needed in engineering operations of wide extent; and for use in the solution of certain scientific problems concerning gravity and the tides. The transference of such accurate levels involves first, finding what mean sea level is at a given point by means of automatic tide-gauge records for several years, and then running a line of precise spirit levels from this gauge inland and connecting with the points whose elevations

[1] For a historical treatment of precise levelling see Report of the U. S. Coast and Geodetic Survey, 1893, Part 2, p. 304.

[2] The total accumulative error of the U. S. Coast and Geodetic Survey transcontinental line of precise levels from New York to San Francisco was only 0.6 foot.

are required. Most European countries have inaugurated such systems of geodetic levelling, this work being considered an integral part of the trigonometrical survey of those countries. In the United States this grade of work was begun on the U. S. Lake Survey in 1875, by carrying a duplicate line of levels from a known elevation at Albany, N. Y., and connecting with each of the Great Lakes.

The importance and extent of this work will be seen from the following statement of the organizations in the United States and miles of precise levels involved in the 1907 [1] level net adjustment.

Precise levelling by the U. S. C. and G. S.....	10,231	miles
Precise levelling by U. S. Geological Survey.....	2,849	"
Precise levelling by U. S. Lake Survey.....	627	"
Water levelling (Great Lakes)	2,720	"
Precise levelling, Corps of Engineers, U.S.A., Mississippi River Commission [2] and Missouri River Commission (Kern levels).....	5,103	"
	21,530	miles [3]

484. Levelling Instruments. Until 1899 some type of the wye level was in general use both in United States and in Europe. The most popular form of such an instrument (made by J. Kern of Aarau, Switzerland) is shown in Fig. 245.

By turning the head so as to bring the eyes of the observer into a vertical line, and observing with both eyes simultaneously,

[1] See valuable report by John F. Hayford, U. S. Coast and Geodetic Survey, entitled " Precise Leveling in the United States, 1903–1907, with its Adjustment." This gives the adjusted elevations of many thousands of precise bench marks widely distributed in the United States.

[2] Professor J. B. Johnson personally conducted some six hundred miles of this work. See map by J. F. Hayford in Trans. Am. Soc. C. E., Vol. XLV, p. 148.

[3] Besides this total 2204 miles of miscellaneous levels were included.

the image of the bubble, as seen in the mirror, will appear projected upon the rod, and its stability and position can be noted while reading the rod. The bubble is placed in the middle of the scale of the tube, and *held there* during the reading of the rod

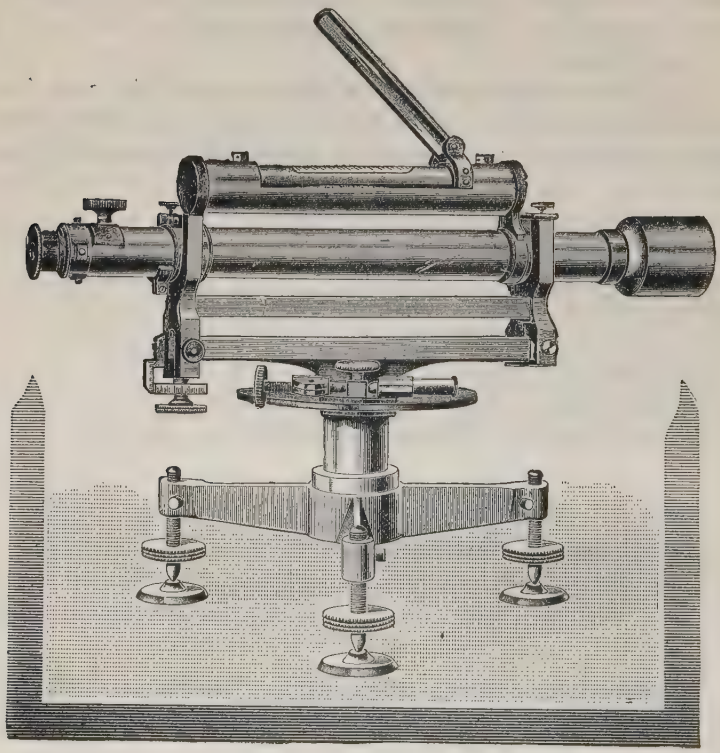


FIG. 245.—Kern Precise Level.

by means of the delicate gradienter screw placed under the eye-end wye. The bubbles in these instruments are very delicate, one division (2 mm.) on the tube having a value of from two to four seconds of arc. They are always made with three levelling-screws, widely spread, and a watch-bubble to assist

in setting. The bubble-tube is chambered at one end so as to maintain a bubble of the most efficient length at all temperatures. The magnifying power is about 45 diameters.

In the precise level shown in Fig. 246 the telescope rests in

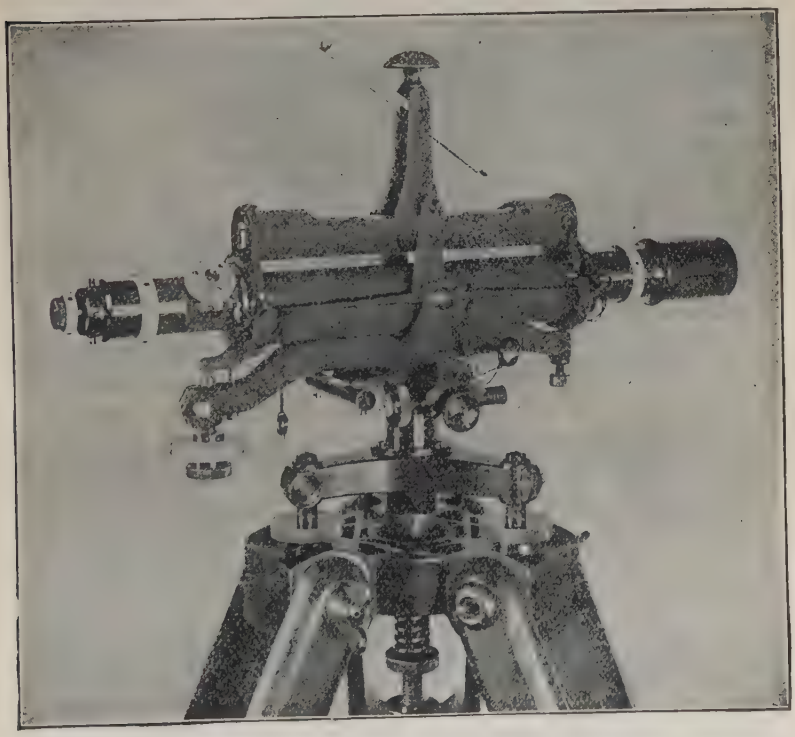


FIG. 246.—Mendenhall Precise Level.

a cradle which allows it to rotate about a horizontal axis near the centre of the instrument. This instrument was designed[1] by Dr. T. C. Mendenhall (then Supt. U. S. Coast Survey and Geodetic Survey).

[1] For a full description of this instrument, see Trans. Am. Soc. C. E., Vol. XXXIX, p. 390.

485. The Coast Survey Precise Level. The most modern type of precise level is the new instrument designed by Mr. E. G. Fischer of the U. S. Coast and Geodetic Survey and shown in Fig. 247.

In the language of Mr. Fischer,[1] "the aims in designing the new precise level were to select the material with a view to the smallness of its expansion coefficient, to protect the vital parts against sudden and unequal changes of temperature, to reduce to the smallest possible dimension the linear distance between level vial and line of collimation, to insure stability by reducing the distance between the centre of gravity and the plane of support, and to enable the observer to obtain the rod-reading, as nearly as possible, simultaneously with the setting of the level."

The bubble readings are made by means of an upper mirror throwing the images of the bubble ends upon two prismatic reflectors set in an auxiliary tube alongside the telescope, and thence to the left eye of the observer. By standing erect the observer then sees both the rod and the bubble at the same time, and can hold the bubble to a central position while reading the rod. A simple lens at the eye end of this tube (changed for different eyes) brings these images into the range of distinct vision. This device enables the bubble to be read simultaneously with the rod, without parallax, and with a normal or erect position of the body. It requires a taller tripod, however, than is customary with other forms of levels. This is probably the most perfect form of precise level ever devised, judged both from the merits of its design and from its remarkable field performance. (See Art. 495). It differs from the older types of levels chiefly because of two things: 1st, it is a dumpy level, and 2d, its level is nonreversible, and is in close juxtaposition to the line of sight. See Fig. 248. Other points of excellence are (3d) its broad base, low centre of gravity and rigid tripod, thus securing a stable instrument; (4th) the construction of the telescope and

[1] Trans. Am. Soc. C. E., Vol. XLV, p. 128.

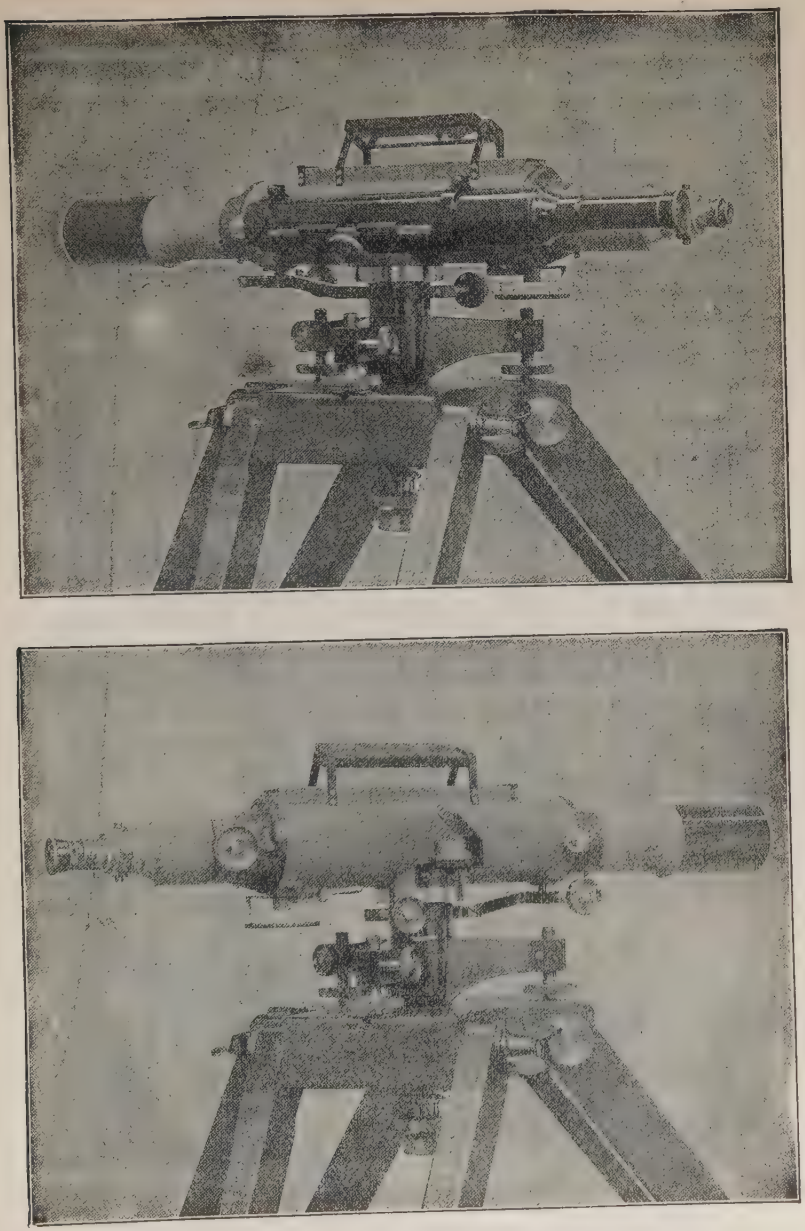


FIG. 247.—Two Views of the Coast and Geodetic Survey Level of 1900.

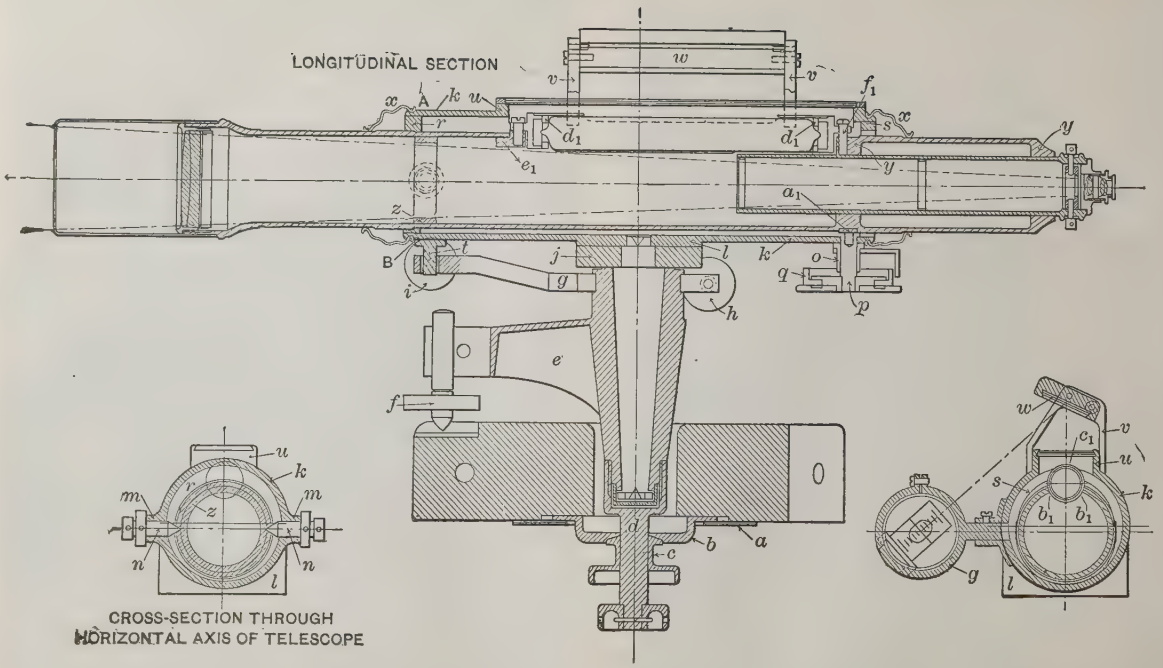


FIG. 248.—Sectional View of Coast Survey Level.

adjacent parts of nickel cast iron with small coefficient of expansion; (5th) the use of the binocular prism and the high tripod, which for work insure comfortable conditions, and hence favorable to accuracy. It has been found that one of the chief sources of error in some precise levelling was due to unequal changes of temperature in the telescope and parts connecting it with the level vial. These errors have not been found in work with this type of instrument nor with the Kern level, but in a form of wye level used on the Coast Survey before 1899.

486. Level Rods. Since 1899 a form of speaking rod has been in general use, but previous to that time a target rod was used by the U. S. Coast and Geodetic Survey. The use of the speaking rod and a simpler programme has resulted in greatly increasing the accuracy and decreasing the cost of precise work.

The design of the rod should insure the accurate reading of the rod both on long and short distances. The material is usually white pine which has been boiled in paraffine to prevent its length being affected by absorbing moisture. The metric unit is nearly universally used.[1] The U. S. Coast and Geodetic Survey rods have the metre graduations on platinum plugs. Probably the most perfect rod ever devised is the one shown in plan and cross-section in Figs. 249 and 250.[2] All such rods are provided with a watch-glass level to keep them plumb, and with a thermometer to keep a record of the rod temperatures.

Another form of rod used with the Kern level is shown in Fig. 252. It is kept plumb by means of a watch level.

487. Rod Supports. A foot plate provided with a spherical hole has been in common use both in United States and in

[1] The U. S. Geological Survey have used rods divided into yards since 1906.

[2] Designed by Professor David S. Molitor, M. Am. Soc. C.E., and fully described in Trans. Am. Soc. C. E., Vol. XLV, p. 12.

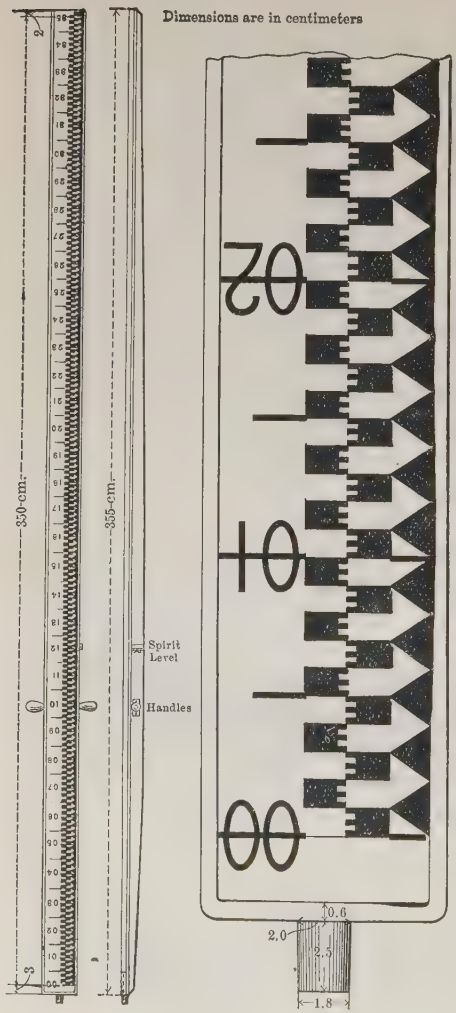


FIG. 249.—Molitor's Precise level Rod.

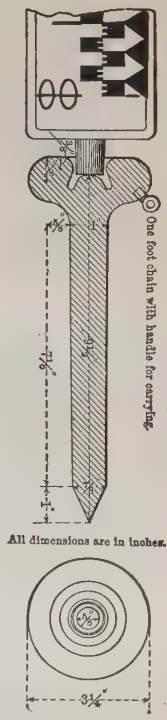


FIG. 251.

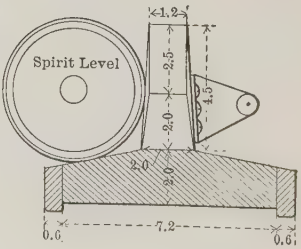


FIG. 250.—Cross-section.

(All dimensions in centimetres except those on the pin, which are in inches.)

FIG. 252.

Europe. This, however, has been found under certain conditions less reliable than some form of steel pin. An excellent design is the pin shown in Fig. 251.[1] Its surface, which supports the rod, is convex upwards, the lower groove being useful to receive any sand and dirt which may be blown in by the winds. Much evidence has accumulated which goes to show that rod supports under certain circumstances are subject to a systematic rising or falling, largely responsible for the accumulation of discrepancy between the forward and backward run over the same line. See Art. 491.

In August, 1903, a U. S. Coast and Geodetic Survey party made a series of experiments[2] to determine the effect of using a point marked on top of the rail of a railroad track as a rod support. The experiment proved that the use of such a support increased both the accuracy and speed of levelling and the practice has since been adopted in all U.S. Coast and Geodetic Survey level-work. The rail is not used just previous or just subsequent to the passage of a train, as it was found that a short time was needed before the rail rose to its former level.

---

[1] Designed by Professor David A. Molitor, Trans. Am. Soc. C. E., Vol. XLV, p. 17. The steel pin was first used by J. B. Johnson in 1881 when U. S. Assistant Engineer.

[2] See pp. 416–18, Appendix 6, Report 1904, for details.

Adjustments of Precise Level

488. Adjustments of the Wye Type of Precise Level.[1] (1) *Level Vial Adjustment.* To make the axis of the striding level parallel to the plane of its supports. This adjustment is made by reversing the striding level on its supports and correcting one half the movement of the bubble by the adjusting screw under one end of the bubble tube. (2) *Adjustment for Wind.* To make the bubble axis parallel to the line joining the centres of the points of support. This adjustment is made the same as explained in Art. 43. (3) *Collimation Adjustment.* To make the middle line of sight lie in the geometrical axis of the telescope rings. This is accomplished in the same way as the engineer's level, Art. 42 (*b*).

Thus, let R_1 = rod-reading for telescope normal;
R_2 = " " " " inverted;
d = distance of rod from instrument.

Then the correction $c = \dfrac{R_2 - R_1}{2d}$.

(4) *Wye Adjustment.* To make the axis of the level vial perpendicular to the vertical axis of the instrument. This is tested after levelling up the instrument by turning the telescope 180° about the vertical axis of the instrument. If in adjustment, the bubble will remain in the centre. Otherwise the run of the bubble is twice the true angular error. This true error is corrected by moving the micrometer screw near the eye-piece until the bubble returns half way to the centre and finally correcting the remaining half by the levelling screws.

[1] For an excellent analysis of the construction and adjustment of this type of instrument, see Trans. Am. Soc. C. E., Vol. XLV, pp. 20–73, by David A. Molitor.

489. Adjustment of the U. S. Coast Survey Dumpy Level. This type of precise level requires only one adjustment, all others being permanently made by the makers. This adjustment consists in making the axis of the bubble tube parallel to the line of collimation. This relation is tested by a modification of the ,'peg" method. Thus, set two turning points about 100 metres apart. Then with the instrument placed between these points and 10 metres from A read all three wires on both A and B. Then with the instrument still between A and B and about 10 metres from B read all three wires on the rod held on both A and B. (See p. 703 for sample of such field notes.) From this field work we have the level error thus determined.

$$C = \frac{(\text{sum of near-rod readings}) - (\text{sum of distant-rod readings})}{(\text{sum of distant-rod intervals}) - (\text{sum of near-rod intervals})}.$$

If C does not exceed 0.01 no adjustment will be necessary. The line of sight will be looking down when C is positive and up when C is negative. If C is greater than 0.01 the line of sight, here taken as the middle wire, can be made truly horizontal by raising or lowering it on the distant rod by C times the length of such sight expressed as rod interval. The constant C may be considered the error per unit of distance in a single sight. Note that the line of sight is here changed by the micrometer screw under the eye end of instrument and *not by the reticule adjusting screws.*

The operation of thus making the line of sight truly horizontal will of course cause the bubble to run off center. If it be now brought to the centre by the special wrench provided for this purpose the axis of the bubble will also be horizontal and hence parallel with the line of sight. Considerable care and skill is required of the beginner in making this adjustment, because of the extreme sensitiveness of the level bubble. The

vertical axis should be clamped. This adjustment should be tested daily.

The constant C will now be re-determined and used to multiply any excess of interval between the backsight and foresights in subsequent lines of levels.

Such a product is obviously the correction to be made to the final bench mark elevation obtained in any subsequent leveling work. If the backsight intervals are in excess the correction has the same sign as C, and the opposite sign when the foresights are in excess.

490. Instrumental Constants. In addition to the above adjustments of the precise wye level, the following instrumental constants need to be determined, usually at the beginning and close of a season:[1]

1. The angular value of one division on the bubble tube.
2. The inequality in the size of the pivot rings.[2]
3. The angular value of the wire interval, or the ratio of the intercepted portion on the rod to the distance of the rod from the instrument.
4. Magnifying power of the telescope.
5. The absolute lengths of the levelling rods.

These constants may be determined as follows:

First. *The value of one division of the bubble* may be readily found by sighting the telescope on the rod, which is set at a known distance from the instrument, and running the bubble from end to end of its tube, taking rod readings for each position of the bubble. The bubble graduations are supposed to be numbered from the centre towards the ends.

Readings should be taken only for extreme positions of the bubble, and not for central or intermediate positions, as those

[1] For a complete analysis of constants, see excellent paper by David A. Molitor, Trans. Am. Soc. C. E., Vol. XLV, pp. 61–73.

[2] This constant obviously does not apply to the U. S. Coast and Geodetic precise level.

would have little weight in fixing the average value of one division. To test the *uniformity* of the curvature of the bubble, however, readings should be taken for movements of a single division, back and forth, until many rod readings have been taken for each bubble position. Then average rod readings could be found for average bubble positions, differing by about one division only, and thence the uniformity of the curvature of the tube determined.

Let E_1 = *mean* of all the eye-end readings of the bubble when it was run to the eye end of its tube;
E_2 = same for bubble at object end of tube;
O_1 = *mean* of all the object end readings when bubble was at eye end of tube;
O_2 = same for bubble at object end of tube;
R_1 = *mean* reading of rod for bubble at eye end;
R_2 = same for bubble at object end;
D = distance from instrument to rod;
v = value of one division of the bubble (sine of the angle) at a unit's distance.

Then
$$v = \frac{R_2 - R_1}{D\left(\frac{E_1 - O_1}{2} - \frac{E_2 - O_2}{2}\right)}. \quad . \quad . \quad . \quad . \quad (1)$$

In seconds of arc we would have

$$v \text{ (in seconds)} = \frac{R_2 - R_1}{D \sin 1'' \left(\frac{E_1 - O_1}{2} - \frac{E_2 - O_2}{2}\right)}. \quad . \quad . \quad (2)$$

The value of a division of the level can also be most conveniently and accurately determined by the use of a level trier.

If a table is to be prepared for corrections to the rod readings for various distances and deviations of the bubble from

the centre of its tube, then the value as given by equation (1) is most convenient to use. The value of one division of a level bubble should be constant, but it is affected by its rigid fastenings, which change their form from changes in temperature.[1]

A sensibility corresponding to 2″ of arc for 2mm. of length is commonly used at present. This is from six to ten times as sensitive as ordinary wye levels.

Second. *The inequality in the size of the rings* is found by reversing the bubble on the rings, and also reversing the telescope in the wyes. The bubble is reversed only in order to eliminate its

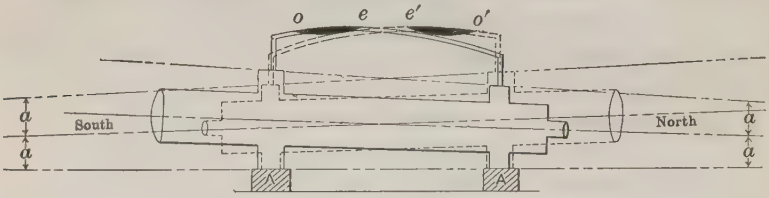


FIG. 253.—The Inequality of the Rings.

error of adjustment. Fig. 253 shows graphically the manner of making the test. A and A' are the tops of the telescope wyes.

In Fig. 253 let o and e, o' and e' represent the readings of the object and eye ends of the level bubble, direct and reversed respectively. Then $\frac{o-e}{2}$ = position of centre of bubble when objective end is south, also $-\frac{o'-e'}{2}$ = position of centre of bubble when the objective end is north. The total angular change of the level due to placing the larger ring where the smaller was before, is $\left(\frac{o-e}{2}\right)+\left(\frac{o'-e'}{2}\right)$. This is evidently twice the angle between top and bottom elements of the circumscribed cone determined by the rings. Now the angle desired is only half

[1] See Trans. Am. Soc. C. E., Vol. XLV, p. 22, by David A. Molitor.

of this, viz., the angle between the tops of the rings and the line of sight, since the latter is the axis of the cone. Hence this inclination is $\frac{1}{2}\left(\frac{o-e}{2}\right)+\left(\frac{o'-e'}{2}\right)$.

If the object end is the larger, the line of sight will be downward when the upper surfaces of the rings are horizontal. The following example will assist in understanding the method of making the observations and the reductions.

For reliable results the instrument should be mounted on a masonry pier, firmly clamped, and the level protected from the sun and atmospheric changes. The mean of two or more determinations should be taken.

	Bubble Readings.		
	North.		South.
Tel. eye end north. Lev. direct.	4.3		5.5
" " " " reversed.	4.7		5.2
	9.0	(−1.7)	10.7
		−0.42	
Tel. eye end south. Lev. direct.	6.2		3.7
" " " " reversed.	6.6		3.3
	12.8	(+5.8)	7.0
		+1.45	
Tel. eye end north. Lev. direct.	4.4		5.5
" " " " reversed.	4.8		5.2
	9.2	(−1.5)	10.7
		−0.38	

Mean reading north = −0.40
" " south = +1.45

North minus south = −1.85

That is to say, the bubble moves 1.85 divisions *toward the object end* when the telescope is reversed in the wyes. This is evidently *twice* the inequality of the pivot rings; and since the axis of a cone is inclined to one of its elements by one-half the angle at

the apex, so the line of sight is inclined to the tops of the rings by one-fourth of 1.85 divisions, or 0.46 divisions of the bubble. It is also evident that the eye-end ring is the smaller, and that therefore when the top surfaces of the rings are horizontal the line of sight inclines downward from the instrument. The correction is therefore positive. This is called the *pivot correction*, and changes only with an unequal wear in the pivot rings.

Third. *The angular value of the wire interval* is found by measuring a base on level ground of about 300 feet from an initial point f in front of the objective. This is fully explained in Art. 213. A stadia reduction table should be prepared as in Art. 215 if distances are required.

Fourth. *Determination of Magnifying Power of Telescope.* The following method is based on the fact that the ratio of the size of an object, as seen by the unaided eye to the object viewed through the eye-piece is the magnifying power of the eye-piece. The instrument is sighted at a rod held as closely as the instrument can be accurately focussed, and then the portion of the rod intercepted by the extreme wires is noted by one eye while the other views the rod directly. Thus if the rod read through the eye-piece was $^{m}0.47$, and the apparent interval projected on the rod was $2^{m}.350$, we have the magnifying power $= \frac{2.350}{47} = 50$ diameters. The magnifying power should not be less than 45 diameters for the best work with a corresponding objective of about $1\frac{5}{8}$ inches.

Fifth. *Standardization of Rod.* The rods should be compared with a standard for total length uniformity of graduation, and should have its coefficient of expansion determined. The distance between the foot of the rod and the zero point of the graduation, called the index correction, should also be determined and applied whenever the rod is used an odd number of times between bench marks. The level on the rod should be adjusted so as to indicate a vertical rod.

It should be noted that all the corrections, due to either the

constants or the lack of adjustments, are applied only to the excess of the sum of the back-sight distances over the sum of the fore-sight distances; the corrections are made out in the form of tables for each metre of distance.

491. Field Methods. The great accuracy attained in precise levelling is due quite as much to the methods used and precautions taken in making the observations as to the instrumental means employed. Aside from errors of observation and instrumental errors, we have two other general classes of errors, which can be avoided only by proper care being used in doing the work. These two classes are *errors from unstable supports* and *atmospheric errors.*

Any settling of the rod between the fore and back readings upon it will result in the final elevation being too high, while any settling of the instrument between the back and fore readings from it will also result in too high a final elevation. Such errors are therefore cumulative, but they can be eliminated by duplicating the work over the same ground *in the opposite direction.* A settlement of the instrument between back sight and fore sight can also be eliminated by taking every other fore sight before the corresponding back sight. As a general precaution, the duplicate line should always be run in the opposite direction. This will result in larger *discrepancies* than if both are run in the same direction, but the mean is nearer the truth.

Atmospheric errors may come from wind, heated air-currents causing the object sighted to tremble or "dance," or from variable refraction. Even with high winds the instrument may be shielded by a screen or tent without affecting the accuracy of the work. To avoid the evil effects of an unsteady atmosphere the length of the sights is shortened; but when a reading cannot be well taken at a distance of about 150 feet, or 50 metres, it would be better to stop, since the errors arising from a number of stations occupied would make the work poor. At about 8 o'clock A.M. and 4 P.M. very large changes in the refraction have been observed on lines over ground which is passing from sun

to shade, or vice versa, when the image was apparently very steady. It is the present practice in the U. S. Coast and Geodetic Survey parties to work all day except the noon hour, instead of three or four hours as was formerly the custom.

In making the observations the bubble is brought exactly to the centre of its tube, the observer being able to do this by means of the thumbscrew under one wye, and the mirror which reflects the image of the bubble to the observer at the eye-piece. If there is no mirror to the bubble, then it is brought approximately to the centre, and the recorder reads it while the observer is reading the three horizontal wires. In any case the bubble reading is recorded in the note book, and if it was not in the middle a correction is made for the eccentric position by means of a table prepared for the purpose. The mean of the three wire readings is taken as the reading of that rod, the observer estimating the tenths of the centimetre spaces, thus reading each wire to the nearest millimetre. The wires should be about equally spaced so that the mean of the three wires coincides very nearly with the middle wire. The differences between the middle and extreme wire readings are also taken out to give the distance, as well as to check the readings themselves by noting the relation of the two intervals. If they are not about equal, then one or more of the three readings is erroneous. This is a most important check, and constitutes an essential feature of the method.

The instrument is always shaded from the sun, both while standing and while being carried between stations. It is absolutely necessary to do this in order to keep the adjustments approximately constant, and the bubble from continually moving.

The following field notes will show clearly the present method of keeping notes in the U. S. Coast and Geodetic Survey:[1]

[1] Formerly a very much more complicated but less accurate programme, was used.

SPIRIT LEVELLING [1]

Date: June 13, 1910. Forward—Backward.
Sun: Cloudy. (Strike out one word.)
From B.M.: No. 5, Madison, Wis. To B.M.: No. 6, U. W. Campus
Wind: Light.

No. of Station.	Thread Reading Back Sight.	Mean.	Thread Interval	Sum of Intervals.	Rod and Temp.	Thread Reading Fore Sight.	Mean.	Thread Interval	Sum of Intervals.
	1.518					1.295			
B.M. 5	1.535	1.535	17		26	1.395	1.395	100	
	1.552		17	34		1.495		100	200
	1.513					1.555			
	1.527	1.527	14			1.656	1.6556	101	
	1.542		15	29	26	1.756		100	201

Corr. for curv. and Ref., -0.7. $C = +.03$.

492. General Instructions for Precise Levelling on the U. S. Coast and Geodetic Survey.[2]

1. Except when specific instructions are given to proceed otherwise, all lines are to be levelled independently in both the forward and the backward direction.

2. It is desirable that the backward measurement on each section should be made under different atmospheric conditions from those which occurred on the forward measurement. It is especially desirable to make the backward measurement in the afternoon if the forward measurement was made in the forenoon, and vice versa. The observer is to secure as much difference of conditions between the forward and backward measurements as is possible without materially delaying the work for that purpose.

3. On all sections upon which the forward and backward measures differ by more than 4.0 mm. $\sqrt{K}$ (in which K is the distance levelled between adjacent bench marks, in kilometres), both the forward and backward measures are to be repeated until two such measures fall within the limit.

4. The programme of observation at each station is to be as follows:

Set up and level the instrument. Read the three lines of the diaphragm as seen projected against the front (or rear) rod, each reading being taken to the nearest millimetre (estimated), and bubble being held continuously in the middle of the tube (i.e., both ends reading the same). As soon as possible thereafter

[1] These field notes furnish the field data for determining the constant C, but are not normal field notes, because the length of sights are not equal.

[2] Part of the specifications as prepared by Mr. John F. Hayford, U. S. Coast and Geodetic Survey.

read the three lines of the diaphragm as seen projected against the rear (or front) rod, estimating to millimetres as before, and holding the bubble continuously in the middle of the tube.

5. At each rod station, the rod thermometer is to be read and the temperature recorded.

6. At stations of odd numbers the back sight is to be taken before the fore sight, and at the even stations the fore sight is to be taken before the back sight.

7. The maximum difference in length between a fore sight and the corresponding back sight is to be 10 m. The actual difference is to be made as small on each pair of sights as is feasible by the use of good judgment without any expenditure of time for this particular purpose.

8. The recorder shall keep a record of the rod intervals subtended by the extreme lines of the diaphragm on each back sight, together with their continuous sum between bench marks. A similar record shall be kept for the fore sights. The two continuous sums shall be kept as nearly equal as is feasible without the expenditure of extra time for that purpose by setting the instrument beyond (or short of) the middle point between the back and front rods. The two continuous sums shall not be allowed to differ by more than a quantity corresponding to a distance of 20 m.

9. Once during each day of observation the error of the level should be determined in the regular course of the levelling, and recorded in a separate opening of the record book as follows: The ordinary observations at an instrument station being completed transcribe the last fore-sight reading as part of the error determination, call up the back rod and have it placed about 10 metres back of the instrument, read the rod, move the instrument to a position about 10 metres behind the front rod, read the front rod and then the back rod. The rod readings must be taken with the bubble in the middle of its tube. The required constant C to be determined, namely, the ratio of the required correction to any rod reading to the corresponding subtended interval is

$$C = \frac{(\text{sum of near rod readings}) - (\text{sum of distant rod readings})}{(\text{sum of distant rod intervals}) - (\text{sum of near rod intervals})}.$$

The level should not be adjusted if C is less than 0.005. If a new adjustment of the level is made, C should at once be redetermined. It is desirable to have the determinations of level error made under the ordinary conditions as to length of sight, character of ground, elevation of line of sight about the ground, etc.

10. Notes for future use in studying levelling errors shall be inserted in the record indicating the time of beginning and ending of the work of each half day, indicating the weather conditions, especially as to cloudiness and wind, indicating whether each portion of the line is run toward or away from the sun, and such other notes as promise to be of value in studying errors.

11. The instrument shall be shaded from the direct rays of the sun both during the observations and the movement from station to station.

12. The maximum length of sight shall be 150 metres, and the maximum is to be attained only under the most favorable circumstances.

13. At the beginning and end of the season, and at least twice each month during the progress of the observations, the three-metre interval between metallic plugs on the face of each level rod shall be measured carefully with a steel tape which shall be used for that purpose alone.

493. Bench Marks. These may be of two classes, depending upon whether they are permanent or temporary. The latter may be nails or spikes driven into telegraph poles or trees, and are intended only for a check on the levels. Permanent bench marks perpetuate for all time the results of the levelling and should therefore be placed where they are likely to be undisturbed. Public buildings, especially if of masonry, stone or concrete piers and abutments and ledges of rock or large boulders are suitable places for locating bench marks. Where these are not available suitable cut stones may be used. In every case the bench marks should be most carefully referenced and described so that a stranger could find the points without difficulty. To guard against the possibility of change, it is usual to determine two or more B.M's. in each locality. Permanent bench marks should be placed at average intervals of from one to two miles and at maximum intervals of ten miles.[1]

494. Limits of Error. This is usually expressed in terms of the discrepancy between the forward and backward run of the same section which is allowed without rerunning the line. This limit is as follows:[2]

[1] For descriptions and elevations of 16,000 B.M's. located in United States determined or adjusted by the U. S. Coast and Geodetic Survey see Appendix 3, Report U. S. Coast and Geodetic Survey for 1903, and special report of 1909, all by Mr. John F. Hayford, Asst. U. S. Coast and Geodetic Survey.

[2] These limits were formerly as follows:

U. S. C. and G. S. before 1899, $E = 5 \text{ mm.}\sqrt{2K}$

U. S. Lake Survey before 1880, $E = 10 \text{ mm.}\sqrt{K}$

Miss. River Commission before 1892, $E = 5 \text{ mm.}\sqrt{K}$.

U. S. C. and G. Survey,	4 mm.$\sqrt{K}$=0.017 ft.$\sqrt{\text{miles}}$
U. S. G. Survey,	4 mm.$\sqrt{K}$
Miss. River Commission,	3 mm.$\sqrt{K}$=0.013 ft.$\sqrt{\text{miles}}$
U. S. Lake Survey	3 mm.$\sqrt{K}$=0.013 ft.$\sqrt{\text{miles}}$

The formula for the probable error of the mean of several observations is $E=\pm.674\sqrt{\frac{\Sigma(R^2)}{n(n-1)}}$ where R is obtained by subtracting the mean from each observed value, and n is the number of such observed values.[1] Where there are but two observations, as in precise levelling, this formula reduces to $E=\pm\frac{1}{3}R$, where R is the discrepancy between the two results.

The International Geodetic Association has fixed on the following limits for the probable error of the mean result per kilometer: $\pm$3mm. per kilometer is tolerable; $\pm$5 mm. per kilometer is too large; $\pm$2mm. per kilometer is fair; $\pm$1 mm. per kilometer is a very high degree of precision. The U. S. Coast and Geodetic Survey levelling since 1900 has shown a probable error between 0.9 mm. and 0.6 mm. per kilometer, this error being determined by dividing the total discrepancy on the main line by the square root of the length of the line.[2]

495. Speed and Cost. Evidently the speed and cost of levelling will vary considerably, depending upon the character of the topography, the weather, number of B.M's. set and the remoteness of the locality, etc. Between 1900 and 1910 the U. S. Coast and Geodetic Survey levelled about 5000 miles at an average speed of 65 miles of computed line per month. In exceptional cases 121 miles per month have been run.[3] A speed

[1] See Appendix D for explanation of probable error. It must be kept in mind that a computed probable error is a true measure of the errors known as compensating, but not the systematic or accumulative errors.

[2] For a tabulation of the accuracy of principal precise levelling work in United States see Trans. Am. Soc. C. E., Vol. XLV, p. 105, and also p. 119.

[3] This corresponds to 242 miles of single line per month, or 9.7 miles per day

of 2.5 miles per hour has even been made, and an average of 1.4 miles of single line per hour for an entire month.

The cost of precise levelling by the U. S. Coast and Geodetic Survey 1903–7, including B.M's. averaged $10.00 per completed mile; Deep Water Survey (Molitor), $20.00 per mile; U. S. Engineer Corps, $19.00 to $32.00; Mississippi River Commission averaged (511 miles) $15.00; New French levelling, $10.00 per mile; Germany, $9.50.

496. Adjustment of a Precise Level Net. The rigid least square method (see Wright's "Adjustment of Observations" for complete exposition, p. 377) should be used in adjusting a large and complicated level net like that involved in the 25,000 miles of precise levelling in the United States.[1] In the case of a much simpler net all run by same methods the following method should give fair results: Certain elevations, one or more being known at the outset, determine by inspection which of the unknown points has its elevation fixed by the levelling with the greatest accuracy. From the two or more lines connecting this point most directly with the points of known elevation compute two or more values for the elevation of this point. Estimate the relative weights of these values and compute the weighted mean as the final accepted elevation of this point. Some of the factors which should be considered in weighting the various values are, shortness of the path, closing error of the circuit involving the line in question; any strengthening of such lines by connecting circuits, etc. Next determine by inspection which of the elevations still unknown is determined with greatest accuracy, obtain two or more elevations of such point, take the weighted mean as before for the adopted elevation and repeat the process till all points are fixed. Under favorable circumstances this method has been known to give results in close agreement with

[1] For description of method employed in working this problem, see Mr. John F. Hayford's discussion in 1903, Report U. S. Coast and Geodetic Survey, Appendix 3.

those derived from the least-square method.[1] The character of the results obtained will depend upon the judgment shown in the above analysis.

497. Elevation of Mean Sea Level. On account of variation in sea level due to tides, storms, currents, and irregularities in the shape of the earth (Art. 387) the determination of its mean elevation is a difficult problem. An accurate solution involves continuous observations of an automatic self-registering tide gauge for several years. The construction and operation of a tide gauge is discussed in Art. 258.

In order that the gauge may record the height of the water in the open ocean, the tide gauge should be placed at a locality free from currents and from the abnormal piling up of the water due to narrow or shallow channels.

The following instance of the variation in the tides at St. Augustine, Fla., emphasizes the errors introduced by using short periods for the determination of mean sea level. Although the average rise and fall is here less than 5 feet, the mean of the hourly ordinates for two different lunar months differed by 1.32 feet, while the mean of the hourly ordinates on one day differed from that of another by 3.2 feet.

REFERENCES ON PRECISE LEVELLING

1. David Molitor. Adjustments, Constants, and Field Methods. Trans. Am. Soc. C. E., Vol. XLV, pp. 62–81.
2. Accuracy of Precise Levels. Same reference, pp. 102–11.
3. Speed and Cost of Precise Levels. Trans. Am. Soc. C. E., Vol. XLV, pp. 112, 171, 178.
4. Precision Leveling in Argentine Republic. Anal. Sociedad Cien Argen., Sept., Oct., and Nov., 1906.
5. H. M. Wilson. Spirit Leveling on the U. S. Geological Survey. Trans. Am. Soc., Vol. XXXIX, 1898, p. 339.
6. Cost and Speed of Precise Leveling on U. S. Geological Survey, U. S. Coast and Geodetic Survey, and U. S. Eng. Corps. Trans. Am. Soc. C. E., Vol. XXXIX, p. 377.

[1] This method was suggested by Mr. John F. Hayford.

7. Instructions for Leveling, U. S. Geological Survey, Am. Soc. C. E., Vol. XXXIX, p. 377.
8. Instruments and Methods. U. S. Coast and Geodetic Survey, before 1899. U. S. Coast and Geodetic Survey Reports for 1879, p. 202. and 1880, p. 137.
9. J. B. Johnson. Sources of error in Precise Leveling. Jour. Asso. Eng. Soc., Vol. II, p. 152.
10. John F. Hayford. Errors in Old U. S. Coast and Geodetic Survey Levels. U. S. Coast and Geodetic Survey Report, 1899.
11. History of Precise Leveling in Foreign Countries. U. S. Coast and Geodetic Survey, Report 1893, pp. 307–20.
12. Ferguson. Method and Results and Cost of Precise Leveling on Mississippi River. Com. Jour. Asso. of Eng. Soc., Vol. XII, 1893, pp. 350–73.
13. U. S. Lake Survey. Precise Leveling, U. S. Lake Survey, Report Chief Engineer, U. S. A., Appendix OO, pp. 2426–29.
14. E. G. Fisher. The New Coast Survey Level. U. S. Coast and Geodetic Survey Report, 1900, p. 526. Also Trans. Am. Soc. C. E., Vol. XLV, p. 127.
15. Precise Dumpy Leveling. Bench-Level Operations on the Catskill Aqueduct Line. Eng. News, Feb. 20, 1908, p. 186.
16. The Precise Level a Possible Successor to the Wye Level. J. F. Hayford. Eng. News, July 2, 1903, p. 2.
17. Experience of U. S. Geological Survey with the Prism Level. E. M. Douglas. Methods and Cost. Eng. News, May 17, 1906.
18. German Precise Leveling. (*a*) Eng. News, 1893, Vol. I, p. 117. (*b*) Eng. News, 1893, p. 317–20. (*c*) German Levels. Andrews. Trans. Am. Soc. C. E., Vol. XXXIX, p. 407. (*d*) Lectures on German Geodetic Surveys. Zeitschrift f. Vermessungswessen, 1888, pp. 310–25.
19. A Connection by Precise Leveling between the Atlantic and Pacific Oceans. J. F. Hayford. Science, Vol. XXI, pp. 673–74, April 28, 1905.
20. Precise Level Operations on the Los Angeles Aqueduct. C. H. Lee. Eng. News, Sept. 17, 1908.
21. Use of a Railroad Rail as a Rod Support in Precise Leveling. Eng. Record, April 29, 1905.

CHAPTER XVIII

PROJECTION OF MAPS, MAP-LETTERING, AND TOPOGRAPHICAL SYMBOLS

I. PROJECTION OF MAPS

498. General Statement. The only *absolutely true map* is a model of the terrestrial globe. But as globes are too awkward for general use, recourse is had to various forms of work projections, all of which are representations of the spheroidal surface upon some plane surface.

The particular method that should be employed in representing portions of the earth's surface on a plane sheet or map depends, *first*, on the extent of the region to be represented; *second*, on the use to be made of the map or chart; and *third*, on the degree of accuracy desired.

Thus, a given kind of projection may suffice for a small region, but the approximation may become too inaccurate when extended over a large area. It is quite impossible to represent a spherical surface on a plane without sacrificing something in the accuracy of the relative distances, courses, or areas; and the use to which the chart is to be put must determine which of these conditions should be fulfilled at the expense of the others. A great many methods have been proposed and used for accomplishing various ends, some of which will be described.

499. Cylinder Projection. We here imagine a right cylinder passed through the middle parallel of the area to be mapped whose elements are parallel to the earth's axis. (Fig. 254.) This cutting cylinder becomes a tangent cylinder if the parallel be the equator.

The simplest form of this projection is known as the *equidistant flat projection.* Here the lengths of a degree on all the parallels and meridian are considered equal.

The result is a great distortion as the pole is approached, because while the length of a degree of latitude remains nearly constant the length of a degree of longitude dwindles to zero.

Mercator's cylinder projection avoids this error by increasing the distances between the parallels toward the poles by the same ratio that the parallels decrease compared to the equator. (See Fig. 255.) Because any direction on this projection may be represented as a straight line it is well adapted for sea charts.

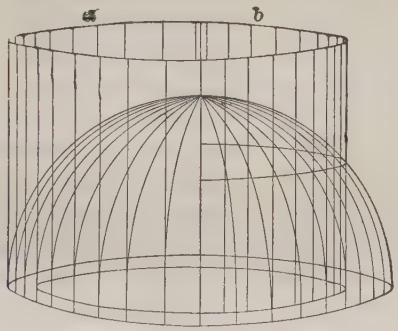


FIG. 254.—Cylinder Projection.

In still another modification of the cylinder projection, sometimes called the *rectangular*, the meridians are drawn as before, straight and parallel, but the parallels are shown as follows: A central meridian is drawn, and divided into minutes of latitude according to the value of these at that latitude as given in Table VIII. Through these points of division draw the parallels of latitude as right lines perpendicular to the central meridian. On the central parallel lay off the minutes of longitude, according to their value for the given latitude, by Table VIII.; and through these points of division draw the other meridians parallel with the first.

The largest error here is in assuming the meridians to be parallel. For the latitude of 40°, two meridians a mile apart will converge at the rate of about a foot per mile. A knowledge of this fact will enable the draughtsman to decide when this method is sufficiently accurate for his purpose. Thus, for an area of 10 miles square, the distortion at the extreme corners in longitude, with reference to the centre of the maps as an origin of co-ordinates, will be about 25 feet. At the equator this method is strictly correct.

FIG. 255.—Mercator's Cylinder Projection.

In this kind of projection, whether plotted from polar or rectangular co-ordinates, or from latitudes and longitudes, all straight lines of the survey, whether determined by triangulation or run out by a transit on the ground, will be straight on the map; that is, the fore and back azimuth of a line is the same, or, in other words, a straight line on the drawing gives a constant angle with all the meridians.

This method may be used on field sheets, where the survey has all been referred to a single meridian.

500. Trapezoidal Projection. Here the meridians are made to converge properly, but both they and the parallels are straight lines. A central meridian is first drawn, and graduated to degrees or minutes; and through these points parallels are drawn, as before. Two of these parallels are selected: one about

one-fourth the height of the map from the bottom, and the other the same distance from the top. These parallels are then subdivided, according to their respective latitudes, from Table VIII.; and through the corresponding points of division the remaining meridians are drawn as straight lines. The map is thus divided into a series of trapezoids. The parallels are perpendicular to but one of the meridians. The principal distortion comes from the parallels being drawn as straight lines, and amounts to about 32 feet in 10 miles in latitude 40°, and is nearly proportional to the square of the distance east or west from the central meridian.

The work should be plotted from computed latitudes and longitudes. The method is adapted to a scheme which has a system of triangulation for its basis, the geodetic position of the stations having been determined. These conditions would be fulfilled in a State topographical or geological survey for the separate sheets, each sheet covering an area of not more than 25 miles square.

501. Conic Projections. The conic projection is similar to the cylinder projection. The tangent cylinder is here replaced by either a tangent cone, Fig. 256, or an intersecting cone, Fig. 257. The parallels are drawn on the surface of the cone as parallel conic circles and the meridians are drawn as straight lines.

In the *simple conic projection* the apex of the cone is taken in the extended axis of the earth, at such an altitude that the cone becomes tangent to the earth's surface at the middle parallel of the map. When this conical surface is developed into a plane, the meridians are straight lines converging to the apex of the cone, and the parallels are arcs of concentric circles about the apex as the common centre.

The sheet is laid out as follows: Draw a central meridian, and graduate it to degrees or minutes, according to their true values as given in Table VIII. Through these points of division draw parallel circular arcs, using the apex of the cone as

the common centre. For values of the length of the side of the tangent cone, which is the length of the central meridian

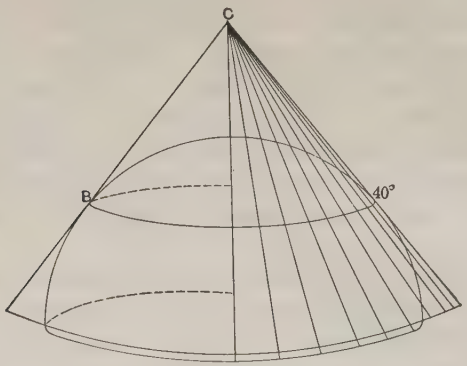


FIG. 256.—The Tangent Cone.

above, see Table VIII. The rectangular co-ordinates of points in these curves are also given in the same table.

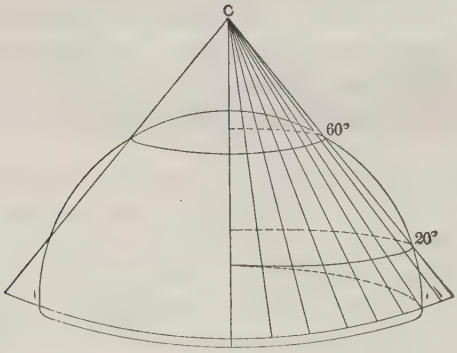


FIG. 257.—The Intersecting Cone.

On the middle parallel of the map the degrees or minutes of longitude are laid off, and through these are drawn the remaining meridians as straight lines radiating from the apex

of the tangent cone. It will be seen that the latitudes are correctly laid off, and the degrees of longitude will be sufficiently accurate for a map covering an area of several hundred miles square. The meridians and parallels are at right angles.

In this projection the degrees of longitude on all parallels, except the middle one, are too great; and therefore the area represented on the map is greater than the corresponding area on the sphere.

The chart should be plotted from computed latitudes and longitudes.

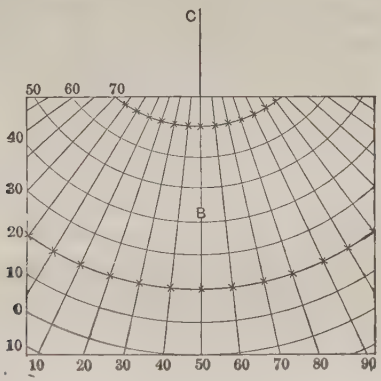


FIG. 258.

502. Mercator's Conic Projection. This is very similar to the above, except that two parallels, one at one-fourth, and one at three-fourths the height of the map, are properly graduated, and the meridians drawn as straight lines through these points of division. The parallels are drawn as concentric circles, as in the simple conic projection. (See Fig. 258.) The cone here is no longer tangent, but intersects the sphere in the two graduated parallels. In this case the region between the parallels of intersection is shown too small, and that oustide these lines is shown too large; so that the area of the whole map will correspond very closely to the corresponding area on the

sphere. This projection is generally used in atlases and geographies. When these parallels are so selected that these areas will be to each other exactly as the scale of the drawing, then it is called "Lambert's projection." This is accomplished by increasing the distances between the parallels at such a rate that the areas included between them and the meridians contain the same area as on the sphere.

503. Bonne's Projection. This differs from the simple conic in this—that all the parallels are properly graduated, and the meridians drawn to connect the corresponding points of division in the parallels. These latter are, however, still concentric circles. The meridians are at right angles to the parallels only in the middle portion of the map. The same scale applies to all parts of the chart. There is a slight distortion at the extreme corners, from the parallels being arcs of concentric circles. The proportionate equality of areas is preserved. A rhumb-line appears as a curve; but when once drawn, its length may be properly scaled.

It will be noted that the last three methods involve the use of but one tangent or intersecting cone.

504. The Polyconic Projection. For very large areas it is preferable to have each parallel the development of the base of a cone tangent in the plane of the given parallel. This projection differs from Bonne's only in the fact that the parallels are no longer concentric arcs, but each is drawn with a radius equal to the side of the cone which is tangent at that latitude. These, of course, decrease towards the pole; and therefore the parallels diverge from each other towards the edge of the chart. The result of this is that a degree of latitude at the side of the map is not equal to a degree on the central meridian; or, in other words, the same scale cannot be applied to all parts of the map. These defects appear, however, only on maps representing very large areas. The whole of North America could be represented by this method without any material distortion.

This method of projection is exclusively used on the U. S.

Coast and Geodetic Survey, and for all other maps and charts of large areas in this country. Extensive tables are published by the War and Navy Departments, and also by the Coast Survey,[1] to facilitate the projection of maps by the polyconic system. Table VIII. gives in a condensed form the rectangular co-ordinates of the points of intersection of the parallels and meridians referred to the intersection of the several parallels with the central meridian as the several origins.

The following is a detail description of the manner of using these tables to construct the projection:[2]

"The location, size and scale of the projection having been determined, draw a line representing the parallel nearest the centre of the sheet, and where it would intersect the most central meridian construct a perpendicular.

"The number of minutes to the extreme parallel, above and below the *middle* one being known, take out their values from the table (for the latitude of the projection), under the head of 'meridian arcs'; lay off the distances on the central meridian, and with the same values strike arcs at each end of the parallel, above and below; tangent to these arcs and through the points on the meridian draw straight lines, forming the extreme parallels of the sheet.

"From the tables as above, under the head of 'arcs of the parallel in meters,' take out the value corresponding to the number of minutes east and west of the central meridian, and lay off the distances, in subdivisions of threes, on each of the three parallels constructed, and then subdivide these spaces, thus securing greater accuracy; through the points thus determined draw the meridians, which will appear as straight lines except

[1] Tables for the Projection of Maps, Based upon the Polyconic Projection of Clarke's Spheroid of 1866, and Computed from the Equator to the Poles. Special Publication, No. 5, U. S. Coast and Geodetic Survey, U. S. Govt. Printing Office, 1900.

[2] Authority, Mr. E. H. Fowler, Chief Draftsman, U. S. Coast and Geodetic Survey.

on very small scales, where the value of x under the head of 'co-ordinates of curvature' in the Tables must be used. For a large-scale projection (1:10000) the value of y, under the head of 'co-ordinates of curvature' is only applied to each end of the central parallel, perpendicular to it, toward the pole, connecting the position with the central meridian by a straight line; but in a small-scale projection the value is laid off for each subdivision from the central meridian and connected by a curved line; from this adjusted central parallel lay off the values for each

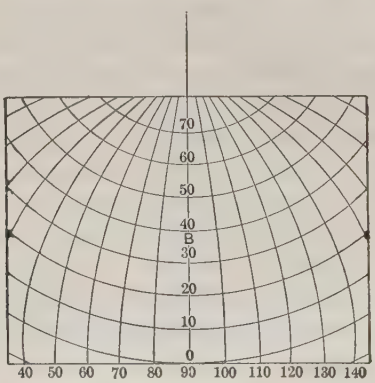


FIG. 259.

subdivision, north and south, as taken from the tables under the head off 'meridian arcs,' and through these points draw the various parallels to complete the projection."

Fig. 259 illustrates the polyconic map on a small scale. Plate I is also a polyconic projection on a larger scale.

505. Formulas Used in the Projection of Maps. The fundamental relations on which the method of polyconic projection rests are given in the following formulas:

Normal, being the radius of curvature of a section perpendicular to the meridian at a given point.....$N = \frac{R_e}{(1 - e^2 \sin^2 L)^{\frac{1}{2}}}$. . (1)

where R_e is the equational radius,
e is the eccentricity,
and L is the latitude.

Radius of the meridian $\ldots\ldots\ldots R_m = N^3 \frac{(1-e^2)}{R_e^2}$ . . (2)

Radius of the parallel. $\ldots\ldots\ldots\ldots R_p = N \cos L$ . . . (3)

Degree of the meridian. $\ldots\ldots\ldots\ldots D_m = \frac{\pi}{180} R_m$ . . . (4)
$= 3600 R_m \sin 1''.$

Degree of the parallel. $\ldots\ldots\ldots\ldots D_p = \frac{\pi}{180} R_p$ . . . (5)
$= 3600 R_p \sin 1''.$

Radius of the developed parallel, or side of tangent cone. $\ldots\ldots\ldots\ldots r = N \cot L$ . . . (6)

If n be any arc of a parallel, in degrees, or any difference of longitude from the central meridian of the drawing, and if θ be the corresponding angle, in degrees, at the vertex of the tangent cone, subtended by the developed parallel, then, since the angular value of arcs of given lengths are inversely as their radii, we have

$$\frac{\theta}{n} = \frac{R_p}{r} = \sin L,$$

or $$\theta = n \sin L \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (8)$$

Since the developed parallels are circular arcs, the rectangular co-ordinates of any point an angular distance of θ from the central meridian is,

$$\left.\begin{array}{ll} \text{Meridian distance,} & d_m = x = r \sin \theta \\ \text{Divergence of parallels,} & d_p = y = r \text{ vers } \theta \\ & \quad = x \tan \frac{1}{2}\theta \end{array}\right\} \quad . \ . \quad (9)$$

For arcs of small extent, the parallel may be considered coincident with its chord; but the angle between the axis of x and the chord is $\frac{1}{2}\theta$. If, then, the length of the arc, which is nD_p, be represented by the chord, we may write

$$\left.\begin{aligned} d_m &= \text{meridian distance} &&= x = nD_p \cos \tfrac{1}{2}\theta \\ \text{and} \quad d_p &= \text{divergence of parallels} &&= y = nD_p \sin \tfrac{1}{2}\theta \end{aligned}\right\} \quad . \quad . \quad (10)$$

If, now, d_{m1} = meridian distance for 1 degree of longitude,
and d_{mn} = meridian distance for n degrees of longitude,

we have
$$\frac{d_{mn}}{d_{m1}} = \frac{nD_p \cos \tfrac{1}{2}\theta_n}{D_p \cos \tfrac{1}{2}\theta_1}.$$

But $\theta = n \sin L$, so that $\theta_1 = 1^\circ \times \sin L = 38'$ for latitude 40°. Therefore

$$\cos \tfrac{1}{2}\theta_1 = \cos 19' = 1, \text{ nearly};$$

so that
$$\frac{d_{mn}}{d_{m1}} = n \cos \tfrac{1}{2}(n \sin L), \text{ nearly.} \quad . \quad . \quad . \quad (11)$$

For $L = 30^\circ$, we have $\sin L = \frac{1}{2}$. Therefore, for latitude 30°,

$$\frac{d_{mn}}{d_{m1}} = n \cos \tfrac{1}{4}n = n \cos (0.25n), \text{ nearly.}$$

If we had obtained the meridian distance, d_m, for 1 degree of longitude, and wish to obtain it for n degrees in latitude 30°, we have but to multiply the distance for 1 degree by $n \cos (0.25n)$.

506. Explanation of Table VIII. The meridian distances are here given, at various latitudes, for a difference of longitude of 1 degree. To find the meridian distance for any number of degrees or parts of degrees, multiply the distance for 1 degree by the factor there given for the given latitude. The factor given in the table for latitude 30° is $n \cos (0.288n)$, in place of $n \cos (0.25n)$, as obtained above. The difference is a correction which has been introduced to compensate the

error made in assuming that the chord was equal in length to its arc. The corrected factors enable the table to be used without material error up to 25 degrees longitude either side of the central meridian.

To obtain the divergence of the parallels for differences of longitude more or less than 1 degree, multiply the divergence for 1 degree by the square of the number of degrees. It is evident that this rule is based on the assumption that the arc of developed parallel is a parabola, and so it may be considered for a distance of 25 degrees either side of the central meridian between the latitudes 30° and 50° without material error.

If the whole of the United States were projected by this table, using the factors given, to a scale of 1 to 1,500,000, thus giving a map some 8 by 10 feet, the maximum deviation of the meridians and parallels from their true positions (which would be at the upper corners) would be but about 0.02 inch.

Thus, for a map of this size, covering 20 degrees of latitude and 50 degrees of longitude, the geodetic lines would have their true position within the width of a fine pencil line, by the use of Table VIII. Fig. 260 will illustrate the use of the table in projecting a map by the polyconic method. The map covers 30 degrees in latitude (30° to 60°) and 60 degrees in longitude. The straight line O_3O_6 is first drawn through the centre of the map, and graduated according to the lengths of 1 degree of latitude, as given in the second column of Table VIII. Through these points of division the lines m'_3m_3, are drawn in pencil at right angles to the central meridian. On these lines the points m_1, m_2, etc., are laid off by the aid of the first part of Table VIII. This table gives the meridian distances when n is less than 1 degree, as well as when n is greater. From the points m_1, m_2, etc., the divergence of the parallel is laid off above the lines Om,

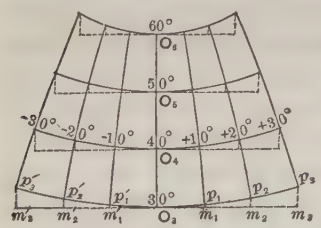


FIG. 260.

by the aid of the second portion of Table VIII., thus obtaining the positions of the points p_1, p_2, etc. The points p mark the intersection of the meridians and parallels; and these may be drawn as straight lines between these points, provided a sufficient number of such points have been located. The map is then to be plotted upon the chart from computed latitudes and longitudes.

507. Summary. We have seen that there are, in general, two ways of plotting a map or chart, and two corresponding uses to which it may be put:

First. We may plot by a system of plane co-ordinates, either polar (azimuth and distance) or rectangular (latitudes and departures). This gives a map from which distance-azimuth (referred to the meridian of the map), and areas are correctly determined.

Second. We may plot the map by computed latitudes and longitudes, and determine from it the relative position of points in terms of their latitude and longitude.

The first system is adapted to small field sheets and detail charts for which the notes were taken by referring all points to a single point and meridian. For this purpose the system of rectangular projection should be selected, as long as the area of the chart is not more than about 100 square miles. If it be larger than this, the trapezoidal system should be used. In case this is done, the work is still plotted as before, provided it has all been referred to a given meridian in the field work, and then converging meridians are drawn as described above. From such a chart, not only the azimuth (referred to the central meridian) and distances may be determined, but the correct longitude and nearly correct latitude are given.

In the case of topographical charts, based on a system of triangulation, each sheet is referred to a meridian passing through a triangulation station on that sheet, or near to it, and the rectangular system used.

In the case of a survey of a long and narrow belt, as for a river, railroad, or canal, if the survey was based on a system of triangulation, the convergence of meridians has been looked

after in the computation of the geodetic positions of these stations, and each is plotted by the rectangular system, being referred to the meridian through the adjacent triangulation station. When many of these are combined into a single map on a small scale, then they must be plotted on the condensed map by latitudes and longitudes, these being taken from the small rectangular projections, and plotted on the reduced chart in polyconic projection, the meridians and parallels having been laid out as shown above.

In case the belt extends mostly east and west, and is not based on a triangulation scheme, then observations for azimuth should be made as often as every 50 miles. It will not do to run on a given azimuth for this distance, however; for there has been a change in the direction of the parallel (or meridian) in this distance, in latitude 40°, of about 40 minutes. According to the accuracy with which the work is done, therefore, when running wholly by back azimuths, the setting of the instrument must be changed. Thus, if in going 1 degree (53 miles), east or west, in latitude 40°, the meridian has shifted 40 minutes, then in going 13 miles east or west the meridian has changed 10 minutes; and this is surely a sufficiently large correction to make it worth while to apply it.

When running west, this correction is applied in the direction of the hands of a watch, and when running east, in the opposite direction; that is, having run west 13 miles by back azimuth, then the pointing which appears north is really 10 minutes west of north, and the telescope must be shifted 10 minutes around to the right.

If the azimuth be corrected in this way, a survey can be carried by back azimuth an indefinite distance, and still have the entire survey referred to the true meridian.

508. The Angle of Convergence of Meridians is the angle θ in the equations given in the above formula. Then

$$\theta = n \sin L,$$

where n is the angular change in degrees of longitude, and L is the latitude of the place.

For $L=30°$, sin $L=\frac{1}{2}$; or, in latitude 30° a change of longitude of 1 degree changes the direction of the meridian by 30 minutes.

For $L=40°$, sin $L=0.643$; or, a change of longitude of 1 degree changes the direction of the meridian by 0.643 of 60 minutes, or 38.6 minutes, being approximately 40 minutes.

For $L=50°$, sin $L=0.766$; or, in going east or west 1 degree, the meridian changes 0.766×60 minutes $=46$ minutes, or approximately 50 minutes.

Therefore we may have the approximate rule, that a change of longitude of 1 degree changes the azimuth by as many minutes as equals the degrees of latitude of the place. This rule results very near the truth between plus and minus 40° latitude, that is, over an equatorial belt 80 degrees in width.

II. Map-Lettering and Topographical Symbols

509. Map-Lettering. The best-drawn map may have its appearance ruined by the poor skill or bad taste displayed in the lettering. The letters should be simple, neat, and dignified in appearance, and have their size properly proportioned to the subject. The map is lettered before the topographical symbols are drawn. When a map is drawn for popular display, some ornamentation may be allowed in the title; but even then the lettering on the map itself should be plain and simple. When the map is for official or professional use, even the title should be made plain.

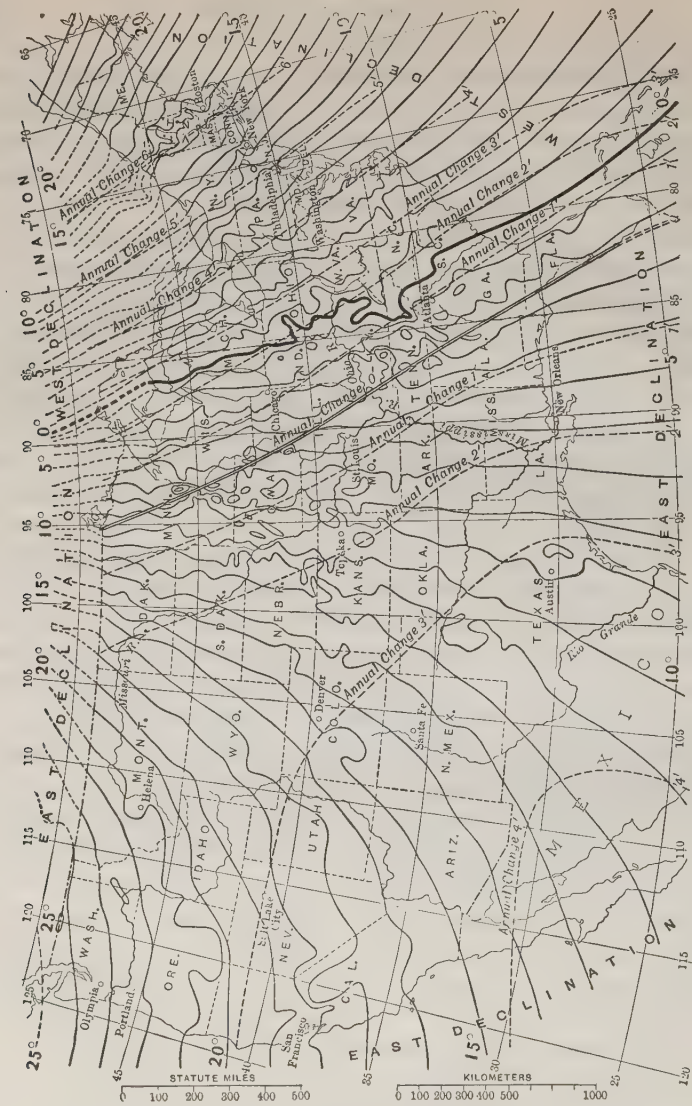
On Plates V and VI are given several sets of alphabets which are well adapted to map work, taken from plates of the U. S. G. S. Of course the size should vary according to the scale of the map and the subject, as shown on Plates II and III. It is a good rule to make all words connected with water in italics.

510. Topographical Symbols.[1] In topographical representation, where elevations have been taken sufficiently numerous and accurate, the outline of the ground should be represented by contours rather than by hachures, or hill shading, which simply gives an approximate notion of the slope of the ground, but no indication of its actual elevation. Where the ground has so steep a slope that the contour lines would fall one upon another, it is well here to put in shading-lines. The water surfaces and streams may be water-lined in blue, or left white. The contour lines over alluvial ground should be brown (crimson and burnt sienna), while those over rocky and barren ground should be in black. This is a very simple and effective method of showing the character of the soil. Plates II and III show some of the topographical symbols used by the U. S. G. S.

The practices of the government surveys should be followed in the matter of conventional surface representation, such as meadow, swamp, woodland, prairie, cane-brake, etc., with all their varieties. Some of these are given in the United States Coast Survey Report for 1879 and 1883, while Plate III shows most of those used on the Mississippi River Survey.

Those shown on Plates III and IV will serve as an example of the best practice.

[1] "Topographical Drawing," by Lieut. Smith, published by John Wiley & Sons, is an excellent text on this subject.

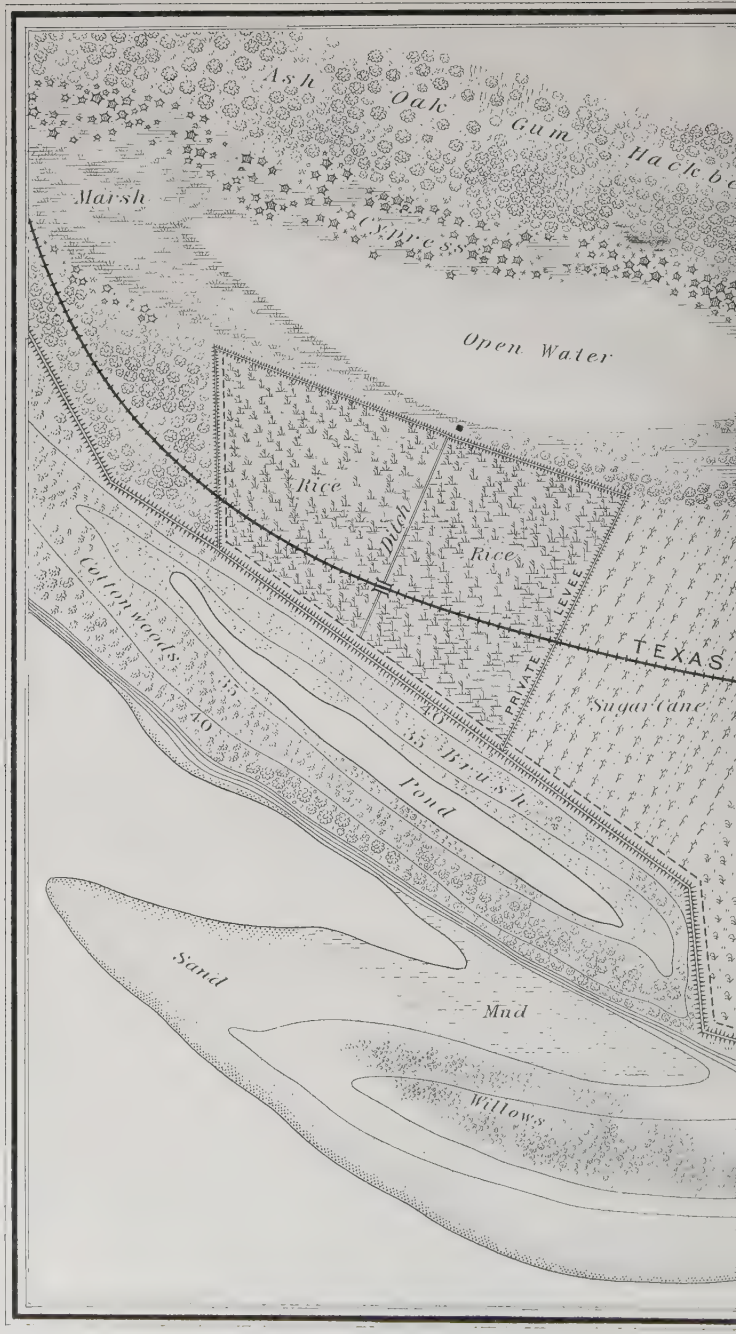


LINES OF EQUAL MAGNETIC DECLINATION AND OF EQUAL ANNUAL CHANGE IN THE UNITED STATES FOR 1915.

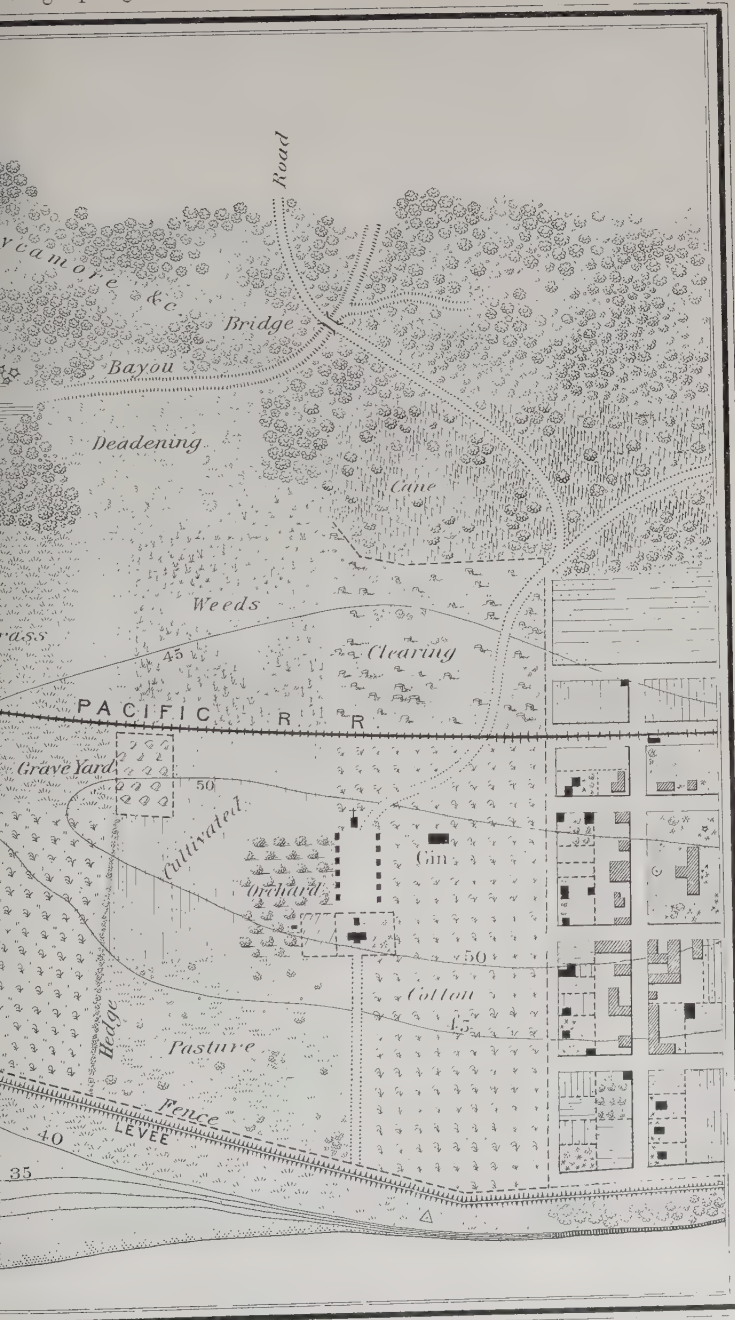
The lines of equal magnetic declination (full lines) apply to January 1, 1915. East of the line marked 0 (agonic line) the north end of the compass needle points west of north, west of that line it points east of north.

The north end of the compass needle is moving to the westward for places east of the line of no change and to the eastward for places west of that line at an annual rate indicated by the lines of equal annual change (broken lines).

TOPOGR
s
Designed



Draw


Road
Sycamore &c.
Bridge
Bayou
Deadening
Cane
Weeds
Clearing
45
PACIFIC R R
Grave Yard
50
Cultivated
Gin
Orchard
50
Cotton
Pasture
Hedge
Fence
LEVEE
40
35

MPLE PORTION OF THE CONTOUR MAPS PUBLISHED BY THE U. S. GEOLOGICAL SURVEY.

SHOWING THE ADVANTAGE OF COLORED CONTOUR AND STREAM LINES.

PLATE III

A PORTION OF THE CUMBERLAND PLATEAU, IN W. VA.

SCALE $\frac{1}{125,000}$

CONTOUR INTERVAL 100 FEET.

APPENDICES

APPENDIX A

THE JUDICIAL FUNCTIONS OF SURVEYORS

BY JUSTICE COOLEY OF THE MICHIGAN SUPREME COURT

When a man has had a training in one of the exact sciences, where every problem within its purview is supposed to be susceptible of accurate solution, he is likely to be not a little impatient when he is told that under some circumstances he must recognize inaccuracies, and govern his action by facts which lead him away from the results which theoretically he ought to reach. Observation warrants us in saying that this remark may frequently be made of surveyors.

In the State of Michigan all our lands are supposed to have been surveyed once or more, and permanent monuments fixed to determine the boundaries of those who should become proprietors. The United States, as original owner, caused them all to be surveyed once by sworn officers, and as the plan of subdivision was simple, and was uniform over a large extent of territory, there should have been, with due care, few or no mistakes; and long rows of monuments should have been perfect guides to the place of any one that chanced to be missing. The truth unfortunately is that the lines were very carelessly run, the monuments inaccurately placed; and, as the recorded witnesses to these were many times wanting in permanency, it is often the case that when the monument was not correctly placed it is impossible to determine by the record, with the aid of anything on the ground, where it was located. The incorrect record of course becomes worse than useless when the witnesses it refers to have disappeared.

It is, perhaps, generally supposed that our town plats were more accurately surveyed, as indeed they should have been, for in general there can have been no difficulty in making them sufficiently perfect for all practical purposes. Many of them, however, were laid out in the woods; some of them by proprietors themselves, without either chain or compass, and some by imperfectly trained surveyors, who, when land was cheap, did not appreciate the importance of having correct lines to determine boundaries when land should become dear. The fact probably is that town surveys are quite as inaccurate as those made under authority of the general government.

It is now upwards of fifty years since a major part of the public surveys in what is now the State of Michigan were made under authority of the United States. Of the lands south of Lansing, it is now forty years since the major part were sold and the work of improvement begun. A generation has passed away since they were converted into

cultivated farms, and few if any of the original corner and quarter stakes now remain.

The corner and quarter stakes were often nothing but green sticks driven into the ground. Stones might be put around or over these if they were handy, but often they were not, and the witness trees must be relied upon after the stake was gone. Too often the first settlers were careless in fixing their lines with accuracy while monuments remained, and an irregular brush fence, or something equally untrustworthy, may have been relied upon to keep in mind where the blazed line once was. A fire running through this might sweep it away, and if nothing were substituted in its place, the adjoining proprietors might in a few years be found disputing over their lines, and perhaps rushing into litigation, as soon as they had occasion to cultivate the land along the boundary.

If now the disputing parties call in a surveyor, it is not likely that any one summoned would doubt or question that his duty was to find, if possible, the place of the original stakes which determined the boundary line between the proprietors. However erroneous may have been the original survey, the monuments that were set must nevertheless govern, even though the effect be to make one half-quarter section ninety acres and the one adjoining but seventy; for parties buy or are supposed to buy in reference to those monuments, and are entitled to what is within their lines, and no more, be it more or less. McIver *v.* Walker, 4 Wheaton's Reports, 444; Land Co. *v.* Saunders, 103 U. S. Reports, 316; Cottingham *v.* Parr, 93 Ill. Reports, 233; Bunton *v.* Cardwell, 53 Texas Reports, 408; Watson *v.* Jones, 85 Penn. Reports, 117.

While the witness trees remain there can generally be no difficulty in determining the locality of the stakes. When the witness trees are gone, so that there is no longer record evidence of the monuments, it is remarkable how many there are who mistake altogether the duty that now devolves upon the surveyor. It is by no means uncommon that we find men whose theoretical education is supposed to make them experts who think that when the monuments are gone, the only thing to be done is to place new monuments where the old ones should have been, and where they would have been if placed correctly. This is a serious mistake. The problem is now the same that it was before; to ascertain, by the best lights of which the case admits, where the original lines were. The mistake above alluded to is supposed to have found expression in our legislation; though it is possible that the real intent of the act to which we shall refer is not what is commonly supposed.

An act passed in 1869, Compiled Laws, § 593, amending the laws respecting the duties and powers of county surveyors, after providing for the case of corners which can be identified by the original field notes or other unquestionable testimony, directs as follows: [1]

"*Second.* Extinct interior-section corners must be re-established at the intersection of two right lines joining the nearest known points on the original section lines east and west and north and south of it.

"*Third.* Any extinct quarter-section corner, except on fractional lines, must be re-established equidistant and in a right line between the section corners; in all other cases at its proportionate distance between the nearest original corners on the same line."

[1] For the U. S. rules governing this subject, see Appendix I, page 817.

The corners thus determined, the surveyors are required to perpetuate by noting bearing trees when timber is near.

To estimate properly this legislation, we must start with the admitted and unquestionable fact that each purchaser from government bought such land as was within the original boundaries, and unquestionably owned it up to the time when the monuments became extinct. If the monument was set for an interior-section corner, but did not happen to be "at the intersection of the two right lines joining the nearest known points on the original section lines east and west and north and south of it," it nevertheless determined the extent of his possessions, and he gained or lost according as the mistake did or did not favor him.

It will probably be admitted that no man loses title to his land or any part thereof merely because the evidences become lost or uncertain. It may become more difficult for him to establish it as against an adverse claimant, but theoretically the right remains; and it remains as a potential fact so long as he can present better evidence than any other person. And it may often happen that, notwithstanding the loss of all trace of a section corner or quarter stake, there will still be evidence from which any surveyor will be able to determine with almost absolute certainty where the original boundary was between the government subdivisions.

There are two senses in which the word extinct may be used in this connection: one of the sense of physical disappearance; the other the sense of loss of all reliable evidence. If the statute speaks of extinct corners in the former sense, it is plain that a serious mistake was made in supposing that surveyors could be clothed with authority to establish new corners by an arbitrary rule in such cases. As well might the statute declare that if a man lose his deed he shall lose his land altogether.

But if by extinct corner is meant one in respect to the actual location of which all reliable evidence is lost, then the following remarks are pertinent:

1. There would undoubtedly be a presumption in such a case that the corner was correctly fixed by the government surveyor where the field notes indicated it to be.

2. But this is only a presumption, and may be overcome by any satisfactory evidence showing that in fact it was placed elsewhere.

3. No statute can confer upon a county surveyor the power to "establish" corners, and thereby bind the parties concerned. Nor is this a question merely of conflict between State and Federal law; it is a question of property right. The original surveys must govern, and the laws under which they were made must govern, because the land was bought in reference to them: and any legislation, whether State or Federal, that should have the effect to change these, would be inoperative, because disturbing vested rights.

4. In any case of disputed lines, unless the parties concerned settle the controversy by agreement, the determination of it is necessarily a judicial act, and it must proceed upon evidence, and give full opportunity for a hearing. No arbitrary rules of survey or of evidence can be laid down whereby it can be adjudged.

The general duty of a surveyor in such a case is plain enough. He is not to assume that a monument is lost until after he has thoroughly sifted the evidence and found himself unable to trace it. Even then he should hesitate long before doing anything to the disturbance of settled

possessions. Occupation, especially if long continued, often affords very satisfactory evidence of the original boundary when no other is attainable; and the surveyor should inquire when it originated, how, and why the lines were then located as they were, and whether a claim of title has always accompanied the possession, and give all the facts due force as evidence. Unfortunately, it is known that surveyors sometimes, in supposed obedience to the State statute, disregard all evidences of occupation and claim of title, and plunge whole neighborhoods into quarrels and litigation by assuming to "establish" corners at points with which the previous occupation cannot harmonize. It is often the case that where one or more corners are found to be extinct, all parties concerned have acquiesced in lines which were traced by the guidance of some other corner or landmark, which may or may not have been trustworthy; but to bring these lines into discredit when the people concerned do not question them not only breeds trouble in the neighborhood, but it must often subject the surveyor himself to annoyance and perhaps discredit, since in a legal controversy the law as well as common sense must declare that a supposed boundary line long acquiesced in is better evidence of where the real line should be than any survey made after the original monuments have disappeared. Stewart *v.* Carleton, 31 Mich. Reports, 270; Diehl *v.* Zanger, 39 Mich. Reports, 601; Dupont *v.* Starring, 42 Mich. Reports, 492. And county surveyors, no more than any others, can conclude parties by their surveys.

The mischiefs of overlooking the facts of possession must often appear in cities and villages. In towns the block and lot stakes soon disappear; there are no witness trees and no monuments to govern except such as have been put in their places, or where their places were supposed to be. The streets are likely to be soon marked off by fences, and the lots in a block will be measured off from these, without looking farther. Now it may perhaps be known in a particular case that a certain monument still remaining was the starting point in the original survey of the town plat; or a surveyor settling in the town may take some central point as the point of departure in his surveys, and assuming the original plat to be accurate, he will then undertake to find all streets and all lots by course and distance according to the plat, measuring and estimating from his point of departure. This procedure might unsettle every line and every monument existing by acquiescence in the town; it would be very likely to change the lines of streets, and raise controversies everywhere. Yet this is what is sometimes done; the surveyor himself being the first person to raise the disturbing questions.

Suppose, for example, a particular village street has been located by acquiescence and use for many years, and the proprietors in a certain block have laid off their lots in reference to this practical location. Two lot owners quarrel, and one of them calls in a surveyor that he may be sure that his neighbor shall not get an inch of land from him. This surveyor undertakes to make his survey accurate, whether the original was, or not, and the first result is, he notifies the lot owners that there is error in the street line, and that all fences should be moved, say, one foot to the east. Perhaps he goes on to drive stakes through the block according to this conclusion. Of course, if he is right in doing this, all lines in the village will be unsettled; but we will limit our attention to the single block. It is not likely that the lot owners generally will

allow the new survey to unsettle their possessions, but there is always a probability of finding someone disposed to do so. We shall then have a lawsuit; and with what result?

It is a common error that lines do not become fixed by acquiescence in a less time than twenty years. In fact, by statute, road lines may become conclusively fixed in ten years; and there is no particular time that shall be required to conclude private owners, where it appears that they have accepted a particular line as their boundary, and all concerned have cultivated and claimed up to it. McNamara *v.* Seaton, 82 Ill. Reports, 498; Bunce *v.* Bidwell, 43 Mich. Reports, 542. Public policy requires that such lines be not lightly disturbed, or disturbed at all after the lapse of any considerable time. The litigant, therefore, who in such a case pins his faith on the surveyor, is likely to suffer for his reliance, and the surveyor himself to be mortified by a result that seems to impeach his judgment.

Of course nothing in what has been said can require a surveyor to conceal his own judgment, or to report the facts one way when he believes them to be another. He has no right to mislead, and he may rightfully express his opinion that an original monument was at one place, when at the same time he is satisfied that acquiescence has fixed the right of parties as if it were at another. But he would do mischief if he were to attempt to "establish" monuments which he knew would tend to disturb settled rights; the farthest he has a right to go, as an officer of the law, is to express his opinion where the monument should be, at the same time that he imparts the information to those who employ him, and who might otherwise be misled, that the same authority that makes him an officer and entrusts him to make surveys, also allows parties to settle their own boundary lines, and considers acquiescence in a particular line or monument, for any considerable period, as strong, if not conclusive, evidence of such settlement. The peace of the community absolutely requires this rule. Joyce *v.* Williams, 26 Mich. Reports, 332. It is not long since that, in one of the leading cities of the State, an attempt was made to move houses two or three rods into a street, on the ground that a survey under which the street had been located for many years had been found on more recent survey to be erroneous.

From the foregoing it will appear that the duty of the surveyor where boundaries are in dispute must be varied by the circumstances. 1. He is to search for original monuments, or for the places where they were originally located, and allow these to control if he finds them, unless he has reason to believe that agreements of the parties, express or implied, have rendered them unimportant. By monuments in the case of government surveys we mean of course the corner and quarter stakes: blazed lines or marked trees on the lines are not monuments; they are merely guides or finger-posts, if we may use the expression, to inform us with more or less accuracy where the monuments may be found. 2. If the original monuments are no longer discoverable, the question of location becomes one of evidence merely. It is merely idle for any State statute to direct a surveyor to locate or "establish" a corner, as the place of the original monument, according to some inflexible rule. The surveyor, on the other hand, must inquire into all the facts; giving due prominence to the acts of parties concerned, and always keeping in mind, *first*,

that neither in his opinion nor his survey can be conclusive upon parties concerned; *second*, that courts and juries may be required to follow after the surveyor over the same ground, and that it is exceedingly desirable that he govern his action by the same lights and rules that will govern theirs. On town plats if a surplus or deficiency appears in a block, when the actual boundaries are compared with the original figures, and there is no evidence to fix the exact location of the stakes which marked the division into lots, the rule of common sense and of law is that the surplus or deficiency is to be apportioned between the lots, on an assumption that the error extended alike to all parts of the block. O'Brien *v.* McGrane, 29 Wis. Reports, 446; Quinnin *v.* Reixers, 46 Mich. Reports, 605.

It is always possible when corners are extinct that the surveyor may usefully act as a mediator between parties, and assist in preventing legal controversies by settling doubtful lines. Unless he is made for this purpose an arbitrator by legal submission, the parties, of course, even if they consent to follow his judgment, cannot, on the basis of mere consent, be compelled to do so; but if he brings about an agreement, and they carry it into effect by actually conforming their occupation to his lines, the action will conclude them. Of course it is desirable that all such agreements be reduced to writing: but this is not absolutely indispensable if they are carried into effect without.

Meander Lines. The subject to which allusion will now be made is taken up with some reluctance, because it is believed the general rules are familiar. Nevertheless it is often found that surveyors misapprehend them, or err in their application; and as other interesting topics are somewhat connected with this, a little time devoted to it will probably not be altogether lost. The subject is that of meander lines. These are lines traced along the shores of lakes, ponds, and considerable rivers as the measures of quantity when sections are made fractional by such waters. These have determined the price to be paid when government lands were bought, and perhaps the impression still lingers in some minds that the meander lines are boundary lines, and all in front of them remains unsold. Of course this is erroneous. There was never any doubt that, except on the large navigable rivers, the boundary of the owners of the banks is the middle line of the river; and while some courts have held that this was the rule on all fresh-water streams, large and small, others have held to the doctrine that the title to the bed of the stream below low-water mark is in the State, while conceding to the owners of the banks all riparian rights. The practical difference is not very important. In this State the rule that the centre line is the boundary line is applied to all our great rivers, including the Detroit, varied somewhat by the circumstance of there being a distinct channel for navigation in some cases with the stream in the main shallow, and also sometimes by the existence of islands.

The troublesome questions for surveyors present themselves when the boundary line between two contiguous estates is to be continued from the meander line to the centre line of the river. Of course the original survey supposes that each purchaser of land on the stream has a water front of the length shown by the field notes; and it is presumable that he bought this particular land because of that fact. In many cases it now happens that the meander line is left some distance from the shore

by the gradual change of course of the stream or diminution of the flow of water. Now the dividing line between two government subdivisions might strike the meander line at right angles, or obliquely; and in some cases, if it were continued in the same direction to the centre line of the river, might cut off from the water one of the subdivisions entirely, or at least cut it off from any privilege of navigation, or other valuable use of the water, while the other might have a water front much greater than the length of a line crossing it at right angles to its side lines. The effect might be that, of two government subdivisions of equal size and cost, one would be of very great value as water-front property, and the other comparatively valueless. A rule which would produce this result would not be just, and it has not been recognized in the law.

Nevertheless it is not easy to determine what ought to be the correct rule for every case. If the river has a straight course, or one nearly so, every man's equities will be preserved by this rule: Extend the line of division between the two parcels from the meander line to the centre line of the river, as nearly as possible at right angles to the general course of the river at that point. This will preserve to each man the water front which the field notes indicated, except as changes in the water may have affected it, and the only inconvenience will be that the division line between different subdivisions is likely to be more or less deflected where it strikes the meander line.

This is the legal rule, and it is not limited to government surveys, but applies as well to water lots which appear as such on town plats. Bay City Gas Light Co. *v.* The Industrial Works, 28 Mich. Reports, 182. It often happens, therefore, that the lines of city lots bounded on navigable streams are deflected as they strike the bank, or the line where the bank was when the town was first laid out.

When the stream is very crooked, and especially if there are short bends, so that the foregoing rule is incapable of strict application, it is sometimes very difficult to determine what shall be done; and in many cases the surveyor may be under the necessity of working out a rule for himself. Of course his action cannot be conclusive; but if he adopts one that follows, as nearly as the circumstances will admit, the general rule above indicated, so as to divide as near as may be the bed of the stream among the adjoining owners in proportion to their lines upon the shore, his division, being that of an expert, made upon the ground and with all available lights, is likely to be adopted as law for the case. Judicial decisions, into which the surveyor would find it prudent to look under such circumstances, will throw light upon his duties and may constitute a sufficient guide when peculiar cases arise. Each riparian lot owner ought to have a line on the legal boundary, namely, the centre line of the stream, proportioned to the length of his line on the shore; and the problem in each case is, how this is to be given him. Alluvion, when a river imperceptibly changes its course, will be apportioned by the same rules.

The existence of islands in a stream, when the middle line constitutes a boundary, will not affect the apportionment unless the islands were surveyed out as government subdivisions in the original admeasurement. Wherever that was the case, the purchaser of the island divides the bed of the stream on each side with the owner of the bank, and his rights also extend above and below the solid ground, and are limited by the

peculiarities of the bed and the channel. If an island was not surveyed as a government subdivision previous to the sale of the bank, it is of course impossible to do this for the purposes of government sale afterwards, for the reason that the rights of the bank owners are fixed by their purchase: when making that, they have a right to understand that all land between the meander lines, not separately surveyed and sold, will pass with the shore in the government sale; and having this right, anything which their purchase would include under it cannot afterward be taken from them. It is believed, however, that the federal courts would not recognize the applicability of this rule to large navigable rivers, such as those uniting the great lakes.

On all the little lakes of the State which are mere expansions near their mouths of the rivers passing through them—such as the Muskegon, Père Marquette, and Manistee—the same rule of bed ownership has been judicially applied, that is, applied to the rivers themselves; and the division lines are extended under the water in the same way. Rice *v.* Ruddiman, 10 Mich., 125. If such a lake were circular, the lines would converge to the centre; if oblong or irregular, there might be a line in the middle on which they would terminate, whose course would bear some relation to that of the shore. But it can seldom be important to follow the division line very far under the water, since all private rights are subject to the public rights of navigation and other use, and any private use of the lands inconsistent with these would be a nuisance, and punishable as such. It is sometimes important, however, to run the lines out for some considerable distance, in order to determine where one may lawfully moor vessels or rafts, for the winter, or cut ice. The ice crop that forms over a man's land of course belongs to him. Lorman *v.* Benson, 8 Mich., 18; People's Ice Co. *v.* Steamer Excelsior, recently decided.

What is said above will show how unfounded is the notion, which is sometimes advanced, that a riparian proprietor on a meandered river may lawfully raise the water in the stream without liability to the proprietors above, provided he does not raise it so that it overflows the meander line. The real fact is that the meander line has nothing to do with such a case, and an action will lie whenever he sets back the water upon the proprietor above, whether the overflow be below the meander lines or above them.

As regards the lakes and ponds of the State, one may easily raise questions that it would be impossible for him to settle. Let us suggest a few questions, some of which are easily answered, and some not:

1. To whom belongs the land under these bodies of water, where they are not mere expansions of a stream flowing through them?

2. What public rights exist in them?

3. If there are islands in them which were not surveyed out and sold by the United States, can this be done now?

Others will be suggested by the answers given to these.

It seems obvious that the rules of private ownership which are applied to rivers cannot be applied to the great lakes. Perhaps it should be held that the boundary is at low-water mark, but improvements beyond this would only become unlawful when they became nuisances. Islands in the great lakes would belong to the United States until sold, and

might be surveyed and measured for sale at any time. The right to take fish in the lakes, or to cut ice, is public, like the right of navigation, but is to be exercised in such manner as not to interfere with the rights of shore owners. But so far as these public rights can be the subject of ownership, they belong to the State, not to the United States; and, so it is believed, does the bed of a lake also. Pollard *v.* Hagan, 3 Howard's U. S. Reports. But such rights are not generally considered proper subjects of sale, but, like the right to make use of the public highways, they are held by the State in trust for all the people.

What is said of the large lakes may perhaps be said also of many of the interior lakes of the State; such, for example, as Houghton, Higgins, Cheboygan, Burt's, Mullet, Whitmore, and many others. But there are many little lakes or ponds which are gradually disappearing, and the shore proprietorship advances *pari passu* as the waters recede. If these are of any considerable size—say, even a mile across—there may be questions of conflicting rights which no adjudication hitherto made could settle. Let any surveyor, for example, take the case of a pond of irregular form, occupying a mile square or more of territory, and undertake to determine the rights of the shore proprietors to its bed when it shall totally disappear, and he will find he is in the midst of problems such as probably he has never grappled with, or reflected upon before. But the general rules for the extension of shore lines, which have already been laid down, should govern such cases, or at least should serve as guides in their settlement.[1]

Where a pond is so small as to be included within the lines of a private purchase from the government, it is not believed the public have any rights in it whatever. Where it is not so included, it is believed they have rights of fishery, rights to take ice and water, and rights of navigation for business or pleasure. This is the common belief, and probably the just one. Shore rights must not be so exercised as to disturb these, and the States may pass all proper laws for their protection. It would be easy with suitable legislation to preserve these little bodies of water as permanent places of resort for the pleasure and recreation of the people, and there ought to be such legislation.

If the State should be recognized as owner of the beds of these small lakes and ponds, it would not be owner for the purpose of selling. It would be owner only as a trustee for the public use; and a sale would be inconsistent with the right of the bank owners to make use of the water in its natural condition in connection with their estates. Some of them might be made salable lands by draining; but the State could not drain, even for this purpose, against the will of the shore owners, unless their rights were appropriated and paid for.

Upon many questions that might arise between the State as owner of the bed of a little lake and the shore owners, it would be presumptuous to express an opinion now, and fortunately the occasion does not require it.

---

[1] Since this address was delivered some of these questions have received the attention of the Supreme Court of Michigan in the cases of Richardson *v.* Prentiss, 48 Mich. Reports, 88, and Backus *v.* Detroit, Albany Law Journal, vol. 26, p. 428.

I have thus indicated a few of the questions with which surveyors may now and then have occasion to deal, and to which they should bring good sense and sound judgment. Surveyors are not and cannot be judicial officers, but in a great many cases they act in a *quasi*-judicial capacity with the acquiescence of parties concerned; and it is important for them to know by what rules they are to be guided in the discharge of their judicial functions. What I have said cannot contribute much to their enlightenment, but I trust will not be wholly without value.

APPENDIX B

MANUAL OF INSTRUCTIONS FOR THE SURVEY OF THE MINERAL LANDS OF THE UNITED STATES[1]

GENERAL PROVISIONS

1. Under section 2334, U. S. Rev. Stats. (see Appendix B hereof), the United States Surveyor-General "may appoint in each land district containing mineral lands as many competent surveyors as shall apply for appointment to survey mining claims."

2. Capable parties desiring such appointments should therefore file their applications with the Surveyor-General for the district wherein appointment is asked, who will furnish all information necessary.

3. Deputies may at the same time hold commissions in more than one State or land district. (20 L. D., 163.)

4. All appointments of deputy mineral surveyors must be submitted to the Commissioner of the General Land Office for approval.

5. The Surveyors-General have authority to suspend or revoke the commissions of their deputy mineral surveyors for cause. Before final action, however, the matter should be submitted to the Commissioner of the General Land Office for approval.

Deputies will be allowed the right of appeal from the action of the Surveyors-General in the usual manner. Such appeal should be filed with the Surveyor-General, who will at once transmit the same, with a full report, to the General Land Office. (20 L. D., 283.)

6. Neither the Surveyor-General nor the Commissioner of the General Land Office has jurisdiction to settle differences, relative to the payment of charges for field work between deputy mineral surveyors and claimants. These are matters of private contract and must be enforced in the ordinary manner, i.e., in the local courts.

The Department has, however, authority to investigate charges affecting the official actions of deputy mineral surveyors, and will, on sufficient cause shown, suspend or revoke absolutely the commission of the deputy.

7. The Surveyors-General should appoint "as many" competent deputy mineral surveyors as apply for appointment, in order that claimants may have a choice of deputies, and be enabled to have their work done on the most advantageous terms.

8. The schedule of charges for office work should be as low as is possible. No additional charges should be made for orders for amended

[1] Reprinted in full from the official publication issued in 1895 by the Commissioner of the General Land Office, Washington, D. C.

surveys, unless the necessity therefor is clearly the fault of the claimant, or considerable additional office work results therefrom.

In cases where the error in the original survey is due to the carelessness or neglect of the deputy mineral surveyor who made it, he should be required to make the necessary corrections in the field at his own expense, and the Surveyor-General should advise him that the penalty for failure to comply with instructions within a specified time will be the suspension or revocation of his commission.

9. These instructions are subject to the limitations of section 2324, U. S. Rev. Stats., so far as the same refers to local laws and customs.

INSTRUCTIONS FOR SURVEYS

1. All official communications must be addressed to the Surveyor-General.[1] You will always refer to the date and subject-matter of the letter to which you reply, and when a mineral claim is the subject of correspondence, you will give the name and survey number.

2. You should keep a complete record of each survey made by you, and the facts coming to your knowledge at the time, as well as copies of all your field notes, reports, and official correspondence, in order that such evidence may be readily produced when called for at any future time.

3. Field notes and other reports must be written in a clear and legible hand or typewritten, in non-copying ink, and upon the proper blanks furnished you gratuitously by the Surveyor-General's Office upon application therefor. *No interlineations or erasures* will be allowed; and no abbreviations or symbols must be used, except such as are indicated in the specimen field notes.

4. No return by you will be recognized as official, unless it is over your signature as a U. S. deputy mineral surveyor, and made in pursuance of a special order from the Surveyor-General's Office. After you have received an order for survey, you are required to make the survey, and return correct field notes thereof to the Surveyor-General's Office without delay.

5. The claimant is required, in all cases, to make satisfactory arrangements with you for the payment for your services and those of your assistants in making the survey, as the United States will not be held responsible for the same. You will call the attention of applicants for mineral-survey orders to the requirements of paragraph 14 of the circular, Appendix A. (Sec. 2334, U. S. Rev. Stats.; Par. 98, Mining Circular, December 10, 1891—see Appendix B hereof.)

6. You will promptly notify the Surveyor-General's Office of any change in your post-office address. (20 L. D., 163.)

NOT TO ACT AS ATTORNEY

7. You are precluded from acting, either directly or indirectly, as attorney in mineral claims. Your duty in any particular case ceases when you have executed the survey and returned the field notes and preliminary plat, with your report, to the Surveyor-General. You will not be allowed to prepare for the mining claimant the papers in support of his application for patent, or otherwise perform the duties of an

[1] For list of Surveyors-General in mining districts, see page 781.

attorney before the land office in connection with a mining claim. You are not permitted to combine the duties of surveyor and notary public in the same case by administering oaths to the parties in interest, but as a notary public you may administer the oaths to your assistants in making the survey. Otherwise you must have absolutely nothing to do with the case, except in your official capacity as surveyor. You will make no survey of a mineral claim in which you hold an interest, nor will you employ chainmen interested therein in any manner. (Par. 101, Mining Circular, December 10, 1891—see Appendix B hereof; 13 C. L. O., 608.)

THE FIELD WORK

8. The survey made and reported must, in every case, be an actual survey on the ground in full detail, made by you in person after the receipt of the order, and without reference to any knowledge you may have previously acquired by reason of having made the location survey or otherwise, and must show the actual facts existing at the time. This precludes you from calculating the connections to corners of the public survey and location monuments, or any other lines of your survey through prior surveys made by others, unless it is satisfactorily shown in your report that you have retraced such lines and found them to be correct. (6 L. D., 718; 7 L. D., 81.)

The term *survey* in these instructions applies not only to the usual field work, but also to the examinations required for the preparation of your affidavits of five hundred dollars expenditure, descriptive reports on placer claims, and all other reports.

SURVEY AND LOCATION

9. The survey of a mining claim may consist of several contiguous locations, but such survey must, in conformity with statutory requirements, distinguish the several locations and exhibit the boundaries of each. (5 L. D., 199; 6 L. D., 808.)

10. The survey must be made in strict conformity with, or be embraced within, the lines of the location upon which the order is based. If the survey and location are identical, that fact must be clearly and distinctly stated in your field notes. If not identical, a bearing and distance must be given from each established corner of the survey to the corresponding corner of the location, and the location corner must be fully described, so that it can be identified. The lines of the location, as found upon the ground, must be laid down upon the preliminary plat in such a manner as to contrast and show their relation to the lines of survey. (1 L. D., 581.)

11. In accordance with the principle that courses and distances must give way when in conflict with fixed objects and monuments, you will not, under any circumstances, change the corners of the location for the purpose of making them conform to the description in the record. If the difference from the location be slight, it may be explained in the field notes.

"The act of Congress of May 10, 1872, expressly provides that 'the location must be distinctly marked upon the ground so that its boundaries can be readily traced,' and 'that all records of mining claims hereafter made shall contain the name or names of the locators, the date of the location, and such a description of the claim or claims located, by

reference to some natural object or permanent monument, as will identify the claim.'" (Sec. 2324, U. S. R. S.; see Appendix B herewith.)

"These provisions of the law must be strictly complied with in each case to entitle the claimant to a survey and patent, and, therefore, should a claimant under a location made *subsequent* to the passage of the act of May 10, 1872, *who has not complied with said requirements in regard to marking the location upon the ground and recording the same, apply for a survey, you will decline to make it.*" (1 L. D., 581.)

You will then report the facts to this office and await further instructions.

Should the survey be applied for under a location made prior to May 10, 1872, or under section 2332, U. S. Rev. Stats., in making the survey thereof you will be governed by the special instructions accompanying the order for survey.

No mining claim located subsequent to May 10, 1872, should exceed the statutory limit in width on each side of the centre of vein or 1500 feet in length, and all surveys must close within 50–100 feet in one thousand feet, and the error must not be such as to make the location exceed the statutory limit, and in absence of other proof the discovery point is held to be the centre of the vein on the surface. The course and length of the vein should be marked upon the plat.

INSTRUMENT

12. All mineral surveys must be made with a transit, provided with a solar attachment, by which the meridian can be determined independently of the magnetic needle, and all courses must be referred to the true meridian. The variation should be noted at each corner of the survey.

THE TRUE MERIDIAN

12*b*. The true course of at least one line of each survey must be ascertained by astronomical observations made at the time of the survey; the data for determining the same and details as to how these data were arrived at must be given. Or, in lieu of the foregoing, the survey must be connected with some line the true course of which has been previously established beyond question, and in a similar manner, by yourself, and, when such lines exist, it is desirable in all cases that they should be used as a proof of the accuracy of subsequent work. (For methods for determining the true meridian see pages 84 to 119, inclusive, General Surveying Manual, 1894.)

CONNECTIONS

13. Connect corner No. 1 of each location embraced in your survey by course and distance with nearest corner of the public survey or with a United States location monument, if the claim lies within two miles of such corner or monument. If both are within the required distance, you must connect with the nearest corner of the public survey. (7 L. D., 475; Par. 45, Circular December 10, 1891—Appendix B herewith.)

(*a*) You will make surveys and connections of mineral claims in suspended townships so long as they remain suspended, in the same manner as though the claims were upon unsurveyed land, except as hereinafter specified, by connecting them with independent mineral monuments. At the same time, you will note the position of any public-

land corner which may be found in the neighborhood of the claim, so that, in case of the release of the township plat from suspension, the position of the claim can be shown on the plat.

(*b*) A mineral survey must not be returned with its connection made only with a corner of the public survey, where the survey of the township within which it is situated is under suspension, nor connected with a mineral monument alone, when situated within the limits of a township the regularity and correctness of the survey of which is unquestioned.

(*c*) In making an official survey hereafter you will establish corner No. 1 of each location embraced in your survey at the end nearest the corner of the public survey or locating monument, unless good cause is shown for its being placed otherwise. If connections are given to both a corner of the public survey and location monument, corners Nos. 1 should be placed at the end nearest the corner of the public survey.

14. When a boundary line of a claim intersects a section line, give courses and distances from point of intersection to the Government corners at each end of the half mile of section line so intersected.

LOCATION MONUMENTS

15. In case your survey is situated in a district where there are no corners of the public survey and no monuments within the prescribed limits, you will proceed to establish a mineral monument, in the location of which you will exercise the greatest care to insure permanency as to site and construction.

The site, when practicable, should be some prominent point, visible for a long distance from every direction, and should be so chosen that the permanency of the monument will not be endangered by snow-, rock-, or landslides, or other natural causes.

16. The location monument should consist of a stone not less than 30 inches long, 20 inches wide, and 6 inches thick, set halfway in the ground, with a conical mound of stone 4 feet high and 6 feet base alongside. The letters U. S. L. M., followed by the consecutive number of the monument in the district, must be plainly chiselled upon the stone. If impracticable to obtain a stone of required dimensions, then a post 8 feet long, 6 inches square, set 3 feet in the ground, scribed as for a stone monument, protected by a well-built conical mound of stone of not less than 3 feet high and 6 feet base around it, may be used. The exact point for connection must be indicated on the monument by an X chiselled thereon; if a post is used, then a tack must be driven into the post to indicate the point.

17. From the monument, connections by course and distance must be taken to two or three bearing trees or rocks, and to any well known and permanent objects in the vicinity, such as the confluence of streams, prominent rocks, buildings, shafts, or mouths, of adits. Bearing trees must be properly scribed B. T., and bearing rocks chiselled B. R., together with the number of the location monument; the exact point on the tree or stone to which the connection is taken should be indicated by a cross or other unmistakable mark. Bearings should also be taken to prominent mountain peaks, and the approximate distance and direction ascertained from the nearest town or mining camp. A detailed description of the locating monument, with a topographical map of its location, should be furnished this office.

CORNERS

18. Corners may consist of—

First—A stone at least 24 inches long set 12 inches in the ground, with a conical mound of stone 1½ feet high, 2 feet base, alongside, and state kind of stone set for corner. A stone should always be used for a corner when possible.

Second—A post at least 3 feet long by 4 inches square, set 18 inches in the ground and surrounded by a substantial mound of stone or earth.

Third—A rock in place.

19. All corners must be established in a permanent and workmanlike manner, and the corner and survey number must be neatly chiselled or scribed on the sides facing the claim. The *exact* corner point must be permanently indicated on the corner. When a rock in place is used its dimensions above ground must be stated, and a cross chiselled at the exact corner point.

20. In case the point for the corner be inaccessible or unsuitable, you will establish a witness corner, which must be marked with the letters W. C. in addition to the corner and survey number. The witness corner should be located upon a line of the survey and as near as possible to the true corner, with which it must be connected by course and distance. The reason why it is impossible or impracticable to establish a true corner must always be stated in the field notes, and in running your next course state whether you start from the true place for corner or from witness corner.

21. The identity of all corners should be perpetuated by taking courses and distances to bearing trees, rocks, and other objects, as prescribed in the establishment of location monuments, and when no bearings are given, state "no bearings available." Permanent objects should be taken whenever possible.

22. If an official survey has been made within a reasonable distance in the vicinity, you will run a connecting line to some corner of the same, and connect in like manner with all conflicting surveys and claims, and describe corner with which connection is made.

23. In survey of contiguous locations which are part of a consoldiated claim, where corners are common, mention bearings but once, but where a corner is common to different claims, the required number of bearings will be taken from each claim.

TOPOGRAPHY

24. Note carefully all topographical features of the claim, taking distances on your lines to intersections with all streams, gulches, ditches, ravines, mountain ridges, roads, trails, etc., with their widths, courses, and other data that may be required to map them correctly. If the claim lies within a town site, locate all municipal improvements, such as blocks, streets, and buildings.

CONFLICTS

25. If, in running the exterior boundaries of a claim, you find that two surveys conflict, you will determine the courses and distances from the established corners thereof, *situate within the boundaries of your survey*, at which the exterior boundaries intersect each other, and run all lines necessary for the determination of the areas in conflict, both with surveyed and unsurveyed claims. You will not, however, show conflicts with unsurveyed claims unless the same are to be excluded.

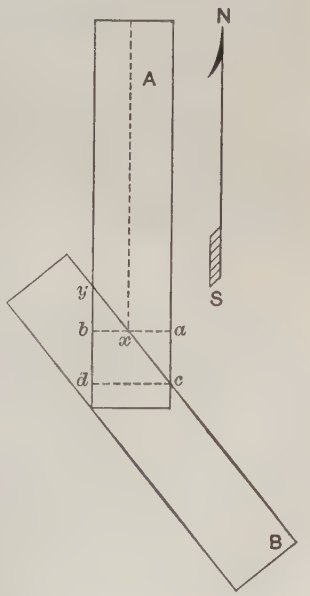
26. Your attention is directed to paragraphs 50 and 51 of General Land Office circular, dated December 10, 1891, as amended by circular of November 7, 1895:

"50. The rights granted to locators under section 2322, Revised Statutes, are restricted to such locations on veins, lodes, or ledges as may be 'situated on the *public domain.*' In applications for lode claims where the survey conflicts with a prior valid lode claim, and the ground in conflict is excluded, the applicant not only has no rihht to the excluded ground, but he has no right to that portion of any vein or lode, the top or apex of which lies within such excluded ground, unless his location was prior to May 10, 1872. His right to the lode claimed terminates where the lode; in its onward course or strike, intersects the exterior boundary of such excluded ground and passes within it. The end line of his survey should not, therefore, be established beyond such intersection.

"51. Where, however, the lode claim for which survey is being made, was located prior to the conflicting claim, and such conflict is to be excluded, in order to include all ground not so excluded the end line of the survey may be established within the conflicting lode claim, but the line must be so run as not to extend any further into such conflicting claim than may be necessary to make such end line parallel to the other end line, and at the same time embrace the ground so held and claimed. The useless practice in such cases of extending *both* the side lines of a survey into the conflicting claim, and establishing an end line wholly within it, beyond a point necessary under the rule just stated, will be discontinued."

EXPLANATION

Location *A* in the diagram represents a location for which survey is applied for. As a location it conflicts with location *B*, and the claimant of *A* lays no claim to the conflicting area. In accordance with the definition of locators' rights under Sec. 2322, U. S. R. S., as given in paragraph 50 of said circular of December 10, 1891, if location *B* was a valid and subsisting location at the time location *A* was made, the locator of *A* has no right to extend the survey of his claim beyond the point where his lode in its onward course intersects the location *B*, and in such case the end lines of the survey should be run through such point of intersection, as represented by lines *a*, *b* on diagram.



If it is more desirable, however, the south end line may be the side line of *B* within the side lines of *A* as represented by *c*, *y*, with which the north end line must be made parallel.

This circular applies also to contiguous locations belonging to one owner and surveyed as a single claim.

If, on the contrary, when location *A* was made the location *B* was not existent, and there was no other conflict, the abandonment of the conflict with the latter location, *B*, brings the case within paragraph 51 of said circular.

To illustrate: *A* is the first and valid location, afterward the *B* location is made, and for reasons satisfactory to *A* the ground in conflict is relinquished to *B*, yet the claimant of *A* claims all the land outside of the conflict included in his location.

The end line contemplated by the circular to include all ground claimed as aforesaid, should be run as represented by the line *cd*, the rest of the lines of conflict being eliminated entirely. The survey must in this case stop at the line *cd*.

27. It will be particularly observed by you that the above provisions of General Land Office circular dated December 10, 1891, are just as applicable in the case of conflicting and overlapping locations embraced in one survey as though the several locations were embraced in separate and distinct surveys.

28. A lode claim that is divided into two parts by an intersecting patented mill site, placer, or agricultural entry, must be confined to that part which contains the discovery shaft and improvements. (13 L. D., 146.)

29. The exterior lines of placer claims or mill sites cannot be extended over other claims and the conflicting areas excluded, as with lode claims, it being the surface ground only, with side lines taken perpendicularly downward, for which application is made. The survey must accurately define the boundaries of the *claim*.

30. If, by reason of intervening surveys or claims, a placer survey should be divided into separate tracts, you will preserve a separate series of numbers for the corners of each location, and a consecutive series of numbers for the corners of the tracts embraced in each; distinguishing the detached portions as Tract A, Tract B, etc., connecting by course and distance a corner of each tract with some corner of one previously described. The provisions of this paragraph will also apply to the surveys of mill sites.

LODE AND MILL SITE

31. A lode and mill site claim in one survey will be distinguished by the letters A and B following the number of the survey. The corners of the mill site will be numbered independently of those of the lode. Corner No. 1 of the mill site must be connected with a corner of the lode claim as well as with a corner of the public survey or United States location monument.

FIELD NOTES

32. In order that the results of your survey may be reported in a uniform manner, you will prepare your field notes and preliminary plat in strict conformity with the specimen field notes and plats which are made part of these instructions. They are designed to furnish you all the needed information concerning the manner of describing the boundaries, corners, connections, intersections, conflicts, and improvements, and stating the variation, area, location, and other data connected with the survey of mineral claims, and contain forms of affidavits for the deputy surveyor and his assistants.

33. When a placer claim includes lodes, or when several contiguous placer or lode locations are included as one claim in one survey, you will give to the corners of each location constituting the same a separate consecutive numerical designation, beginning with Corner No. 1 in each case. In the former case you will first describe the placer claim in your field notes.

34. Throughout the description of the survey, after each reference to

the lines or corners of a location, give the name thereof, and if unsurveyed state the fact. If reference is made to a location included in a prior official survey, the survey number must be given, followed by the name of the location. Describe your corners once only.

35. The total area of each location in a claim embraced by its exterior boundaries, and also the area in conflict with each intersecting survey or claim, should be stated; also the total area claimed. But when locations of the survey conflict with each other, such conflicts should only be stated in connection with the location from which the conflicting area is excluded.

36. You will state particularly whether the claim is upon surveyed or unsurveyed public lands, giving in the former case the quarter section, township, and range in which it is located, and in the latter, the township, as near as can be determined. When upon surveyed lands the section lines should be indicated by full lines and the quarter-section lines by dotted lines.

37. The title page must contain the post-office address of the claimant or his authorized agent.

EXPENDITURE OF FIVE HUNDRED DOLLARS

38. In making out your certificate of the value of the improvements, you will follow the form prescribed in the specimen field notes.

39. Only *actual* expenditures and *mining* improvements made by the claimant or his grantors, having a direct relation to the development of the claim, can be included in your estimate. "Labor or improvements within the meaning of the statute are deemed to have been had on a mining claim, whether it consists of one location or several, when the labor is performed or the improvements are made for its development, that is, to facilitate the extraction of the metals it may contain." (6 L. D., 222.)

40. The expenditures required may be made from the surface or in running a tunnel, drifts, or cross-cuts, for the development of the claim. Improvements of any other character, such as buildings, machinery, or roadways, must be excluded from your estimate unless you show clearly that they are associated with actual excavations, such as cuts, tunnels, shafts, etc., and are essential to the practical development of the surveyed claim.

41. You will locate all mining and other improvements upon the claim by courses and distances from corners of the survey, or from points on the centre or side lines, specifying with particularity and detail the dimensions and character of each, and the improvements upon each location should be numbered consecutively, the point of discovery being always No. 1.

42. You will give in detail the value of each mining improvement included in your estimate of expenditure, and when a tunnel or other improvement has been made for the development of other claims in connection with the one for which survey is made, you must give the name, ownership, and survey number, if any, of each claim to which a portion or interest is credited, and the value of the portion or interest credited to the claim. The value of improvements made upon other locations, or by a former locator who has abandoned his claim, cannot

be included in your estimate, but should be described and located in your notes and plat.

43. In case of a lode and mill-site claim in the same survey, an expenditure of five hundred dollars must be shown upon the lode claim only.

44. When a survey embraces several locations held in common constituting one entire claim, whether lode or placer, an expenditure of five hundred dollars upon such entire claim embraced in the survey will be sufficient, but in preparing your estimate of five hundred dollars' expenditure thereon you will observe the requirements of the decision of the Commissioner of the General Land Office, dated June 11, 1890, quoted below for your information:

"When two or more lode locations are embraced in one entry, and the improvements on each lode are not of the value of $500, it must be shown that a sum equal in value, of labor or improvements, has been expended for the common benefit of all those of which the improvements do not equal that sum, with a satisfactory explanation of how and in what manner such improvements tend to a common benefit." (See also 20 L. D., 394.)

The explanatory statement in such cases should be given in your field notes, or affidavit, at the conclusion of the description of the improvements included in the estimate of expenditure, and should be as full and explicit as the facts in the case warrant, dealing only with the improvements, conditions, and circumstances, as they actually existed at the time of making the survey or examination, without reference to what is possible or what the claimants may intend to do.

45. Following your certificate you will locate and describe all other improvements made by the claimant within the boundaries of the survey. Those made by other parties, if any, whose names should be mentioned, will be given in a separate description, following those of the claimants.

46. If the value of the labor and improvements upon a mineral claim is less than five hundred dollars at the time of survey, you are authorized to file your affidavit of five hundred dollars expenditure at any time. If the affidavit is made subsequent to the period of publication, it should be shown, if practicable, when the improvements were made. The information on which to base this affidavit must be derived by the deputy who makes the actual survey from a careful examination upon the premises.

PRELIMINARY PLAT

47. You will file with your field notes a preliminary plat on blank sent you for that purpose, protracted on a scale of 200 feet to an inch, if practicable, in conformity with the specimen plat herewith. In preparing plats make the top north. Copy of your calculations of areas by double meridian distances, and of all triangulations or traverse lines, must also be furnished. The lines of the claim surveyed, on this plat and on all plats of *approved* surveys, should be heavier and show a contrast with conflicting claims.

ERRORS

48. You will also mention in your notes the discovery of any material errors in prior official surveys, stating explicitly what lines are found to

be in error, and giving in express terms the courses and lengths thereof as determined by you.

49. Whenever a survey has been reported in error, the deputy surveyor who made it will be required to promptly make a thorough examination, upon the premises, and report the result, under oath, to the Surveyor-General's office. In case he finds his survey in error, he will report in detail all discrepancies with the original survey, and submit any explanation he may have to offer as to the cause. If, on the contrary, he should report his survey correct, a joint survey will be ordered to settle the differences with the surveyor who reported the error.[1]

JOINT SURVEY

50. A joint survey must be made within ten days after the date of order, unless satisfactory reasons are submitted, under oath, for a postponement.

51. The field work must in every sense of the term be a *joint* and not a separate survey, and the observations and measurements taken with the same instrument and chain, previously tested and agreed upon.

52. The deputy surveyor found in error or, if both are in error, the one who reported the same will make out the field notes of the joint survey, which, after being duly signed and sworn to by both parties, must be transmitted to the Surveyor-General's office. The surveyor found in error will be required to pay all expenses of the joint survey and ten dollars per day to the surveyor whose work is found to be substantially correct, and either deputy shall have the right to require the other to deposit the estimated amount of this expense with the Surveyor-General before the joint work is begun.

AMENDED SURVEYS

53. Inasmuch as amended surveys are ordered only by special instructions from the General Land Office, and the conditions and

[1] The following circular recently issued (1890) by the Hon. Commissioner of the General Land Office has somewhat changed the practice of reporting out surveys in error:

"When a mining claim has been surveyed and patented in accordance therewith, the land described therein is disposed of, and so long as the patent is outstanding the jurisdiction of the Department in regard to that particular tract is terminated. It therefore follows that land thus patented cannot be properly included in a subsequent patent, merely because years afterwards a deputy mineral surveyor in making a subsequent survey reports to have found the true corners of the old survey to occupy a different position from that reported in the survey which was the basis for patent of the old claim. And the same thing is true as to reported discrepancies as to the length and courses of lines of prior approved surveys.

"Where such a state of things actually exists the owner of the new claim applied for, who desires to include an area in his claim, conveyed in a patent of an older claim, which as a matter of fact is not embraced in the lines of the old claim as staked upon the ground, should procure the surrender of the old patent by the proper method, through the courts if necessary, and then show in a new patent of the old claim in its true position as staked and thus eliminate from the patent the areas desired not in conflict."

As a result of the above circular the deputy when he finds a prior approved survey in error cannot report it out, but must give a tie to the position of the corners as they appear by the records of the Surveyor-General's Office and not as they actually are on the ground. At present the practice is to calculate ties to prior official surveys through the section corner connections.

This ruling seems to a certain extent to be a violation of the old-established principle that "monuments hold over descriptions."

circumstances peculiar to each separate case, and the object sought by the required amendment, alone govern all special matters relative to the manner of making such survey and the form and subject matter to be embraced in the field notes thereof, but few general rules applicable to cases can be laid down.

54. The amended survey must be made in strict conformity with, or be embraced within, the lines of the original survey. If the amended and original surveys are identical, that fact must be clearly and distinctly stated in your field notes. If not identical, a bearing and distance must be given from each established corner of the amended survey to the corresponding corner of the original survey. The lines of the original survey, as found upon the ground, must be laid down upon the preliminary plat in such manner as to contrast and show their relation to the lines of the amended survey.

55. The field notes of the amended survey must be prepared on the same size and form of blanks as are the field notes of the original survey, and the word "amended" must be used before the word "survey" wherever it occurs in the field notes.

DESCRIPTIVE REPORTS ON PLACER CLAIMS

56. By General Land Office circular approved December 10, 1891, par. 63 (see Appendix B hereof), you are required to make a full examination of all placer claims at the time of survey, and file with your field notes a descriptive report, in which you will describe:

(*a*) The quality and composition of the soil, and the kind and amount of timber, and other vegetation.

(*b*) The *locus* and size of streams, and such other matter as may appear upon the surface of the claims.

(*c*) The character and extent of all surface and underground workings, whether placer or lode, for mining purposes, locating and describing them, as required by section 41.

(*d*) The proximity of centres of trade or residence.

(*e*) The proximity of well-known systems of lode deposits or of individual lodes.

(*f*) The use or adaptability of the claim for placer mining, and whether water has been brought upon it in sufficient quantity to mine the same, or whether it can be procured for that purpose.

(*g*) What works or expenditures have been made by the claimant or his grantors for the development of the claim, and their situation and location with respect to the same as applied for.

(*h*) The true situation of all mines, salt licks, salt springs, and mill sites, which come to your knowledge, or report that none exist on the claim, as the facts may warrant.

(*i*) Said report must be made under oath, and duly corroborated by one or more disinterested persons.

57. Descriptive reports on placer claims taken by legal subdivisions are authorized only by special order, and must contain a description of the claim in addition to the foregoing requirements.

PRACTICE

58. The practice of employing the claimants, their attorneys, or parties in interest, as assistants in making surveys of mineral claims, will not be allowed.

59. Your field work must be accurately and properly performed and your returns made in conformity with the foregoing instructions. Errors in the survey must be corrected at your own expense, and if the time required in the examination of your returns is increased by reason of your neglect or carelessness, you will be required to make an additional deposit for office work. You will be held to a strict accountability for the faithful discharge of your duties, and will be required to observe fully the requirements and regulations in force as to making mineral surveys. If found incompetent as a surveyor, careless in the discharge of your duties, or guilty of a violation of said regulations, your appointment will be promptly revoked.

APPLICATION TO UNITED STATES SURVEYOR-GENERAL FOR SURVEY OF MINING CLAIM

(4—689.)

DENVER, COLO., *January* 25, 1893.

UNITED STATES SURVEYOR-GENERAL,
Denver, Colorado.

Sir: T. E. Jenkins *et al.*, claimants, hereby make application for an official survey, under the provisions of Chapter Six, Title Thirty-two, of the Revised Statutes of the United States, and regulations and instructions thereunder, of the mining claim known as the Cumro Placer and Poorman, Hawley, Ætna, and Podunk lodes and the Poorman mill site, situate in Pike's Peak Mining District, El Paso County, Colorado, in Sections 17, 19, and 20, Township No. 14 S., Range No. 69 W. Said claim is based upon valid locations made on various dates, 18——, and duly recorded on various dates, 18——, and is fully described in the duly certified copies of the record of the location certificates filed herewith. Said certificates contain the name of the locators, the dates of location, and such a definite description of the claim by reference to natural objects or permanent monuments as will identify the claim, and said locations have been distinctly marked by monuments on the ground, so that their boundaries can be readily traced.

I request that you will send me an estimate of the amount required to defray the expenses of platting and other work in your office, required under the regulations, that I may make proper deposit therefor, and that thereupon you will cause the survey to be made by A. L. Hawley, United States deputy mineral surveyor, and proper action to be taken thereon by your office, as required by the United States mining laws and regulations thereunder.

J. E. JENKINS,
For himself and co-claimants.

P. O. address: Denver,
Arapahoe County, Colorado.

(4—682.)

ORDER FOR MINERAL SURVEY

DEPARTMENT OF THE INTERIOR,
OFFICE OF U. S. SURVEYOR-GENERAL,
DENVER, COLO., *February* 6, 1893.

To A. L. HAWLEY,
U. S. Deputy Mineral Surveyor,
Denver, Colorado.

Sir: Application has been filed in this office by T. E. Jenkins *et al.*, dated January 25, 1893, for an official survey of the mining claim of T. E. Jenkins *et al.*, known as the Cumro Placer and Poorman, Hawley, Ætna, and Podunk lodes and Poorman mill site, situate in Pike's Peak Mining District, El Paso County, in Sections 17, 19, and 20, Township No. 14 S., Range No. 69 W., which claim is based upon locations made on May 1, 1892, May 4, 1892, May 4, 1888, June 4, 1892, June 14, 1891, and December 5, 1891, respectively, and duly recorded various dates, 18—, and is fully described in the duly certified copies of the record of the location certificates filed by the applicants for said survey, copies of which are herewith inclosed. You are hereby directed to make the survey of said claim in strict conformity with existing laws, official regulations, and instructions thereunder, and to make proper return to this office. Said survey will be designated as Survey No. 8000, A. and B.

Very respectfully,

U. S. Surveyor-General
for Colorado.

SPECIMEN FIELD NOTES

(4—683.)

[Mineral Survey No. .... 8000, A and B....
..... Pueblo ..... Land District.

FIELD NOTES

OF THE SURVEY OF THE MINING CLAIM OF

.....T. E. Jenkins, *et al.*,.....

KNOWN AS THE

.... Cumro Placer and Poorman, Hawley, Ætna, and Podunk Lodes,
..... and Poorman Mill Site .....
.....Pike's Peak .....Mining District,.....
..... El Paso .....County, ...Colorado .....
Sections ..... 17, 19, and 20, ...Township .....14 S.,
Range.....69 W.....

Surveyed under instructions dated.....February 6, 1893.....

by ..... A. L. HAWLEY, ...
U. S. Deputy Mineral Surveyor.

Claim located....., 18....
Survey commenced.....February 9, 1893....
Survey completed.....February 12, 1893....
Address of claimants:
.....Denver,.....
.....Colorado.....

FEET.

SURVEY 8000 A

CUMRO PLACER

TRACT A

Beginning at Cor. No. 1.

Identical with the SW. Cor of the location and with the SW. Cor. of sec. 17, T. 14 S., R. 69 W. of the 6th Principal Meridian.

A pine post, 4½ ft. long, 4 ins. square, set 2 ft. in the ground, with mound of stone, alongside the section corner, scribed 1–8000 A, whence

A spruce, 17 ins. diam., bears N. 8° 41′ W. 7 ft., and a spruce, 14 ins. diam., bears S. 68° 14′, E. 18.5 ft., each blazed and scribed B. T. 1–8000 A.

James Peak bears N. 52° 21′ W.

Hahns Peak bears N. 29° 28′ W.

Thence North.
Va. 14° 22′ E.

660. To Cor. No. 2.

Identical with a corner of the location.

A pine post, 4½ ft. long, 4 ins. square, set 18 ins. in the ground, with mound of earth and stone, scribed 2–8000 A, whence

A spruce, 18 ins. diam., blazed and scribed B. T. 2–8000 A, bears S. 14° 47′, E. 17.3 ft.

Thence N. 89° 50′ E.
Va. 14° 28′ E.

182.3 To Cor. No. 3.

On line 1–2 Hawley lode of this survey.

A cedar post, 5 ft. long, 4 ins. square, set 2 ft. in the ground, with mound of earth, scribed 3–8000 A, whence

A corner of the location bears N. 89° 50′ E. 1126.7 ft.

Thence S. 3° 48′ E.
Va. 14° 28′ E.

84.5 To Cor. No. 4.

A pine post, 4½ ft. long, 4 ins. square, set 2 ft. in the ground, with mound of stone, scribed 4–1–8000 A, whence

A pine, 14 ins. diam., bears S. 21° 47′ E. 14.3 ft., and a spruce, 11 ins. diam., bears N. 14° 52′ E. 6 ft., each blazed and scribed B. T. 4–1–8000 A.

Thence N. 86° 12′ E.
Va. 14° 28′ E.

67.2 To Cor. No. 5.

On line 1–2 Poorman lode of this survey.

A cottonwood post, 5 ft. long, 4 ins. square, set 2 ft. in the ground, with mound of stone, scribed 5–8000 A.

Feet.	
	Thence S. 17° 38′ W. Va. 14° 36′ E.
243.4	To Cor. No. 6. A pine post, 4½ ft. long, 4 ins. square, set 2 ft. in the ground, with mound of earth, scribed 6–1–8000 A, whence A high Mt. bears N. 51° 14′ E.
	Thence S. 41° 14′ E. Va. 14° 30′ E.
350.48	To Cor. No. 7. A granite bowlder, 27×12×9 ins., set 16 ins. in the ground, chiselled 7–6–8000 A.
	Thence N. 17° 38′ E. Va. 14° 30′ E.
542.38	To Cor. No. 8. On line 4–1 Hawley lode of this survey. A pine post, 4⅛ ft. long, 5 ins. square, set 2 ft. in the ground, scribed 8–8000 A.
	Thence N. 86° 12′ E. Va. 14° 28′ E.
416.8	To Cor. No. 9. On line 3–4 Podunk lode of this survey. A granite stone, 26×16×6 ins., set 18 ins. in the ground, with mound of stone, chiselled 9–8000 A.
	Thence S. 41° 14′ E. Va. 14° 28′ E.
555.44	To Cor. No. 10. On line 4–1 Sur. No. 7000, Ajax lode, claimant unknown. A pine post, 4½ ft. long, 4 ins. square, set 18 ins. in the ground, with mound of earth, scribed 10–8000 A.
	Thence S. 7° 45′ W. Va. 14° 25′ E.
210.	To Cor. No. 11. On line 4–1 Sur. No. 7000, Ajax lode, at N. 75° 45′ E., 10.73 ft. from Cor. No. 4. A pine post, 5 ft. long, 4 ins. square, set 2 ft. in the ground, with mound of earth and stone, scribed 11–8000 A, whence A cottonwood post, 8 ins. diam., blazed and scribed B. T. 11–8000 A, bears N. 33° 27′ W. 5 ft.
	Thence S. 89° 50′ W. Va. 14° 25′ E.
1329.42	To Cor. No. 1, the place of beginning.

TRACT B.

Beginning at Cor. No. 12.

At intersection of lines 3–4, Ætna lode of this survey, and Aztec lode, unsurveyed, John Doe, claimant.

FEET.	
	A spruce post, 4½ ft. long, 4 ins. square, set 18 ins. in the ground, scribed 12–8000 A, whence
	A pine stump, 18 ins. diam., 3 ft. high, blazed and scribed B. S. 12–8000 A, bears N. 89° 11′ E. 9.4 ft.
	Thence S. 41° 14′ E. Va. 14° 30′ E.
397.33	To Cor. No. 13.
	A sandstone, 36×20×4 ins., set 16 ins. in the ground with mound of stone, chiselled 13–3–8000 A.
	Thence S. 48° 46′ W. Va. 14° 30′ E.
300.	Cor. Nos. 2, Ætna and Podunk lodes of this survey, a point in Cumro Creek, 4 ft. wide, flows west.
330.7	To Cor. No. 14.
	A rock in place 6×4×2 ft. above the general surface, chiselled cross (×) at corner point and 14–W. C. 2–2–8000 A, whence,
	Cor. No. 11, Tract A, of this survey, bears S. 89° 50′ W. 539 ft.
	Thence N. 89° 50′ E. Va. 14° 30′ E.
769.6	To Cor. No. 15.
	Identical with the S. ¼ Cor. of said Sec. 17, and with the SE. Cor. of the location.
	A granite stone, 12×10×6 ins. above the ground, chiselled 15–8000 A.
	Thence North. Va. 14° 30′ E.
265.	Cumro Creek, 4 ft. wide, flows S. 65° W.
758.1	To Cor. No. 16.
	On line 3–4, Aztec lode, unsurveyed, at S. 72° 43′ W. 715.6 ft. from Cor. No. 3.
	A spruce post, 4½ ft. long, 4 ins. square, set 6 ins. in the ground to bed rock, with mound of earth and stone, scribed 16–8000 A.
	Thence S. 72° 43′ W. Va. 14° 28′ E.
819.1	To Cor. No. 12, the place of beginning.

TRACT C.

Beginning at Cor. No. 17.

On line 1–2, Aztec lode, unsurveyed, at S. 72° 43′ W. 22.26 ft. from Cor. No. 2.

A pine post, 4½ ft. long, 4 ins. square, set 12 ins. in the ground to bed rock, with mound of stone, scribed 17–8000 A, whence,

Cor. No. 16, Tract B, of this survey, bears S 314.2 ft.

FEET.	
	Thence S. 72° 43′ W. Va. 14° 40′ E.
938.26	To Cor. No. 18. At intersection of lines 1–2 and 2–3, Aztec lode, unsurveyed, and Sur. No. 7000, Ajax lode, respectively. A cross (×) at corner point and 18–8000 A chiselled on a granite rock in place, showing 10×3×2 ft. above the general surface.
	Thence N. 7° 45′ E. Va. 14° 30′ E.
530	To Cor. No. 19. On line 2–3, Sur. No. 7000, Ajax lode, at S. 7° 45′ W. 116.4 ft. from Cor. No. 2. A granite stone, 28×10×3 ins., set 12 ins. in the ground, chiselled 19–8005, whence A corner of the location bears S. 89° 50′ W. 484.6 ft.
	Thence N. 89° 50′ E. Va. 14° 30′ E.
824.43	To Cor. No. 20. A granite rock, 30×20×16 ins., set 16 ins. in the ground, chiselled 20–8000 A, whence A cross (×) and B. R. 20–8000 A, chiselled 4 ft. above the ground on a limestone cliff 20 ft. high, bears S. 83° 11′ E. 45.6 ft.
	Thence South. Va. 14° 30′ E.
247.72	To Cor. No. 17, the place of beginning.

AREA.

Tract A, containing	14.660 acres.
Tract B, containing	9.858 "
Tract C, containing	7.532 "
Total area Cumro placer	32.070 acres.

POORMAN LODE.

	Beginning at Cor. No. 1. Identical with Cor. No. 6 Cumro placer of this survey, whence The SW. Cor. Sec. 17, T. 14 S., R. 69 W. of the 6th P. M., bears S. 27° 28′ W. 393.26 ft.
	Thence N. 17° 38′ E.
243.4	Cor. No. 5 Cumro placer and intersect line 4–1 Hawley lode, both of this survey.
565.7	Intersect line 2–3 Hawley lode of this survey.
831.4	To Cor. No. 2. A granite rock, 30×20×16 ins., set 16 ins. in the ground, chiselled 2–8000 A.

FEET.	
	Thence N. 48° 46′ E.
661.57	To Cor. No. 3. A pine post, 5 ft. long, 4 ins. square, set 22 ins. in the ground. to bed rock, scribed 3–8000 A.
	Thence S. 41° 14′ E.
300	To Cor. No. 4. A cedar post, 4½ ft. long, 5 ins. square, set 18 ins. in the ground, scribed 4–4–8000 A, whence Cor. No. 1 Sur. No. 7000, Ajax lode, bears N. 72° 22′ E. 422.6 ft.
	Thence S. 48° 46′ W.
300	Cor. Nos. 1 Ætna and Podunk lodes of this survey.
578	To Cor. No. 5. A cedar stump, 3 ft. high, hewed to 4 ins. square, surrounded by mound of stone, scribed 5–4–8000 A, whence A cross (×) and B. R. 5–4–8000 chiselled on a porphyry stone, showing 9×6×4 ft. above the ground, bears N. 75° 14′ E. 32.7 ft.
	Thence S. 17° 38′ W.
64.4	Intersect line 2–3 Hawley lode of this survey.
386.7	Cor. No. 8 Cumro placer and intersect line 4–1 Hawley lode, both of this survey.
929.04	To Cor. No. 6. Identical with Cor. No. 7 Cumro placer of this survey.
	Thence N. 41° 14′ W.
350.48	To Cor. No. 1, the place of beginning.

HAWLEY LODE.

Beginning at Cor. No. 1.
Identical with Cor. No. 1 of the location and with Cor. No. 4 Cumro placer of this survey, whence
The SW. Cor. Sec. 17, T. 14 S., R. 69 W. of the 6th P. M. bears S. 18° 4′ W. 606.1 ft.
Cor. No. 1 Poorman lode of this survey bears S. 1° 41′ W. 227.6 ft.

	Thence N. 3° 48′ W. Va. 14° 28′ E.
84.5	Cor. No. 3 Cumro placer of this survey.
300	To Cor. No. 2. A sandstone, 30×12×2 ins., set 14 ins. in the ground, chiselled 2–8000 A, whence Cor. No. 4 of the location bears N. 45° W. 28.5 ft.
	Thence N. 86° 12′ E. Va. 14° 25′ E.
185	Intersect line 1–2 Poorman lode of this survey.
507.3	Intersect line 5–6 Poorman lode of this survey.

FEET.	
567.6	Intersect line 3–4 Podunk lode of this survey.
926.7	Intersect lines 1–2 Podunk and Ætna lodes of this survey.
1264.7	To Cor. No. 3.
	A granite stone, 26×14×8 ins., set 12 ins. in the ground with mound of stone, chiselled 3–8000 A, whence
	Cor. No. 3 of the location bears N. 86° 12′ E. 235.3 ft.
	Cor. No. 1 Sur. No. 7000, Ajax lode, bears N. 11° E. 529.9 ft.
	Cor. No. 1 Aztec lode, unsurveyed, bears S. 32° 30′ W. 378.4 ft.
	Thence S. 3° 48′ E.
	Va. 14° 26′ E.
150	Intersect line 4–1 Sur. No. 7000, Ajax lode, at S. 7° 45′ W. 676 ft. from Cor. No. 1.
250.6	Intersect line 1–2 Aztec lode, unsurveyed, at N. 72° 43′ E. 229.8 ft. from Cor. No. 1.
300	To Cor. No. 4.
	A pine post, 4½ ft. long, 4 ins. square, set 18 ins. in the ground, scribed 4–8000 A, whence
	Cor. No. 4 of the location bears N. 86° 12′ E. 235.3 ft.
	Thence S. 86° 12′ W.
	Va. 14° 15′ E.
30.8	Intersect line 4–1 Sur. No. 7000, Ajax lode, at S. 7° 45′ W. 829.1 ft. from Cor. No. 1.
108.3	Intersect lines 1–2 Ætna and Podunk lodes of this survey.
206	Intersect line 1–2 Aztec lode, unsurveyed, at N. 72° 43′ E. 18.1 ft. from Cor. No. 1.
458.4	Cor. No. 9 Cumro placer and intersect line 3–4 Podunk lode, both of this survey.
875.2	Cor. No. 8 Cumro placer and intersect line 5–6 Poorman lode, both of this survey.
1197.5	Cor. No. 5 Cumro placer and intersect line 1–2 Poorman lode, both of this survey.
1264.7	To Cor. No. 1, the place of beginning.

ÆTNA LODE

Beginning at Cor. No. 1.

On line 4–5 Poorman lode of this survey.

A pine post, 4½ ft. long, 4 ins. square, set 18 ins. in the ground, scribed 1–1–8000 A, whence

The SW. Cor. Sec. 17, T. 14 S., R. 69 W. of the 6th P. M., bears S. 38° 2′ W. 1465 ft.

Cor. No. 1 Aztec lode, unsurveyed, bears S. 32° 19′ E. 607.76 ft.

Cor. No. 3 Hawley lode of this survey, bears S. 69° 46′ E. 561.9 ft.

Thence S. 41° 14′ E.

288.3	Intersect line 2–3 Hawley lode of this survey.
641.2	Intersect line 1–2 Aztec lode, unsurveyed, at N. 72° 43′ E. 103.1 ft. from Cor. No. 1.

Feet.	
666.1	Intersect line 4–1 Hawley lode of this survey.
766.9	Intersect line 4–1 Sur. No. 7000, Ajax lode, at S. 7° 45′ W. 910.9 ft. from Cor. No. 1.
969.5	Intersect line 3–4 Aztec lode, unsurveyed, at N. 72° 43′ E. 236.3 ft. from Cor. No. 4.
1164.7	Intersect line 2–3 Sur. No. 7000, Ajax lode, at N. 7° 45′ E. 328.2 ft. from Cor. No. 3.
1500	To Cor. No. 2.
	On line 13–14 Cumro placer of this survey.
	Not set, as it falls in the centre of Cumro Creek, where permanent corner could not be established, whence
	Witness corner to Cor. No. 2, identical with Cor. No. 14 Cumro placer of this survey, bears S. 48° 46′ W. 30.7 ft.
	Cor. No. 3 Sur. No. 7000, Ajax lode, bears S. 74° 38′ W. 275.2 ft.
	Thence N. 48° 46′ E.
300	To Cor. No. 3.
	Identical with Cor. No. 13 Cumro placer of this survey.
	Thence N. 41° 14′ W.
397.33	Cor. No. 12 Cumro placer of this survey and intersect line 3–4 Aztec lode, unsurveyed, at N. 72° 43′ E. 564.6 ft. from Cor. No. 4.
596.43	Intersect line 2–3 Sur. No. 7000, Ajax lode, at N. 7° 45′ E. 725.8 ft. from Cor. No. 3.
725.6	Intersect line 1–2 Aztec lode, unsurveyed, at N. 72° 43′ E. 431.3 ft. from Cor. No. 1.
994	Intersect line 4–1 Sur. No. 7000, Ajax lode, at S. 7° 45′ W. 513.2 ft. from Cor. No. 1.
1500	To Cor. No. 4.
	Identical with Cor. No. 4 Poorman lode of this survey.
	Thence S. 48° 46′ W.
300	To Cor. No. 1, the place of beginning.
	PODUNK LODE
	Beginning at Cor. No. 1.
	Identical with Cor. No. 1 Ætna lode of this survey, whence Cor. No. 2 of the location bears N. 48° 46′ E. 22 ft.
	The SW. Cor. Sec. 17, T. 14 S., R. 69 W. of the 6th P. M., bears S. 38° 2′ W. 1465 ft.
	Thence S. 41° 14′ E.
288.3	Intersect line 2–3 Hawley lode of this survey.
641.2	Intersect line 1–2 Aztec lode, unsurveyed, at N. 72° 43′ E. 103.1 ft. from Cor. No. 1.
666.1	Intersect line 4–1 Hawley lode of this survey.
766.9	Intersect line 4–1 Sur. No. 7000, Ajax lode, at S. 7° 45′ W 910.9 ft. from Cor. No. 1.
969.5	Intersect line 3–4 Aztec lode, unsurveyed, at N. 72° 43′ E. 236.3 ft. from Cor. No. 4.

Feet.	
1164.7	Intersect line 2–3 Sur. No. 7000, Ajax lode, at N. 7° 45′ E. 328.2 ft. from Cor. No. 3.
1500	To Cor. No. 2. In Cumro creek. Identical with Cor. No. 2 Ætna lode of this survey, whence Cor. No. 3 of the location bears N. 48° 46′ E. 22 ft.
	Thence S. 48° 46′ W.
30.7	Witness corner to Cor. No. 2. Identical with witness corner to Cor. No. 2 Ætna lode, and with Cor. No. 14 Cumro placer, both of this survey.
250	Witness corner to Cor. No. 3. A granite stone, 30×20×4 ins., set 14 ins. in the ground, with mound of stone, chiselled W. C. 3–8000 A, whence A cedar stump, 14 ins. diam., 2 ft. high, blazed and scribed B. S. W. C. 3–8000 A, bears N. 7° 56′ W. 8.4 ft. Pike's Peak bears N. 5° E.
278	To Cor. No. 3. On face of inaccessible granite cliff. Identical with Cor. 4 of the location.
	Thence N. 41° 14′ W.
37	Cumro creek, 4 ft. wide, course S. 80° W.
155	Intersect line 3–4 Sur. No. 7000, Ajax lode, at N. 82° 15′ W. 46.3 ft. from Cor. No. 3.
491.3	Cor. No. 10 Cumro placer of this survey and intersect line 4–1. Sur. No. 7000, Ajax lode, at N. 7° 45′ E. 220.7 ft. from Cor. No. 4.
1046.7	Cor. No. 9 Cumro placer and intersect line 4–1 Hawley lode, both of this survey.
1424.5	Intersect line 2–3 Hawley lode of this survey.
1500	To Cor. No. 4. Identical with Cor. No. 5 Poorman lode of this survey and with Cor. No. 1 of the location.
	Thence N. 48° 46′ A.
278	To Cor. No. 1, the place of beginning.
	Variation at all corners of the Poorman, Ætna, and Podunk lodes, 14° 30′ E.

AREA

Total area Hawley lode	8.710 acres.
Area in conflict with	
Poorman lode of this survey	2.220 "
Sur. No. 7000, Ajax lode	.053 "
Aztec lode, unsurveyed	.117 "
Aztec lode, unsurveyed (exclusive of its conflict with Sur. 7000, Ajax lode)	.089 "
Total area Hawley lode	8.710 acres

Less area in conflict with		
Poorman lode of this survey	2.220 acres.	
Sur. No. 7000, Ajax lode	.053 "	
Aztec lode, unsurveyed	.089 "	=2.362 acres.
Net area Hawley lode		6.348 acres.
Total area Ætna lode		10.331 acres.
Area in conflict with		
Hawley lode of this survey		1.537 "
Sur. No. 7000, Ajax lode		2.738 "
Sur. No. 7000, Ajax lode (exclusive of its conflict with Hawley lode of this survey)		2.167 "
Aztec lode, unsurveyed		2.261 "
Aztec lode, unsurveyed (exclusive of its conflict with Hawley lode of this survey)		2.167 "
Aztec lode, unsurveyed (exclusive of its conflict with Sur. No. 7000, Ajax lode)		.484 "
Aztec lode, unsurveyed (exclusive of its conflict with Hawley lode of this survey and Sur. No. 7000, Ajax lode)		.418 "
Total area Ætna lode		10.331 acres.
Less area in conflict with		
Hawley lode of this survey	1.537 acres.	
Sur. No. 7000, Ajax lode	2.685 "	
Aztec lode, unsurveyed	.418 "	=4.640 "
Net area Ætna lode		5.691 acres.
Total area Podunk lode		9.573 acres.
Area in conflict with		
Hawley lode of this survey		2.411 "
Sur. No. 7000, Ajax lode		2.526 "
Aztec lode, unsurveyed		1.169 "
Aztec lode, unsurveyed (exclusive of its conflict with Hawley lode of this survey)		1.147 "
Aztec lode, unsurveyed (exclusive of its conflict with Sur. No. 7000, Ajax lode)		.811 "
Aztec lode, unsurveyed (exclusive of its conflict with Hawley lode of this survey and Sur. No. 7000, Ajax lode)		.789 "
Total area Podunk lode		9.573 acres.
Less area in conflict with		
Hawley lode of this survey	2.411 acres.	
Sur. No. 7000, Ajax lode	2.516 "	
Aztec lode, unsurveyed	.789 "	=5.716 "
Net area Podunk lode		3.857 acres.
Net area Hawley lode		6.348 "
Net area Ætna lode		5.691 "
Total and net area Poorman lode		10.331 "
Net area lode claim		26.227 acres.

Feet.	SURVEY No. 8000 B
	POORMAN MILL SITE
	Beginning at Cor. No. 1.
	A spruce post, 5 ft. long, 4 ins. square, set 18 ins. in the ground, scribed 1–8000 B, whence
	Cor. No. 6 Sur. No. 8000 A, Poorman lode, bears N. 50° 8′ E. 3782 ft.
	The NE. Cor. Sec. 19, T. 14 S., R. 69 W. of the 6th P. M., bears N. 46° 48′ E. 3416.9 ft.
	Thence S. 85° 51′ W.
65	Cumro creek, 4 ft. wide, flows N. 65° W.
858	Cumro creek, 4 ft. wide, flows South.
921.2	To Cor. No. 2.
	A granite stone, 28×12×10 ins., set 12 ins. in the ground, with mound of stone, chiselled 2–8000 B, whence
	A pine, 12 ins. diam., blazed and scribed B. T. 2–8000 B, bears W. 9.5 ft.
	Thence N. 37° 55′ E.
467.66	To Cor. No. 3.
	A granite stone, 30×12×8 ins., set 18 ins. in the ground, with mound of stone, chiselled 3–8000 B.
	Thence S. 64° 25′ E.
700	To Cor. No. 1, the place of beginning.
	Containing 3.671 acres.
	Variation at all corners, 14° 45′ E.

The surveys of the Poorman and Ætna lodes and the Poorman mill site are identical with the respective locations.

LOCATION

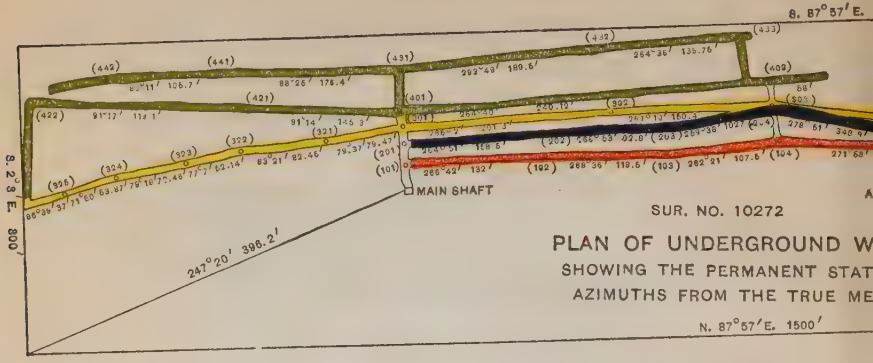
This claim is located in the SW. ¼ of Sec. 17, the NE. ¼ of Sec. 19, and the NW. ¼ of Sec. 20, T. 14 S., R. 69 W. of the 6th P. M.

EXPENDITURE OF FIVE HUNDRED DOLLARS

I certify that the value of the labor and improvements upon this claim, placed thereon by the claimants and their grantors, is not less than five hundred dollars, and that said improvements consist of:

Placer workings, the centre of the northeasterly end of which bears from Cor. No. 15, Cumro placer, N. 46° W. 285 ft., averaging 40 ft. wide and 8 ft. deep, and extending S. 62° W. 120 ft. along the bed of Cumro creek. Value, $800.

The discovery shaft of the Poorman lode, which bears from Cor. No. 5 N. 56° 48′ W. 955.7 ft., 6×4 ft., 12 ft. deep. Value, $100.



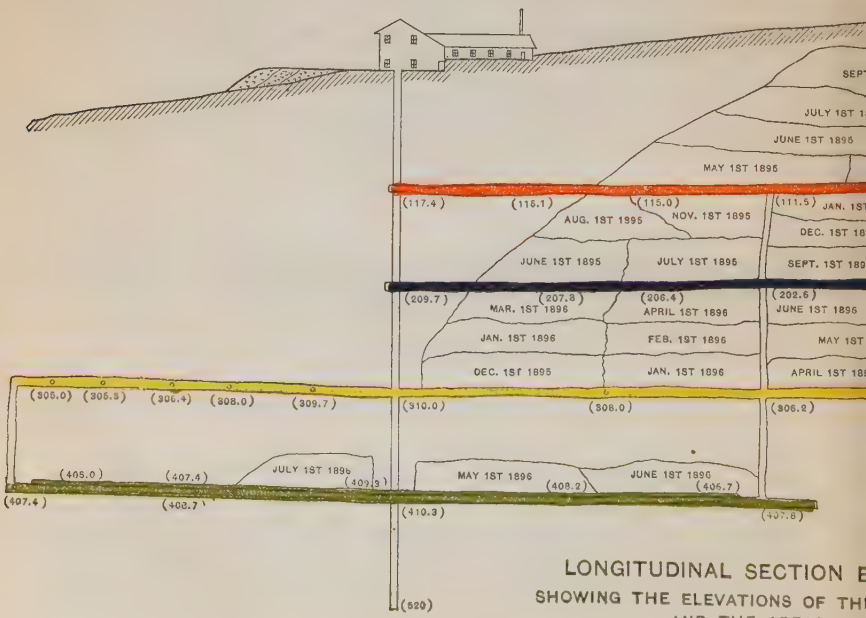
SUR. NO. 10272

PLAN OF UNDERGROUND W

SHOWING THE PERMANENT STAT

AZIMUTHS FROM THE TRUE ME

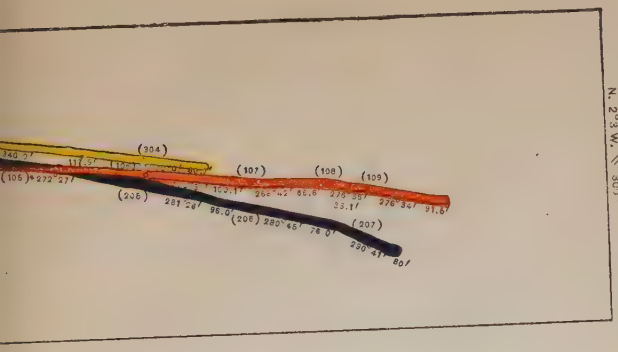
N. 87°57′E. 1500′

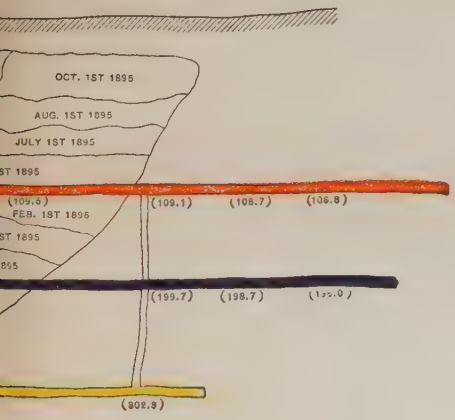


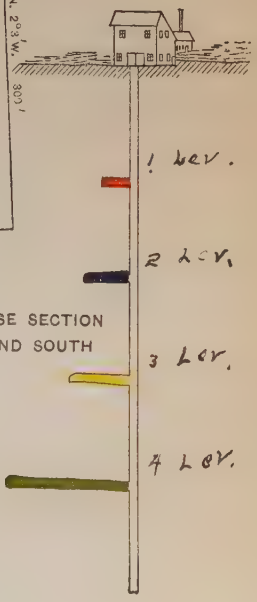
LONGITUDINAL SECTION E

SHOWING THE ELEVATIONS OF TH

AND THE AREAS STO







TRANSVERSE SECTION
NORTH AND SOUTH

WEST.
T STATIONS

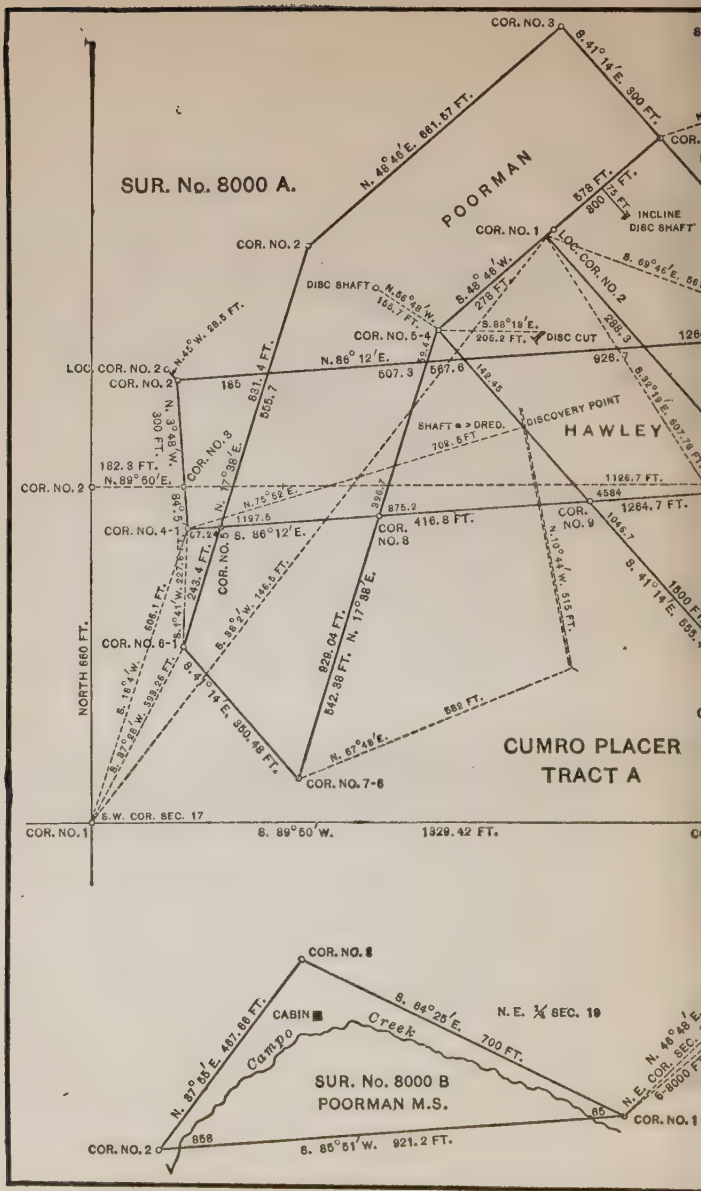
MAP OF

THE LINCOLN MINE

JEFFERSON COUNTY, COLORADO

SURVEYED BY ROBERT S. STOCKTON, SEPT. 10TH 1896
ORIGINAL SCALE: 100 FT. TO 1 INCH
SCALE OF REDUCED PLATE: 185 FT. TO 1 INCH

[To face page 762]


SUR. No. 8000 A.
POORMAN
HAWLEY
CUMRO PLACER TRACT A
SUR. No. 8000 B POORMAN M.S.
COR. NO. 1
COR. NO. 2
COR. NO. 3
COR. NO. 4-1
COR. NO. 5
COR. NO. 6-1
COR. NO. 7-8
COR. NO. 8
COR. NO. 9
COR. NO. 5-4
INCLINE DISC SHAFT
DISC SHAFT
DISC CUT
DISCOVERY POINT
SHAFT ● = DRED.
Creek
CABIN
N. E. ¼ SEC. 19
N. E. COR. SEC. 19
S. W. COR. SEC. 17
NORTH 660 FT.
S. 89° 50' W. 1329.42 FT.
N. 48° 46' E. 681.67 FT.
S. 41° 14' E. 300 FT.
S. 48° 46' W. 278 FT.
N. 86° 12' E. 607.3
S. 86° 12' E. 1197.6
S. 88° 19' E. 205.2 FT.
N. 17° 38' E. 829.04 FT.
S. 17° 38' E. 842.88 FT.
N. 67° 45' E. 589 FT.
S. 41° 14' E. 380.48 FT.
S. 64° 25' E. 700 FT.
N. 87° 35' E. 487.88 FT.
S. 85° 51' W. 921.2 FT.
N. 10° 44' W. 515 FT.
S. 41° 14' E. 1500 FT.
1264.7 FT.
1126.7 FT.
416.8 FT.
875.2
926.7
858

SURVEY No. 8000 A. & B.

PUEBLO — LAND DISTRICT

SURVEYED FEBRUARY 9-12, 1893

by, A. L. Hawley,

U.S. Dep. Min. Surveyor

Rah

[*To face page 762*]

The discovery cut of the Podunk lode, the face of which bears from Cor. No. 4 S. 88° 13′ E. 205.2 ft., 5 ft. wide, 10 ft. face, running N. 50° E. 24 ft. in earth and rock. Value, $110.

The incline discovery shaft of the Ætna lode, the mouth of which is on the centre line 75 ft. from the centre of line 4–1, 4½×6 ft., 24 ft. deep in rock, timbered, course S. 48°, dip. 60°. Value, $250.

The last 120 ft. of a tunnel, the mouth of which bears from Cor. No. 6, Poorman lode, N. 67° 48′ E. 528 ft., 5×6 ft. running N. 10° 44′ W. 515 ft. to breast. The point of discovery of the Hawley lode is in this tunnel 475 ft. from the mouth, and bears from Cor. No. 1, N. 75° 52′ E. 702.5 ft. Value of last 120 ft., $2,300.

This tunnel is in course of construction for the development of this claim and Surs. Nos. 6582 and 6583, Roy and Raymond lodes, claimants unknown. An undivided half interest in the first 375 ft. of this tunnel has been credited to each of the last two mentioned surveys.

A shaft on the centre line of the Hawley lode 672 ft. from the centre of line 1–2, 3½×6 ft., 20 ft. deep in earth and rock, timbered, at the bottom of which is a drift 4×6 ft. running N. 86° E. 18 ft.

Value of shaft and drift, $300.

The surface embraced by this claim ascends rapidly from the mouth of the tunnel towards Cor. No. 3 of the Poorman lode, the northerly ends of the Poorman, Ætna, and Podunk lodes being from 300 to 500 ft. higher than the mouth of the tunnel. The veins of the Ætna and Podunk lodes dip about 60° to the S.W. The tunnel, described and included in the estimate of expenditure, continued in its present course, will cut the veins of the several locations at great depth, whereby by one system of workings and one plant of machinery the entire claim can be most advantageously and economically developed.

OTHER IMPROVEMENTS

A shaft which bears from Cor. No. 20 Cumro placer, S. 48° 30′ W. 305 ft., 3×5 ft., 12 ft. deep in earth and gravel. A. K. Smith, claimant.

A log cabin, the W. corner of which bears from Cor. No. 13 Cumro placer, S. 40° E. 120 ft., 12×16 ft., course of long sides N. 44° E.

A log cabin, the N.W. Cor. of which bears from Cor. No. 3 Sur. No. 8000 B, Poorman mill site, S. 10° 40′ E. 107 ft., 16 ft. square, course of sides S. 3° E. Said cabins belong to claimants herein.

INSTRUMENT

This survey was made with a Gurley Light Mountain Transit. The courses were deflected from the true meridian as determined by direct solar observations. The distances were measured with 50-ft. and 500-ft. steel tapes.

NOTE.—The disagreements between these field notes and the location certificates of the Hawley and Ætna lodes and the Poorman mill site, with regard to the position of the discovery point and the course of the boundary lines, are due to errors in the latter.

NOTE.—Neither E. E. Ames nor myself, who appear as locators of the Hawley lode, held any interest, directly or indirectly, in this claim at the time making the survey, having sold our interests in June, 1890.

(4—685.)

FINAL OATHS FOR SURVEYS

LIST OF NAMES

A list of the names of the individuals employed by....A. L. Hawley...., United States deputy mineral surveyor, to assist in running, measuring and marking the lines, corners, and boundaries described in the foregoing field notes of the survey of the mining claim of....T. E. Jenkins *et al*....., known as the....Cumro placer and Poorman, Hawley, Ætna, and Podunk lodes, and Poorman mill site...., and showing the respective capacities in which they acted:

....., *Chainman.*
.....E. E. Ames....., *Chainman.*
.....G. W. Trommlitz....., *Axeman.*
....., *Flagman.*

FINAL OATHS OF ASSISTANTS

We,....E. E. Ames....and....G. W. Trommlitz...., do solemnly swear that we assisted....A. L. Hawley...., United States deputy mineral surveyor, in marking the corners and surveying the boundaries of the mining claim of....T. E. Jenkins *et al*....., known as the....Cumro placer and Poorman, Hawley, Ætna, and Podunk lodes, and Poorman mill site.... represented in the foregoing field notes as having been surveyed by said deputy-mineral surveyor and under his direction; and that said survey has been in all respects, to the best of our knowledge and belief, faithfully and correctly executed, and the corner and boundary monuments established according to law and the instructions furnished by the United States Surveyor-General for....Colorado.....

....., *Chainman.*
.....E. E. Ames....., *Chainman.*
.....G. W. Trommlitz..... *Axeman.*
....., *Flagman.*

Subscribed and sworn to by the above-named persons before me this ....13th.....day of..... Februar....1893.

[SEAL.] .....A. L. Hawley.....,
.....*Notary Public*.....,
.....*El Paso County, Colorado.*

My commission expires July 28th, 1896.

(4—686.)

FINAL OATH OF UNITED STATES DEPUTY MINERAL SURVEYOR

I, ...A. L. Hawley...., United States deputy mineral surveyor, do solemnly swear that, in pursuance of instructions received from the United States Surveyor-General for....Colorado...., dated....February

6th, 1893...., I have, in strict conformity to the laws of the United States, the official regulations and instructions thereunder, and the instructions of said Surveyor-General, faithfully and correctly executed the survey of the mining Claim of....T. E. Jenkins *et al*....., known as the....Cumro placer and Poorman, Hawley, Ætna, and Podunk lodes, and Poorman mill site...., situate in ....Pike's Peak....Mining District..... El Paso ....County...., Colorado...., in Sections 17, 19, and 20...., Township No.....14 S....Range No.....69 W. of the 6th P. M..... and designated as Survey No.....8000 A and B...., as represented in the foregoing field notes, which accurately show the boundaries of said mining claim as distinctly marked by monuments on the ground, and described in the attached copy of each location certificate, which was received by me from the Surveyor-General with said instructions, and that all the corners of said survey have been established and perpetuated in strict accordance with the law, official regulations and instructions thereunder; and I do further solemnly swear that the foregoing are the true and original field notes of said survey and my report therein, and that the labor expended and improvements made upon said mining claim by claimants or....their.... grantors are as therein fully stated, and that the character, extent, location, and itemized value thereof are specified therein with particularity and full detail, and that no portion of said labor or improvements so credited to this claim has been included in the estimate of expenditures upon any other claim.

.....A. L. Hawley.....,
United States Deputy Mineral Surveyor.

Subscribed and sworn to by the said....A. L. Hawley...., United States deputy mineral surveyor, before me....a Justice of the Peace in and for El Paso County, Colorado...., this....20th....day of....February..... 1893.

.....Geo. K. Kimball.....,
Justice of the Peace.

LOCATION CERTIFICATE—PLACER CLAIM

Know all men by these presents, That I, T. E. Jenkins, the undersigned citizen of the United States, resident of the County of Arapahoe and State of Colorado, having complied with the provisions of Chapter 6, Title XXXII, of the Revised Statutes of the United States, and with local customs, laws, and regulations, claim by right of discovery and location, as a placer claim, the following-described premises, situate, lying and being in Pike's Peak mining district, County of El Paso, and State of Colorado, to wit:

The SE. ¼ of the SW. ¼, and the S. ½ of the SW. ¼ of the SW. ¼ of Sec. 17, T. 14 S., R. 69 W. of the 6th P. M. To be known as the Cumro Placer.

Located May 1st, 1892.

Date of certificate, June 4th, 1892.

T. E. Jenkins.

LOCATION CERTIFICATE—LODE CLAIM

STATE OF COLORADO, } ss.:
County of.. El Paso.., }

KNOW ALL MEN BY THESE PRESENTS, That....O. F. Shattuck...., the undersigned, has this....4th....day of....May...., 1892, located and claimed, and by these presents does locate and claim, by right of discovery and location, in compliance with the mining acts of Congress, approved May 10, 1872, and all subsequent acts, and with local customs, laws, and regulations, ...1500....linear feet and horizontal measurement on the....Poorman....lode, vein, ledge, or deposit along the vein thereof, with all its dips, angles, and variations, as allowed by law, together with....150....feet on the....westerly....side and....150 feet on the....easterly....side of the middle of said vein at the surface, so far as can be determined from present developments; and all veins, lodes, ledges, or deposits and surface ground within the lines of said claim, ...620....feet running N. 48° 46′ E.....from centre of discovery.... shaft....and....880....feet running....S. 17° 38′ W.....from centre of discovery....shaft....; said discovery....shaft....being situate upon said lode, vein, ledge, or deposit, and within the lines of said claim, in....Pike's Peak....Mining District, County of....El Paso....and State of Colorado, described by metes and bounds as follows, to wit:

Beginning at Corner No. 1,....Whence Cor. to Secs. 17, 18, 19, and 20, T. 14, S., R. 69 W. bears S. 27° 28′ E. 393.26 ft., thence N. 17° 38′ E. 831.34 ft. to Cor. No. 2, thence N. 48° 46′ E. 661.7 ft. to Cor. No, 3, thence S. 41° 14′ E. 300 ft. to Cor. No. 4, thence S. 48° 46′ W. 578 ft. to Cor. No. 5, thence S. 17° 38′ W. 929.04 ft. to Cor. No. 6, thence N. 41° 14′ W. 350.48 ft. to Cor. No. 1, the place of beginning.....

Said lode was discovered on the 12th day of....April...., A.D. 1892. .....O. F. SHATTUCK.....[SEAL.]
Attest:..... .....[SEAL.]
Date of location.....May 4th,..... A.D. 1892. .....[SEAL.]
Date of certificate.....June 1st....A.D. 1892. .....[SEAL.]
.....[SEAL.]
.....[SEAL.]

AMENDED LOCATION CERTIFICATE—LAW OF 1889

STATE OF COLORADO, } ss.:
County of..El Paso.., }

KNOW ALL MEN BY THESE PRESENTS, That ....A. L. Hawley and E. E. Ames...., the undersigned, have this....4th day of....May...., 1888, amended, located, and claimed, and by these presents do amend, locate, and claim, by right of discovery and amended location, in compliance with the mining acts of Congress, approved May 10, 1872, and all subsequent acts, and with section 2409 of the general laws, of Colorado, and with local customs, laws, and regulations....1500....linear feet and horizontal measurement, on the....Hawley....lode, vein, ledge, or deposit, along the vein thereof, with all its dips, angles, and variations, as allowed by law, together with....150 feet on each side of the middle of said vein at the surface, so far as can be determined from present

developments, and all veins, lodes, ledges, or deposits and surface ground within the lines of said claim....800 feet running....easterly....from centre of discovery....point in tunnel...., and....700 feet running ....westerly....from centre of discovery....point...., said discovery ....point....being situate upon said lode, vein, ledge, or deposit, and within the lines of said claim in....Pike's Peak Mining District, County of....El Paso...., and State of Colorado, described by metes and bounds as follows, to wit:

Beginning at Corner No. 1.... whence Cor. to Secs. 17, 18, 19, and 20, T. 15 S., R. 69 W. bears S. 18° 4′ E. 606.1 ft., thence N. 3° 48′ E. 300 ft. to Cor. No. 2, thence N. 87° E. 1500 ft. to Cor. No. 3, thence S. 3° 48′ W. 300 ft. to Cor. No. 4, thence S. 87° W. 1500 ft. to Cor. No. 1, the place of beginning..... This being the same lode originally located on the.... 17th....day of....September...., 1886, and recorded on the....12th ....day of....December...., 1886, in Book....4...., Page....48...., in the office of the Recorder of....El Paso....County. This further and amended certificate of location is made without waiver of any previously acquired rights, but for the purpose of correcting any errors in the original location, description, or record.....

Said lode was discovered the....12th.... day of....Sept.....A.D. 1886.	.....A. L. HAWLEY.....	[SEAL.]
	.....E. E. AMES.....	[SEAL.]
Attest:.....	.....	[SEAL.]
Date of amended location....May 4th.... ....., A.D. 1888.	.....	[SEAL.]
	.....	[SEAL.]
Date of amended certificate.....July 14th ....., A.D. 1888.	.....	[SEAL.]
	.....	[SEAL.]

LOCATION CERTIFICATE—LODE CLAIM

STATE OF COLORADO, } *ss.:*
County of..El Paso.., }

KNOW ALL MEN BY THESE PRESENTS, That....Grant Safely...., the undersigned, has this....4th....day of....June...., 1892, located and claimed and by these presents does locate and claim, by right of discovery and location, in compliance with the mining acts of Congress, approved May 10, 1872, and all subsequent acts, and with local customs, laws, and regulations, ...1500 linear feet and horizontal measurements on the....Ætna....lode, vein, ledge, or deposit, along the vein thereof, with all its dips, angles, and variations, as allowed by law, together with....150....feet on the....southwesterly....side, and....150 ....feet on the....northeasterly....side of the middle of said vein at the surface, so far as can be determined from present developments; and all veins, ledges, lodes, or deposits and surface ground within the lines of said claim....75....feet running....N. 40° W.....from centre of discovery....cut....; and 1425....feet running S. 40° E....from the centre of discovery....cut; said discovery....cut....being situate upon said lode, vein, ledge, or deposit, and within the lines of said claim in.... Pike's Peak....Mining District, County of....El Paso...., and State of Colorado, described by metes and bounds as follows, to wit:

Beginning at Corner No. 1....whence S.W. Cor. Sec. 17, T. 15 S. R. 69 W. bears S. 38° 2′ E. 1465 ft., thence S. 40° 1500 ft. to Cor. No. 2,

thence N. 50° E. 300 ft. to Cor. No. 3, thence N. 40° W. 1500 ft. to Cor. No. 4, thence S. 50° W. 300 ft. to Cor. No. 1, the place of beginning.....

Said lode was discovered on the...1st... day of....May...., A.D. 1892. Attest:..... Date of location.....June 4th....., A.D. 1892. Date of certificate....July 6th....., A.D. 1892.	.....	GRANT SAFELY..... [SEAL.] [SEAL.] [SEAL.] [SEAL.] [SEAL.] [SEAL.] [SEAL.]

ADDITIONAL AND AMENDED LOCATION CERTIFICATE

LAW OF 1889

STATE OF COLORADO, } ss.:
County of..El Paso.., }

KNOW ALL MEN BY THESE PRESENTS, That....John H. Routt...., the undersigned, has this....4th....day....of....May...., 1891, amended, located, and claimed, and by these presents does amend, locate, and claim, by right of the original discovery and this additional and amended location certificate, in compliance with the mining acts of Congress, approved May 10, 1872, and all subsequent acts, and with section 2409 of the General Statutes of Colorado, and with local customs, laws, and regulations ....1500....linear feet and horizontal measurement on the ....Podunk ....lode, vein, ledge, or deposit, along the vein thereof, with all its dips, angles, and variations, as allowed by law, together with ....150....feet on each side of the middle of said vein at the surface, so far as can be determined from present developments, and all veins, lodes, ledges, or deposits and surface ground within the lines of said claim ....140....feet running....N. 41° 14′ W.....from centre of discovery ....cut....and....1360....feet running....S. 41° 14′ E.....from centre of discovery....cut...., said discovery....cut....being situate upon said lode, vein, ledge, or deposit, and within the lines of said claim in....Pike's Peak....Mining District. County of....El Paso...., and State of Colorado, described by metes and bounds as follows, to wit:

Beginning at Cor. No. 1....whence the S.W. Cor. Sec. 17, T. 15 S., R. 69 W. bears S. 38° 2′ W. 1465 ft., thence S. 41° 14′ E. 1500 ft. to Cor. No. 2, thence S. 48° 46′ W. 300 ft. to Cor. No. 3, thence N. 41° 14′ W. 1500 ft. to Cor. No. 4, thence N. 48° 46′ E. 300 ft. to Cor. No. 1, the place of beginning.....This being the same lode originally located on the....6th....day of....April...., 1888, and recorded on the....14th....day of....June...., 1888, in Book....3...., Page....48...., in the office of the Recorder of....El Paso....County. This further additional and amended certificate of location is made without waiver of any previously acquired rights, but for the purpose of correcting any errors in the original location, description, or record, and of taking in and acquiring all forfeited or abandoned overlapping ground, and of taking in any part of any overlapping claim which has been abandoned, and of securing all the benefits of said section 2409 of the General Statutes of Colorado.

Said lode was discovered the....1st.... day of....April...., A.D. 1888. Attest:..... Date of additional and amended certificate .....June 14th....., 1891.	.....JOHN H. ROUTT.....	[SEAL.] [SEAL.] [SEAL.] [SEAL.] [SEAL.] [SEAL.]

LOCATION CERTIFICATE OF MILL SITE

To all whom these presents may concern:

KNOW YE, That I, A. E. Lowe, of the County of Arapahoe, and State of Colorado, do hereby declare and publish as a legal notice to all the world, that I have a valid right to the occupation, possession, and enjoyment of all and singular that tract or parcel of land, not exceeding five acres, situate, lying and being in the County of El Paso, and State of Colorado, bounded and described as follows:

POORMAN MILL SITE

Beginning at Cor. No. 1, whence the N.E. Cor. Sec. 19, T. 14 S., R. 69 W. bears N. 46° 48′ E. 3416.9 ft., thence S. 85° 50 W. 921.2 ft. to Cor. No. 2, thence N. 37° 00′ E. 467.66 ft. to Cor. No. 3, thence S. 64° 00′ E. 700 ft. to Cor. No. 1, the place of beginning.

Containing 4 acres, more or less.

Variation 14° 45′ E.

Together with all and singular the hereditaments and appurtenances thereto belonging or in any wise appertaining.

Witness my hand and seal this 5th day of December, A.D. 1891.

[SEAL.] A. E. LOWE.

(4—694.)

SURVEY NO. 8000 A

TITLE PAGE TO REPORT UNDER CIRCULAR "N" OF SEPT. 23, 1882

REPORT

Under General Land Office Circular "N" of September 23, 1882, upon the Placer Mining Claim known as the....Cumro placer...., claimed by....T. E. Jenkins *et al*....., situate in....Pike's Peak....Mining District...., El Paso....County...., Colorado...., embracing....32.07 acres, and forming a portion of the S. half of the S.W. quarter....in Sec. ....17...., Town....14 S..... Range.... 69 W. of the 6th P. M.....

Examination made....February 15th...., 1893.

.....By A. L. HAWLEY.....,

U. S. Deputy Mineral Surveyor.

SURVEY NO. 8000 A

CUMRO PLACER

a The soil embraced in this claim consists of decomposed mineral-bearing granite on the mountain slopes, and auriferous sand and gravel along the creek bottom, all covered with a thin layer of loam and alluvium supporting a scant growth of grass and sage brush, with scattering pine, spruce, cedar, and cottonwood timber.

b The only stream passing through this claim is Cumro creek, 4 ft. wide and about 2 ft. deep, which crosses the extreme southeast corner.

A log cabin, the west corner of which bears from Cor. No. 13 S. 40° E. 120 ft. 12×16 ft., course of long sides N. 44° E.

c The surface and underground workings on this claim consists of:

A tunnel, the mouth of which bears from Cor. No. 7 N. 67° 48′ E. 582 ft., 5×6 ft., running N. 10° 44′ W. 515 ft. to breast.

A shaft, which bears from Cor. No. 20 S. 48° 30′ W. 305 ft., 3×5 ft., 12 ft. deep in earth and rock.

Placer workings, the centre of the northeasterly end of which bears from Cor. No. 15 N. 46° W. 285 ft., averaging 40 ft. wide and 8 ft. deep, and extending S. 62° W. 120 ft. along the bed of Cumro creek.

d The nearest post-office to the claim is Jamestown, a mining camp of about 300 population, located on Brush creek, about 2 miles south of the claim. The nearest railroad station is Tie Siding, a spur and flag station on the Denver, Apex and Western R. R., at the confluence of Cumro and Plum creeks, about 6 miles southwesterly from the claim.

e Other than the system of lode deposits adjoining and forming a part of this claim, there are none nearer than Carbonate, situate about 4 miles to the northeast.

f This claim is peculiarly adapted for placer-mining purposes, inasmuch as the contour of the surface and the character and nature of the soil are such that it can be most advantageously and cheaply worked by hydraulic gaints and the tailings be rapidly and easily disposed of. Cumro creek carries about 50 cu. ft. of water per second during the dry season, being an abundance of water for working the claim. As yet no water has been taken upon the claim for its development, except in washing the placer workings hereinbefore described; but, by a survey, it has been found that by a ditch not over one mile in length water can be taken from Cumro creek onto the highest portions of the claim; it being the express intention of the claimants to work the claim in this manner.

g The works and expenditures made by the claimants for the development of the claim consist of the placer workings described under paragraph *c* of this report.

h There are no mines, salt licks, salt springs, or mill sites upon this claim.

(4—695.)

OATH OF UNITED STATES DEPUTY MINERAL SURVEYOR

UNDER GENERAL LAND OFFICE CIRCULAR "N" OF SEPT. 23, 1882

I,....A. L. Hawley...., U. S. deputy mineral surveyor, do solemnly swear that in pursuance of an order received from the U. S. Surveyor-General for.... Colorado...., dated....February 6th...., 1893, I have made, under the provisions of General Land Office Circular "N", approved September 23, 1882, a personal and thorough examination, upon the premises, of the placer claim mining of....T. E. Jenkins *et al*...., known as the....Cumro placer...., situate in....Pike's Peak....Mining District, ...El Paso....County,....Colorado...., embracing....32.07 acres and forming a portion of the S. ½ of the S.W. ¼ of Sec. 17...., in Township No. 14 S...., Range No.....69 W. of the 6th P. M....., and that my report of such examination, hereto attached, is specific and in detail, and is a full and true statement of the facts upon all the points specified in said circular.

.....A. L. Hawley....., *U. S. Deputy Mineral Surveyor.*

Subscribed and sworn to by the said....A. L. Hawley...., U. S. deputy mineral surveyor, before me, ...a notary public in and for El Paso County, Colorado...., this....20th....day of....February, 1893.

[SEAL.] .....B. F. Clark....., ...*Notary Public*.....

My commission expires December 20, 1893.

(4—696.)

CORROBORATIVE AFFIDAVIT UNDER GENERAL LAND OFFICE CIRCULAR "N" OF SEPT. 23, 1882

State of Colorado, } *ss.:*
County of ..El Paso.., }

..... W. H. Wilson....and.... J. P. Thompson...., being first duly sworn, severally depose and say that he is personally and well acquainted with the placer mining claim of....T. E. Jenkins *et al*...., known as the....Cumro placer...., situate in....Pike's Peak....Mining District ...., El Paso....County, Colorado...., embracing.... 32.07 acres and forming a portion of the S. ½ of the S.W. ¼ of Sec. 17, ...in Township No.....14 S.....Range No.....69 W. of the 6th P. M.....; and also with the character of all the land included in said claim, and has been so acquainted for....10 and 12....years last past; that his knowledge of said claim and land is derived from....prospecting the ground nd working the claim...., and is such as to enable him to testify understandingly with regard thereto; that he has carefully read the foregoing report of....A. L. Hawley....U. S. deputy mineral surveyor, and that

to his own personal knowledge said report is in all respects true and accurate.

.....W. H. WILSON.....
.....J. P. THOMPSON.....

Subscribed and sworn to by the above-named persons this....20th.... day of.... February...., 1893.

[SEAL.] ....B. F. CLARK....., ...*Notary Public*.....

My commission expires December 20, 1893.

(4—687.)

SURVEYOR-GENERAL'S CERTIFICATE OF APPROVAL OF FIELD NOTES AND SURVEY OF MINING CLAIM

DEPARTMENT OF THE INTERIOR,
OFFICE OF U. S. SURVEYOR-GENERAL,

————————18—.

I, U. S. Surveyor-General for ———, do hereby certify that the foregoing and hereto attached field notes and return of the survey of the mining claim of ——— ———, known as the ———, situate in ——— mining district,——— County, ———, in Section ———, Township No. ———, Range No. ———, designated as Survey No. ———, executed by ———, U. S. deputy mineral surveyor, ———. 18———, under my instructions dated ———, 18——, have been critically examined and the necessary corrections and explanations made, and the said field notes and return, and the survey they describe, are hereby approved. A true copy of the copy of the location certificate filed by the applicant for survey is included in the field notes.

U. S. Surveyor-General
For ————————

(4—688.)

UNITED STATES SURVEYOR-GENERAL'S FINAL CERTIFICATE ON FIELD NOTES

DEPARTMENT OF THE INTERIOR,
OFFICE OF U. S. SURVEYOR-GENERAL,

————————18—.

I, U. S. Surveyor-General for———, do hereby certify that the foregoing transcript of the field notes, return, and approval of the survey of the mining claim of ———, known as the ———, situate in ——— mining district. ——— County, ———, in Section———, Township No. ———, Range No. ———, and designated as Survey No. ———, has been correctly copied from the originals on file in this office; that said field notes

furnish such an accurate description of said mining claim as will, if incorporated into a patent, serve fully to identify the premises, and that such reference is made therein to natural objects or permanent monuments as will perpetuate and fix the *locus* thereof.

And I further certify that five hundred dollars' worth of labor has been expended in improvements made upon said mining claim by claimant or ——— grantors, and that said improvements consist of———, and that no portion of said labor or improvements has been included in the estimate of expenditures upon any other claim.

I further certify that the plat hereof, filed in the U. S. Land Office at ———, is correct and in conformity with the foregoing field notes.

U. S. Surveyor-General,
For ———

APPENDIX A

CIRCULAR TO APPLICANTS

To Applicants for Mineral Survey Orders:

You will observe the following requirements in the conduct of your business with the Surveyor-General's Office, the same being based upon the United States mining laws and circular and special instructions from the Commissioner of the General Land Office:

1. All applications for survey orders, descriptive reports on placer claims, or certificates of five hundred dollars' expenditure, should be addressed to the Surveyor-General[1] and be signed by the claimants, their agent or attorney.

2. Each application should contain:

(*a*) The name of the claimant in full, and as it is desired to appear in the application for patent.

(*b*) The name of each location embraced in the claim.

(*c*) The name of the land and mining districts in which the claim is located.

(*d*) The name of the United States deputy mineral surveyor to whom it is desired the order shall be issued.

(For form of application see page 751).

3. You are required to file with each application for survey order a copy of the record of location of the claim, properly certified by the recorder of the county or mining district where the claim is situate.

4. The deputy mineral surveyor is required to survey the claim in *strict conformity* with or *within* the lines of the location upon which the order of survey is based. You are, therefore, advised before filing your application to see that your location has been made in compliance with the law and regulations, and that it properly describes the claim for which the patent is sought.

[1] See page 781 for list of offices of U. S. Surveyors-General.

The act of Congress of May 10, 1872, expressly provides that "the location must be distinctly marked on the ground, so that its boundaries can be readily traced," and "that all records of mining claims hereafter made shall contain the name or names of the locators, the date of location, and such a description of the claim or claims, located by reference to some natural object or permanent monument, as will identify the claim."

"These provisions of the law must be strictly complied with in each case to entitle a claimant to a survey and patent, and therefore should a claimant under a location made *subsequent* to the passage of the mining act of May 10, 1872, who has not complied with said requirements in regard to marking the location upon the ground, and recording the same apply for a survey, you will decline to make it."

"The only relief for a party under such circumstances, will be to make a new location in conformity to law and regulations, as no case will be approved by this office, unless these and all other provisions of law are substantially complied with." (See General Land Office circular dated Nov. 20, 1873.)

5. Par. 99, General Land Office circular, of Dec. 10, 1891, edition Dec. 1, 1894, relating to the expense of office work connected with the survey of mineral claims, reads as follows:

"With regard to the *platting* of the claim and other *office work* in the Surveyor-General's office, that officer will make an estimate of the cost thereof, which amount the claimant will deposit with any assistant United States treasurer, or designated depository, in favor of the United States treasurer, to be passed to the credit of the fund created by 'individual depositors for surveys of the public lands,' and file with the Surveyor-General duplicate certificates of such deposits in the usual manner."

6. The various Surveyors-General have adopted schedules of rates for office work, and an estimate of the cost in any particular case may be had upon application.

Should an applicant deem an estimate excessive, he will be allowed the right of appeal to the General Land Office in the usual manner.

In transmitting such an appeal the Surveyor-General should transmit therewith a full report.

7. Should the office work in any case amount to more than the estimate, or if an amended order is issued, an additional deposit will be required.

8. In districts where there are no United States depositories, you should deposit with the nearest assistant United States treasurer, or depository, and in all cases immediately forward the original certificate to the Secretary of the Treasury and the duplicate to the Surveyor-General's office, retaining the triplicate for your own use and security. Under no circumstances will the deposit be made by the Surveyor-General. (See paragraph 5, preceding.)

9. An application for an amended survey order must be accompanied with a statement setting forth fully the reasons for the proposed amendment and all the material facts in the matter.

10. If, after having obtained a survey order, you should abandon your purpose of having a survey made, you can apply the deposit, less the amount estimated for office expenses already incurred, on a new survey if one is desired.

11. Upon discovery of any error or defect in a order you are requested to return it to the Surveyor-General's Office for correction or amendment.

12. If, after having obtained an order for survey, you should find that the record of location does not practically describe the location as staked upon the ground, you should file a certified copy of an amended location certificate, correctly describing the claim, and obtain an *amended order* for survey. If a relocation of the claim is made embracing ground not included in the original order, or other material change is made, you will abandon the original number of the order for survey, and a new order will be issued in which a number in the current series will be substituted.

13. The order of approval of surveys of mineral claims is prescribed by General Land Office circular dated March 3, 1881, as follows:

"The mining survey first applied for shall have the priority of action in all its stages in the office of the Surveyor-General, including the delivery thereof, over any other survey of the same ground or any portion thereof.

"The Surveyor-General should not order or authorize a survey of a claim which conflicts with one previously applied for until the survey first applied for has been completed, examined, approved, and platted, and the plats delivered.

"When the conflict does not appear until the field notes of the respective surveys are returned, then the survey first applied for should be first examined, approved, and platted, and the plats delivered before the field notes of the survey last applied for are taken up for examination or plats constructed.

"When the survey first authorized is not returned within a reasonable period, and the applicant for a conflicting survey makes affidavit that he believes (stating the reasons for his belief) that such first applicant has abandoned his purpose of having a survey made, or is deferring it for vexatious purposes, to wit, to postpone the subsequent applicant, the Surveyor-General shall give notice of such charges to such first applicant, and call upon him for an explanation under oath of the delay. He shall also require the deputy mineral surveyor to make a full statement in writing, explanatory of the delay; and if the Surveyor-General shall conclude that good and sufficient reasons for such delay do not exist, he shall authorize the applicant for the conflicting survey to proceed with the same; otherwise the order of proceedings shall not be changed.

"Whenever an applicant for a survey shall have reason to suppose that a conflicting claimant will also apply for a survey for patent, he may give a notice in writing to the Surveyor-General particularly describing such conflicting claim, and file a copy of the notice of location of such conflicting claim. In such case the Surveyor-General will not order or authorize any survey of such conflicting claim until the survey first applied for has been examined, completed, approved, and platted, and the plats delivered."

14. You have the option of employing any United States deputy mineral surveyor in the district to execute the order of survey, and must make satisfactory arrangements with such surveyor for the payment for his services and those of his assistants in making the survey, as the United States will be not held responsible for the payment of the same. The duty of the deputy surveyor in any particular case ceases when he

has executed the survey and returned the same to this office. He is not allowed to prepare fort he mining claimant the papers in support of an application for patent, being precluded from acting either directly or indirectly as attorney in mineral claims. (Sec. 2334, United States Revised Statutes; see Appendix B hereof.)

15. You are advised of your right to appeal to the Commissioner of the General Land Office from the approval or disapproval of the survey of your claim. The appeal must be in writing or in print, should set forth in brief and clear terms the specific points of exception to the ruling appealed from, and should be transmitted through the Surveyor-General's Office.

APPENDIX B

SECTIONS OF THE U. S. REVISED STATUTES AND PARAGRAPHS OF THE MINING CIRCULAR OF DECEMBER 10, 1891, RELATIVE TO SURVEYS OF MINING CLAIMS.

Sec. 2325. A patent for any land claimed and located for valuable deposits may be obtained in the following manner: Any person, association, or corporation authorized to locate a claim under this chapter, having claimed and located a piece of land for such purposes, who has, or have, complied with the terms of this chapter, may file in the proper land-office an application for a patent, under oath, showing such compliance, together with a plat and field notes of the claim or claims in common, made by or under the direction of the United States Surveyor-General, showing accurately the boundaries of the claim or claims, which shall be distinctly marked by monuments on the ground. . . .

PAR. 28. The claimant is required in the first place to have a correct survey of his claim made under authority of the Surveyor-General of the State or Territory in which the claim lies; such survey to show with accuracy the exterior surface boundaries of the claim, which boundaries are required to be distinctly marked by monuments on the ground. Four plats and one copy of the original field notes, in each case, will be prepared by the Surveyor-General; one plat and the original field notes to be retained in the office of the Surveyor-General, one copy of the plat to be given the claimant for posting upon the claim, one plat and a copy of the field notes to be given the claimant for filing with the proper register, to be finally transmitted by that officer, with other papers in the case, to this office, and one plat to be sent by the Surveyor-General to the register of the proper land district to be retained on his files for future reference. As there is no resident Surveyor-General for the State of Arkansas, applications for the survey of mineral claims in said State should be made to the Commissioner of this office, who, under the law, is *ex officio* the U. S. Surveyor-General.

45. The Surveyors-General should designate all surveyed mineral claims by a progressive series of numbers, beginning with survey No. 37, irrespective as to whether they are situated on surveyed or unsurveyed lands, the claim to be so designated at date of issuing the order therefor,

in addition to the local designation of the claim; it being required in all cases that the plat and field notes of the survey of a claim must, in addition to the reference to permanent objects in the neighborhood, describe the locus of the claim, with reference to the lines of public surveys, by a line connecting a corner of the claim with the nearest public corner of the United States surveys, unless such claim be on unsurveyed lands at a distance of more than two miles from such public corner, in which latter case it should be connected with a United States mineral monument. Such connecting line must not be more than *two miles* in length and should be measured on the ground direct between the points, or calculated from actually surveyed traverse lines if the nature of the country should not permit direct measurement. If a regularly established survey corner is within two miles of a claim situated on unsurveyed lands, the connection should be made with such corner in preference to a connection with a United States mineral monument. The connecting line must be surveyed by the deputy mineral surveyor at the time of his making the particular survey, and be made a part thereof.

46. Upon the approval of the survey of a mining claim made upon surveyed lands, the Surveyor-General will prepare and transmit to the local land office and to this office a diagram tracing showing the portions of legal 40-acre subdivisions made fractional by reason of the mineral survey, designating each of such portions by the proper lot number, beginning with No. 1 in each section and giving the area of each lot.

47. The survey and plat of mineral claims, required by section 2325, Revised Statutes of the United States, to be filed in the proper land office with application for patent, must be made subsequent to the recording of the location of the mine; and when the original location is made by survey of a United States deputy surveyor, such location survey cannot be substituted for that required by the statute, as above indicated.

48. The Surveyor-General should derive his information upon which to base his certificate as to the value of labor expended or improvements made from his deputy who makes the actual survey and examination upon the premises, and such deputy should specify with particularity and full detail the character and extent of such improvements.

49. The following particulars should be observed in the survey of every mining claim.

(1) The exterior boundaries of the claim should be represented on the plat of survey and in the field notes.

(2) The intersection of the lines of the survey with the lines of conflicting prior surveys should be noted in the field notes and represented upon the plat.

(3) Conflicts with unsurveyed claims, where the applicant for survey does not claim the area in conflict, should be shown by actual survey.

(4) The total area of the claim embraced by the exterior boundaries should be stated, and also the area in conflict with each intersecting survey, substantially as follows:

	Acres.
Total area of claim	10.50
Area in conflict with survey No. 302	1.56
Area in conflict with survey No. 948	2.33
Area in conflict with Mountain Maid lode mining claim, unsurveyed	1.48

It does not follow that because mining surveys are required to exhibit all conflicts with prior surveys the areas of conflict are to be excluded.

The field notes and plat are made a part of the application for patent, and care should be taken that the description does not inadvertently exclude portions intended to be retained. It is better that the application for patent should state the portions to be excluded in express terms. A survey executed as in the example given will enable the applicant for patent to exclude such conflicts as may seem desirable. For instance, the conflict with survey No. 302 and with the Mountain Maid lode claim might be excluded and that with survey No. 948 included.

(For paragraphs 50 and 51, see page 749.)

SEC. 2327. The description of vein or lode claims, upon surveyed lands, shall designate the location of the claim with reference to the lines of the public surveys, but need not conform therewith; but where a patent shall be issued for claims upon unsurveyed lands, the surveyor-general, in extending the surveys, shall adjust the same to the boundaries of such patented claim, according to the plat or description thereof, but so as in no case to interfere with or change the location of any such patented claim. (See paragraph 46, page 748.)

SEC. 2331. Where placer claims are upon surveyed lands, and conform to legal subdivisions, no further survey or plat shall be required, and all placer-mining claims located after the tenth day of May, eighteen hundred and seventy-two, shall conform as near as practicable with the United States system of public-land surveys, and the rectangular subdivisions of such surveys, and no such location shall include more than twenty acres for each individual claimant; but where placer claims cannot be conformed to legal subdivisions, survey and plat shall be made as on unsurveyed lands; and where by the segregation of mineral lands in any legal subdivision a quantity of agricultural land less than forty acres remains, such fractional portion of agricultural land may be entered by any party qualified by law, for homestead or pre-emption purposes.

SEC. 2334. The Surveyor-General of the United States may appoint in each land district containing mineral lands as many competent surveyors as shall apply for appointment to survey mining claims. The expenses of the survey of vein or lode claims, and the survey and subdivision of placer claims into smaller quantities than one hundred and sixty acres, together with the cost of publication of notices, shall be paid by the applicants, and they shall be at liberty to obtain the same at the most reasonable rates, and they shall also be at liberty to employ any United States deputy surveyor to make the survey. The Commissioner of the General Land Office shall also have power to establish the maximum charges for surveys. . . .

98. The Surveyors-General of the several districts will, in pursuance of said law, appoint in each land district as many *competent* deputies for the survey of mining claims as may seek such appointment: it being distinctly understood that all expenses of these notices and surveys are to be borne by the mining claimants and not by the United States; the system of making *deposits* for mineral surveys, as required by previous instructions, being hereby revoked as regards *field work;* the claimant having the option of employing *any* deputy surveyor within such district to do his work in the field.

99. With regard to the *platting* of the claim and other *office work* in the Surveyor-General's Office, that officer will make an estimate of the cost thereof, which amount the claimant will deposit with any

assistant United States treasurer or designated depository in favor of the United States Treasurer, to be passed to the credit of the fund created by "individual depositors for surveys of the public lands," and file with the Surveyor-General duplicate certificates of such deposit in the usual manner.

100. The Surveyors-General will endeavor to appoint mineral deputy surveyors, so that one or more may be located in each mining district for the greater convenience of miners.

101. The usual oaths will be required of these deputies and their assistants as to the correctness of each survey executed by them.

The duty of the deputy mineral surveyor ceases when he has executed the survey and returned the field notes and preliminary plat thereof with his report to the Surveyor-General. He will not be allowed to prepare for the mining claimant the papers in support of an application for patent, or otherwise perform the duties of an attorney before the land office in connection with a mining claim.

The Surveyors-General and local land officers are expected to report any infringement of this regulation to this office.

DESCRIPTIVE REPORTS ON PLACERS

Par. 63, Mining Circular

(2) Section 2395. Revised Statutes (subdivision 7), requires the surveyor to "note in his field books the true situation of all mines, salt licks, salt springs, and mill seats which come to his knowledge;" also "all watercourses over which the lines he runs may pass." It further requires him to "note the quality of the lands." These descriptive notes are required by subdivision 8 to be incorporated in the plat by the Surveyor-General.

(3) If these duties have been performed, the public surveys will furnish a reasonable guide to the district officers and to claimants in prosecuting their applications. But experience has shown that great neglect has resulted from inattention to the law in this respect, and the regular plats are of very little value in the matter. It will, therefore, be required in the future that deputy surveyors shall, at the expense of the parties, make full examinations of all placer claims, surveyed by them, and duly note the facts as specified in the law, stating the quality and composition of the soil, the kind and amount of timber and other vegetation, the locus and size of streams, and such other matters as may appear upon the surface of the claim. This examination should include the character and extent of all surface and underground workings, whether placer or lode, for mining purposes.

(4) In addition to these data, which the law requires to be shown in all cases, the deputy should report with reference to the proximity of centres of trade or residence; also of well-known systems of lode deposit of or individual lodes. He should also report as to the use or adaptability of the claim for placer mining; whether water has been brought upon it in sufficient quantity to mine the same, or whether it can be procured for that purpose; and, finally, what works or expenditures have been made

by the claimant or his grantors for the development of the claim, and their situation and location with respect to the same as applied for.

(5) This examination should be reported by the deputy under oath to the Surveyor-General, and duly corroborated; and a copy of the same should be furnished with the application for patent to the claim, constituting a part thereof, and included in the oath of the applicant.

CONTESTS BETWEEN MINERAL AND AGRICULTURAL CLAIMANTS

Segregation Surveys

114. When the case comes before this office [General Land Office], such decision will be made as the law and the facts may justify; and in cases where a survey is necessary to set apart the mineral from the agricultural land, the necessary instructions will be given to enable the proper party, *at his own expense*, to have the work done, at his option, either by United States deputy, county, or other local surveyor; the survey in such case, where the claims to be segregated are vein or lode claims, must be executed in such manner as will conform to the requirements in section 2320, U. S. Revised Statutes, as to length and width and parallel end lines.

115. Such survey when executed must be properly sworn to by the surveyor, either before a notary public, officer of a court of record, or before the register or receiver, the deponent's character and credibility to be properly certified to by the officer administering the oath.

116. Upon the filing of the plat and field notes of such survey, duly sworn to as aforesaid, you [Register and Receiver] will transmit the same to the Surveyor-General for his verification and approval; who, if he finds the work correctly performed, will properly mark out the same upon the original township plat in his office, and furnish authenticated copies of such plat and description both to the proper local land office and to this office, to be affixed to the duplicate and triplicate township plats respectively.

117. With a copy of plat and description furnished the local office and this office, must be a diagram tracing, verified by the Surveyor-General, showing the claim or claims segregated, and designating the separate fractional agricultural tracts in each 40-acre legal subdivision by the proper lot number, beginning with No. 1 in each section, and giving the area in each lot, the same as provided in paragraph 45, in the survey of mining claims on surveyed lands.

SURVEY OF ADVERSE CLAIMS

Sec. 2326. Where an adverse claim is filed during the period of publication, it shall be upon oath of the person or persons making the same, and shall show the nature, boundaries, and extent of such adverse claim. . . .

86. In order that the "*boundaries*" and "*extent*" of the claim may be shown, it will be incumbent upon the adverse claimant to file a plat showing his entire claim, its relative situation or position with the one against

which he claims, and the extent of the conflict. This plat must be made from an actual survey by a United States deputy surveyor, who will officially certify thereon to its correctness; and in addition there must be attached to such plat of survey a certificate or sworn statement by the surveyor as to the approximate value of the labor performed or improvements made upon the claim by the adverse party or his predecessors in interest, and the plat must indicate the position of any shafts, tunnels, or other improvements, if any such exist, upon the claim of the party opposing the application, and by which party said improvements were made: *Provided, however*, That, if the application for patent describes the claim by legal subdivisions, the adverse claimant, if also claiming by legal subdivisions, may describe his adverse claim in the same manner without further survey or plat.

MINING DISTRICTS AND SURVEYORS-GENERAL

Alaska, Sitka	Williams, Louis L.
Arizona, Tucson	Manning, Levi H.
California, San Franscisco	Green William S.
Colorado, Denver	Robinson, Thomas D.
Idaho, Boise City	Straughan, Joseph C.
Montana, Helena	Neill, John S. M.
Nevada, Reno	Belknap, Clayton.
New Mexico, Santa Fe	Easley, Charles F.
Oregon, Portland	Arnold, John C.
South Dakota, Huron	Hughes, Richard B.
Utah, Salt Lake City	Snow, George W.
Washington, Olmypia	Watson, William P.
Wyoming, Cheyenne	Thompson, John C.

APPENDIX C

FINITE DIFFERENCES

THE CONSTRUCTION OF TABLES

In the accompanying figure the ordinates are spaced at the uniform distance l apart. Let the successive values of these ordinates, and their several orders of differences, be represented by the following notation:

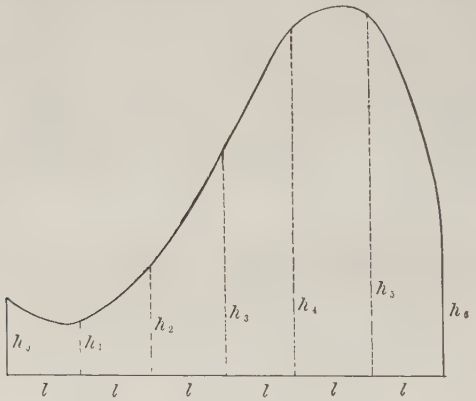


FIG. 261.

Values of the function,	h_0,	h_1,	h_2,	h_3,	h_4,	h_5,	h_6.
First order of differences,	$\Delta' h_0$,	$\Delta' h_1$,	$\Delta' h_2$,	$\Delta' h_3$,	$\Delta' h_4$,	$\Delta' h_5$.	
Second " "	$\Delta'' h_0$,	$\Delta'' h_1$,	$\Delta'' h_2$,	$\Delta'' h_3$,	$\Delta'' h_4$.		
Third " "	$\Delta''' h_0$,	$\Delta''' h_1$,	$\Delta''' h_2$,	$\Delta''' h_3$.			
Fourth " "	$\Delta^{\text{iv}} h_0$,	$\Delta^{\text{iv}} h_1$,	$\Delta^{\text{iv}} h_2$.				
etc.,	.	etc.					

We may now write

$$\left.\begin{aligned}
h_1 &= h_0 + \Delta' h_0;\\
h_2 &= h_1 + \Delta' h_1 = h_0 + \Delta' h_0 + \Delta' h_0 + \Delta'' h_0 = h_0 + 2\Delta' h_0 + \Delta'' h_0;\\
h_3 &= h_2 + \Delta' h_2 = h_0 + 3\Delta' h_0 + 3\Delta'' h_0 + \Delta''' h_0;\\
h_4 &= h_0 + 4\Delta' h_0 + 6\Delta'' h_0 + 4\Delta''' h_0 + \Delta^{\mathrm{iv}} h_0;\\
h_n &= h_0 + n\Delta' h_0 + \frac{n(n-1)}{1\cdot 2}\Delta'' h_0 + \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}\Delta''' h_0 + \text{etc.}
\end{aligned}\right\} \quad . \quad . \quad (1)$$

It is to be observed that the coefficients follow the law of the binomial development. It is also seen that the *first* of the successive orders of differences are alone sufficient to enable any term of the function to be computed. We will now proceed to find these first terms of the several orders of differences for any given equation.

Almost all functions of a single variable can be developed by the aid of Maclaurin's Formula, in the form

$$y_0 = C_0 + C_1 x_0 + C_2 x_0^2 + C_3 x_0^3 + C_4 x_0^4 + \text{etc.} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If x take an increment Δx, thus becoming x_1, the *change* in y_0 will be represented by $\Delta' y_0$ and its value will be the new value of the function minus its initial value, or $\Delta' y_0 = y_1 - y_0$. By putting $x + \Delta x$ for x in the above equation, developing, subtracting the original equation, and reducing, we would obtain

$$\begin{aligned}
y_1 - y_0 = \Delta' y_0 = {} & (C_1 + 2C_2 x_0 + 3C_3 x_0^2 + 4C_4 x_0^3)\,\Delta x\\
& + (C_2 + 3C_3 x_0 + 6C_4 x_0^2)\,\Delta^2 x + (C_3 + 4C_4 x_0)\,\Delta^3 x + C_4 \Delta^4 x, \quad . \quad (3)
\end{aligned}$$

assuming that the function stops with $C_4 x_0^4$.

If x_1 should now take another increment Δx equal to the previous one, we would have $x_2 = x_1 + \Delta x$ and $y_2 = y_1 + \Delta' y_1$. Now $\Delta' y_1$ is the value $\Delta' y$ when x_0 has become x_1, and the *difference* between $\Delta' y_0$ and $\Delta' y_1$ is the *change* in the value of $\Delta' y_0$ due to this change in x.

Hence $$\Delta' y_1 - \Delta' y_0 = \Delta'' y_0.$$

To find the value of $\Delta'' y_0$, substitute $x + \Delta x$ for x in equation (3), develop, subtract equation (3), reduce, and obtain

$$\Delta'' y_0 = (2C_2 + 6C_3 x_0 + 12C_4 x_0^2)\,\Delta^2 x + (6C_3 + 24C_4 x_0)\,\Delta^3 x + 14C_4 \Delta^4 x. \quad . \quad (4)$$

Similarly we find

$$\Delta''' y_0 = (6C_3 + 24C_4 x_0)\,\Delta^3 x + 36C_4 \Delta^4 x. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$\Delta^{\mathrm{iv}} y_0 = 24C_4 \Delta^4 x \text{ (a constant)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

From the above development we see—

1. That *the number of orders of differences is equal to the highest exponent of the variable involved*, the last difference being a constant.

2. That if any initial value (x_0) of the variable be taken, the first of the several orders of differences can be obtained in terms of this initial value, its constant increment, and the constant coefficients. This furnishes a ready means of computing a table of values of the function, if it can be represented in the form of equation (1). Evidently if the initial value of the variable (x_0) be taken as zero, the evaluation for the several initial differences is much simplified, for then all the terms in x disappear. If the constant increment be also taken as unity, the labor is still further reduced.

EXAMPLE.—Construct a table of values of the function

$$y = 50 - 40x + 20x^2 + 4x^3 - x^4. \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Let the initial value of the variable be zero and the increments unity. Evaluating the initial differences by equations (3) to (6), we find, for $x_0 = 0$, and $\Delta x = 1$,

$$\begin{aligned} y_0 &= +50; \\ \Delta' y_0 &= C_1 + C_2 + C_3 + C_4 = -17; \\ \Delta'' y_0 &= 2C_2 + 6C_3 + 14C_4 = +50; \\ \Delta''' y_0 &= 6C_3 + 36C_4 = -12; \\ \Delta^{iv} y_0 &= 24C_4 = -24. \end{aligned}$$

From these initial values we may readily construct the following table:

Values of x.	Values of y[1].	1st Differences. $\Delta' y$.	2d Differences. $\Delta'' y$.	3d Differences. $\Delta''' y$.	4th Differences. $\Delta^{iv} y$.
0	50				
		−17			
1	33		+ 50		
		+ 33		− 12	
2	66		+ 38		− 24
		+ 71		− 36	
3	137		+ 2		− 24
		+ 73		− 60	
4	210		− 58		− 24
		+ 15		− 84	
5	225		−142		− 24
		−127		−108	
6	98		−150		− 24
		−377		−132	
7	−279		−182	etc.	etc.
		−759			
8	−1038	etc.	etc.		
etc.	etc.				

[1] Fig. 152 is the locus of this curve, the ordinates being taken from this column.

The initial values in all the columns being given, the table is made by continual additions, one column after another, working from right to left. Thus, the 4th difference being constant, the initial value, −24, is simply repeated indefinitely. The column of 3d differences is now computed by adding continuously −24 to the preceding value. The column of 2d differences is next made out, the quantity to be added each

time being the intervening 3d difference, which is not constant. In a similar manner proceed with the column of 1st differences, and finally with the values of the function itself.

The above formulæ apply to all functions of a single variable not higher than the fourth degree. Evidently any of the C coefficients may be zero, and so cause one or more of the powers of x to entirely disappear. If the variable is involved to a higher degree than the fourth, a new development may be made, or the initial values of the successive orders of differences may be determined by simply evaluating the function for a series of successive values of the variable, one more in number than the degree of the equation, and then working out the successive columns of differences from these until the last, or constant, difference is found. The table may then be continued by combining these differences, as before. Thus in the above example of the first five values of y might have been found by direct evaluation of the function for the corresponding values of x, and then the successive differences taken out until the constant fourth difference, -24, was found. This can always be done without resorting to any algebraic discussion as given above.

THE EVALUATION OF IRREGULAR AREAS

The ordinates to any curve, as that in Fig. 152 for instance, may be represented by such an equation as the last of equations (1), where the length of any ordinate is given in terms of its number from the initial ordinate, the value of this first ordinate, and the first of the successive orders of differences. This equation is

$$h_n = h_0 + n\Delta' h_0 + \frac{n(n-1)}{1\cdot 2}\Delta'' h_0 + \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}\Delta''' h_0 + \text{etc.},$$

where h_n is the nth, and therefore any ordinate to the curve. The constant distance between the ordinates apparently does not enter the equation, but it is really represented in the several Δ's.

By the calculus the area of any figure included between any curve, the axis of abscissas, and two extreme ordinates is $A = \int_0^x h dx$, where h is the general value of an ordinate, $= h_n$ in the above equation, where it is shown to be a function of n. Also $x = nl$ where l is the constant distance between ordinates, whence $dx = l dn$. Substituting these values of h and dx, we have

$$A = \int_0^x h dx = \int_0^n h_n l dn = l\int_0^n h_n dn = l\left[h_0\int_0^n dn + \Delta' h_0\int_0^n n dn\right.$$

$$+ \frac{\Delta'' h_0}{1\cdot 2}\int_0^n n(n-1)dn + \frac{\Delta''' h_0}{1\cdot 2\cdot 3}\int_0^n n(n-1)(n-2)dn$$

$$\left. + \frac{\Delta^{iv} h_0}{\lfloor 4}\int_0^n n(n-1)(n-2)(n-3)dn + \text{etc.}\right]. \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Integrating this equation, we obtain

$$A = l\left\{\begin{array}{l} nh_0 + \frac{n^2}{2}\Delta' h_0 + \left(\frac{n^3}{6} - \frac{n^2}{4}\right)\Delta'' h_0 + \left(\frac{n^4}{24} - \frac{n^3}{6} + \frac{n^2}{6}\right)\Delta''' h_0 \\ + \left(\frac{n^5}{120} - \frac{n^4}{16} + \frac{11n^3}{72} - \frac{n^2}{8}\right)\Delta^{\text{iv}} h_0 + \left(\frac{n^6}{720} - \frac{n^5}{60} + \frac{7n^4}{96} - \frac{5n^3}{36} + \frac{n^2}{10}\right)\Delta^{\text{v}} h_0 \\ + \left(\frac{n^7}{5040} - \frac{n^6}{288} + \frac{17n^5}{720} - \frac{5n^4}{64} + \frac{137n^3}{1080} - \frac{n^2}{12}\right)\Delta^{\text{vi}} h_0 + \text{etc.} \end{array}\right\} \quad . \quad . \quad (9)$$

From the schedule of differences on p. 605, we may at once find the initial values of the several orders of differences in terms of the successive values of the function. Thus

$$\Delta' h_0 = h_1 - h_0;$$
$$\Delta'' h_0 = \Delta' h_1 - \Delta' h_0 = h_2 - 2h_1 + h_0;$$
$$\Delta''' h_0 = \Delta'' h_1 - \Delta'' h_0 = \Delta' h_2 - 2\Delta' h_1 + \Delta' h_0 = h_3 - 3h_2 + 3h_1 - h_0;$$
$$\Delta^{\text{iv}} h_0 = \Delta''' h_1 - \Delta''' h_0 = \Delta'' h_2 - 2\Delta'' h_1 + \Delta'' h_0 = \Delta' h_3 - 3\Delta' h_2 + 3\Delta' h_1 - \Delta' h_0$$
$$= h_4 - 4h_3 + 6h_2 - 4h_1 + h_0.$$

Again, the coefficients follow the law of the binomial development, and we may write

$$\Delta^n h_0 = h_n - nh_{n-1} + \frac{n(n-1)}{1 \cdot 2} h_{n-2} - \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} h_{n-3} + \text{etc.} \quad . \quad (10)$$

By the aid of this equation we may now substitute for the several initial differences in equation (9) their values in terms of the successive values of the function. Also for any area divided into n sections by ordinates, uniformly spaced a distance l apart, equation (9) will give the area in terms of l, n, and the several ordinates, when these latter are substituted for the Δ's by means of equation (10).

Thus, for $n = 1$, equation (9) becomes

$$A = l(h_0 + \tfrac{1}{2}\Delta' h_0) = \frac{l}{2}(h_0 + h_1), \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

which is the *Trapezoidal Rule.*

For $n = 2$,

$$A = l(2h_0 + 2\Delta' h_0 + (\tfrac{8}{6} - 1)\Delta'' h_0) = l(\tfrac{1}{3}h_0 + \tfrac{4}{3}h_1 + \tfrac{1}{3}h_2) = \frac{l}{3}(h_0 + 4h_1 + h_2), \quad (12)$$

which is called *Simpson's $\frac{1}{3}$ Rule.*

If $l' = 2l$ = total length of figure, this formula becomes

$$A = \frac{l'}{6}(h_0 + 4h_1 + h_2), \quad . \quad . \quad . \quad . \quad . \quad . \quad (12a)$$

which is the well-known *form* of the *Prismoidal Formula*, and it would be that formula if areas were substituted for ordinates.

If $n=3$,

$$A=\frac{3l}{8}(h_0+3h_1+3h_2+h_3), \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

which is called *Simpson's $\frac{3}{8}$ Rule*.

If $n=4$,

$$A=\frac{2l}{45}[7(h_0+h_4)+32(h_1+h_3)+12h_2]. \quad . \quad . \quad . \quad . \quad (14)$$

If $n=6$,

$$A=l[6h_0+18\Delta'h_0+27\Delta''h_0+24\Delta'''h_0+\tfrac{123}{10}\Delta^{\mathrm{vi}}h_0+\tfrac{33}{10}\Delta^{\mathrm{v}}h_0+\tfrac{41}{140}\Delta^{\mathrm{iv}}h_0].$$

If now the coefficient of $\Delta^{\mathrm{vi}}h_0$ be changed from $\frac{41}{140}$ to $\frac{42}{140}$, which would not affect curves of a degree less than the sixth, the resulting equation, when the h's are substituted for the Δ's, takes the following very simple form:

$$A=\frac{3l}{10}[h_0+h_2+h_4+h_6+5(h_1+h_3+h_5)+h_3], \quad . \quad . \quad . \quad (15)$$

which is called *Weddel's Rule*.

For a greater number of ordinates than seven, it is best to use either equation (12), (13), or (15) several times, as the formulæ become very complicated for $n>6$.

APPENDIX D

(From Tracy's "Plane Surveying," by permission of the Author.[1])

ERRORS

In the first part of this chapter the aim is to define the different kinds of error, and to show how these errors affect measurements; also to explain how the most probable or best value of a quantity may be obtained, and how the precision of measurements may be judged. In the second part of the chapter a brief review of the method of least squares is given in so far as that method is likely to be used in plane surveying.

Methods of preventing or eliminating errors, and the relative importance of errors from different sources, will be treated more in detail in subsequent chapters.

Error. No measurement in surveying can be exact. The difference between a measurement and the true value of the quantity measured is the *true* error of that measurement. *The true error of a measurement is never known*, for the true value of the quantity measured is never known. How then can a surveyor judge of the precision of his measurements, and how can he determine the most probable or best values of the quantities measured? These are the main questions to be considered in any discussion of errors.

Sources of Error. (1) Natural; (2) Instrumental; (3) Personal.

(1) *Natural.* Temperature, wind, refraction, action of gravity, obstacles to measurement.

(2) *Instrumental.* Imperfections in the construtcion and adjustment of instruments. Expansion, contraction, and other possible changes in the instruments.

(3) *Personal.* Imperfections of human sight and touch. Actual mistakes.

Three Classes of Errors. (1) *Mistakes;* (2) *Constant errors;* (3) *Accidental errors.* If a measurement of a quantity is greater than the true value of that quantity, the error is *plus;* if less than the true value, the error is *minus.* The total error is not due to any one cause, but it is the *algebraic* sum of errors due to different causes.

Mistakes are errors which have their source in the mind of the observer.

Constant errors are those which occur from well-understood causes and can be eliminated.

Accidental errors are those which remain after mistakes and constant errors have been eliminated.

---

[1] This excellent text may be obtained from the publishers, John Wiley & Sons, New York.

An accidental error is as likely to be plus as minus, but a constant error under given conditions always has the same sign and the same magnitude.

Thus, for example, if a tape is $\frac{1}{4}$ inch too long because of high temperature, this is a constant error, because under the same conditions, i.e., the same temperature, the tape will always be $\frac{1}{4}$ inch too long. Under *changed* conditions, i.e., low temperature, the tape may become too short; then the sign of the error is changed. This illustrates how a *constant* error from a given source may be plus or it may be minus, but under the *same* conditions it cannot be plus *or* minus. Suppose, on the other hand, that an observer in measuring between two points makes an error, in reading the tape, so small that he cannot detect it. This *accidental* error is as likely to be plus as minus, and if he repeats the measurement he is liable to make a similar error and that, too, is as likely to be plus as minus.

If a measurement is repeated a number of times under precisely the same conditions, the *constant* error is the same for each measurement, both in sign and magnitude. It is not reduced or affected in any way by taking the mean of all the measurements. The accidental error of each measurement, on the contrary, is as likely to be plus as minus, and the accidental error of the mean, therefore, is likely to be less than the accidental error of any one measurement. Theoretically, that part of the total error due to accidental errors would entirely disappear from the means of an indefinitely large series of observations, taken under precisely the same conditions; that part due to constant errors would still remain.

(*a*) *Illustrations.* A few illustrations of the three classes of errors will help to make the distinctions clear:

Mistakes. Errors due to carelessness, inexperience, and mental confusion, such as mistakes in reading the tape, the vernier, or the level rod; keeping the wrong count of tape lengths; turning the wrong tangent screw; sighting at the wrong station.

Constant errors. Errors due to such well-understood sources as temperature, sag, pull, imperfect graduation, and imperfect adjustments of instruments.

Accidental errors. Errors due to imperfections of human sight and touch which render it impossible to make exact readings, to know when the cross-hair exactly bisects a signal, to handle instruments delicately; sudden and imperceptible changes in an instrument itself; indeterminate *variations* in pull, temperature, and certain other sources of constant errors.

(*b*) *Remark.* When mistakes are very small it is sometimes difficult to distinguish them from accidental errors. It is to be noticed also that while temperature is a source of constant error, *variations* of temperature too small to be measured may be classed as accidental errors; the same is true of small variations in some of the other sources of constant errors.

Notice also that constant errors from different sources tend to balance each other when some are plus, and some are minus; as, for example, when the error in chaining due to high temperature tends to balance error due to sag. Again, constant errors from the same source may tend to balance each other in the mean of measurements taken under *different* conditions, as, for example, when, in chaining, part of the measurements of a line are made when the temperature is above the standard temperature for the tape, and part when it is below.

(*c*) *Cumulative and compensating errors.* When the value of a quantity is found by adding together the measurements of several smaller quantities, any source of constant error becomes a source of *cumulative* errors, because, under given conditions, a constant error always has the same sign; on the other hand, any source of accidental error becomes a source of *compensating* errors, since for each of the separate measurements the sign is as likely to be plus as minus. *Compen-*

sating errors tend to balance each other, while the cumulative errors from any one source affect the total result in the same way.

(*d*) *Illustrations.* Any constant error in the length of a tape becomes cumulative when the tape is applied several times in measuring a line, i.e., when the tape lengths are added together, for the total error due to that cause will vary in direct proportion to the number of tape lengths. On the other hand, errors which occur in marking where the end of the tape comes each time that it is stretched are compensating, for the chances are that if one tape length is marked too long the error will be balanced by another tape length marked too short.

Errors due to the settling of the instrument in levelling are cumulative, because the effect is always the same: it makes the calculated elevations of succeeding points too high. Errors due to the difficulty of telling when the cross-hair exactly bisects the target are compensating.

Discrepancies. In ordinary surveying most of the distances, angles, and elevations are measured but once. An important measurement, however, is often repeated, and if the discrepancy is too large the quantity is measured a third time.

A large discrepancy between two measurements of the same quantity indicates a *mistake* in the work, but it does not follow that because the discrepancy is small the *error* is small. The discrepancy is not an error, it is simply the difference between two measurements each of which contains an error that *may* be very large. This is an important point too often overlooked.

(*a*) *Illustration.* A surveyor measures a line, the true length of which is 400 feet, with a tape which, unknown to him, is ½ foot too short. The first time he gets 402 feet and the second time 402.05. The discrepancy is only 0.05 foot, indicating that he has made no serious mistake, but the true error is 2 feet for the first measurement and 2.05 feet for the second.

Elimination of Constant Errors. The discrepancy between the two measurements in the preceding illustration did not indicate the true error, because of a *constant* error in the length of the tape which no amount of repeating the measurement under the same conditions could reveal. The surveyor should have known that his tape was ½ foot too short, and have corrected his measurements accordingly; then the discrepancy would have indicated more nearly the precision of his work. This illustrates why it is the first business of the surveyor to eliminate constant errors as far as practicable. A mistake, as revealed by a discrepancy, may then be corrected by again repeating the measurement. Constant errors and mistakes having been eliminated only accidental errors remain, and these may be reduced by taking the arithmetical mean.

As pointed out above under certain conditions constant errors become *cumulative*, while accidental errors are *compensating;* hence the obvious importance of eliminating constant errors in chaining, in levelling, and in all similar work involving the summation of separate measurements.

Prevention and Elimination of Errors. (*a*) *Mistakes* are best discovered by systematic checks. *Checking* work, an essential part of all surveying, consists in comparing results with known conditions.

Illustrations. The discrepancy between duplicate measurements should be small; the sum of two or more measurements should equal another measurement, as, for example, when the sum of two angles should equal a third angle; the sum of two or more measurements should fulfil some known condition, as, for example, the interior angles of a triangle should add up 180°.

(*b*) *Constant errors* are prevented or eliminated partly by systematic methods of field work, partly by calculation and correction.

Illustrations. Errors due to imperfect adjustment can be eliminated by certain systematic methods in the use of instruments, such, for example, as the method of double reverse or making back sights and fore sights equal in levelling. Reading both verniers on a transit eliminates error due to eccentricity. Each of these examples illustrates how *by taking observations in pairs* a plus error from a certain source of constant error may be offset by a minus error from the same source.

Errors in chaining due to temperature can be calculated and corrected; so can errors due to sag and pull, though it is better to prevent them by taking proper precautions. (See Chapter V.)

(*c*) *Accidental errors* cannot be eliminated, but they follow a mathematical law,—the law of probability,—according to which they may be greatly reduced by comparing the results of a series of measurements. (See p. 15.)

Relative Importance of Errors from Different Sources. It is evident that constant and cumulative errors as a rule are more important than accidental and compensating errors, and that the more measurements there are of a quantity the less the mean is affected by accidental errors. Beyond this it is difficult to make any general statement. Any error large enough to be *appreciable* should usually be avoided or corrected, but appreciable is a relative term, and an error which is appreciable under certain conditions is inappreciable under other conditions. Thus, for example, the error is chaining due to changes in temperature is an appreciable error provided the precision required warrants the use of a thermometer, otherwise it may be considered inappreciable. Errors in plane surveying so small that they are ignored would often be considered of great magnitude in geodetic surveying.

(*a*) *Remark.* The more the surveyor knows about the sources and nature of errors the more likely he is to judge correctly of their relative importance. He must know that some sources of constant or cumulative errors are more important than others, and that certain errors, though they may be compensating, are not to be ignored. A more thorough treatment of these questions is given in the chapters on *Errors in Linear Measurement*, *Errors in Angular Measurement*, and *Errors in Levelling*.

Probable Values and the Adjustment of Measurements. Probable values are found from observed values. When the relations between observed values are complicated a knowledge of the "theory of least squares" is necessary. Some simple cases which frequently occur in surveying may be given, however, without entering upon that subject.

There are two general cases, viz.: (1) When the measurements are of one and the same quantity; (2) when the measurements are of different but related quantities.

(*a*) *Several measurements of the same quantity.* The most probable value of the quantity is the arithmetical mean of all the measurements, *provided the measurements are made with equal care.*

Illustrations. Three measurements of a line made with equal care are 160.01, 160.1, and 160.03. The most probable value is $(160.01 + 160.1 + 160.03) \div 3 = 160.047$. Four measurements of an angle are 30° 29′, 30° 31′, 30° 31′, and 30° 30′. The most probable value is $(30° 29' + 30' 31' + 30° 31' + 30° 30') \div 4 = 30° 30' 15''$.

(*b*) *Measurements of several related quantities.* When quantities are related because of *two or more* conditions which they must fulfil, the problem of finding the most probable values of these related quantities properly belongs to the "theory of least squares." When, how-

ever, there is only *one* condition fixing the relations between the quantities, the most probable values, as a rule, can be easily found. Two cases under this head which often arise in surveying are: (1) when the sum of several measurements should equal an *exact* quantity, and (2) when the sum should equal some other *measurement*. In the first case the difference between the sum and the *exact* quantity is a true error; in the second case the difference between the sum and a *measured* quantity is a *discrepancy*.

(*c*) Case I. *When the sum of several measurements should equal an exact quantity distribute the true error equally among all the measurements.* The results will be the most probable values of the measurements.

Illustration. The three angles of a triangle add up 179° 59′. The *exact* quantity fixing the relations between the measurements is 180°. The *true* error is 1′ or 60″. Hence add to each of the three angles one-third of 60″, or 20″.

In the same way the most probable values of the interior angles of any polygon may be found by finding the true error of their sum and distributing this error equally among all the angles. Notice, however, that the true error of any one angle is not known.

(*d*) Case II. *When the sum of several measurements should equal some other measurement distribute the amount of the discrepancy equally among all of the measurements, including that one to which the sum should be equal.* In this case the check measurement (i.e., the total measurement) cannot be exact, and, if all the measurements are made with the same care, it is likely to contain an error equal to that contained in each of the smaller measurements. *If, however, the correction is added to each of the smaller measurements, it must be subtracted from the total measurement and vice versa.*

(*e*) *Illustration.* If in the illustration of Case I the three angles had been the three parts of a *fourth* angle instead of the three angles of a triangle, and if the measured value of the *fourth* angle were 180°, then $180° - 179° 59′ = 1′ = 60″$ is the discrepancy. The correction is $60″ \div 4 = 15″$ instead of 20″, and this correction is added to each of the three smaller angles but subtracted from the total angle. The most probable value of the latter is, therefore, $180° - 15″ = 179° 59′ 45″$.

(*f*) *Illustration.* The measured value of an angle is 40°. When divided into two angles the measurements are respectively 24° 30′ and 15° 29′. The discrepancy is $40° - (24° 30′ + 15° 29′) = 1′ = 60″$. Three measurements of equal care are involved, hence the correction is $60″ \div 3 = 20″$ and the most probable values are 24° 30′ 20″, 15° 29′ 20″, and 39° 59′ 40″.

(*g*) *Remark.* The essential difference between Case I and Case II is that in Case I the quantity used as a check has a *true* value which cannot be changed, while in Case II the check value is itself a measurement, and liable, therefore, to contain an error equal to that contained in each of the smaller measurements. The method in Case II does not hold for a number of linear measurements of different lengths.

THE THEORY OF ERRORS OR METHOD OF LEAST SQUARES.[1] The "method of least squares" is a method of determining the most probable or best result of observations. It is based on the assumption that accidental errors will entirely disappear from the mean of an indefinitely large series of independent observations of the same quantity made under precisely the same conditions, and that therefore *if constant errors are eliminated from these observations* the arithmetical mean is the most *probable value* of that quantity. In practice, however, (1) an *indefinitely* large number of observations cannot be made, (2) no

[1] It is the purpose of this article to give only so much of the theory of errors as may be needed in the simple cases which arise in plane surveying. For a more comprehensive treatment of the subject consult one of the standard works on the method of least squares.

two observations can be made under *exactly* the same conditions, and (3) constant errors cannot be *entirely* eliminated. If follows, therefore, that the most *probable value* of a quantity is strictly a *theoretical* value, and that the greater the number of observations, and the more completely constant errors are eliminated from these observations, the nearer the most *probable* value of a quantity will approach the *true* value as a limit. It never reaches this limit (or if it did, there would be no means of knowing it), and, therefore, no matter how accurate the observations may be, the probable value always contains an error.

(*a*) *A residual* is the difference between the value of a quantity as obtained by a single observation and the most probable value of that quantity as obtained by a number of observations. In any given case there are as many residuals as there are observations.

(*b*) *The probable error* of a quantity is a magnitude such that the chances are even that the true error contained in that quantity is greater or less than the probable error.

Illustration. A series of measurements of the length of a line gives for the mean or most probable value 528.91 feet,a nd the probable error of that mean, determined by a method to be explained later, is ±0.034 foot. This means that it is an even wager that the true error contained in 528.91 feet does not exceed ±0.034 foot. Notice that this is quite different from saying that the true error does not exceed ±0.034 foot.

Use Made of Most Probable Values and Probable Errors. *Most probable values* are used in surveying for the adjustment of measurements, and *probable errors* to indicate the precision of measurements.

Most probable value. The simplest and most common cases of adjusting measurements by finding the arithmetical mean have already been given on p. 791.

(*a*) *Probable error of the mean.* Suppose that a second series of measurements of the length of the line in the preceding article gives 528.94 feet for the mean and ±0.065 feet for the probable error of that mean. The results of the two sets of measurements are:

Most Probable Value.	Probable Error.
528.91	±0.034
528.94	±0.065

The probable error of the mean of all the measurements in the first series is approximately half that of the mean as determined by the second series. This indicates that the precision of the first result (528.91) may be regarded as twice that of the second (528.94).

(*b*) The *probable error of a single observation* is found from the residuals of a number of observations, as explained in the next article. If it is necessary to take a number of observations in order to determine the probable error of a single observation, why not be content to determine the probable error of the mean of all the observations and stop there? Of what use is the probable error of a single observation? The answer is that, having determined the probable error of a single observation under given conditions, this probable error indicates the precision which may be expected in any single observation made under the same conditions.

Illustration. In measuring an angle a number of times the probable error of a single observation is found to be ±8″. This means that for any other angle measured by the same observer with the same transit under practically the same conditions a prob-

able error may be expected of about $\pm 8''$. If this is well within the allowable error, the observer is justified in proceeding with single observations of all angles that can be measured under the given conditions.

(*c*) *Probable errors and weights.* Probable errors afford a means of finding the weights which should be given different sets of observations, for it has been found that the weights of observations vary inversely as the squares of their probable errors.

Illustration. On p. 796 the probable error of the mean of the first set of observations is $2''.8$, and of the second set $4''.6$. Hence the weights are as $\frac{1}{(2.8)^2}$ to $\frac{1}{(4.6)^2}$, or about as 3 to 1.

To Determine Probable Errors. The formulas given below for calculating probable errors are based on the following assumptions:

(1) That small errors are more frequent than large ones.

(2) That positive and negative errors are equally frequent.

(3) That very large errors do not occur.

Additional points to be kept in mind are: (1) Constant errors must be eliminated as far as practicable. (2) Observations should be made under as nearly the same conditions as possible. (3) At least three observations should be made. If there are only two observations, the *discrepancy* will indicate the precision as truly as will probable error. The more observations there are the more significance may be attached to the probable error, although a small number of observations will give surprisingly satisfactory results if the other conditions already mentioned are fulfilled.

(*a*) *Remark.* The student is warned against attaching importance to probable errors unless he is reasonably sure that the above conditions are fulfilled, especially that pertaining to the elimination of constant errors. Since the method of least squares is based on the arithmetical mean as the most probable value, and since, as has been pointed out, constant errors cannot be eliminated by taking the mean of observations, the more completely the constant errors are eliminated, or corrected, the more applicable is the method of least squares. If, for example, a tape is too long, all measurements made with it will be too short, and there is no magic in least squares which will correct the results. The probable error in this case loses its significance unless each of the separate measurements from the mean of which the probable error is derived is first corrected for the constant error in the length of the tape.

Formulas for Probable Errors.[1]

Let n = the number of the observations;

v = the difference between any one observation and the mean of all the observations, i.e., the *residual error* (p. 793);

E = the probable error of a single observation;

E_0 = the probable error of the mean of all the observations;

Σ = a symbol signifying *sum* of.

Then, from the theory of least squares,

$$E = 0.6745\sqrt{\frac{\Sigma v^2}{n-1}}, \quad \text{or} \quad \frac{0.6745}{\sqrt{n-1}}\sqrt{\Sigma v^2}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$E_0 = \frac{E}{\sqrt{n}} = 0.6745\sqrt{\frac{\Sigma v^2}{n(n-1)}}, \quad \text{or} \quad \frac{0.6745}{\sqrt{n(n-1)}}\sqrt{\Sigma v^2}. \quad . \quad . \quad (2)$$

[1] See any treatise on the Theory of Least Squares.

Let Σp = sum of the weights and R_0 the probable error of the general or weighted mean; then

$$R_0 = 0.6745 \sqrt{\frac{\Sigma pv^2}{(n-1)\Sigma p}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Let Rs = the probable error of the sum or difference of several independent quantities z, z_1, z_2, . . . z_n.

r, r_1, r_2, . . . r_n = the probable errors of z_1, z_2, . . . z_n, respectively.

$$R_s = \sqrt{r^2 + r_1^2 + r_2^2 \ . \ . \ . \ + r_n^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The probable error of a product Az, where A is a known quantity, z an observed quantity, and r the probable error of z, is

$$R_p^2 = A^2 r^2, \quad \text{or} \quad R_p = Ar \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Let SS_1 = an area found by multiplying together the measured sides S and S_1;

r and r_1 = the probable errors of S and S_1 respectively. Then the probable error of the computed area SS_1 is

$$R_a = \sqrt{S^2 r_1^2 + S_1^2 r^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Abbreviated Methods. In applying the formulas of the preceding article approximate values will often be all that are required. For this purpose the two following equations may be substituted for equations (1) and (2) respectively:

$$E = \frac{0.8453\,\Sigma v}{\sqrt{n(n-1)}} \quad . \quad (7) \qquad E_0 = \frac{0.8453\,\Sigma v}{n\sqrt{n-1}} \quad . \quad (8)$$

When there are only two observations and D is the discrepancy between them,

$$E = 0.47D \text{ or roughly } \tfrac{1}{2}D \text{ and } E_0 = 0.337 \text{ or roughly } \tfrac{1}{3}D. \quad . \quad . \quad (9)$$

In formulas (1) and (2) work may be saved by using the coefficient given in the following table:

n	Single Observations, $\frac{0.6745}{\sqrt{n-1}}$	Series of Observations, $\frac{0.6745}{\sqrt{n(n-1)}}$	n	Single Observations, $\frac{0.6745}{\sqrt{n-1}}$	Series of Observations, $\frac{0.6745}{\sqrt{n(n-1)}}$
			11	0.2133	0.0643
2	0.6745	0.4769	12	0.2034	0.0587
3	0.4769	0.2754	13	0.1947	0.0540
4	0.3894	0.1947	14	0.1871	0.0500
5	0.3372	0.1508	15	0.1803	0.0465
6	0.3016	0.1231	16	0.1742	0.0435
7	0.2754	0.1041	17	0.1686	0.0409
8	0.2549	0.0901	18	0.1636	0.0386
9	0.2385	0.0795	19	0.1590	0.0365
10	0.2248	0.0711	20	0.1547	0.0346

A table of squares and square roots greatly facilitates the work. Other tables similar to that given above will be found in standard text-books on least squares.

ILLUSTRATION OF THE METHOD OF LEAST SQUARES APPLIED TO ANGLES.

First Observer.				Second Observer.			
No.	Angle.	v	v^2	No.	Angle.	v	v^2
1	60° 20′ 20″	4	16	1	60° 20′ 50″	14	196
2	30	6	36	2	21 00	24	576
3	10	14	196	3	20 40	4	16
4	20	4	16	4	20	16	256
5	40	16	256	5	30	6	36
6	10	14	196	6	10	26	676
7	20	4	16	7	20	16	256
8	40	16	256	8	21 00	24	576
	8)100		$\Sigma v^2 = 988$		8)162 170		$\Sigma v^2 = 2588$
	Mean = 60° 20′ 24″ −				Mean = 60° 20′ 36″ +		
	$E_0 = \frac{0.6745}{\sqrt{n(n-1)}}\sqrt{\Sigma v^2}$				$E_0 = \frac{0.6745}{\sqrt{n(n-1)}}\sqrt{\Sigma v^2}$		
	$= 0.0901 \times (\pm 31.4) = \pm 2.8''$				$= 0.0901 \times (\pm 50.9) = \pm 4.6''$		
	$E = E_0\sqrt{n} = \pm 2.8'' \times 2.8 = \pm 8.0''$				$E = E_0\sqrt{n} = \pm 4.6'' \times 2.8 = \pm 12.9''$		

Illustration. The results given above were obtained by two different observers using different transits. The probable error of the mean in the first case is $\pm 2.''8$, while in the second it is $\pm 4''.6$. This indicates that the relative precision of the work of the two observers is as 2.8 to 4.6, or nearly as 5 to 3.

To find the most probable value of the angle by comparing the mean values of the two series of observations it is necessary to weight their mean values. Since weights are inversely proportional to the squares of probable errors, p. 793, the weights are as $\frac{1}{2.8^2}$ to $\frac{1}{4.6^2}$, or nearly as 3 to 1.

The most probable value is $60° 20' + \frac{24'' \times 3 + 36'' \times 1}{4} = 60° 20' 27''$.

The probable error of the above value may be obtained as follows:

$24'' = z_1$ and $36'' = z_2$, $\pm 2''.8 = r_1$ and $\pm 4''.6 = r_2$, $\frac{3}{4} = A_1$ and $\frac{1}{4} = A_2$.

From (5) $\begin{cases} \text{Probable error of } A_1z_1 \text{ or } 24 \times \frac{3}{4} = A_1r_1 = (\frac{3}{4} \times \pm 2''.8) = \pm 2''.1 = R_p, \\ \text{Probable error of } A_2z_2 \text{ or } 36 \times \frac{1}{4} = A_2r_2 = (\frac{1}{4} \times \pm 4''.6) = \pm 1''.15 = R_{p1}. \end{cases}$

The probable error of the sum of $(24'' \times \frac{3}{4}) + (36'' \times \frac{1}{4})$, is from (4),

$$R_S = \sqrt{R_p^2 + R_{p1}^2} = \sqrt{2.1^2 + 1.15^2} = \pm 2''.4.$$

APPENDIX E

GEOGRAPHICAL POSITIONS OF BASE-LINES AND PRINCIPAL MERIDIANS GOVERNING THE PUBLIC SURVEYS

Since the adoption of the rectangular system of public surveys, May 20, 1785, twenty-four initial points, or the intersection of the principal bases with surveying meridans, have been brought into requisition to secure the certainty and brevity of description in the transfer of public lands to individual ownership. From the principal bases townships of six miles square are run out and established, with regular series of numbers counting north and south thereof, and from the surveying meridians a like series of ranges are numbered both east and west of the principal meridians.

During the period of one hundred years since the organization of the system the following numerical and independent principal meridians and bases have been initiated, to wit:

The first principal meridian divides the States of Ohio and Indiana, having for its base the Ohio River, the meridian being coincident with 84° 51′ of longitude west from Greenwich. The meridian governs the surveys of public lands in the State of Ohio.

The second principal meridan coincides with 86° 28′ of longitude west from Greenwich, starts from the confluence of the Little Blue River with the Ohio, runs north to the northern boundary of Indiana, and governs the surveys in Indiana, and a portion of those in Illinois.

The third principal meridian starts from the mouth of the Ohio River and extends to the northern boundary of the State of Illinois, and governs the surveys in said State *east* of the meridian, with the exception of those projected from the second meridian, and the surveys on the *west* to the Illinois River. This meridian coincides with 89° 10′ 30″ of longitude west from Greenwich.

The fourth principal meridian begins in the middle of the channel of the mouth of the Illinois River, in latitude 38° 58′ 12″ north and longitude 90° 28′ 45″ west from Greenwich, and governs the surveys in Illinois west of the Illinois River and west of the third principal meridian lying north of the river. It also extends due north through Wisconsin and northeastern Minnesota, governing all the surveys in the former and those in the latter State lying east of the Mississippi and the third guide meridian (west of the fifth principal meridian) north of the river.

The fifth principal meridian starts from the mouth of the Arkansas River, and, with a common base-line running due west from the mouth of the Saint Francis River, in Arkansas, governs the surveys in Arkansas,

Missouri, Iowa, Minnesota west of the Mississippi, and the third guide meridian north of the river, and in Dakota Territory east of the Missouri River. This meridian is coincident with 90° 58′ longitude west from Greenwich.

The sixth principal meridian coincides with longitude 97° 22′ west from Greenwich, and, with the principal base-line intersecting it on the 40th degree of north latitude, extends north to the intersection of the Missouri River and south to the 37th degree of north latitude, controlling the surveys in Kansas, Nebraska, that part of Dakota lying south and west of the Missouri River, Wyoming, and Colorado, excepting the valley of the Rio Grande del Norte, in southwestern Colorado, where the surveys are projected from the New Mexico meridian.

In addition to the foregoing six principal meridians and bases governing public surveys, there have been established the following meridians and bases, viz.:

The Michigan meridian, in longitude 84° 19′ 09″ west from Greenwich, with a base-line on a parallel seven miles north of Detroit, governing the surveys in Michigan.

The Tallahassee meridian, in longitude 84° 18′ west from Greenwich, runs due north and south from the point of intersection with the base-line at Tallahassee, and governs the surveys in Florida.

The Saint Stephen's meridian, longitude 88° 02′ west from Greenwich, starts from Mobile, passes through Saint Stephen's, intersects the base-line on the 31st degree of north latitude, and controls the surveys of the southern district in Alabama and of the Pearl River district lying east of the river and south of township 10 north in the State of Mississippi.

The Huntsville meridian, longitude 86° 31′ west from Greenwich, extends from the northern boundary of Alabama as a base, passes through the town of Huntsville, and governs the surveys of the northern district in Alabama.

The Choctaw meridian, longitude 89° 10′ 30″ west from Greenwich, passes two miles west of the town of Jackson, in the State of Mississippi, starting from the base-line twenty-nine miles south of Jackson, and terminating on the south boundary of the Chickasaw cession, controlling the surveys east and west of the meridian and north of the base.

The Washington meridian, longitude 91° 05′ west from Greenwich, seven miles east of the town of Washington, in the State of Mississippi, with the base-line corresponding with the 31st degree of north latitude, governs the surveys in the southwestern angle of the State.

The Saint Helena meridian, 91° 11′ longitude west from Greenwich, extends from the 31st degree of north latitude, as a base, due south, and passing one mile east of Baton Rouge, controls the surveys in the Greensborough and the southeastern districts of Louisiana, both lying east of the Mississippi.

The Louisiana meridian, longitude 92° 20′ west from Greenwich, intersects the 31st degree north latitude at a distance of forty-eight miles west of the eastern bank of the Mississippi River, and with the base-line coincident with the said parallel of north latitude, governs the surveys in Louisiana west of the Mississippi.

The New Mexico meridian, longitude 105° 52′ 09″ west from Greenwich, intersects the principal base-line on the Rio Grande del Norte, about ten miles below the mouth of the Puerco River, on the parallel of 34° 19′

north latitude, and controls the surveys in New Mexico, and the valley of the Rio Grande del Norte, in Colorado.

The Great Salt Lake meridian, longitude 111° 53′ 47″ west from Greenwich, intersects the base-line at the corner of Temple Block, in Salt Lake City, Utah, on the parallel of 40° 46′ 04″ north latitude, and governs the surveys in the Territory of Utah.

The Boisé meridian, longitude 116° 20′ west from Greenwich, intersects the principal base between the Snake and Boisé Rivers, in latitude 43° 26′ north. The initial monument, at the intersection of the base and meridian, is nineteen miles distant from Boisé City, on a course of south 29° 30′ west. This meridian governs the surveys in the Territory of Idaho.

The Mount Diablo meridian, California, coincides with longitude 121° 54′ west from Greenwich, intersects the base-line on the summit of the mountain from which it takes its name, in latitude 37° 53′ north, and governs the surveys of all central and northeastern California and the entire State of Nevada.

The San Bernardino meridian, California, longitude 116° 56′ west from Greenwich, intersects the base-line at Mount San Bernardino, latitude 34° 06′ north, and governs the surveys in southern California lying east of the meridian, and that part of the surveys situated west of it which are south of the eighth standard parallel south of the Mount Diablo base-line.

The Humbolt meridian, longitude 124° 11′ west from Greenwich, intersects the principal base-line on the summit of Mount Pierce, in latitude 40° 25′ 30″ north, and controls the surveys in the northwestern corner of California lying west of the coast range of mountains and north of township 5 south of the Humboldt base.

The Willamette meridian is coincident with longitude 122° 44′ west from Greenwich, its intersection with the base-line is on the parallel of 45° 30′ north latitude, and it controls the public surveys in Oregon and Washington Territory.

The Montana meridian extends north and south from the initial monument established on the summit of a limestone hill, eight hundred feet high, longitude 111° 40′ 54″ west from Greenwich. The base-line runs east and west from the monument on the parallel of 45° 46′ 27″ north latitude. The surveys for the entire Territory of Montana are governed by this meridian.

The Gila and Salt River meridian intersects the base-line on the south side of the Gila River, opposite the mouth of Salt River, in longitude 112° 15′ 46″ west from Greenwich, and latitude 33° 22′ 57″ north, and governs the public surveys in the Territory of Arizona.

The Indian meridian intersects the base-line at Fort Arbuckle, Indian Territory, in longitude 97° 15′ 56″ west from Greenwich, latitude 34° 31′ north, and governs the surveys in that Territory.

APPENDIX F

SOME SUGGESTIONS ON THE DESIGN AND TEST OF AN ENGINEER'S TRANSIT AND LEVEL, WITH STANDARD SPECIFICATIONS FOR SAME [1]

LET us first inquire what are the features of the usual complete engineer's transit or tachymeter which are most open to criticism, and what are the proper remedies. It is the writer's observation that the performances of the vertical circle and telescope level fall far short of what reasonably should be expected, and thereby greatly limit its usefulness as a topographic instrument. The causes of this deficiency may be stated as follows:

(*a*) Too small a vertical circle.

(*b*) The omission of a sensitive level from the vertical circle for the control of its zero.

(*c*) The use of irregular and non-sensitive telescope level vials.

It is a well-known fact that an error of several feet in the horizontal location of a topographic point would scarcely be discernible on a contour map of ordinary scale, while an equally large error in its vertical location might entirely vitiate the map for its designed purpose. In view of this fact, the time-honored custom of using transits with large horizontal circles, frequently reading to fractions of a minute, results in simply strengthening the work at a point already the strongest, while the custom which sanctions the use of small vertical circles weakens the work at a point where the greatest accuracy is needed. With a transit provided with horizontal and vertical circles of *equal* size, the elevations will still constitute the weakest part of the work.

The usual excuse for the common practice of furnishing transits with smaller vertical than horizontal circles is found in the supposed danger of injuring the larger-sized circle. The writer's experience under the most trying circumstances, incidental to rugged mountain climbing, would indicate that this danger is greatly overestimated. In fact, if the circle is protected with a cast metal guard (and it always should be so protected), there can be little danger.

Whether an arc or a circle be furnished, its vernier should be provided with an accurate and easy method of adjusting the zero, a tangent screw being preferred. Few American transits are so provided.

---

[1] Abstract of discussion by Professor L. S. Smith of paper, Some Devices for Increasing the Accuracy or Rapidity of Surveying Operation. . Trans. Am. Soc. C. E., Vol. XLVIII, 1902, p. 130.

One objection to the substitution of the vertical arc for the complete circle is, that it precludes the *reversion* of the circle for the elimination of errors of adjustment in such problems as, for example, the determination of azimuth by the direct solar observation, now so commonly used in the West.

Although the small vertical circles generally furnished by makers ostensibly read to single minutes, experienced observers are well aware that this accuracy is seldom realized, because of the uncertainty of the reading. Such circles should be graduated to half minutes and provided with permanently mounted magnifying glasses.

A second element which introduces error into the vertical work of a transit is the common practice of depending upon cheap non-sensitive plate levels for the control of the vertical circle zero. If not specified makers send out plate levels with a sensitiveness between 100 and 360 seconds per one-tenth inch of level. Now every topographer knows that after the orienting back sight has been taken and before the fore sight can be taken, the instrument will get out of level, a condition indicated by the plate bubbles getting off centre. This introduces errors in the vertical circle readings, it may be of equal angular amounts. Nor can the plate levels be relevelled without introducing an error in the orientation which will affect all subsequent azimuths. But even if this displacement does not always take place, a sensibility of 100 to 360 seconds is quite inadequate to control the vertical circle. The proper remedy for this evil is found by placing an extra level on the standard or upon the vertical circle guard, made adjustable so as to read zero when the telescope level is horizontal.

The writer would advocate two double verniers on the vertical circle even if the second vernier is used only to check eccentricity. The entire vertical circle should be protected by the guard, the verniers being covered by glass windows, the same as the verniers on the horizontal limb.

A third suggested change in the construction of the telescope has for its object suitable provision for making the optical centre of the object glass describe a straight line coincident with the geometrical axis of the telescope. This requirement is often lacking and yet without it the other specifications will fail to insure accurate work. The most frequent causes of the failure of the above requirements are due to either too short an object-glass slide or an imperfect or inadequate fitting of the movement. This point is discussed more fully under the head of the wye level.

A fourth needed change in the design of the transit and one certain to improve the accuracy of its vertical work is found in a rational increase in the sensibility of its telescope level.

Except in very mountainous regions a properly run topographic traverse by the "transit and stadia method" allows a considerable proportion of the vertical angle reading on turning points to be replaced by level readings, thereby correspondingly reducing the amount of required office work, while at the same time insuring a higher degree of accuracy in the resulting levels. This would be notably the case of a combined hydrographic and topographic survey of a river and its adjacent valley. This point was first brought to the writer's attention while making a survey of the Colorado River for a distance of about

25 miles near Yuma, Arizona. At this point the river flows in a wide sandy valley of light gradient, so that practically all elevations were taken by level readings. Although this survey was executed under the most trying conditions of weather, and with an average length of sight of about 300 metres, its levels checked on bench marks determined by accurate spirit levelling within about one-half a foot.

Equally close results have since been observed on topographic lines which lead the writer to the conclusion that a properly devised level on the transit telescope is quite sufficient to secure results under favorable circumstances comparable to that of common wye levels, and expressed by the formula, error = 0.05 feet × the square root of distance in miles as the highest limit, and perhaps twice this for lowest limit, the latter being expected only under very unfavorable circumstances of length of sight and weather conditions.

If the above statement of the accuracy of transit levels be accepted as true, and a fast-increasing number of topographers are accepting it, the field for the engineer's wye level is small indeed. However, such high-grade work cannot be expected with transits furnished with the non-sensitive and irregularly ground level so frequently furnished the unsuspecting purchaser. After considerable experimental work to determine this point, the writer would suggest a sensibility corresponding to one-tenth inch, equal to from 15 to 20 seconds of arc. Of course the sensibility of the levels and the magnifying power of the telescope are inter-related.

Other important requisites of a rationally designed engineer's transit, not always complied with, may be considered under the following heads: (*a*) Magnifying power, (*b*) Chromatic and spherical aberration, (*c*) Definition, (*d*) Flatness of field, (*e*) Illumination, (*f*) Centring of lenses. The limits of this paper will allow only a very general treatment of the above points. Preliminary to testing a transit, the owner would do well to look carefully into the telescope barrel through the objective in order to discover the presence of an interposed diaphragm, sometimes inserted by the maker to prevent the discovery of the many evils resulting from poor lenses and poor workmanship.

Such a diaphragm should never be tolerated, because it must reduce the effective diameter of the objective correspondingly, and thereby seriously interfere with its function as a light gatherer. The presence of such a diaphragm may always be taken as *prima facie* evidence of poor workmanship, *unless inserted near the recticule.*

The magnifying power of the eye-piece should depend in part upon the least count of its vernier and the size of the objective. An ideal magnification would be one such that a movement of the alidade corresponding to the least count of its vernier, should cause an easily perceptible movement of the line of sight on the object sighted. As the diameter of the objective largely determines the illumination of the image, it should be as large as possible and be consistent with other details. It should also be transparent and highly polished. If d be the diameter of the objective in millimetres, and m the magnifying power of the telescope in diameters, then the illuminating power of the telecope $i = \frac{d^2}{5m^2}$; the natural illumination being taken as unity.

The flatness of the field depends largely on the sphericity of the eye-

piece. This may be tested as follows: Draw a heavy-lined square eight inches on a side, and place it at such a distance from the instruments that it shall nearly fill the entire field. When the telescope is focussed its side should appear perfectly straight, though a slight curve at the *extreme* limits of the field should cause no appreciable error in stadia work.

The definition of a telescope depends upon three conditions—the accuracy of the curvature of the various lenses, upon their centring, and upon their movement in focussing. If the eccentricity of the eye-piece is large or if the optical axis of the lens be not parallel to the line of sight, the definition may be so poor as to seriously effect the focussing of the extreme stadia wires. Displacements of the objective are even more serious in producing poor definition of the image.

Lateral displacements of either eye-pieces or objective may be discovered by taking the telescope of the transit out of the standards, and, after focussing the threads and objective on some distant point, turn the telescope in improvised wooden wyes. The image and the threads will appear to move in the arc of a circle above the telescope axis even when the wire or recticule adjustment has been perfectly made. Not infrequently makers send out even wye levels whose lenses, when tested as above, show very large eccentricities. Such an instrument should be promptly returned to the maker.

SPECIFICATIONS FOR COMPLETE ENGINEER'S TRANSITS FURNISHED TO THE UNIVERSITY OF WISCONSIN

General Conditions. All constructive details shall be of superior workmanship and design. Weight not essential to the stiffness of the instrument shall be avoided by the standard rib construction. The instrument without tripod should weigh not more than ten pounds. Especial attention shall be given to making extra strong all parts likely to be injured by accident. Only the hardest bell metal and phosphor bronze shall be used for centres and telescope axis, and soft brass shall be everywhere avoided. Tangent screws and head of tripod shall be made of aluminum bronze and provided with German silver spiral springs. Prices shall be F. O. B., Madison, Wis.

Horizontal Circle. The diameter at the edge of graduation shall be five (5) inches. The graduation shall be to half degrees on solid silver, frosted finish, and every ten degrees shall be distinctly numbered from zero to 360 in both directions, the numbers sloping in the direction they are increasing.

The circle shall be provided with two double verniers, A and B, exactly 180 degrees apart, and reading to single minutes.

The zeros of the verniers shall be placed at an angle of 35° to the line of sight, and shall be provided with ground-glass reflectors. Eccentricity of vertical axis and of all verniers must be as perfectly eliminated as the highest mechanical skill will allow.

Complete Vertical Circle. Transit shall be provided with a complete vertical circle with a diameter not less than 5 inches, graduated on solid silver and read to single minutes by two double verniers. The verniers shall be provided with an easy and reliable means of adjustment. The vertical circle shall be furnished with a cast-metal guard

to protect the entire edge of the graduation. An extra level vial sensiblity 1-10 inch = 20 seconds with suitable means of adjustment, shall be firmly mounted on this guard so as to control the zeros of the verniers. The horizontal axis shall have a radial clamp and slow motion screw with a gradienter.

Magnetic Needle. Instrument shall be provided with the usual compass box and a sensitive and accurate needle about $3\frac{1}{2}$ inches long.

Telescope. The objective shall have a clear aperture of not less than $1\frac{3}{8}$ inches in diameter, perfectly achromatic and with a flat field, suitable for accurate stadia measurements. Both the objective and ocular lenses shall be mounted so that their optical centres lie in the geometrical centre of the telescope tube, and provision shall be made to insure that the optical centres shall describe straight lines when focussing.

Eye-piece shall be inverting and magnifying between 22 and 28 diameter and be free from chromatic aberration. It shall also be provided with a prismatic eye-piece with a solar shade.

The regular eye-piece shall have the usual focussing screw or worm and a dust guard.

The horizontal axis of the telescope shall be provided with the most improved means of adjustment and shall be fitted with a suitable base for the attachment of a Saegmüller solar attachment.

Stadia wires. Two extra fixed stadia wires, very fine and opaque, shall be furnished and placed equidistant from the middle crosswire. The total distance apart of the stadia wires shall be 1–100 of the focal length of the objective. They shall be of such size as not to cover more than 0.02 of a foot on a rod held at a distance of 1000 feet.

Telescope level. The level shall be ground to a uniform curvature of $\frac{1}{10}$ inch = 15 to 20 seconds. The graduations shall extend over the entire exposed length and shall be numbered on the vial both ways from zero at the centre.

Plate levels. Both plate levels shall be accurately fixed to the transit with the most approved and stable provision for their adjustment. Both levels shall be graduated and their curvature shall be $\frac{1}{10}$ inch = 40 seconds.

Clamps. All clamps shall be of the radial pattern.

Finish. The telescope, telescope level, vertical circle guard, and standards shall have the so-called "cloth finish," the remainder any good standard finish.

Tripod shall be of split-leg pattern and provided with shifting centre. Its weight shall not exceed eight pounds. The steel shoe of tripod shall be provided with a shoulder for forcing it into the ground.

Box. The instrument shall be furnished in a neat hardwood box with lock, together with a sun shade, reflector, magnifying glass, plumb bob, wrench, adjusting pins, and silk or waterproof bag. The box shall have a leather strap and rubber cushion on the bottom, and shall be so constructed as to allow of transit being packed flat instead of upright.

ENGINEER'S LEVELLING INTRUMENTS

A discussion of the relative merits of "wye" and "dumpy" levels is aside from the main purposes of this paper, and yet it may not be out of place to briefly state the writer's veiws on this point. The Amer-

ican engineer is clearly behind his European brother, who has long recognized the merits of the dumpy type of levelling instruments. The greater steadiness claimed for the wye level is more than offset in the dumpy level by its simplicity of construction and adjustment, with the resulting certainty of the relation between the line of sight and the axis of the level vial. The supporters of the wye level have little ground on which to stand. The most perfectly constructed levelling instrument ever made, both from the theoretical standpoint as well as evidenced by its completed work, is the latest type of precise level designed by the U. S. Coast and Geodetic Survey. This level is of the dumpy type. With it 250 miles of single level lines have been run per month, and that too with greatly inrceased accuracy over that claimed for commercial levelling.[1]

The following brief discussions include the important points in the specifications for a wye level used at the University of Wisconsin.

1. *Telescope.*

(*a*) The object glass should have a clear aperture of not less than 1¾ inches and should have the other requisites given in the discussion of the transit. It is doubly important in the level that the object glass should be so securely and accurately mounted that when focussed at different distances its optical centre shall describe a straight line coincident with the geometrical axis of the telescope. Any wobbling is fatal to accurate work. Stable conditions can be best secured by the use of long object-glass slide, extending nearly to the reticule. In general it appears that this detail is most often slighted by the makers.

(*b*) It is imperative that the diameters of the rings or collars should be the same if the axis of the bubble is to be made parallel to the line of sight by the "indirect method." The possibility of making the above adjustment by the indirect method is sometimes given as one of the arguments for the use of the "wye" level, and yet if the diameters of the rings differ by 1–5000 part of an inch the resulting error in a rod reading 300 feet distant would amount to about .01 of a foot even when the instrument appeared to be in perfect adjustment.

(*c*) The eye-piece should be inverting, magnifying 35 diameters, and protected by dust shade.

2. *Level vial.* The level vial should be carefully ground to a uniform curvature of 1–10 inch equal to 10 seconds of arc. The graduations should be etched on to the bubble tube itself and not attached to a scale, as is frequently done.

3. A clamp and slow motion should be provided to the vertical axis. Tripod specifications are similar to those of the transit with this addition.. The accuracy of any work is in large measure dependent upon the mental state of the observer. If he can observe in a comfortable position in the long run his work will show an increased accuracy over that performed under uncomfortable circumstances. For this reason it would be well to have the length of the tripod legs increased so that the engineer could observe in a nearly upright position. If a tripod be provided like that of the U. S. Coast and Geodetic Survey precise level, no unsteadiness will result from this increased length.

[1] See Eng. News, July 2, 1903, for an article by J. F. Hayford, on this subject, giving the experience of the U. S. Coast and Geodetic Survey.

Finish. The cloth finish should be specified on telescope and level vial. Frequently it would be convenient if instrument-makers would provide interchangeable tripods so that both the transit and level would fit on same tripod.

Dumpy level. The only modifications in the specifications suggested in the dumpy level would be to decrease the magnifying power to 30 diameters and the sensibility of the level vial to $\frac{1}{10}$ inch = 12 to 15 seconds of arc. The eye-piece should be inverting. The small extra charge for the clamp and slow motion to the vertical axis is well worth the cost.

The present type of "dumpy" level made in this country would be greatly improved by the adoption of some of the improvements made on the new U. S. Coast and Geodetic Survey precise level. Briefly stated these are as follows:

(*a*) A micrometer for making the final pointings of the line of sight after the level is approximately level.

(*b*) Inserting the more sensitive level vial in thet op of the telescope and as near to the line of sight as possible.

(*c*) The use of a reflector which will allow a view of the level bubble at the same time as the rod reading is made.

APPENDIX G

THE OWNERSHIP OF SURVEYS, AND WHAT CONSTITUTES A SURVEY AND MAP[1]

A GREAT difference of opinion seems to exist among surveyors as to how much of the information obtained and work done in making a survey should be furnished to the individual for whom the survey is made. It is believed that some surveyors have mistaken notions as to what constitutes a survey and map and as to the ownership of the same. Many surveyors keep what are called private notes. All men doing business as surveyors must keep notes of all surveys in a convenient form for ready reference. The extent to which these notes are "private," however, seems to the writer not to have been fully comprehended by all surveyors, and hence has arisen the difference of opinion mentioned.

The present article is an attempt to present a side of this question that has not, so far as the observation of the writer has extended, been heretofore fully considered. An endeavor has also been made to point out to the young surveyor a line of action which it is believed he will find to his advantage to follow, as well as to that of the community in which he works.

In this discussion, the question of what constitutes a survey arises at once, and the answer obviously depends on the object of the survey. The discussion will be confined to land surveys; that is, surveys made for the purpose of subdividing a large tract of land into smaller parcels to be sold, or surveys made for the purpose of determining the boundary of a tract the description of which is known, or surveys made to determine the description when the boundaries are known.

The principle to be enunciated applies to any other survey as well, be it railroad, canal, bridge, or topographical survey. Indeed, it is well understood in all such surveys, but seems to be ignored by many engineers having to do with land surveys.

A survey is "the operation of finding the contour, dimensions, position, or *other particulars* of any part of the earth's surface, . . tract of land, etc., and *representing the same on paper*."

In making a survey it is necessary to set certain points, called monuments or corners, and to determine a description of these points. These items therefore become a part of the survey.

---

[1] By William G. Raymond, C.E., Dean of Scientific School, University of Iowa. The original paper, of which this is a slight modification, appeared in *The Polytechnic*, the student journal of the Rensselaer Polytechnic Institute, January, 1894.

Then a map must be drawn. This map, to be a faithful representation of the ground and the work done, should, together with the notes, show all of the items mentioned.

The object of establishing monuments or corners and describing them is a double one, viz.: the marking on the ground of the boundaries of the tract, and the securing of definite information as to the location of the tract with reference to other points or tracts, so that from this information the land may at a future time be found.

The survey is not complete, therefore, till the corners are fixed, information that will preserve their location obtained, and the same delineated on a map and its accompanying notes.

The doing of all this, then, constitutes the survey. To whom belongs the survey?

It would appear to be evident that it belongs to the individual who pays to have it made. It is not readily seen in what way the survey, or any part of it, becomes the sole property of the surveyor.

The surveyor may keep a copy of his notes to facilitate his future work, but he has not the shadow of a claim to a single note the time for taking which has been paid for by his employer.

If his charge for his work is on a time basis, there can be no question as to the correctness of the above proposition. If he takes the work for a definite sum for the entire job, he may take as much time as he likes and as many private notes, but he is bound in honor to return to his employer the complete survey, and if he does so, it is not obvious that the private notes would thereafter be of great assistance to him in securing further work, particularly when it is remembered that professional men of repute do not bid against each other for professional work.

His reputation for accuracy and honesty will be far a more potent factor in securing employment than any set of private notes fairly obtained.

It is true that a great many surveyors hold a different opinion, and purposely return their maps and notes in such condition that, while they may answer the purpose for which they are primarily made, they do not tell the whole story, nor enough to make it easily possible for another surveyor to relocate the tract surveyed. When this is done, the person ordering the survey does not receive what he pays for. Something is withheld. It seems to need no argument to show that this is radically wrong.

But there is another reason for condemning this practice.

The correct and permanent location of all public land lines, as streets, alleys, etc., as well as the permanent location of party lines between private owners, is a matter of the gravest importance, and no information that will at all serve to definitely fix such lines in their correct positions for all time should be withheld from the owner who pays for the survey, be it private citizen, municipality, county, or State.

The records of monuments and street lines made by a city engineer are no more his private property than are the records in the offices of the clerk, auditor, or treasurer the property of the individuals who hold office at the time the records are made.

The correctness of the position assumed has been indicated by court decisions. A great deal of laxity is shown in the conduct of offices of city engineers and county surveyors.

The methods of regulating the pay of these officers has doubtless had

much to do with this. It is not uncommonly the case that the surveyor receives no salary, but is allowed to collect certain specified fees for work performed, and this gives color to his claim that his work is private work and belongs to him.

That this is not true concerning the public work he does, is believed to be evident from what has preceded. That the records of work done for private citizens are not the property of the public, needs no demonstration, but it is true that such work belongs to those citizens for whom it was done.

The writer believes that a different policy should be pursued with regard to these offices.

He believes that in every case such office should be a salaried one, with such salaried assistants as may be necessary, and that certain fees should be prescribed for performing the various kinds of work that the surveyor may be called upon to do within the limits of the territory of the political division whose servant he is. These fees should cover all work connected with public construction or public or private land lines, and should be returned to the public treasury.

Their amount may be regulated from time to time so that they shall aggregate a sum sufficient to pay the expenses of the office. They should of course, not cover work of a private character, not having to do with land lines. But the entire public is interested in the permanency of land lines, and all records concerning them made by a public official should become public property. The writer has had in the past some experience in this class of work, and never declined to furnish a competitor with any information in his possession that would help the competitor to arrive at the truth in surveys he might have under way.

The writer believes that the permanency of land lines is too important a matter to be subject to avaricious and jealous rivalry, and he believes that all the suveyors in a given district should co-operate to preserve in their correct places all lines within the district.

To this end the returns of every surveyor made to the owner should be thoroughly complete. Maps made for filing as public records should be so finished as to enable any surveyor to relocate the land without the least uncertainty as to the correctness of his work. That this is done in very few instances is well known to every surveyor who has had occasion to examine public records for data for surveys he has been called upon to make.

Because of the fact that in most cases neither owners nor attoneys have been fully posted, nor could they be expected to be, as to what constitutes a complete description, sufficient for relocation, and because surveyors have been willing to let matters stand as they were, great carelessness has arisen in the practice of making and filing maps for record.

[1] While in some States good laws exist prescribing what shall appear on a map before it will be received as a public record, in more States there is nothing whatever to guide either owner, surveyor, attorney, or recorder in the matter. In the county records in such States anything that is made up of lines and figures, either drawn by hand, photo-lithographed,

[1] What follows is a modification of some notes on this subject prepared by the writer for the Technical Society of the Pacific Coast.

or simply printed with "rule" and type and labelled "This is a map," is considered a sufficient basis for the correct description and location of the property it purports to represent. The records are full of auctioneer's circulars, manufactured in a printing-office from information, coming from nobody knows where, filed at the request of the auctioneer's clerk, with no name of owner or other interested party attached, except as the name of the auctioneer appears in the accompanying advertisement. Further than this, these maps are frequently purposely distorted to create a favorable impression of the property to be sold. Wide streets are shown where only narrow ones exist, streets opened for the full width where they have been opened for but half their width, rectangular subdivisions that really may not be even parallelograms, etc., etc. Such maps as these frequently form the only basis for the description and location of the property they are supposed to represent. This circular business is bad, *very* bad for those who buy; but is the information given by these circulars much worse that than furnished by many of the maps made by surveyors and filed at the request of the owners?

On these plats, if of "additions," we find lines indicating the boundaries of blocks and lots, all of which blocks and lots are numbered; the names of streets appear in neat letters; a few dimensions, possibly all linear dimensions, will be given; the streets or blocks may be tinted with soft and delicate tints, and the whole set off with an elegant border and title.

As an exhibition of the draughtsman's skill these maps are perhaps valuable. As a source of information as to the location of the lines they purport to show, they are worth about as much as the auctioneer's circular.

Perhaps they have a few more figures, and the presumption may be a little stronger that the figures are correct.

Examine one of these maps closely. There will be found no evidence that a monument has been set in the field; not an angle recorded, though the lines may cross at all sorts of angles; and dimensions given that do not agree among themselves, so that the angles cannot be calculated.

There will be found no name signed, except, possibly, that of the surveyor, who thus advertises either his stupidity or something worse. Let us be kindly, and call it stupidity.

Frequently no monuments are set except small stakes at the corners of the blocks; but the fact that even such stakes have been set is not recorded on the plat.

One who is acquainted with the practice of surveyors in a given district knows at what points to look for such stakes, and if they have been set and not pulled out to make room for a fence post or building, he may succeed in finding them. Some surveyors have a practice of setting stakes a certain distance away from the point the stake is supposed to mark, but no mention of this fact appears on the map. In fact the map is so drawn that no one but a surveyor who made it can write a description of any one of the parcels of land shown, nor correctly locate it on the ground. Furthermore, the surveyor himself finds it impossible, after the lapse of a few years and the destruction of his "private marks," to rerun any one of the lines exactly as originally laid out.

It is easy to see to what this leads: impossible descriptions of property, giving opportunity for differences in judgment as to interpretation of what was intended; disputes as to position of party lines; costly litigation and expensive movement of structures begun or completed, and the

actual shifting of lines back and forth by different surveyors, or even the same surveyor, honestly trying to locate the lines properly.

The writer has seen enough of trouble of this sort to indicate to him that a radical change is needed in the field work and mapping of cities, towns, and additions, not to mention farms and other tracts of land that that it may be necessary to lay out and describe.

So long as fallible man is responsible for the accuracy of surveys, maps, and descriptions of properties, so long will there be errors; but that it is possible to greatly reduce their number by proper regulation, the writer is fully persuaded. What we have been describing are not maps at all, or at most they are very imperfect maps, and "What constitutes a map" thus seems to be a very pertinent question.

A map of a city, town, or addition, or other tract of land, serving as a basis for the description of property, should furnish all the information necessary to the proper description and location of the various parcels shown, and also of the whole piece. It should further show the exact location of the whole tract to the lands immediately adjoining; particularly should this be done when an offset or angle in a street line occurs. To accomplish these things there should appear on the map the following items:

1. The lengths of all lines shown.
2. The exact angle made by all intersecting lines.
3. The exact position and character of all monuments set, with notes of reference points.
4. The number of each block and lot.
5. The names of all streets, streams or bodies of water, and recognized landmarks.
6. The scale.
7. The direction of the meridian, and a note as to whether the true or magnetic meridian is shown—it should be the true meridian.
8. The angles of intersection made by the lines of adjoining property with the boundaries of the tract mapped.
9. The exact amount of offset in lines that may extend from the outside through the tract mapped.
10. A simple, complete, and explicit title, including the date and the name of the surveyor.

Thus much to make the map valuable for *description and location* of the property it represents.

Of course monuments will not be shown if none have been set, and very frequently none are set, either from carelessness on the part of the surveyor, or an unwillingness on the part of the owner to pay their cost. Monuments of a permanent character should be set at each corner of a tract surveyed, and at least two, visible the one from the other, on the line of each street. If these monuments are not placed on the centre lines of the streets, they should be placed at *uniform* distances from the centre or property lines. If placed with reference to the centre line, they should all be placed on the same side of the centre. In streets extending east and west the monuments should all be on the north of the centre, or they should all be on the south, and at uniform distance. In streets extending north and south the monuments should all be on the east of the centre, or all on the west.

Uniformity in such practice saves a vast amount of time.

Monuments may be set at uniform distances from the block lines, in the sidewalk area, and this is an excellent practice.

The stakes or monuments set at the corners of the blocks in additions, or town sites, should never be the *only* stakes or monuments set in the tract.

That the map may be reliable there should appear on it the following:

1. The certificate of the surveyor that he has carefully surveyed the land, that the map is a correct representation of the tract, and that he has set monuments (to be described) at the points indicated on the map.

2. The acknowledged signature of all persons possessing title to any of the land shown in the tract, and, if possible, those of adjoining owners.

3. If of an addition, the acknowledged dedication to public use forever of all areas shown as streets or roads.

4. If a street of full width, whose centre line is a boundary of the tract, is shown, the acknowledged signature of the owner of the adjoining property, unless his half of the street has been previously dedicated.

It has been already stated that, in some States, a map be may filed at the request of any person, and without signature.

This practice frequently leads to trouble. The writer knows of cases in which owners of large tracts of land have had those tracts subdivided and have taken land of adjoining non-resident owners for street purposes without the consent or knowledge of those owners. When at a later day the owners of the land so taken have objected and attempted to close half of the street, trouble of a serious character has arisen. The same trouble has occurred where streets have been run through narrow gores of land and have subsequently been completely closed, leaving houses built on the mapped property without outlet. Time and again have cases of this sort come to the knowledge of the writer.

Having pointed out certain evils, it remains to suggest a remedy. It lies in the enactment of a law in each State governing these matters.

There should appear on the statutes of every State a law explicitly defining what shall appear on every map filed for reference, and making it a misdemeanor to file a map that does not strictly conform to the definition.

In the absence of such laws it is believed that the young surveyor can assist greatly in a much-needed reform, by following the principles suggested in this paper as the correct ones, and avoiding the errors here indicated.

It is hoped that those graduates of our engineering schools who drift into this class of work will be guided by a higher principle than that which actuates the surveyor who covers up his tracks, at the expense of his employer, in order to secure a monopoly of the business of his locality.

The young surveyor can spend his energies to greater advantage in devising new and better methods of work than in inventing ways for hiding information that it has been endeavored to show belongs to his employer.

Certainly a thorough education should so broaden the young man's views as to make it impossible for him to be controlled by those meaner instincts, which, indulged, lead ever to narrow his vision and prevent him from perceiving the greater problems that continually present themselves for solution.

APPENDIX H

INSTRUCTIONS RELATIVE TO MAKING AND FILING OF TOWN, CITY, AND VILLAGE PLATS IN THE STATE OF MICHIGAN.

AUDITOR-GENERAL'S OFFICE,
Lansing, Mich.,.....191..

By act No. 309, Laws of 1887, important amendments have again been made to Sections 1 and 2 of Chapter 32 of Howell's Annotated Statutes in regard to the recording of town plats and the vacating of the same. These amendments took effect September 28, 1887, and all plats made on or after that date must be in conformity therewith.

1. In making the SURVEY it is required that "permanent monuments shall be located in the ground at all angles in the boundaries of the land platted, and at all the intersections of streets or streets and alleys, as shown on the map or plat, and when there are permanent objects in the vicinity of such monuments the bearings and distances of such objects shall be noted. The character of the monuments and the bearings and distances of such witness points or objects shall be distinctly given in the most convenient manner on the plat." The exact position of the monuments should by indicated on the plat be a small circle or cross.

2. If the plat be of a town, city or village, the full name of such town, city or village must appear as the title or name of the plat; if the land platted be an addition to or a subdivision of a town, city or village already platted, then let the title of the plat include, with the name of such addition or subdivision, the name of the town, city or village, as the case may be, of which such platted land is a subdivision, or to which it is an addition.

3. The plat must be on a scale showing not more than two hundred feet to an inch, and on good muslin-backed paper, 18×24 inches in size; all certificates must be written or printed on the paper on which the plat is made, and on the same side of the sheet.

4. The sections and parts of sections platted must be designated by lines with appropriate letters and figures. In case of a subdivision of lots or blocks of a previous survey, the outlines of the original or previous lots or blocks so subdivided must be designated by lines which must be marked with appropriate letters and figures. This must be done in such a manner as to show, without reference to the written description, the starting point and the course and length of each of the outlines. Where any of the outlines are identical and coterminous with the lines of a previous survey or plat, it will be sufficient to give the destination of such outlines as given in such previous survey or plat.

5. The land platted is to be fully described *in writing or printing* upon the paper on which the plat is drawn. This description must be so com-

plete that from it, without reference to the plat, the starting point can be determined and the outlines run. In connection with the description should be a short and simple form of dedication, which must be signed by the proprietors and their wives, whose signatures must be witnessed, and whose execution of the dedication must be acknowledged as deeds conveying land are required to be witnessed and acknowledged.

6. There must be drawn upon the plat a plain designation of the cardinal points and a correct scale.

7. Where all the lots in any block are of the same dimensions, it shall be sufficient to mark the precise length and width upon one tier thereof, but all gores, triangles, or other lots, which are not either squares or parallelograms, shall have the length of their sides plainly defined by figures.

8. The streets must be named or numbered and their course and width designated. All public grounds and alleys must be properly designated.

9. The surveyor must certify that the plat is a correct one and that the monuments described in it have been planted as therein described.

10. Detached parcels cannot be included in one plat, nor can more than one plat be made on one sheet. Contiguous parcels owned by different parties may be embraced in one plat, all joining in the execution and acknowledgment; it is not necessary to specify the particular parcels belonging to each.

11. Before a plat shall be recorded, and before any copies are made therefrom, the "original" must be forwarded to the Auditor-General for his approval; if found, in his opinion, to conform to law, it will be endorsed as approved and returned to the person sending it.

12. For the purpose of recording, an exact copy is to be made from the original after it has been approved by the Auditor-General, which copy must have copied upon it the Auditor-General's endorsement of approval and then be certified as follows:

STATE OF MICHIGAN, }
County of..... } ss.

We.....Register of Deeds, and.....Surveyor, hereby certify that we have each carefully compared this copy with the original plat of .....and that it is an exact copy thereof and of
[Title of plat.]
the whole of such original map or plat.

.....Register of Deeds.
.....Surveyor.

13. The "copy" so certified is to be delivered to the Register of Deeds and filed as the record, and must have the proper endorsement of record made upon it. No "copy" can be received for record unless the Auditor-General's certificate of approval is copied thereon. The transcript from the record that is to be filed in the office of the Auditor-General should have copied upon it the above certificate of comparison of the "copy" with the original, the approval of the Auditor-General, and the certificate of the Register of Deeds as to the time, volume and page of the record. The following certificate of comparison with the record must then be made upon the transcript, which should at once be filed with the Auditor-General.

STATE OF MICHIGAN, }
County of..... } ss.

We.....Register of Deeds, and.....Surveyor, hereby certify that

we have each carefully compared the annexed copy with the plat of .....now of record in the office of said Register
[Title of plat.]
of Deeds, and that it is true transcript therefrom and of the whole of such record.

.....Register of Deeds.
.....Surveyor.

14. The Register of Deeds is entitled to a fee of $2 for each plat recorded by him, and each plat filed with the Auditor-General must be accompanied by the legal fee of $1 for the benefit of the State.

15. The foregoing is not intended to be a perfect manual containing all that is embodied in the law, but to call attention to points in which plats are most likely to be defective.

16. Every plat sent to the Auditor-General, either for approval or for filing in his office, should be accompanied by the name and postoffice address of the person sending it, to insure proper return.

17. The law of 1887 is appended for convenient reference; *it should be carefully studied and all its requirements observed.* (Omitted in this Appendix.)

18. Observe the requirements of Section 135, General Tax Law of 1893, which is hereto appended.

19. The following resolutions were adopted by the Michigan Engineering Society at their annual meeting held at Ann Arbor in January, 1886, which are worthy of attention as expressing the views of an association composed of the most intelligent and competent surveyors of the State: *First*, That the written description of the land platted should be clear and distinct, describing it in as brief a manner as is consistent with accuracy, so that there shall be no misunderstanding as to what land the plat is intended to cover; that the outlines of the plat itself shall be marked with appropriate letters and figures corresponding with the written description indicating the courses and length of those lines; that if any lots lying within the outlines of the plat are not intended to be considered a part of the plat they should not be numbered or lettered, and the fact that they are excepted be noted in the written description. The courses and length of the lines of such lots should be marked on the plat by appropriate letters and figures, and the lots themselves marked on the plat as excepted. *Second*, Use a short and simple form of acknowledgment. . . . *Third*, Use but one unit of measurement in the plat. Make a diagram on the plat of the scale used, with appropriate letters to show what the scale is. *Fourth*, The four cardinal points be indicated in a simple manner by an arrow or *fleur de lis*, with appropriate letters. This we understand to be intended merely to indicate in a general way the points of compass on the map. Whenever practicable, give the courses in the written description and on the plat from the true meridian, and also, when practicable, that the angle of intersecting lines be given on the plat. *Fifth*, That we esteem it of the first importance that permanent monuments be located in the ground at all important points in the plats, and that the character and location of such monuments, by their bearing trees or points, be distinctly given on the plat.

FORMS OF DEDICATION, DESCRIPTION, AND SURVEYOR'S CERTIFICATE.

AUDITOR-GENERAL'S OFFICE,
Lansing, Mich.,.....189..

In response to frequent requests for forms of Dedication, Description and Surveyor's Certificate to be observed in making plats, the following have been prepared; while they are short and simple, it is believed they meet the requirements of law.

STANLEY W. TURNER, Auditor-General.

☞ (The Attorney-General advises that if the proprietor is a widower or bachelor the fact should be stated in the dedication to account for the absence of the signature of the wife.)

DEDICATION.

Know all men by these presents, That we.....as proprietor, and.....his wife, have caused the land embraced in the annexed plat to be surveyed, laid out and platted, to be known as.....and that the streets and alleys as shown
(Insert title of Plat.)
on said plat are hereby dedicated to the use of the public.

Signed and sealed in presence of

.....[L. S.]
.....[L. S.]

STATE OF MICHIGAN, } ss.
County of..... }

On this.....day of....189.. before me, a Notary Public in and for said county, personally came the above named.....and.....his wife, known to me to be the persons who executed the above dedication and acknowledged the same to be their free act and deed.

Notary Public.....Co., Mich.

.....

DESCRIPTION OF LAND PLATTED.

The land embraced in the annexed plat of..... is described as follows:

.....
.....
.....
.....

SURVEYOR'S CERTIFICATE.

I hereby certify that the plat hereon delineated is a correct one and that permanent monuments, consisting of.....
(Describe the monuments.)
have been planted at points marked thus.....as thereon
(Using some symbol such as a small circle (**O**), or a (**X**), or letters, or numerals, to indicate the exact location.)
shown at all angles in the boundaries of the land platted, and at all intersections of streets and alleys.

.....Surveyor.

APPENDIX I

RESTORATION OF LOST OR OBLITERATED CORNERS AND SUBDIVISIONS OF SECTIONS

(*Circular issued by the General Land Office of the Department of the Interior, October 16, 1896.*)

The increasing number of letters from county and local surveyors received at this office making inquiry as to the proper method of restoring to their original position lost or obliterated corners marking the survey of the public lands of the United States, or such as have been wilfully or accidentally moved from their original position, have rendered the preparation of the following general rules necessary, particularly as in a very large number of cases the immediate facts necessary to a thorough and intelligent understanding are omitted. Moreover, surveys having been made under the authority of different acts of Congress, different results have been obtained, and no special law has been enacted by that authority covering and regulating the subject of the above-named inquiries. Hence, the general rule here given must be considered merely as an expression of the opinion of this office on the subject, based, however, upon the spirit of the several acts of Congress authorizing the surveys, as construed by this office, and by United States court decisions. When cases arise which are not covered by these rules, and the advice of this office is desired, the letter of inquiry should always contain a description of the particular corner, with reference to the township, range, and section of the public surveys, to enable this office to consult the record.

An *obliterated* corner is one where no visible evidence remains of the work of the original surveyor in establishing it. Its location may, however, have been preserved beyond all question by acts of landowners, and by the memory of those who knew and recollect the true situs of the original monument. In such cases it is not a lost corner.

A *lost* corner is one whose position cannot be determined, beyond reasonable doubt, either from original marks or reliable external evidence.

Surveyors sometimes err in their decision whether a corner is to be treated as *lost* or only *obliterated*.

Surveyors who have been United States deputies should bear in mind that in their private capacity they must act under somewhat different rules of law from those governing original surveys, and should carefully distinguish between the provisions of the statute which guide a Government deputy and those which apply to retracement of lines once surveyed. The failure to observe this distinction has been prolific of erroneous work and injustice to landowners.

To restore extinct boundaries of the public lands correctly, the surveyor must have some knowledge of the manner in which townships were subdivided by the several methods authorized by Congress. Without this knowledge he may be greatly embarrassed in the field, and is liable to make mistakes invalidating his work, and leading eventually to serious litigation. It is believed that the following synopsis of the several acts of Congress regulating the surveys of the public lands will be of service to county surveyors and others, and will help to explain many of the difficulties encountered by them in the settlement of such questions.

Compliance with the provisions of Congressional legislation at different periods has resulted in *two* sets of corners being established *on township lines* at one time; at other times *three* sets of corners have been established on *range lines;* while the system now in operation makes but *one* set of corners on *township boundaries*, except on standard lines—i.e., base and correction lines, and in some exceptional cases.

The following brief explanation of the modes which have been practiced will be of service to all who may be called upon to restore obliterated boundaries of the public land surveys:

Where two sets of corners were established on township boundaries, one set was planted at the time the exteriors were run, those on the north boundary belonging to the sections and quarter sections north of said line, and those on the west boundary belonging to the sections and quarter sections west of that line. The other set of corners was established when the township was subdivided. This method, as stated, resulted in the establishment of two sets of corners on all four sides of the townships.

Where three sets of corners were established on the range lines, the subdivisional surveys were made in the above manner, except that the east and west section lines, instead of being closed upon the corners previously established on the east boundary of the township, were run due east from the last interior section corner, and new corners were erected at the points of intersection with the range line.

The method now in practice requires section lines to be initiated from the corners on the south boundary of the township, and to close on existing corners on the east, north, and west boundaries of the township, except when the north boundary is a base line or standard parallel.

But in some cases, for special reasons, an opposite course of procedure has been followed, and subdivisional work has been begun on the north boundary and has been extended southward and eastward or southward and westward.

In the more recent general instructions, greater care has been exercised to secure rectangular subdivisions by fixing a strict limitation that no new township exteriors or section lines shall depart from a true meridian or east and west line more than twenty-one minutes of arc; and that where a random line is found liable to correction beyond this limit, a true line on a cardinal course must be run, setting a closing corner on the line to which it closes.

This produces, in new surveys closing to irregular old work, a great number of exteriors marked by a double set of corners. All retracing surveyors should proceed under these new conditions with full knowledge of the field notes and exceptional methods of subdivision.

SYNOPSIS OF ACTS OF CONGRESS.

The first enactment in regard to the surveying of the public lands was an ordinance passed by the Congress of the Confederation May 20, 1785, prescribing the mode for the survey of the "Western Territory," and which provided that said territory should be divided into "townships of six miles square, by lines running due north and south, and others crossing them at right angles" as near as might be.

Ordinance of the Congress of the Confederation of May 20, 1785. U. S. Land Laws, p. 349. Edition 1828.

It further provided that the first line running north and south should begin on the Ohio River, at a point due north from the western terminus of a line run as the south boundary of the State of Pennsylvania, and the first line running east and west should begin at the same point and extend through the whole territory. In these initial surveys only the exterior lines of the townships were surveyed, but the plats were marked by subdivisions into sections 1 mile square, numbered from 1 to 36, commencing with No. 1 in the southeast corner of the township, and running from south to north in each tier to No. 36 in the northwest corner of the township; mile corners were established on the township lines. The region embraced by the surveys under this law forms a part of the present State of Ohio, and is generally known as "the Seven Ranges."

The Federal Congress passed a law, approved May 18, 1796, in regard to surveying the public domain, which applied to "the territory northwest of the River Ohio, and above the mouth of the Kentucky River."

Act of May 18, 1796. U. S. Statutes at Large, vol. 1, p. 465. Section 2395. U. S. Revised Statutes.

Section 2 of said act provided for dividing such lands as had not been already surveyed or disposed of "by north and south lines run according to the true meridian, and by others crossing them at right angles, so as to form townships of 6 miles square," etc. It also provided that "one-half of said townships, taking them alternately, should be subdivided into sections containing, as nearly as may be, 640 acres each, by running through the same each way parallel lines at the end of every two miles; and by making a corner on each of said lines at the end of every mile." The act also provided that "the sections shall be numbered, respectively, beginning with the number one in the northeast section, and proceeding west and east alternately through the townhsip, with progressive numbers till the thirty-sixth be completed." This method of numbering sections is still in use.

An act amendatory of the foregoing, approved May 10, 1800, required the "townships west of the Muskingum, which are directed to be sold in quarter townships, to be subdivided into half sections of 320 acres each, as nearly as may be, by running parallel lines through the same from east to west, and from south to north, at the distance of one mile from each other, and marking corners, at the distance of each half mile on the lines running from east to west, and at the distance of each mile on those running from south to north. And the

Act of May 10, 1800, U. S. Statutes at Large, vol. 2, p. 73. Section 2395, U. S. Revised Statutes.

interior lines of townships intersected by the Muskingum, and of all townships lying east of that river, which have not been heretofore actually subdivided into sections, shall also be run and marked . . . And in all cases where the exterior lines of the townships thus to be subdivided into sections or half sections shall exceed or shall not extend six miles, the excess of deficiency shall be specially noted, and added to or deducted from the western or northern ranges of sections or half sections in such townships, according as the error may be in running the lines from east to west or from south to north." Said act also provided that the northern and western tiers of sections should be sold as containing only the quantity expressed on the plats, and all others as containing the complete legal quantity.

Act of June 1, 1796. U. S. Statutes at Large, vol. 1, p. 490.

The act approved June 1, 1796, "regulating the grants of land appropriated for military services," etc., provided for dividing the "United States Military Tract," in the State of Ohio, into townships 5 miles square, each to be subdivided into quarter townships containing 4,000 acres.

Act of March 1, 1800. U. S. Statutes at Large, vol. 2, p. 14.

Section 6 of the act approved March 1, 1800, amendatory of the foregoing act, enacted that the Secretary of the Treasury was authorized to subdivide the quarter townships into lots of 100 acres, bounded as nearly as practicable by parallel lines 160 perches in length by 100 perches in width. These subdivisions into lots, however, were made upon the plats in the office of the Secfetary of the Treasury, and the actual survey was only made at a subsequent time when a sufficient number of such lots had been located to warrant the survey. It thus happened, in some instances, that when the survey came to be made the plat and survey could not be made to agree, and that fractional lots on plats were entirely crowded out. A knowledge of this fact may explain some of the difficulties met with in the district thus subdivided.

Act of Feb. 11, 1805, U. S. Statutes at Large, vol. 2, p. 313. Section 2396, U. S. Revised Statutes.

The act of Congress approved February 11, 1805, directs the subdivision of the public lands into quarter sections, and provides that all corners marked in the field shall be established as the proper corners of the sections or quarter sections which they were intended to designate, and that corners of half and quarter sections not marked shall be placed as nearly as possible "equidistant from those two corners which stand on the same line." This act further provides that "the boundary lines actually run and marked" (in the field) "shall be established as the proper boundary lines of the sections, or subdivisions, for which they were intended, and the length of such lines as returned by either of the surveyors aforesaid shall be held and considered as the true length thereof. And the boundary lines which shall not have been actually run and marked as aforesaid shall be ascertained by running straight lines from the established corners to the opposite corresponding corners, but in those portions of the fractional townships where no such opposite or corresponding corners have been or can be fixed, the said boundary lines shall be ascertained by running from the established corners due north and south, or east and west lines, as the case may be, to the watercourse, Indian boundary line, or other external boundary of such fractional township."

The act of Congress approved April 24, 1820, provides for the sale of public lands in half-quarter sections, and requires that "in every case of the division of a quarter section the line for the division thereof shall run north and south," "and fractional sections, containing 160 acres and upwards, shall in like manner, as nearly as practicable, be subdivided into half-quarter sections, under such rules and regulations as may be prescribed by the Secretary of the Treasury; but fractional sections containing less than 160 acres shall not be divided."

Act of April 24, 1820. U. S. Statutes at Large, vol. 3, p. 566. Section 2397, U. S. Revised Statutes.

The act of Congress approved May 24, 1824, provides "that whenever in the opinion of the President of the United States, a departure from the ordinary mode of surveying land on any river, lake, bayou, or watercourse would promote the public interest, he may direct the surveyor-general in whose district such land is situated, and where the change is intended to be made, under such rules and regulations as the President may prescribe, to cause the lands thus situated to be surveyed in tracts of two acres in width, fronting on any river, bayou, lake, or watercourse and running back the depth of forty acres."

Act of May 24, 1824. U. S. Statutes at Large, vol. 4, p. 34.

The act of Congress approved April 5, 1832, directed the subdivision of the public lands into quarter-quarter sections; that in every case of the division of a half-quarter section the dividing line should run east and west, and that fractional sections should be subdivided, under rules and regulations prescribed by the Secretary of the Treasury. Under the latter provision the Secretary directed that fractional sections containing less than 160 acres, or the residuary portion of a fractional section, after the subdivision into as many quarter-quarter sections as it is susceptible of, may be subdivided into lots, each containing the quantity of a quarter-quarter section as nearly as practicable, by so laying down the line of subdivision that they shall be 20 chains wide, which distances are to be marked on the plat of subdivision, as are also the areas of the quarter-quarters and residuary fractions.

Act of April 5, 1832. U. S. Statutes at Large, vol. 4, p. 503. Section 2397, U. S. Revised Statutes.

These two acts last mentioned provided that the corners and contents of half-quarter and quarter-quarter sections should be ascertained as nearly as possible in the manner and on the principles prescribed in the act of Congress approved February 11, 1805.

GENERAL RULES.

From the foregoing synopsis of Congressional legislation it is evident—

1st. That the boundaries of the public lands established and returned by the duly appointed Government surveyors, when approved by the surveyors-general and accepted by the Government, *are unchangeable.*

2d. That the original township section, and quarter-section corners established by the Government surveyors must stand as the true corners which they were intended to represent, whether the corners be in place or not.

3d. That quarter-quarter corners not established by the Government surveyors shall be placed on the straight lines joining the section and quarter-section corners and midway between them, except on the last

half mile of section lines closing on the north and west boundaries of the township, or on other lines between fractional sections.

4th. That all subdivisional lines of a section running between corners established in the original survey of a township must be straight lines, running from the proper corner in one section line to its opposite corresponding corner in the opposite section line.

5th. That in a fractional section where no opposite corresponding corner has been or can be established, any required subdivision line of such section must be run from the proper original corner in the boundary line due east and west, or north and south, as the case may be, to the watercourse, Indian reservation, or other boundary of such section, with due parallelism to section lines.

From the foregoing it will be plain that extinct corners of the Government surveys must be restored to their original locations, whenever it is possible to do so; and hence resort should always be first had to the marks of the survey in the field. The locus of the missing corner should be first identified on the ground by the aid of the mound, pits, line trees, bearing trees, etc., described in the field notes of the original survey.

The identification of mounds, pits, witness trees, or other permanent objects noted on the field notes of survey, affords the best means of relocating the missing corner in its original position. If this cannot be done, clear and convincing testimony of citizens as to the locality it originally occupied should be taken, if such can be obtained. In any event, whether the locus of the corner be fixed by one means of the other, such locus should always be tested and confirmed by measurements to *known* corners. No definite rule can be laid down as to what shall be sufficient evidence in such cases, and much must be left to the skill, fidelity, and good judgment of the surveyor in the performance of his work.

EXCEPTIONAL CASES.

When new measurements are made on a single line to determine the position thereon for a restored lost corner (for example, a quarter-section corner on line between two original section corners), or when new measurements are made between original corners on two lines for the purpose of fixing by their intersection the position of a restored missing corner (for example, a corner common to four sections or four townships), it will almost invariably happen that discrepancies will be developed between the new measurements and the original measurements in the field notes. When these differences occur the surveyor will in all cases establish the missing corner by proportionate measurements on lines conforming to the original field notes and by the method followed in the original survey. From this rule there can be no departure, since it is the basis upon which the whole operation depends for accuracy and truth.

In cases where the relocated corner cannot be made to harmonize with the field notes in all directions, and unexplained error in the first survey is apparent, it sometimes becomes the task of the surveyor to place it according to the requirements of one line and against the calls of another line. For instance if the line between sections 30 and 31 reported 78 chains long, would draw the missing corner on range line

1 chain eastward out of range with the other exterior corners, the presumption would be strong that the range line had been run straight and the length of the section line wrongly reported, because experience shows that west random lines are regarded as less important than range lines and more liable to error.

Again, where a corner on a standard parallel has been obliterated, it is proper to assume that it was placed in line with other corners, and if an anomalous length of line reported between sections 3 and 4 would throw the closing corner into the northern township, a surveyor would properly assume that the older survey of the standard line is to control the length of the later and minor line. The marks or corners found on luch a line closing to a standard parallel fix its *location*, but its *length* should be limited by its actual intersection, at which point the lost closing corner may be placed.

The strict rule of the law that "all corners marked in the field shall be established as the corners which they were intended to designate," and the further rule that "the length of lines returned by the surveyors shall be held and considered as the true length thereof," are found in some cases to be impossible of fulfilment in all directions at once, and a surveyor is obliged to choose, in his own discretion, which of two or more lines must yield, in order to permit the rules to be applied at all.

In a case of an erroneous but existing closing corner, which was set some distance out of the true State boundary of Missouri and Kansas, it was held by this office that a surveyor subdividing the fractional section should preserve the boundary as a straight line, and should not regard said closing corner as the proper corner of the adjacent fractional lots. The said corner was considered as fixing the position of the line between two fractional sections, but that its length extended to a new corner to be set on the true boundary line. The surveyor should therefore preserve such an original corner as evidence of the line; but its erroneous position cannot be allowed to cause a crook between mile corners of the original State boundary.

It is only in cases where it is manifestly impossible to carry out the literal terms of the law, that a surveyor can be justified in making such a decision.

The principle of the preponderance of one line over another of less importance has been recognized in the rule for restoring a section corner common to two townships in former editions of this circular. The new corner should be placed *on* the township line; and measurements to check its position by distances to corners within the townships are useful to confirm it if found to agree well, but should not cause it to be placed off the line if found not to agree, if the general condition of the boundary supports the presumption that it was properly alined.

TO RESTORE LOST OR OBLITERATED CORNERS.

1. *To restore corners on base lines and standard parallels.*—Lost or obliterated standard corners will be restored to their original positions on a base line, standard parallel, or correction line, by proportionate measurements on the line, conforming as nearly as practicable to the original field notes and joining the nearest identified original standard corners on opposite sides of the missing corner or corners, as the case may be.

(*a*) The term "standard corners" will be understood to designate standard township, section, quarter section, and meander corners; and in addition, closing corners, as follows: Closing corners used in the original survey to determine the position of a standard parallel of established during the survey of the same, will with the standard corners, govern the alinement and measurements made to restore lost or obliterated standard corners; but no other closing corners will control in any manner the restoration of standard corners on a base line or standard parallel.

(*b*) A lost or obliterated closing corner from which a standard parallel has been initiated or to which it has been directed will be re-established in its original place by proportionate measurement from the corners used in the original survey to determine its position. Measurements from corners on the opposite side of the parallel will not control in any manner the relocation of said corner.

(*c*) A missing closing corner originally established during the survey of a standard parallel as a corner from which to project surveys *south* will be restored to its original position by considering it a standard corner and treating it accordingly.

(*d*) Therefore, paying attention to the preceding explanations, we have for the restoration of one or several corners on a standard parallel and for general application to all other surveyed lines, the following proportion:

As the original field note distance between the selected known corners *is to* the new measure of said distance, *so is* the original field note length of any part of the line to the required new measure thereof.

The sum of the computed lengths of the several parts of a line must be equal to the new measure of the whole distance.

(*e*) As has been observed, existing original corners cannot be disturbed; consequently, discrepancies between the new and the original field note measurements of the line joining the selected original corners will not in any manner affect measurements beyond said corners, but the differences will be distributed proportionately to the several intervals embraced in the line in question.

(*f*) After having checked each new location by measurement to the nearest known corners, new corners will be established permanently and new bearings and measurements taken to prominent objects, which should be of as permanent a character as possible, and the same recorded for future reference.

2. *Restoration of township corners common to four townships.*—Two cases should be clearly recognized: 1st. Where the position of the original township corner has been made to depend upon measurements on two lines at right angles to each other. 2d. Where the original corner has been located by measurements on one line only; for example, on a guide meridian.

(*a*) For restoration of a township corner originally subject to the first condition: A line will first be run connecting the nearest identified original corners on the meridonal township lines, north and south of the missing corner, and a temporary corner will be placed at the proper proportionate distance. This will determine the corner in a north and south direction only.

Next, the nearest original corners on the latitudinal township lines

will be connected and a point thereon will be determined in a similar manner, independent of the temporary corner on the meridional line. Then through the first temporary corner run a line east (or west) and through the second temporary corner a line north (or south), as relative situations may suggest. The intersection of the two lines last run will define the position of the restored township corner, which may be permanently established.

(*b*) The restoration of a lost or obliterated township corner established under the second conditions, i.e., by measurements, on a single line, will be effected by proportionate measurements on said line, between the nearest identified original corners on opposite sides of the missing township corner, as before described.

3. *Re-establishment of corners common to two townships.*—The two nearest known corners on the township line, the same not being a base or a correction line, will be connected as in case No. 1, by a right line, and the missing corner established by proportionate distance as directed in that case; the location thus found will be checked upon by measurements to nearest known section or quarter-section corners north and south, or east and west, of the township line, as the case may be.

4. *Re-establishment of closing corners.*—Measure from the quarter-section, section, or township corner east or west, as the case may be, to the next preceding or succeeding corner in the order of original establishment, and re-establish the missing closing corner by proportionate measurement. The line upon which the closing corner was originally established should always be remeasured, in order to check upon the correctness of the new location. See page 744 ff. for details.

5. *Re-establishment of interior section corners.*—This class of corners should be re-established in the same manner as corners common to four townships. In such cases, when a number of corners are missing on all sides of the one sought to be re-established, the entire distance must, of course, be remeasured between the nearest existing recognized corners both north and south, and east and west, in accordance with the rule laid down, and the new corner re-established by proportionate measurement. The mere measurement in any one of the required directions will not suffice, since the direction of the several sections lines running northward through a township, or running east and west, are only in the most exceptional cases true prolongations of the alinement of the section lines initiated on the south boundary of the township; while the east and west lines running through the township, and theoretically supposed to be at right angles with the former, are seldom in that condition, and the alinements of the closing lines on the east and west boundaries of the township, in connection with the interior sections lines, are even less often in accord. Moreover, the alinement of the section line itself from corner to corner, in point of fact, also very frequently diverges from a right line, although presumed to be such from the record contained in the field notes and so designated on the plats, and becomes either a broken or a curved line. This fact will be determined, in a timbered country, by the blazes which may be found upon trees on either side of the line, and although such blazed line will not strictly govern as to the absolute direction assumed by such line, it will assist very materially in determining its approximate direction, and should never be neglected in retracements for the re-establishment of lost corners of

any description. Sight trees described in the field notes, together with the recorded distances to same, when fully identified, will, it has been held, in one or more States, govern the line itself, even when not in a direct or straight line between established corners, which line is then necessarily a broken line by passing through said sight trees. Such trees, when in existence and properly identified beyond a question of doubt, will very materially assist in evidencing the correct relocation of a missing corner. It is greatly to be regretted that the earlier field notes of survey are so very meagre in the notation of the topography found on the original line, which might in very many instances materially lessen a surveyor's labors in retracement of lines and re-establishment of the required missing corner. In the absence of such sight trees and other evidence regarding the line, as in an open country, or where such evidence has been destroyed by time, the elements, or the progress of improvement, the line connecting the known corners should be run straight from corner to corner.

6. *Re-establishment of quarter-section corners on township boundaries.*—Only one set of quarter-section corners are actually marked in the field on township lines, and they are established at the time when the township exteriors are run. When double section corners are found, the quarter-section corners are considered generally as standing midway between the corners of their respective sections, and when required to be established or re-established, as the case may be, they should be generally so placed; but great care should be exercised not to mistake the corners belonging to one township for those of another. After determining the proper section corners marking the line upon which the missing quarter-section corner is to be re-established, and measuring said line, the missing quarter-section corner will be re-established in accordance with the requirements of the original field notes of survey, by proportionate measurement between the section corners marking the line.

Where there are double sets of section corners on township and range lines, and the quarter-section corners for sections south of the township or east of the range lines are required to be established in the field, the said quarter-section corners should be so placed as to suit the calculation of areas of the quarter sections adjoining the township boundaries as expressed upon the official township plat, adopting proportionate measurements when the present measurement of the north and west boundaries of the section differ from the original measurements.

7. *Re-establishment of quarter-section corners on closing section lines between fractional sections.*—This class of corners must be re-established according to the original measurement of 40 chains from the last interior section corner. If the measurements do not agree with the original survey, the excess or deficiency must be divided proportionately between the two distances, as expressed in the field notes of original survey. The section corner started from and the corner closed upon should be connected by a right line, unless the retracement should develop the fact that the section line is either a broken or curved line, as is sometimes the case.

8. *Re-establishment of interior quarter-section corners.*—In some of the older surveys these corners are placed at variable distances, in which case the field notes of the original survey must be consulted, and the

quarter-section corner re-established at proportionate distances between the corresponding section corners, in accordance therewith. The later surveys being more uniform and in stricter accordance with law, the missing quarter-section corner must be re-established equidistant between the section corners marking the line, according to the field notes of the original survey. The remarks made under section 5, in relation to section lines, apply with full force here also; the caution there given not to neglect sight trees is equally applicable, since the proper re-establishment of the quarter-section corner may in some instances very largely depend upon its observance, and avoid one of the many sources of litigation.

9. *Where double corners were originally established, one of which is standing, to re-establish the other.*—It being remembered that the corners established when the exterior township lines were run belong to the sections in the townships north and west of those lines, the surveyor must first determine beyond a doubt to which sections the existing corner belongs. This may be done by testing the courses and distances to witness trees or other objects noted in the original field notes of survey and by remeasuring distances to known corners. Having determined to which township the existing corner belongs, the missing corner may be re-established in line north or south of the existing corner, as the case may be, at the distance stated in the field notes of the original survey, by proportionate measurement, and tested by retracement to the opposite corresponding corner of the section to which the missing section corner belongs. These double corners being generally not more than a few chains apart, the distance between them can be more accurately laid off, and it is considered preferable to first establish the missing corner as above, and check upon the corresponding interior corner, than to reverse the proceeding; since the result obtained is very way more accurate and satisfactory.

10. *Where double corners were originally established, and both are missing, to re-establish the one established when the township line was run.*—The surveyor will connect the nearest known corners on the township line by a right line, being careful to distinguish the section from the closing corners, and re-establish the missing corner at the point indicated by the field notes of the original survey by proportionate measurement. The corner thus restored will be common to two sections either north or west of the township boundary, and the section north or west, as the case may be, should be carefully retraced, thus checking upon the re-established corner, and testing the accuracy of the result. It cannot be too much impressed upon the surveyor that any measurements to objects on line noted in the original survey are means of determining and testing the correctness of the operation.

11. *Where double corners were originally established, and both are missing, to re-establish the one established when the township was subdivided.*—The corner to be re-established being common to two sections south or east of the township line, the section line closing on the missing section corner should be first retraced to an intersection with the township line in the manner previously indicated, and a temporary corner established at the point of intersection. The township line will of course have been previously carefully retraced in accordance with the requirements of the original field notes of survey, and marked in such a manner as to be readily identified when reaching the same with the retraced section line.

The location of the temporary corner planted at the point of intersection will then be carefully tested and verified by remeasurements to objects and known corners on the township line, as noted in the original field notes of survey, and the necessary corrections made in such relocation. A permanent corner will then be erected at the corrected location on the township line, properly marked and witnessed, and recorded for future requirements.

12. *Where triple corners were originally established on range lines, one or two of which have become obliterated, to re-establish either of them.*—It will be borne in mind that only two corners were established as actual corners of sections, those established on the range line not corresponding with the subdivisional survey east or west of said range line. The surveyor will, therefore, first proceed to identify the existing corner or corners, as the case may be, and then re-establish the missing corner or corners in line north or south, according to the distances stated in the original field notes of survey in the manner indicated for the re-establishment of double corners, testing the accuracy of the result obtained, as hereinbefore directed in other cases. If, however, the distances between the triple corners are not stated in the original field notes of survey, as is frequently the case in the returns of older surveys, the range line should be first carefully retraced, and marked in a manner sufficiently clear to admit of easy identification upon reaching same during the subsequent proceedings. The section lines closing upon the missing corners must then be retraced in accordance with the original field notes of survey, in the manner previously indicated and directed, and the corners re-established in the manner directed in the case of double corners. The surveyor cannot be too careful, in the matter of retracement, in following closely all the recorded indications of the original line, and nothing, however slight, should be neglected to insure the correctness of the retracement of the original line; since there is no other check upon the accuracy of the re-establishment of the missing corners, unless the entire corresponding section lines are remeasured by proportional measurement and the result checked by a recalculation of the areas as originally returned, which, at best, is but a very poor check, because the areas expressed upon the margin of many plats of the older surveys are erroneously stated on the face of the plats, or have been carelessly calculated.

13. *Where triple corners were originally established on range lines, all of which are missing, to re-establish same.*—These corners should be re-established in accordance with the foregoing directions, commencing with the corner originally established when the range line was run, establishing the same in accordance with previously given directions for restoring section and quarter-section corners; that is to say, by remeasuring between the nearest known corners on said township line, and re-establishing the same by proportionate measurement. The two remaining will then be re-established in conformity with the general rules for re-establishment of double corners.

14. *Re-establishment of meander corners.*—Before proceeding with the re-establishment of missing meander corners, the surveyor should have carefully rechained at least three of the section lines between known corners of the township within which the lost corner is to be relocated, in order to establish the proportionate measurement to be used. This

requirement of preliminary remeasurement of section lines must in no case be omitted; since it gives the only data upon which the fractional section line can be remeasured proportionately, the corner marking the terminus, or the meander corner, being missing, which it is intended to re-establish. The missing meander corner will be re-established on the section or township line retraced in its original location, by the proportionate measurement found by the preceding operations, from the nearest known corner on such township or section line, in accordance with the requirements of the original field notes of survey.

Meander corners hold the peculiar position of denoting a point on line between landowners, without usually being the legal terminus or corner of the lands owned. Leading judicial decisions have affirmed that meander lines are not strictly boundaries, and do not limit the ownership to the exact areas placed on the tracts, but that said title extends to the water, which, by the plat, appears to bound the land.

As such water boundaries are, therefore, subject to change by the encroachment or recession of the stream or lake, the precise location of old meanders is seldom important, unless in States whose laws prescribe that dried lake beds are the property of the State.

Where the United States has disposed of the fractional lots adjacent to shores, it claims no marginal lands left by recession or found by reason of erroneous survey. The lines between landowners are therefore regarded as extended beyond the original meander line of the shore, but the preservation or relocation of the meander corner is important, as evidence of the *position* of the section line.

The different rules by which division lines should be run between private owners of riparian accretions are a matter of State legislation, and not subject to a general rule of this office.

15. *Fractional section lines.*—County and local surveyors being sometimes called upon to restore fractional section lines closing upon Indian, military, or other reservations, private grants, etc., such lines should be restored upon the same principles as directed in the foregoing pages, and checked whenever possible upon such corners or monuments as have been placed to mark such boundary lines.

In some instances corners have been moved from their original position, either by accident or design, and county surveyors are called upon to restore such corners to their original positions, but, owing to the absence of any and all means of identification of such location, are unable to make the result of their work acceptable to the owners of the lands affected by such corner. In such cases the advice of this office has invariably been to the effect that the relocation of such corner must be made in accordance with the orders of a court of competent jurdisdiction, the United States having no longer any authority to order any changes where the lands affected by such corner have been disposed of.

RECORDS.

The original evidences of the public-land surveys in the following States have been transferred, under the provisions of sections 2218, 2219, and 2220, United States Revised Statutes, to the State authorities to

whom application should be made for such copies of the original plats and field notes as may be desired, viz.:

Alabama: Secretary of State, Montgomery.
Arkansas: Commissioner of State Lands, Little Rock.
Illinois: Auditor of State, Springfield.
Indiana: Auditor of State, Indianapolis.
Iowa: Secretary of State, Des Moines.
Kansas: Auditor of State and Register of State Lands, Topeka.
Michigan: Commissioner of State Land Office, Lansing.
Mississippi: Commissioner of State Lands, Jackson.
Missouri: Secretary of State, Jefferson City.
Nebraska: Commissioner of Public Lands and Buildings, Lincoln.
Ohio: Auditor of State, Columbus.
Wisconsin: Commissioners of Public Lands, Madison.

In other public-land States the original field notes and plats are retained in the offices of the United States surveyors-general.

SUBDIVISION OF SECTIONS

This office being in receipt of many letters making inquiry in regard to the proper method of subdividing sections of the public lands, the following general rules have been prepared as a reply to such inquiries. The rules for subdivision are based upon the laws governing the survey of the public lands. When cases arise which are not covered by these rules, and the advice of this office in the matter is desired, the letter of inquiry should, in every instance, contain a description of the particular tract or corner, with reference to township, range, and section of the public surveys, to enable the office to consult the record; also a diagram showing conditions found:

1. *Subdivision of sections into quarter sections.*—Under the provisions of the act of Congress approved February 11, 1805, the course to be pursued in the subdivision of sections into quarter sections is to run straight lines from the established quarter-section corners, United States surveys, to the opposite corresponding corners. The point of intersection of the lines thus run will be the corner common to the several quarter sections, or, in other words, the legal centre of the section.

(*a*) Upon the lines closing on the north and west boundaries of a township, the quarter-section corners are established by the United States deputy surveyors at 40 chains to the north or west of the last interior section corners, and the excess or deficiency in the measurement is thrown into the half mile next to the township or range line, as the case may be.

(*b*) Where there are double sets of sections corners on township and range lines, the quarter corners for the sections south of the township lines and east of the range lines are not established in the field by the United States deputy surveyors, but in subdividing such sections said quarter corners should be so placed as to suit the calculations of the areas of the quarter-sections adjoining the township boundaries as expressed upon the official plat, adopting proportionate measurements where the

new measurements of the north or west boundaries of the section differ from the original measurements.

2. *Subdivision of fractional sections.*—Where opposite corresponding corners have not been or cannot be fixed, the subdivision lines should be ascertained by running from the established corners due north, south, east, or west lines, as the case may be, to the watercourse, Indian boundary line, or other boundary of such fractional section.

(*a*) The law presumes the section lines surveyed and marked in the field by the United States deputy surveyors to be due north and south or east and west lines, but in actual experience this is not always the case. Hence, in order to carry out the spirit of the law, it will be necessary in running the subdivisional line through fractional sections to adopt mean courses where the section lines are not due lines, or to run the subdivision line parallel to the east, south, west, or north boundary of the section, as conditions may require, where there is no opposite section line.

3. *Subdivision of quarter sections into quarter-quarters.*—Preliminary to the subdivision of quarter sections, the quarter-quarter corners will be established at points midway between the second and quarter-section corners, and between quarter corners and the centre of the section, except on the last half mile of the lines closing on the north or west boundaries of a township, where they should be placed at 20 chains, proportionate measurement, to the north or west of the quarter-section corner.

(*a*) The quarter-quarter section corners having been established as directed above, the subdivision lines of the quarter section will be run straight between opposite corresponding quarter-quarter section corners on the quarter-section boundaries. The intersection of the lines thus run will determine the place for the corner common to the four quarter-quarter sections.

4. *Subdivision of fractional quarter sections.*—The subdivision lines of fractional quarter sections will be run from properly established quarter quarter section corners (paragraph 3) due north, south, east, or west, to the lake, watercourse, or reservation which renders such tracts fractional, or parallel to the east, south, west, or north boundary of the quarter section, as conditions may require. (See paragraph 2 (*a*).)

5. *Proportionate measurement.*—By "proportionate measurement," as used in this circular, is meant a measurement having the same ratio to that recorded in the original field notes as the *length of chain* used in the new measurement has to the *length of chain* used in the original survey, assuming that the original and new measurements have been correctly made.

For example: The length of the line from the quarter-section corner on the west side of sec. 2, T. 24 N., R. 14 E., Wisconsin, to the north line of the township, by the United States deputy surveyor's chain, was reported as 45.40 chains, and by the county surveyor's measure is reported as 42.90 chains; then the distance which the quarter-quarter section corner should be located north of the quarter-section corner would be determined as follows:

As 45.40 chains, the Government measure of the whole distance, is to 42.90 chains, the county surveyor's measure of the same distance, so is 20.00 chains, original measurement, to 18.90 chains by the county surveyor's measure, showing that by proportionate measurement in this

case the quarter-quarter section corner should be set at 18.90 chains north of the quarter-section corner, instead of 20.00 chains north of such corner, as represented on the official plat. In this manner the discrepancies between original and new measurements are equitably distributed.

S. W. LAMOREUX,
Commissioner.

DEPARTMENT OF THE INTERIOR,
October 16, 1896.

Approved:
DAVID R. FRANCIS,
Secretary.

TABLES

TABLE I

Trigonometric Formulas

Trigonometric Functions

Let A (Fig. 107) = angle BAC = arc BF and let the radius $AF = AB = AH = 1$. We then have

$\sin A = BC$
$\cos A = AC$
$\tan A = DF$
$\cot A = HG$
$\sec A = AD$
$\operatorname{cosec} A = AG$
$\operatorname{versin} A = CF = BE$
$\operatorname{covers} A = BK = HL$
$\operatorname{exsec} A = BD$
$\operatorname{coexsec} A = EG$
$\operatorname{chord} A = BF$
$\operatorname{chord} 2A = BI = 2BC$

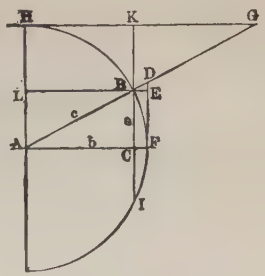


Fig. 262.

In the right-angled triangle ABC (Fig. 107).
Let $AB = c$, $AC = b$, and $BC = a$.
We then have

1. $\sin A = \frac{a}{c} = \cos B$
2. $\cos A = \frac{b}{c} = \sin B$
3. $\tan A = \frac{a}{b} = \cot B$
4. $\cot A = \frac{b}{a} = \tan B$
5. $\sec A = \frac{c}{b} = \operatorname{cosec} B$
6. $\operatorname{cosec} A = \frac{c}{a} = \sec B$
7. $\operatorname{vers} A = \frac{c-b}{c} = \operatorname{covers} B$
8. $\operatorname{exsec} A = \frac{c-b}{b} = \operatorname{coexsec} B$
9. $\operatorname{covers} A = \frac{c-a}{c} = \operatorname{versin} B$
10. $\operatorname{coexsec} A = \frac{c-a}{a} = \operatorname{exsec} B$
11. $a = c \sin A = b \tan A$
12. $b = c \cos A = a \cot A$
13. $c = \frac{a}{\sin A} = \frac{b}{\cos A}$
14. $a = c \cos B = b \cot B$
15. $b = c \sin B = a \tan B$
16. $c = \frac{a}{\cos B} = \frac{b}{\sin B}$
17. $a = \sqrt{(c+b)(c-b)}$
18. $b = \sqrt{(c+a)(c-a)}$
19. $c = \sqrt{a^2+b^2}$
20. $C = 90° = A + B$
21. $\text{Area} = \frac{ab}{2}$

TABLE I.—*Continued*

TRIGONOMETRIC FORMULAS

SOLUTION OF OBLIQUE TRIANGLES

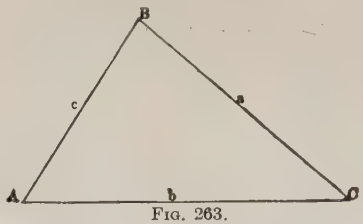


FIG. 263.

	GIVEN.	SOUGHT.	FORMULAS.
22	A, B, a	C, b, c	$C=180°-(A+B)$, $\quad b=\frac{a}{\sin A}\,.\,\sin B$, $\quad c=\frac{a}{\sin A}\sin(A+B)$
23	A, a, b	B, C, c	$\sin B=\frac{\sin A}{a}\,.\,b$, $\quad C=180°-(A+B)$, $\quad c=\frac{a}{\sin A}\,.\,\sin C$
24	C, a, b	$\frac{1}{2}(A+B)$	$\frac{1}{2}(A+B)=90°-\frac{1}{2}C$
25		$\frac{1}{2}(A-B)$	$\tan\frac{1}{2}(A-B)=\frac{a-b}{a+b}\tan\frac{1}{2}(A+B)$
26		A, B	$A=\frac{1}{2}(A+B)+\frac{1}{2}(A-B)$, $B=\frac{1}{2}(A+B)-\frac{1}{2}(A-B)$
27		c	$c=(a+b)\frac{\cos\frac{1}{2}(A+B)}{\cos\frac{1}{2}(A-B)}=\sqrt{a^2+b^2-2ab\cos C}$
28		Area	$K=\frac{1}{2}ab\sin C$
29	a, b, c	A	Let $s=\frac{1}{2}(a+b+c)$; $\quad \sin\frac{1}{2}A=\sqrt{\frac{(s-b)(s-c)}{bc}}$
30			$\cos\frac{1}{2}A=\sqrt{\frac{s(s-a)}{bc}}$; $\quad \tan\frac{1}{2}A=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$
31			$\sin A=\frac{2\sqrt{s(s-a)(s-b)(s-c)}}{bc}$; $\quad \cos A=\frac{b^2+c^2-a^2}{2bc}$
32		Area	$K=\sqrt{s(s-a)(s-b)(s-c)}$
33	A, B, C, a	Area	$K=\frac{a^2\sin B\,.\,\sin C}{2\sin A}$

TABLE I—*Continued*

Trigonometric Formulas

	General Formulas.
34	$\sin A = \frac{1}{\operatorname{cosec} A} = \sqrt{1-\cos^2 A} = \tan A \cos A$
35	$\sin A = 2 \sin \frac{1}{2}A \cos \frac{1}{2}A = \operatorname{vers} A \cot \frac{1}{2}A$
36	$\sin A = \sqrt{\frac{1}{2} \operatorname{vers} 2A} = \sqrt{\frac{1}{2}(1-\cos 2A)}$
37	$\cos A = \frac{1}{\sec A} = \sqrt{1-\sin^2 A} = \cot A \sin A$
38	$\cos A = 1 - \operatorname{vers} A = 2\cos^2 \frac{1}{2}A - 1 = 1 - 2\sin^2 \frac{1}{2}A$
39	$\cos A = \cos^2 \frac{1}{2}A - \sin^2 \frac{1}{2}A = \sqrt{\frac{1}{2} + \frac{1}{2}\cos 2A}$
40	$\tan A = \frac{1}{\cot A} = \frac{\sin A}{\cos A} = \sqrt{\sec^2 A - 1}$
41	$\tan A = \sqrt{\frac{1}{\cos^2 A} - 1} = \frac{\sqrt{1-\cos^2 A}}{\cos A} = \frac{\sin 2A}{1+\cos 2A}$
42	$\tan A = \frac{1-\cos 2A}{\sin 2A} = \frac{\operatorname{vers} 2A}{\sin 2A} = \operatorname{exsec} A \cot \frac{1}{2}A$
43	$\cot A = \frac{1}{\tan A} = \frac{\cos A}{\sin A} = \sqrt{\operatorname{cosec}^2 A - 1}$
44	$\cot A = \frac{\sin 2A}{1-\cos 2A} = \frac{\sin 2A}{\operatorname{vers} 2A} = \frac{1+\cos 2A}{\sin 2A}$
45	$\cot A = \frac{\tan \frac{1}{2}A}{\operatorname{exsec} A}$
46	$\operatorname{vers} A = 1 - \cos A = \sin A \tan \frac{1}{2}A = 2\sin^2 \frac{1}{2}A$
47	$\operatorname{vers} A = \operatorname{exsec} A \cos A$
48	$\operatorname{exsec} A = \sec A - 1 = \tan A \tan \frac{1}{2}A = \frac{\operatorname{vers} A}{\cos A}$
49	$\sin \frac{1}{2}A = \sqrt{\frac{1-\cos A}{2}} = \sqrt{\frac{\operatorname{vers} A}{2}}$
50	$\sin 2A = 2 \sin A \cos A$
51	$\cos \frac{1}{2}A = \sqrt{\frac{1+\cos A}{2}}$
52	$\cos 2A = 2\cos^2 A - 1 = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A$

TABLE I.—*Continued*

TRIGONOMETRIC FORMULAS

	GENERAL FORMULAS
53	$\tan \frac{1}{2}A = \frac{\tan A}{1+\sec A} = \operatorname{cosec} A - \cot A = \frac{1-\cos A}{\sin A} = \sqrt{\frac{1-\cos A}{1+\cos A}}$
54	$\tan 2A = \frac{2 \tan A}{1-\tan^2 A}$
55	$\cot \frac{1}{2}A = \frac{\sin A}{\operatorname{vers} A} = \frac{1+\cos A}{\sin A} = \frac{1}{\operatorname{cosec} A - \cot A}$
56	$\cot 2A = \frac{\cot^2 A - 1}{2 \cot A}$
57	$\operatorname{vers} \frac{1}{2}A = \frac{\frac{1}{2} \operatorname{vers} A}{1+\sqrt{1-\frac{1}{2} \operatorname{vers} A}} = \frac{1-\cos A}{2+\sqrt{2(1+\cos A)}}$
58	$\operatorname{vers} 2A = 2 \sin^2 A$
59	$\operatorname{exsec} \frac{1}{2}A = \frac{1-\cos A}{(1+\cos A)+\sqrt{2(1+\cos A)}}$
60	$\operatorname{exsec} 2A = \frac{\tan^2 A}{1-\tan^2 A}$
61	$\sin (A \pm B) = \sin A \,.\, \cos B \pm \sin B \,.\, \cos A$
62	$\cos (A \pm B) = \cos A \,.\, \cos B \mp \sin A \,.\, \sin B$
63	$\sin A + \sin B = 2 \sin \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B)$
64	$\sin A - \sin B = 2 \cos \frac{1}{2}(A+B) \sin \frac{1}{2}(A-B)$
65	$\cos A + \cos B = 2 \cos \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B)$
66	$\cos B - \cos A = 2 \sin \frac{1}{2}(A+B) \sin \frac{1}{2}(A-B)$
67	$\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin (A+B) \sin (A-B)$
68	$\cos^2 A - \sin^2 B = \cos (A+B) \cos (A-B)$
69	$\tan A + \tan B = \frac{\sin (A+B)}{\cos A \,.\, \cos B}$
70	$\tan A - \tan B = \frac{\sin (A-B)}{\cos A \,.\, \cos B}$

No.	Log.	D.	No.	Log.
10	000	41	**60**	778
11	041	38	61	785
12	079	35	62	792
13	114	32	63	799
14	146	30	64	806
15	176	28	65	813
16	204	26	66	820
17	230	25	67	826
18	255	24	68	833
19	279	22	69	839
20	301	21	**70**	845
21	322	20	71	851
22	342	20	72	857
23	362	18	73	863
24	380	18	74	869
25	398	17	75	875
26	415	16	76	881
27	431	16	77	886
28	447	15	78	892
29	462	15	79	898
30	477	14	**80**	903
31	491	14	81	908
32	505	14	82	914
33	519	13	83	919
34	531	12	84	924
35	544	12	85	929
36	556	12	86	934
37	568	12	87	940
38	580	11	88	944
39	591	11	89	949
40	602	11	**90**	954
41	613	10	91	959
42	623	10	92	964
43	633	10	93	968
44	643	10	94	973
45	653	10	95	978
46	663	9	96	982
47	672	9	97	987
48	681	9	98	991
49	690	9	99	996
50	699	9	**100**	000
51	708	8	101	004
52	716	8	102	009
53	724	8	103	013
54	732	8	104	017
55	740	8	105	021
56	748	8	106	025
57	756	7	107	029
58	763	8	108	033
59	771	7	109	037
60	778		**110**	041

ARC	SIN	Diff.	TAN	Diff.
0°.0				
0.1	7.242	301	7.242	301
0.2	7.543	176	7.543	176
0.3	7.719	125	7.719	125
0.4	7.844	97	7.844	97
0.5	7.941	79	7.941	79
0.6	8.020	67	8.020	67
0.7	8.087	58	8.087	58
0.8	8.145	51	8.145	51
0.9	8.196	46	8.196	46
1.0	8.242	41	8.242	41
1.1	8.283	38	8.283	38
1.2	8.321	35	8.321	35
1.3	8.356	32	8.356	32
1.4	8.388	30	8.388	30
1.5	8.418	28	8.418	28
1.6	8.446	26	8.446	26
1.7	8.472	25	8.472	25
1.8	8.497	24	8.497	24
1.9	8.521	22	8.521	22
2.0	8.543	21	8.543	21
2.1	8.564	20	8.564	21
2.2	8.584	19	8.585	19
2.3	8.603	19	8.604	18
2.4	8.622	18	8.622	18
2.5	8.640	17	8.640	17
2.6	8.657	16	8.657	17
2.7	8.673	16	8.674	15
2.8	8.689	15	8.689	16
2.9	8.704	15	8.705	14
3.0	8.719	14	8.719	15
3.1	8.733	14	8.734	13
3.2	8.747	13	8.747	14
3.3	8.760	13	8.761	13
3.4	8.773	13	8.774	12
3.5	8.786	12	8.786	13
3.6	8.798	12	8.799	12
3.7	8.810	11	8.811	11
3.8	8.821	12	8.822	12
3.9	8.833	11	8.834	11
4.0	8.844	10	8.845	10
4.1	8.854	10	8.855	11
4.2	8.865	11	8.866	10
4.3	8.875	10	8.876	10
4.4	8.885	10	8.886	10
4.5	8.895	9	8.896	10
4.6	8.904	9	8.906	9
4.7	8.913	10	8.915	9
4.8	8.923	9	8.924	9
4.9	8.932	8	8.933	9
5.0	8.940		8.942	
ARC	COS	Diff.	COT	Diff

NOTE: These tables are printed separately and mounted on heav

ARC	ARC	SIN	Diff.	TAN	Diff.	COT	COS	ARC
90°.0	0°							90°
89.9	1	8.242	301	8.242	301	1.758	0.000	89
89.8	2	8.543	176	8.543	176	1.457	0.000	88
89.7	3	8.719	125	8.719	126	1.281	9.999	87
89.6	4	8.844	96	8.845	97	1.155	9.999	86
89.5	5	8.940	79	8.942	80	1.058	9.998	85
89.4	6	9.019	67	9.022	67	0.978	9.998	84
89.3	7	9.086	58	9.089	59	0.911	9.997	83
89.2	8	9.144	50	9.148	52	0.852	9.996	82
89.1	9	9.194	46	9.200	46	0.800	9.995	81
89.0	10	9.240	41	9.246	43	0.754	9.993	80
88.9	11	9.281	37	9.289	38	0.711	9.992	79
88.8	12	9.318	34	9.327	36	0.673	9.990	78
88.7	13	9.352	32	9.363	34	0.637	9.989	77
88.6	14	9.384	29	9.397	31	0.603	9.987	76
88.5	15	9.413	27	9.428	29	0.572	9.985	75
88.4	16	9.440	26	9.457	28	0.543	9.983	74
88.3	17	9.466	24	9.485	27	0.515	9.981	73
88.2	18	9.490	23	9.512	25	0.488	9.978	72
88.1	19	9.513	21	9.537	24	0.463	9.976	71
88.0	20	9.534	20	9.561	23	0.439	9.973	70
87.9	21	9.554	20	9.584	22	0.416	9.970	69
87.8	22	9.574	18	9.606	22	0.394	9.967	68
87.7	23	9.592	17	9.628	21	0.372	9.964	67
87.6	24	9.609	17	9.649	20	0.351	9.961	66
87.5	25	9.626	16	9.669	19	0.331	9.957	65
87.4	26	9.642	15	9.688	19	0.312	9.954	64
87.3	27	9.657	15	9.707	19	0.293	9.950	63
87.2	28	9.672	14	9.726	18	0.274	9.946	62
87.1	29	9.686	13	9.744	17	0.256	9.942	61
87.0	30	9.699	13	9.761	18	0.239	9.938	60
86.9	31	9.712	12	9.779	17	0.221	9.933	59
86.8	32	9.724	12	9.796	17	0.204	9.928	58
86.7	33	9.736	12	9.813	16	0.187	9.924	57
86.6	34	9.748	11	9.829	16	0.171	9.919	56
86.5	35	9.759	10	9.845	16	0.155	9.913	55
86.4	36	9.769	10	9.861	16	0.139	9.908	54
86.3	37	9.779	10	9.877	16	0.123	9.902	53
86.2	38	9.789	10	9.893	15	0.107	9.897	52
86.1	39	9.799	9	9.908	16	0.092	9.891	51
86.0	40	9.808	9	9.924	15	0.076	9.884	50
85.9	41	9.817	9	9.939	15	0.061	9.878	49
85.8	42	9.826	8	9.954	16	0.046	9.871	48
85.7	43	9.834	8	9.970	15	0.030	9.864	47
85.6	44	9.842	7	9.985	15	0.015	9.857	46
85.5	45°	9.849		0.000		0.000	9.849	45°
85.4								
85.3	ARC	COS	Diff.	COT	Diff.	TAN	SIN	ARC
85.2								
85.1								
85.0								
ARC								

CONSTANTS.

Rad. in Deg. = 57°.29578	log. 1.75812	Circ. in Deg. = 360°	log. 2.55630
" " Min. = 3437′ .747	log. 3.53627	" " Min. = 21600′	log. 4.33445
" " Sec. = 206264″.8	log. 5.31443	" " Sec. = 1296000″	log. 6.11261
sin 1′ = 6.464		log π 0.49715	

TABLE II

For Converting Metres, Feet, and Chains

Metres to Feet.		Feet to Metres and Chains.			Chains to Feet.	
Metres.	Feet.	Feet.	Metres.	Chains.	Chains.	Feet.
1	3.28083	1	0.30480	0.0151	0.01	0.66
2	6.56167	2	0.60960	.0303	.02	1.32
3	9.84250	3	0.91440	.0455	.03	1.98
4	13.12333	4	1.21920	.0606	.04	2.64
5	16.40447	5	1.52400	.0758	.05	3.30
6	19.68500	6	1.82880	.0909	.06	3.96
7	22.96583	7	2.13360	.1061	.07	4.62
8	26.24667	8	2.43840	.1212	.08	5.28
9	29.52750	9	2.74321	.1364	.09	5.94
10	32.80833	10	3.04801	.1515	.10	6.60
20	65.61667	20	6.09601	.3030	.20	13.20
30	98.42500	30	9.14402	.4545	.30	19.80
40	131.2333	40	12.19202	.6061	.40	26.40
50	164.0416	50	15.24003	.7576	.50	33.00
60	196.8500	60	18.28804	.9091	.60	39.60
70	229.6583	70	21.33604	1.0606	.70	46.20
80	262.4667	80	24.38405	1.2121	.80	52.80
90	295.2750	90	27.43205	1.3636	.90	59.40
100	328.0833	100	30.48006	1.5151	1	66.00
200	656.1666	200	60.96012	3.0303	2	132
300	984.2500	300	91.44018	4.5455	3	198
400	1312.333	400	121.92024	6.0606	4	264
500	1640.416	500	152.40030	7.5756	5	330
600	1968.500	600	182.88037	9.0909	6	396
700	2296.583	700	213.36043	10.606	7	462
800	2624.666	800	243.84049	12.121	8	528
900	2952.750	900	274.32055	13.636	9	594
1000	3280.833	1000	304.8006	15.151	10	660
2000	6561.667	2000	609.6012	30.303	20	1320
3000	9842.500	3000	914.4018	45.455	30	1980
4000	13123.33	4000	1219.2024	60.606	40	2640
5000	16404.47	5000	1524.0030	75.750	50	3300
6000	19685.00	6000	1828.8037	90.909	60	3960
7000	22965.83	7000	2133.6043	106.06	70	4620
8000	26246.67	8000	2438.4049	121.21	80	5280
9000	29527.50	9000	2743.2055	136.36	90	5940

TABLE III

Logarithms of Numbers. § 173

Nat. Nos.	0	1	2	3	4	5	6	7	8	9	Proportional Parts. 1	2	3	4	5	6	7	8	9
10	.0000	.0043	.0086	.0128	.0170	.0212	.0253	.0294	.0334	.0374	4	8	12	17	21	25	29	33	37
11	.0414	.0453	.0492	.0531	.0569	.0607	.0645	.0682	.0719	.0755	4	8	11	15	19	23	26	30	34
12	.0792	.0828	.0864	.0899	.0934	.0969	.1004	.1038	.1072	.1106	3	7	10	14	17	21	24	28	31
13	.1139	.1173	.1206	.1239	.1271	.1303	.1335	.1367	.1399	.1430	3	6	10	13	16	19	23	26	29
14	.1461	.1492	.1523	.1553	.1584	.1614	.1644	.1673	.1703	.1732	3	6	9	12	15	18	21	24	27
15	.1761	.1790	.1818	.1847	.1875	.1903	.1931	.1959	.1987	.2014	3	6	8	11	14	17	20	22	25
16	.2041	.2068	.2095	.2122	.2148	.2175	.2201	.2227	.2253	.2279	3	5	8	11	13	16	18	21	24
17	.2304	.2330	.2355	.2380	.2405	.2430	.2455	.2480	.2504	.2529	2	5	7	10	12	15	17	20	22
18	.2553	.2577	.2601	.2625	.2648	.2672	.2695	.2718	.2742	.2765	2	5	7	9	12	14	16	19	21
19	.2788	.2810	.2833	.2856	.2878	.2900	.2923	.2945	.2967	.2989	2	4	7	9	11	13	16	18	20
20	.3010	.3032	.3054	.3075	.3096	.3118	.3139	.3160	.3181	.3201	2	4	6	8	11	13	15	17	19
21	.3222	.3243	.3263	.3284	.3304	.3324	.3345	.3365	.3385	.3404	2	4	6	8	10	12	14	16	18
22	.3424	.3444	.3464	.3483	.3502	.3522	.3541	.3560	.3579	.3598	2	4	6	8	10	12	14	15	17
23	.3617	.3636	.3655	.3674	.3692	.3711	.3729	.3747	.3766	.3784	2	4	6	7	9	11	13	15	17
24	.3802	.3820	.3838	.3856	.3874	.3892	.3909	.3927	.3945	.3962	2	4	5	7	9	11	12	14	16
25	.3979	.3997	.4014	.4031	.4048	.4065	.4082	.4099	.4116	.4133	2	3	5	7	9	10	12	14	15
26	.4150	.4166	.4183	.4200	.4216	.4232	.4249	.4265	.4281	.4298	2	3	5	7	8	10	11	13	15
27	.4314	.4330	.4346	.4362	.4378	.4393	.4409	.4425	.4440	.4456	2	3	5	6	8	9	11	13	14
28	.4472	.4487	.4502	.4518	.4533	.4548	.4564	.4579	.4594	.4609	2	3	5	6	8	9	11	12	14
29	.4624	.4639	.4654	.4669	.4683	.4698	.4713	.4728	.4742	.4757	1	3	4	6	7	9	10	12	13
30	.4771	.4786	.4800	.4814	.4829	.4843	.4857	.4871	.4886	.4900	1	3	4	6	7	9	10	11	13
31	.4914	.4928	.4942	.4955	.4969	.4983	.4997	.5011	.5024	.5038	1	3	4	6	7	8	10	11	12
32	.5051	.5065	.5079	.5092	.5105	.5119	.5132	.5145	.5159	.5172	1	3	4	5	7	8	9	11	12
33	.5185	.5198	.5211	.5224	.5237	.5250	.5263	.5276	.5289	.5302	1	3	4	5	6	8	9	10	12
34	.5315	.5328	.5340	.5353	.5366	.5378	.5391	.5403	.5416	.5428	1	3	4	5	6	8	9	10	11
35	.5441	.5453	.5465	.5478	.5490	.5502	.5514	.5527	.5539	.5551	1	2	4	5	6	7	9	10	11
36	.5563	.5575	.5587	.5599	.5611	.5623	.5635	.5647	.5658	.5670	1	2	4	5	6	7	8	10	11
37	.5682	.5694	.5705	.5717	.5729	.5740	.5752	.5763	.5775	.5786	1	2	3	5	6	7	8	9	10
38	.5798	.5809	.5821	.5832	.5843	.5855	.5866	.5877	.5888	.5899	1	2	3	5	6	7	8	9	10
39	.5911	.5922	.5933	.5944	.5955	.5966	.5977	.5988	.5999	.6010	1	2	3	4	5	7	8	9	10
40	.6021	.6031	.6042	.6053	.6064	.6075	.6085	.6096	.6107	.6117	1	2	3	4	5	6	8	9	10
41	.6128	.6138	.6149	.6160	.6170	.6180	.6191	.6201	.6212	.6222	1	2	3	4	5	6	7	8	9
42	.6232	.6243	.6253	.6263	.6274	.6284	.6294	.6304	.6314	.6325	1	2	3	4	5	6	7	8	9
43	.6335	.6345	.6355	.6365	.6375	.6385	.6395	.6405	.6415	.6425	1	2	3	4	5	6	7	8	9
44	.6435	.6444	.6454	.6464	.6474	.6484	.6493	.6503	.6513	.6522	1	2	3	4	5	6	7	8	9
45	.6532	.6542	.6551	.6561	.6571	.6580	.6590	.6599	.6609	.6618	1	2	3	4	5	6	7	8	9
46	.6628	.6637	.6646	.6656	.6665	.6675	.6684	.6693	.6702	.6712	1	2	3	4	5	6	7	7	8
47	.6721	.6730	.6739	.6749	.6758	.6767	.6776	.6785	.6794	.6803	1	2	3	4	5	5	6	7	8
48	.6812	.6821	.6830	.6839	.6848	.6857	.6866	.6875	.6884	.6893	1	2	3	4	4	5	6	7	8
49	.6902	.6911	.6920	.6928	.6937	.6946	.6955	.6964	.6972	.6981	1	2	3	4	4	5	6	7	8
50	.6990	.6998	.7007	.7016	.7024	.7033	.7042	.7050	.7059	.7067	1	2	3	3	4	5	6	7	8
51	.7076	.7084	.7093	.7101	.7110	.7118	.7126	.7135	.7143	.7152	1	2	3	3	4	5	6	7	8
52	.7160	.7168	.7177	.7185	.7193	.7202	.7210	.7218	.7226	.7235	1	2	2	3	4	5	6	7	7
53	.7243	.7251	.7259	.7267	.7275	.7284	.7292	.7300	.7308	.7316	1	2	2	3	4	5	6	6	7
54	.7324	.7332	.7340	.7348	.7356	.7364	.7372	.7380	.7388	.7396	1	2	2	3	4	5	6	6	7

TABLE III.—*Continued*

Logarithms of Numbers

Nat. Nos.	0	1	2	3	4	5	6	7	8	9	Proportional Parts. 1	2	3	4	5	6	7	8	9
55	.7404	.7412	.7419	.7427	.7435	.7443	.7451	.7459	.7466	.7474	1	2	2	3	4	5	5	6	7
56	.7482	.7490	.7497	.7505	.7513	.7520	.7528	.7536	.7543	.7551	1	2	2	3	4	5	5	6	7
57	.7559	.7566	.7574	.7582	.7589	.7597	.7604	.7612	.7619	.7627	1	2	2	3	4	5	5	6	7
58	.7634	.7642	.7649	.7657	.7664	.7672	.7679	.7686	.7694	.7701	1	1	2	3	4	4	5	6	7
59	.7709	.7716	.7723	.7731	.7738	.7745	.7752	.7760	.7767	.7774	1	1	2	3	4	4	5	6	7
60	.7782	.7789	.7796	.7803	.7810	.7818	.7825	.7832	.7839	.7846	1	1	2	3	4	4	5	6	6
61	.7853	.7860	.7868	.7875	.7882	.7889	.7896	.7903	.7910	.7917	1	1	2	3	4	4	5	6	6
62	.7924	.7931	.7938	.7945	.7952	.7959	.7966	.7973	.7980	.7987	1	1	2	3	3	4	5	6	6
63	.7993	.8000	.8007	.8014	.8021	.8028	.8035	.8041	.8048	.8055	1	1	2	3	3	4	5	5	6
64	.8062	.8069	.8075	.8082	.8089	.8096	.8102	.8109	.8116	.8122	1	1	2	3	3	4	5	5	6
65	.8129	.8136	.8142	.8149	.8156	.8162	.8169	.8176	.8182	.8189	1	1	2	3	3	4	5	5	6
66	.8195	.8202	.8209	.8215	.8222	.8228	.8235	.8241	.8248	.8254	1	1	2	3	3	4	5	5	6
67	.8261	.8267	.8274	.8280	.8287	.8293	.8299	.8306	.8312	.8319	1	1	2	3	3	4	5	5	6
68	.8325	.8331	.8338	.8344	.8351	.8357	.8363	.8370	.8376	.8382	1	1	2	3	3	4	4	5	6
69	.8388	.8395	.8401	.8407	.8414	.8420	.8426	.8432	.8439	.8445	1	1	2	2	3	4	4	5	6
70	.8451	.8457	.8463	.8470	.8476	.8482	.8488	.8494	.8500	.8506	1	1	2	2	3	4	4	5	6
71	.8513	.8519	.8525	.8531	.8537	.8543	.8549	.8555	.8561	.8567	1	1	2	2	3	4	4	5	5
72	.8573	.8579	.8585	.8591	.8597	.8603	.8609	.8615	.8621	.8627	1	1	2	2	3	4	4	5	5
73	.8633	.8639	.8645	.8651	.8657	.8663	.8669	.8675	.8681	.8686	1	1	2	2	3	4	4	5	5
74	.8692	.8698	.8704	.8710	.8716	.8722	.8727	.8733	.8739	.8745	1	1	2	2	3	4	4	5	5
75	.8751	.8756	.8762	.8768	.8774	.8779	.8785	.8791	.8797	.8802	1	1	2	2	3	3	4	5	5
76	.8808	.8814	.8820	.8825	.8831	.8837	.8842	.8848	.8854	.8859	1	1	2	2	3	3	4	5	5
77	.8865	.8871	.8876	.8882	.8887	.8893	.8899	.8904	.8910	.8915	1	1	2	2	3	3	4	4	5
78	.8921	.8927	.8932	.8938	.8943	.8949	.8954	.8960	.8965	.8971	1	1	2	2	3	3	4	4	5
79	.8976	.8982	.8987	.8993	.8998	.9004	.9009	.9015	.9020	.9025	1	1	2	2	3	3	4	4	5
80	.9031	.9036	.9042	.9047	.9053	.9058	.9063	.9069	.9074	.9079	1	1	2	2	3	3	4	4	5
81	.9085	.9090	.9096	.9101	.9106	.9112	.9117	.9122	.9128	.9133	1	1	2	2	3	3	4	4	5
82	.9138	.9143	.9149	.9154	.9159	.9165	.9170	.9175	.9180	.9186	1	1	2	2	3	3	4	4	5
83	.9191	.9196	.9201	.9206	.9212	.9217	.9222	.9227	.9232	.9238	1	1	2	2	3	3	4	4	5
84	.9243	.9248	.9253	.9258	.9263	.9269	.9274	.9279	.9284	.9289	1	1	2	2	3	3	4	4	5
85	.9294	.9299	.9304	.9309	.9315	.9320	.9325	.9330	.9335	.9340	1	1	2	3	3	3	4	4	5
86	.9345	.9350	.9355	.9360	.9365	.9370	.9375	.9380	.9385	.9390	1	1	2	2	3	3	4	4	5
87	.9395	.9400	.9405	.9410	.9415	.9420	.9425	.9430	.9435	.9440	0	1	1	2	2	3	3	4	4
88	.9445	.9450	.9455	.9460	.9465	.9469	.9474	.9479	.9484	.9489	0	1	1	2	2	3	3	4	4
89	.9494	.9499	.9504	.9509	.9513	.9518	.9523	.9528	.9533	.9538	0	1	1	2	2	3	3	4	4
90	.9542	.9547	.9552	.9557	.9562	.9566	.9571	.9576	.9581	.9586	0	1	1	2	2	3	3	4	4
91	.9590	.9595	.9600	.9605	.9609	.9614	.9619	.9624	.9628	.9633	0	1	1	2	2	3	3	4	4
92	.9638	.9643	.9647	.9652	.9657	.9661	.9666	.9671	.9675	.9680	0	1	1	2	2	3	3	4	4
93	.9685	.9689	.9694	.9699	.9703	.9708	.9713	.9717	.9722	.9727	0	1	1	2	2	3	3	4	4
94	.9731	.9736	.9741	.9745	.9750	.9754	.9759	.9763	.9768	.9773	0	1	1	2	2	3	3		
95	.9777	.9782	.9786	.9791	.9795	.9800	.9805	.9809	.9814	.9818	0	1	1	2	2	3	3	4	4
96	.9823	.9827	.9832	.9836	.9841	.9845	.9850	.9854	.9859	.9863	0	1	1	2	2	3	3	4	4
97	.9868	.9872	.9877	.9881	.9886	.9890	.9894	.9899	.9903	.9908	0	1	1	2	2	3	3	4	4
98	.9912	.9917	.9921	.9926	.9930	.9934	.9939	.9943	.9948	.9952	0	1	1	2	2	3	3	4	4
99	.9956	.9961	.9965	.9969	.9974	.9978	.9983	.9987	.9991	.9996	0	1	1	2	2	3	3	3	4

TABLE IV

Logarithms of Sines and Tangents

	0°				1°				
	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	
0′		0.0000			8.2419	9.9999	8.2419	1.7581	60′
1	6.4637	.0000	6.4637	3.5363	.2490	.9999	.2491	.7509	59
2	.7648	.0000	.7648	.2352	.2561	.9999	.2652	.7438	58
3	6.9408	.0000	6.9408	3.0592	.2630	.9999	.2631	.7369	57
4	7.0658	.0000	7.0658	2.9342	.2699	.9999	.2700	.7300	56
5	.1627	.0000	.1627	.8373	.2766	.9999	.2767	.7233	55
6	.2419	.0000	.2419	.7581	.2832	.9999	.2833	.9167	54
7	.3088	.0000	.3088	.6912	.2898	.9999	.2899	.7101	53
8	.3668	.0000	.3668	.6332	.2962	.9999	.2963	.7037	52
9	.4180	.0000	.4180	.5820	.3025	.9999	.3026	.6974	51
10	.4637	.0000	.4637	.5363	.3088	.9999	.3089	.6911	50
11	.5051	.0000	.5051	.4949	.3150	.9999	.3150	.6850	49
12	.5429	.0000	.5429	.4571	.3210	.9999	.3211	.6789	48
13	.5777	.0000	.5777	.4223	.3270	.9999	.3271	.6729	47
14	.6099	.0000	.6099	.3901	.3329	.9999	.3330	.6670	46
15	.6398	.0000	.6398	.3602	.3388	.9999	.3389	.6611	45
16	.6678	.0000	.6678	.3322	.3445	.9999	.3446	.6554	44
17	.6942	.0000	.6942	.3058	.3502	.9999	.3503	.6497	43
18	.7190	.0000	.7190	.2810	.3558	.9999	.3559	.6441	42
19	.7425	.0000	.7425	.2575	.3613	.9999	.3614	.6386	41
20	.7648	.0000	.7648	.2352	.3668	.9999	.3669	.6331	40
21	.7859	.0000	.7860	.2140	.3722	.9999	.3723	.6277	39
22	.8061	.0000	.8062	.1938	.3775	.9999	.3776	.6224	38
23	.8255	.0000	.8255	.1745	.3828	.9999	.3829	.6171	37
24	.8439	.0000	.8439	.1561	.3880	.9999	.3881	.6119	36
25	.8617	.0000	.8617	.1383	.3931	.9999	.3932	.6068	35
26	.8787	.0000	.8787	.1213	.3982	.9999	.3983	.6017	34
27	.8951	.0000	.8951	.1049	.4032	.9999	.4033	.5967	33
28	.9109	.0000	.9109	.0891	.4082	.9999	.4083	.5917	32
29	.9261	.0000	.9261	.0739	.4131	.9999	.4132	.5868	31
30	.9408	.0000	.9409	.0591	.4179	.9999	.4181	.5819	30
31	.9551	.0000	.9551	.0449	.4227	.9998	.4229	.5771	29
32	.9689	.0000	.9689	.0311	.4275	.9998	.4276	.5724	28
33	.9822	.0000	.9823	.0177	.4322	.9998	.4323	.5677	27
34	7.9952	.0000	7.9952	2.0048	.4368	.9998	.4370	.5630	26
35	8.0078	.0000	8.0078	1.9922	.4414	.9998	.4416	.5584	25
36	.0200	.0000	.0200	.9800	.4459	.9998	.4461	.5539	24
37	.0319	.0000	.0319	.9681	.4504	.9998	.4506	.5494	23
38	.0435	.0000	.0435	.9565	.4549	.9998	.4551	.5449	22
39	.0548	.0000	.0548	.9452	.4593	.9998	.4595	.5405	21
40	.0658	.0000	.0658	.9342	.4637	.9998	.4638	.5362	20
41	.0765	.0000	.0765	.9235	.4680	.9998	.4682	.5318	19
42	.0870	.0000	.0870	.9130	.4723	.9998	.4725	.5275	18
43	.0972	.0000	.0972	.9028	.4765	.9998	.4767	.5233	17
44	.1072	.0000	.1072	.8928	.4807	.9998	.4809	.5191	16
45	.1169	.0000	.1170	.8830	.4848	.9998	.4851	.5149	15
46	.1265	.0000	.1265	.8735	.4890	.9998	.4892	.5108	14
47	.1358	.0000	.1359	.8641	.4930	.9998	.4933	.5067	13
48	.1450	.0000	.1450	.8550	.4971	.9998	.4973	.5027	12
49	.1539	.0000	.1540	.8460	.5011	.9998	.5013	.4987	11
50	.1627	.0000	.1627	.8373	.5050	.9998	.5053	.4947	10
51	.1713	.0000	.1713	.8287	.5090	.9998	.5092	.4908	9
52	.1797	0.0000	.1798	.8202	.5129	.9998	.5131	.4869	8
53	.1880	9.9999	.1880	.8120	.5167	.9998	.5170	.4830	7
54	.1961	.9999	.1962	.8038	.5206	.9998	.5208	.4792	6
55	.2041	.9999	.2041	.7959	.5243	.9998	.5246	.4754	5
56	.2119	.9999	.2120	.7880	.5281	.9998	.5283	.4717	4
57	.2196	.9999	.2196	.7804	.5318	.9997	.5321	.4679	3
58	.2271	.9999	.2272	.7728	.5355	.9997	.5358	.4642	2
59	.2346	.9999	.2346	.7654	.5392	.9997	.5394	.4606	1
60	8.2419	9.9999	8.2419	1.7581	8.5428	9.9997	8.5431	1.4569	0
	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	
	89°				88°				

TABLE IV—*Continued*

Logarithms of Sines and Tangents

	2°				3°				4°				
	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	
0′	8.5428	9.9997	8.5431	1.4569	8.7188	9.9994	8.7194	1.2806	8.8436	9.9989	8.8446	1.1554	60′
1	.5464	.9997	.5467	.4533	.7212	.9994	.7218	.2782	.8544	.9989	.8465	.1535	59
2	.5500	.9997	.5503	.4497	.7236	.9994	.7242	.2758	.8472	.9989	.8483	.1517	58
3	.5535	.9997	.5538	.4462	.7260	.9994	.7266	.2734	.8490	.9989	.8501	.1499	57
4	.5571	.9997	.5573	.4427	.7283	.9994	.7290	.2710	.8508	.9989	.8518	.1482	56
5	.5605	.9997	.5608	.4392	.7307	.9994	.7313	.2687	.8525	.9989	.8536	.1464	55
6	.5640	.9997	.5643	.4357	.7330	.9994	.7337	.2663	.8543	.9989	.8554	.1446	54
7	.5674	.9997	.5677	.4323	.7354	.9994	.7360	.2640	.8560	.9989	.8572	.1428	53
8	.5708	.9997	.5711	.4289	.7377	.9994	.7383	.2617	.8578	.9989	.8589	.1411	52
9	.5742	.9997	.5745	.4255	.7400	.9993	.7406	.2594	.8595	.9989	.8607	.1393	51
10	.5776	.9997	.5779	.4221	.7423	.9993	.7429	2.2571	.8613	.9989	.8624	.1376	50
11	.5809	.9997	.5812	.4188	.7445	.9993	.7452	.2548	.8630	.9988	.8642	.1358	49
12	.5842	.9997	.5845	.4155	.7468	.9993	.7475	.2525	.8647	.9988	.8659	.1341	48
13	.5875	.9997	.5878	.4122	.7491	.9993	.7497	.2503	.8665	.9988	.8676	.1324	47
14	.5907	.9997	.5911	.4089	.7513	.9993	.7520	.2480	.8682	.9988	.8964	.1306	46
15	.5939	.9997	.5943	.4057	.7535	.9993	.7542	.2458	.8699	.9988	.8711	.1289	45
16	.5972	.9997	.5975	.4025	.7557	.9993	.7565	.2435	.8716	.9988	.8728	.1272	44
17	.6003	.9997	.6007	.3993	.7580	.9993	.7587	.2413	.8733	.9988	.8745	.1255	43
18	.6035	.9996	.6038	.3962	.7602	.9993	.7609	.2391	.8749	.9988	.8762	.1238	42
19	.6066	.9996	.6070	.3930	.7623	.9993	.7631	.2369	.8766	.9988	.8778	.1222	41
20	.6097	.9996	.6101	.3899	.7645	.9993	.7652	.2348	.8783	.9988	.8795	.1205	40
21	.6128	.9996	.6132	.3868	.7667	.9993	.7674	.2326	.8799	.9987	.8812	.1188	39
22	.6159	.9996	.6163	.3837	.7688	.9992	.7696	.2304	.8816	.9987	.8829	.1171	38
23	.6189	.9996	.6193	.3807	.7710	.9992	.7717	.2283	.8833	.9987	.8845	.1155	37
24	.6220	.9996	.6223	.3777	.7731	.9992	.7739	.2261	.8849	.9987	.8862	.1138	36
25	.6250	.9996	.6254	.3476	.7752	.9992	.7760	.2240	.8865	.9987	.8878	.1122	35
26	.6279	.9996	.6283	.3717	.7773	.9992	.7781	.2219	.8882	.9987	.8895	.1105	34
27	.6309	.9996	.6313	.3687	.7794	.9992	.7802	.2198	.8898	.9987	.8911	.1089	33
28	.6339	.9996	.6343	.3657	.7815	.9992	.7823	.2177	.8914	.9987	.8927	.1073	32
29	.6368	.9996	.6372	.3628	.7836	.9992	.7844	.2156	.8930	.9987	.8944	.1056	31
30	.6397	.9996	.6401	.3599	.7857	.9992	.7865	.2135	.8946	.9987	.8960	.1040	30
31	.6426	.9996	.6430	.3570	.7877	.9992	.7886	.2114	.8962	.9986	.8976	.1024	29
32	.6454	.9996	.6459	.3541	.7898	.9992	.7906	.2094	.8978	.9986	.8992	.1008	28
33	.6483	.9996	.6487	.3513	.7918	.9992	.7927	.2073	.8994	.9986	.9008	.0992	27
34	.6511	.9996	.6515	.3485	.7939	.9992	.7947	.2053	.9010	.9986	.9024	.0976	26
35	.6539	.9996	.6544	.3456	.7959	.9992	.7967	.2033	.9026	.9986	.9040	.0960	25
36	.6567	.9996	.6571	.3429	.7979	.9991	.7988	.2012	.9342	.9986	.9056	.0944	24
37	.6595	.9995	.6599	.3401	.7999	.9991	.8008	.1992	.9057	.9986	.9071	.0929	23
38	.6622	.9995	.6627	.3373	.8019	.9991	.8028	.1972	.9073	.9986	.9087	.0913	22
39	.6650	.9995	.6654	.3346	.8039	.9991	.8048	.1952	.9089	.9986	.9103	.0897	21
40	.6677	.9995	.6682	.3318	.8059	.9991	.8067	.1933	.9104	.9986	.9118	.0882	20
41	.6704	.9995	.6709	.3291	.8078	.9991	.8087	.1913	.9119	.9985	.9134	.0866	19
42	.6731	.9995	.6736	.3264	.8098	.9991	.8107	.1893	.9135	.9985	.9150	.0850	18
43	.6758	.9995	.6762	.3238	.8117	.9991	.8126	.1874	.9150	.9985	.9165	.0835	17
44	.6784	.9995	.6789	.3211	.8137	.9991	.8146	.1854	.9166	.9985	.9180	.0820	16
45	.6810	.9995	.6815	.3185	.8156	.9991	.8165	.1835	.9181	.9985	.9196	.0804	15
46	.6837	.9995	.6842	.3158	.8175	.9991	.8185	.1815	.9196	.9985	.9211	.0789	14
47	.6863	.9995	.6868	.3132	.8194	.9991	.8204	.1796	.9211	.9985	.9226	.0774	13
48	.6889	.9995	.6894	.3106	.8213	.9990	.8223	.1777	.9226	.9985	.9241	.0759	12
49	.6914	.9995	.6920	.3080	.8232	.9990	.8242	.1758	.9241	.9985	.9256	.0744	11
50	.6940	.9995	.6945	.3055	.8251	.9990	.8261	.1739	.9256	.9985	.9272	.0728	10
51	.6965	.9995	.6971	.3029	.8270	.9990	.8280	.1720	.9271	.9984	.9287	.0713	9
52	.6991	.9995	.6996	.3004	.8289	.9990	.8299	.1701	.9286	.9984	.9302	.0698	8
53	.7016	.9994	.7021	.2979	.8307	.9990	.8317	.1683	.9301	.9984	.9316	.0684	7
54	.7041	.9994	.7046	.2954	.8326	.9990	.8336	.1664	.9315	.9984	.9331	.0669	6
55	.7066	.9994	.7071	.2929	.8345	.9990	.8355	.1645	.9330	.9984	.9346	.0654	5
56	.7090	.9994	.7096	.2904	.8363	.9990	.8373	.1627	.9345	.9984	.9361	.0639	4
57	.7115	.9994	.7121	.2879	.8381	.9990	.8392	.1608	.9359	.9984	.9376	.0624	3
58	.7140	.9994	.7145	.2855	.8400	.9990	.8410	.1590	.9374	.9984	.9390	.0610	2
59	.7164	.9994	.7170	.2830	.8418	.9989	.8428	.1572	.9388	.9984	.9405	.0595	1
60	8.7188	9.9994	8.7194	1.2806	8.8436	9.9989	8.8446	1.1554	8.9403	9.9983	8.9420	1.0580	0
	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	
	88°				87°				86°				

TABLE IV—*Continued*

Logarithms of Sines and Tangents

Arc	Sin.	Df	Cos.	Df	Tan.	Df	Cot.	Arc	Arc	Sin.	Df	Cos.	Df	Tan.	Df	Cot.	Arc
° ′								° ′	° ′								° ′
5 0	8.9403	142	9.9983	1	8.9420	143	1.0580	85 0	15 0	9.4130	47	9.9849	3	9.4281	50	0.5719	75 0
10	.9545	137	.9982	1	.9563	138	.0437	50	10	.4177	46	.9846	3	.4331	50	.5669	50
20	.9682	134	.9981	1	.9701	135	.0299	40	20	.4223	46	.9843	4	.4381	49	.5619	40
30	.9816	129	.9980	1	8.9836	130	.0164	30	30	.4269	45	.9839	3	.4430	49	.5570	30
40	8.9945	125	.9979	2	8.9966	127	1.0034	20	40	.4314	45	.9836	4	.4479	48	.5521	20
50	9.0070	122	.9977	1	9.0093	123	0.9907	10	50	.4359	44	.9832	4	.4527	48	.5473	10
6 0	.0192	119	.9976	1	.0216	120	.9784	84 0	16 0	.4403	44	.9828	3	.4575	47	.5425	74 0
10	.0311	115	.9975	2	.0336	117	.9664	50	10	.4447	44	.9825	4	.4622	47	.5378	50
20	.0426	113	.9973	1	.0453	114	.9547	40	20	.4491	42	.9821	4	.4669	45	.5331	40
30	.0539	109	.9972	1	.0567	111	.9433	30	30	.4533	43	.9817	3	.4716	46	.5284	30
40	.0648	107	.9971	2	.0678	108	.9322	20	40	.4576	42	.9814	4	.4762	46	.5238	20
50	.0755	104	.9969	1	.0786	105	.9214	10	50	.4618	41	.9810	4	.4808	45	.5192	10
7 0	.0859	102	.9968	2	.0891	104	.9109	83 0	17 0	.4659	41	.9806	4	.4853	45	.5147	73 0
10	.0961	99	.9966	2	.0995	101	.9005	50	10	.4700	41	.9802	4	.4898	45	.5102	50
20	.1060	97	.9964	1	.1096	98	.8904	40	20	.4741	40	.9798	4	.4943	44	.5057	40
30	.1157	95	.9963	2	.1194	97	.8806	30	30	.4781	40	.9794	4	.4987	44	.5013	30
40	.1252	93	.9961	2	.1291	94	.8709	20	40	.4821	40	.9790	4	.5031	44	.4969	20
50	.1345	91	.9959	1	.1385	93	.8615	10	50	.4861	39	.9786	4	.5075	43	.4925	10
8 0	.1436	89	.9958	2	.1478	91	.8522	82 0	18 0	.4900	39	.9782	4	.5118	43	.4882	72 0
10	.1525	87	.9956	2	.1569	89	.8431	50	10	.4939	38	.9778	4	.5161	42	.4839	50
20	.1612	85	.9954	2	.1658	87	.8342	40	20	.4977	38	.9774	4	.5203	42	.4797	40
30	.1697	84	.9952	2	.1745	86	.8255	30	30	.5015	37	.9770	5	.5245	42	.4755	30
40	.1781	82	.9950	2	.1831	84	.8169	20	40	.5052	38	.9765	4	.5287	42	.4713	20
50	.1863	80	.9948	2	.1915	82	.8085	10	50	.5090	36	.9761	4	.5329	41	.4671	10
9 0	.1943	79	.9946	2	.1997	81	.8003	81 0	19 0	.5126	37	.9757	5	.5370	41	.4630	71 0
10	.2022	78	.9944	2	.2078	80	.7922	50	10	.5163	36	.9752	4	.5411	40	.4589	50
20	.2100	76	.9942	2	.2158	78	.7842	40	20	.5199	36	.9748	5	.5451	40	.4549	40
30	.2176	75	.9940	2	.2236	77	.7764	30	30	.5235	35	.9743	4	.5491	40	.4509	30
40	.2251	73	.9938	2	.2313	76	.7687	20	40	.5270	36	.9739	5	.5531	40	.4469	20
50	.2324	73	.9936	2	.2389	74	.7611	10	50	.5306	35	.9734	4	.5571	40	.4429	10
10 0	.2397	71	.9934	3	.2463	73	.7537	80 0	20 0	.5341	34	.9730	5	.5611	39	.4389	70 0
10	.2468	70	.9931	2	.2536	73	.7464	50	10	.5375	34	.9725	4	.5650	39	.4350	50
20	.2538	68	.9929	2	.2609	71	.7391	40	20	.5409	34	.9721	5	.5689	38	.4311	40
30	.2606	68	.9927	3	.2680	70	.7320	30	30	.5443	34	.9716	5	.5727	39	.4273	30
40	.2674	66	.9924	2	.2750	69	.7250	20	40	.5477	33	.9711	5	.5766	38	.4234	20
50	.2740	66	.9922	3	.2819	68	.7181	10	50	.5510	33	.9706	4	.5804	38	.4196	10
11 0	.2806	64	.9919	2	.2887	66	.7113	79 0	21 0	.5543	33	.9702	5	.5842	37	.4158	69 0
10	.2870	64	.9917	3	.2953	67	.7047	50	10	.5576	33	.9697	5	.5879	38	.4121	50
20	.2934	63	.9914	2	.3020	65	6980	40	20	.5609	32	.9692	5	.5917	37	.4083	40
30	.2997	61	.9912	3	.3085	64	.6915	30	30	.5641	32	.9687	5	.5954	37	.4046	30
40	.3058	61	.9909	2	.3149	63	.6851	20	40	.5673	31	.9682	5	.5991	37	.4009	20
50	.3119	60	.9907	3	.3212	63	.6788	10	50	.5704	32	.9677	5	.6028	36	.3972	10
12 0	.3179	59	.9904	3	.3275	61	.6725	78 0	22 0	.5736	31	.9672	5	.6064	36	.3936	68 0
10	.3238	58	.9901	2	.3336	61	.6664	50	10	.5767	31	.9667	6	.6100	36	.3900	50
20	.3296	57	.9899	3	.3397	61	.6603	40	20	.5798	30	.9661	5	.6136	36	.3864	40
30	.3353	57	.9896	3	.3458	59	.6542	30	30	.5828	31	.9656	5	.6172	36	.3828	30
40	.3410	56	.9893	3	.3517	59	.6483	20	40	.5859	30	.9651	5	.6208	35	.3792	20
50	.3466	55	.9890	3	.3576	58	.6424	10	50	.5889	30	.9646	6	.6243	36	.3757	10
13 0	.3521	54	.9887	3	.3634	57	.63667	770	23 0	.5919	29	.9640	5	.6279	35	.3721	67 0
10	.3575	54	.9884	3	.3691	57	.6309	50	10	.5948	30	.9635	6	.6314	34	.3686	50
20	.3629	53	.9881	3	.3748	56	.6252	40	20	.5978	29	.9629	5	.6348	35	.3652	40
30	.3682	52	.9878	3	.3804	55	.6196	30	30	.6007	29	.9624	6	.6383	34	.3617	30
40	.3734	52	.9875	3	.3859	55	.6141	20	40	.6036	29	.9618	5	.6417	35	.3583	20
50	.3786	51	.9872	3	.3914	54	.6086	10	50	.6065	28	.9613	6	.6452	34	.3548	10
14 0	.3837	50	.9869	3	.3968	53	.6032	76 0	24 0	.6093	28	.9607	5	.6486	34	.3514	66 0
10	.3887	50	.9866	3	.4021	53	.5979	50	10	.6121	28	.9602	6	.6520	33	.3480	50
20	.3937	49	.9863	4	.4074	53	.5926	40	20	.6149	28	.9596	6	.6553	34	.3447	40
30	.3986	49	.9859	3	.4127	52	.5873	30	30	.6177	28	.9590	6	.6587	33	.3413	30
40	.4035	48	.9856	3	.4178	51	.5822	20	40	.6205	27	.9584	5	.6620	34	.3380	20
50	.4083	47	.9853	4	.4230	51	.5770	10	50	.6232	27	.9579	6	.6654	33	.3346	10
15 0	9.4130	47	9.9849	3	9.4281	50	0.5719	75 0	25 0	0.6259	27	9.9573	7	9.6687	33	0.3313	65 0
Arc	Cos.	Df	Sin.	Df	Cot.	Df	Tan.	Arc	Arc	Cos.	Df	Sin.	Df	Cot.	Df	Tan.	Arc

TABLE IV—*Continued*

Logarithms of Sines and Tangents

Arc	Sin.	Df	Cos.	Df	Tan.	Df	Cot.	Arc	Arc	Sin.	Df	Cos.	Df	Tan.	Df	Cot.	Arc
° ′								° ′	° ′								° ′
25 0	9.6259	27	9.9573	6	9.6687	33	0.3313	65 0	35 0	9.7586	18	9.9134	9	9.8452	27	0.1548	55 0
10	.6286	27	.9567	6	.6720	32	.3280	50	10	.7604	18	.9125	9	.8479	27	.1521	50
20	.6313	27	.9561	6	.6752	33	.3248	40	20	.7622	18	.9116	9	.8506	27	.1494	40
30	.6340	26	.9555	6	.6785	32	.3215	30	30	.7640	17	.9107	9	.8533	26	.1467	30
40	.6366	26	.9549	6	.6817	33	.3183	20	40	.7657	18	.9098	9	.8559	27	.1441	20
50	.6392	26	.9543	6	.6850	32	.3150	10	50	.7675	17	.9089	9	.8586	27	.1414	10
26 0	.6418	26	.9537	7	.6882	32	.3118	64 0	36 0	.7692	18	.9080	10	.8613	26	.1387	54 0
10	.6444	26	.9530	6	.6914	32	.3086	50	10	.7710	17	.9070	9	.8639	27	.1361	50
20	.6470	25	.9524	6	.6946	31	.3054	40	20	.7727	17	.9061	9	.8666	26	.1334	40
30	.6495	26	.9518	6	.6977	32	.3023	30	30	.7744	17	.9052	10	.8692	26	.1308	30
40	.6521	25	.9512	7	.7009	31	.2991	20	40	.7761	17	.9042	9	.8718	27	.1282	20
50	.6546	24	.9505	6	.7040	32	.2960	10	50	.7778	17	.9033	10	.8745	26	.1255	10
27 0	.6570	25	.9499	7	.7072	31	.2928	63 0	37 0	.7795	16	.9023	9	.8771	26	.1229	53 0
10	.6595	25	.9492	6	.7103	31	.2897	50	10	.7811	17	.9014	10	.8797	27	.1203	50
20	.6620	24	.9486	7	.7134	31	.2866	40	20	.7828	16	.9004	9	.8824	26	.1176	40
30	.6644	24	.9479	6	.7165	31	.2835	30	30	.7844	17	.8995	10	.8850	26	.1150	30
40	.6668	24	.9473	7	.7196	30	.2804	20	40	.7861	16	.8985	10	.8876	26	.1124	20
50	.6692	24	.9466	7	.7226	31	.2774	10	50	.7877	16	.8975	10	.8902	26	.1098	10
28 0	.6716	24	.9459	6	.7257	30	.2743	62 0	38 0	.7893	17	.8965	10	.8928	26	.1072	52 0
10	.6740	23	.9453	7	.7287	30	.2713	50	10	.7910	16	.8955	10	.8954	26	.1046	50
20	.6763	24	.9446	7	.7317	31	.2683	40	20	.7926	15	.8945	10	.8980	26	.1020	40
30	.6787	23	.9439	7	.7348	30	.2652	30	30	.7941	16	.8935	10	.9006	26	.0994	30
40	.6810	23	.9432	7	.7378	30	.2622	20	40	.7957	16	.8925	10	.9032	26	.0968	20
50	.6833	23	.9425	7	.7408	30	.2592	10	50	.7973	16	.8915	10	.9058	26	.0942	10
29 0	.6856	22	.9418	7	.7438	29	.2562	61 0	39 0	.7989	15	.8905	10	.9084	26	.0916	51 0
10	.6878	23	.9411	7	.7467	30	.2533	50	10	.8004	16	.8895	11	.9110	25	.0890	50
20	.6901	22	.9404	7	.7497	29	.2503	40	20	.8020	15	.8884	10	.9135	26	.0865	40
30	.6923	23	.9397	7	.7526	30	.2474	30	30	.8035	15	.8874	10	.9161	26	.0839	30
40	.6946	22	.9390	7	.7556	29	.2444	20	40	.8050	16	.8864	11	.9187	25	.0813	20
50	.6968	22	.9383	8	.7585	29	.2415	10	50	.8066	15	.8853	10	.9212	26	.0788	10
30 0	.6990	22	.9375	7	.7614	30	.2386	60 0	40 0	.8081	15	.8843	11	.9238	26	.0762	50 0
10	.7012	21	.9368	7	.7644	29	.2356	50	10	.8096	15	.8832	11	.9264	25	.0736	50
20	.7033	22	.9361	8	.7673	28	.2327	40	20	.8111	14	.8821	11	.9289	26	.0711	40
30	.7055	21	.9353	7	.7701	29	.2299	30	30	.8125	15	.8810	10	.9315	26	.0685	30
40	.7076	21	.9346	8	.7730	29	.2270	20	40	.8140	15	.8800	11	.9341	25	.0659	20
50	.7097	21	.9338	7	.7759	29	.2241	10	50	.8155	14	.8789	11	.9366	26	.0634	10
31 0	.7118	21	.9331	8	.7788	28	.2212	59 0	41 0	.8169	15	.8778	11	.9392	25	.0608	49 0
10	.7139	21	.9323	8	.7816	29	.2184	50	10	.8184	14	.8767	11	.9417	26	.0583	50
20	.7160	21	.9315	7	.7845	28	.2155	40	20	.8198	15	.8756	11	.9443	25	.0557	40
30	.7181	20	.9308	8	.7873	29	.2127	30	30	.8213	14	.8745	12	.9468	26	.0532	30
40	.7201	21	.9300	8	.7902	28	.2098	20	40	.8227	14	.8733	11	.9494	25	.0506	20
50	.7222	20	.9292	8	.7930	28	.2070	10	50	.8241	14	.8722	11	.9519	25	.0481	10
32 0	.7242	20	.9284	8	.7958	28	.2042	58 0	42 0	.8255	14	.8711	12	.9544	26	.0456	48 0
10	.7262	20	.9276	8	.7986	28	.2014	50	10	.8269	14	.8699	11	.9570	25	.0430	50
20	.7282	20	.9268	8	.8014	28	.1986	40	20	.8283	14	.8688	12	.9595	26	.0405	40
30	.7302	20	.9260	8	.8042	28	.1958	30	30	.8297	14	.8676	11	.9621	25	.0379	30
40	.7322	20	.9252	8	.8070	27	.1930	20	40	.8311	13	.8665	12	.9646	25	.0354	20
50	.7342	19	.9244	8	.8097	28	.1903	10	50	.8324	14	.8653	12	.9671	26	.0329	10
33 0	.7361	19	.9236	8	.8125	28	.1875	57 0	43 0	.8338	13	.8641	12	.9697	25	.0303	47 0
10	.7380	20	.9228	9	.8153	27	.1847	50	10	.8351	14	.8629	11	.9722	25	.0278	50
20	.7400	19	.9219	8	.8180	28	.1820	40	20	.8365	13	.8618	12	.9747	25	.0253	40
30	.7419	19	.9211	8	.8208	27	.1792	30	30	.8378	13	.8606	12	.9772	26	.0228	30
40	.7438	19	.9203	9	.8235	28	.1765	20	40	.8391	14	.8594	12	.9798	25	.0202	20
50	.7457	19	.9194	8	.8263	27	.1737	10	50	.8405	13	.8582	13	.9823	25	.0177	10
34 0	.7476	18	.9186	9	.8390	27	.1710	56 0	44 0	.8418	13	.8569	12	.9848	26	.0152	46 0
10	.7494	19	.9177	8	.8317	27	.1683	50	10	.8431	13	.8557	12	.9874	25	.0126	50
20	.7513	18	.9169	9	.8344	27	.1656	40	20	.8444	13	.8545	13	.9899	25	.0101	40
30	.7531	19	.9160	9	.8371	27	.1629	30	30	.8457	12	.8532	12	.9924	25	.0076	30
40	.7550	18	.9151	9	.8398	27	.1602	20	40	.8469	13	.8520	12	.9949	26	.0051	20
50	.7568	18	.9142	8	.8425	27	.1575	10	50	.8482	13	.8507	13	9.9975	25	.0025	10
35 0	9.7586	18	9.9134	9	9.8452	27	0.1548	55 0	45 0	9.8495		9.8495		0.0000		0.0000	45 0
Arc	Cos.	Df	Sin.	Df	Cot.	Df	Tan.	Arc	Arc	Cos.	Df	Sin.	Df	Cot.	Df	Tan.	Arc

TABLE V [1]

HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS. § 218

Minutes.	0°		1°		2°		3°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	100.00	0.00	99.97	1.74	99.88	3.49	99.73	5.23
2	100.00	0.06	99.97	1.80	99.87	3.55	99.72	5.28
4	100.00	0.12	99.97	1.86	99.87	3.60	99.71	5.34
6	100.00	0.17	99.96	1.92	99.87	3.66	99.71	5.40
8	100.00	0.23	99.96	1.98	99.86	3.72	99.70	5.46
10	100.00	0.29	99.96	2.04	99.86	3.78	99.69	5.52
12	100.00	0.35	99.96	2.09	99.85	3.84	99.69	5.57
14	100.00	0.41	99.95	2.15	99.85	3.90	99.68	5.63
16	100.00	0.47	99.95	2.21	99.84	3.95	99.68	5.69
18	100.00	0.52	99.95	2.27	99.84	4.01	99.67	5.75
20	100.00	0.58	99.95	2.33	99.83	4.07	99.66	5.80
22	100.00	0.64	99.94	2.38	99.83	4.13	99.66	5.86
24	100.00	0.70	99.94	2.44	99.82	4.18	99.65	5.92
26	99.99	0.76	99.94	2.50	99.82	4.24	99.64	5.98
28	99.99	0.81	99.93	2.56	99.81	4.30	99.63	6.04
30	99.99	0.87	99.93	2.62	99.81	4.36	99.63	6.09
32	99.99	0.93	99.93	2.67	99.80	4.42	99.62	6.15
34	99.99	0.99	99.93	2.73	99.80	4.48	99.62	6.21
36	99.99	1.05	99.92	2.79	99.79	4.53	99.61	6.27
38	99.99	1.11	99.92	2.85	99.79	4.59	99.60	6.33
40	99.99	1.16	99.92	2.91	99.78	4.65	99.59	6.38
42	99.99	1.22	99.91	2.97	99.78	4.71	99.59	6.44
44	99.98	1.28	99.91	3.02	99.77	4.76	99.58	6.50
46	99.98	1.34	99.90	3.08	99.77	4.82	99.57	6.56
48	99.98	1.40	99.90	3.14	99.76	4.88	99.56	6.61
50	99.98	1.45	99.90	3.20	99.76	4.94	99.56	6.67
52	99.98	1.51	99.89	3.26	99.75	4.99	99.55	6.73
54	99.98	1.57	99.89	3.31	99.74	5.05	99.54	6.78
56	99.97	1.63	99.89	3.37	99.74	5.11	99.53	6.84
58	99.97	1.69	99.88	3.43	99.73	5.17	99.52	6.90
60	99.97	1.74	99.88	3.49	99.73	5.23	99.51	6.96
$c+f=$ 0.75	0.75	0.01	0.75	0.02	0.75	0.03	0.75	0.05
$c+f=$ 1.00	1.00	0.01	1.00	0.03	1.00	0.04	1.00	0.06
$c+f=$ 1.25	1.25	0.02	1.25	0.03	1.25	0.05	1.25	0.08

[1] This table was computed by Mr. Arthur Winslow of the State Geological Survey of Pennylvania, from equations 7 and 8, p. 383.

TABLE V.—*Continued*

HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS

Minutes.	4°		5°		6°		7°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	99.51	6.96	99.24	8.68	98.91	10.40	98.51	12.10
2	99.51	7.02	99.23	8.74	98.90	10.45	98.50	12.15
4	99.50	7.07	99.22	8.80	98.88	10.51	98.48	12.21
6	99.49	7.13	99.21	8.85	98.87	10.57	98.47	12.26
8	99.48	7.19	99.20	8.91	98.86	10.62	98.46	12.32
10	99.47	7.25	99.19	8.97	98.85	10.68	98.44	12.38
12	99.46	7.30	99.18	9.03	98.83	10.74	98.43	12.43
14	99.46	7.36	99.17	9.08	98.82	10.79	98.41	12.49
16	99.45	7.42	99.16	9.14	98.81	10.85	98.40	12.55
18	99.44	7.48	99.15	9.20	98.80	10.91	98.39	12.60
20	99.43	7.53	99.14	9.25	98.78	10.96	98.37	12.66
22	99.42	7.59	99.13	9.31	98.77	11.02	98.36	12.72
24	99.41	7.65	99.11	9.37	98.76	11.08	98.34	12.77
26	99.40	7.71	99.10	9.43	98.74	11.13	98.33	12.83
28	99.39	7.76	99.09	9.48	98.73	11.19	98.31	12.88
30	99.38	7.82	99.08	9.54	98.72	11.25	98.29	12.94
32	99.38	7.88	99.07	9.60	98.71	11.30	98.28	13.00
34	99.37	7.94	99.06	9.65	98.69	11.36	98.27	13.05
36	99.36	7.99	99.05	9.71	98.68	11.42	98.25	13.11
38	99.35	8.05	99.04	9.77	98.67	11.47	98.24	13.17
40	99.34	8.11	99.03	9.83	98.65	11.53	98.22	13.22
42	99.33	8.17	99.01	9.88	98.64	11.59	98.20	13.28
44	99.32	8.22	99.00	9.94	98.63	11.64	98.19	13.33
46	99.31	8.28	98.99	10.00	98.61	11.70	98.17	13.39
48	99.30	8.34	98.98	10.05	98.60	11.76	98.16	13.45
50	99.29	8.40	98.97	10.11	98.58	11.81	98.14	13.50
52	99.28	8.45	98.96	10.17	98.57	11.87	98.13	13.56
54	99.27	8.51	98.94	10.22	98.56	11.93	98.11	13.61
56	99.26	8.57	98.93	10.28	98.54	11.98	98.10	13.67
58	99.25	8.63	98.92	10.34	98.53	12.04	98.08	13.73
60	99.24	8.68	98.91	10.40	98.51	12.10	98.06	13.78
$c+f=0.75$	0.75	0.06	0.75	0.07	0.75	0.08	0.74	0.10
$c+f=1.00$	1.00	0.08	0.99	0.09	0.99	0.11	0.99	0.13
$c+f=1.25$	1.25	0.10	1.24	0.11	1.24	0.14	1.24	0.16

TABLE V.—*Continued*

HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS

Minutes.	8°		9°		10°		11°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Dift. Elev.
0...	98.06	13.78	97.55	15.45	96.98	17.10	96.36	18.73
2...	98.05	13.84	97.53	15.51	96.96	17.16	96.34	18.78
4...	98.03	13.89	97.52	15.56	96.94	17.21	96.32	18.84
6...	98.01	13.95	97.50	15.62	96.92	17.26	96.29	18.89
8...	98.00	14.01	97.48	15.67	96.90	17.32	96.27	18.95
10...	97.98	14.06	97.46	15.73	96.88	17.37	96.25	19.00
12...	97.97	14.12	97.44	15.78	96.86	17.43	96.23	19.05
14...	97.95	14.17	97.43	15.84	96.84	17.48	96.21	19.11
16...	97.93	14.23	97.41	15.89	96.82	17.54	96.18	19.16
18...	97.92	14.28	97.39	15.95	96.80	17.59	96.16	19.21
20...	97.90	14.34	97.37	16.00	96.78	17.65	96.14	19.27
22...	97.88	14.40	97.35	16.06	96.76	17.70	96.12	19.32
24...	97.87	14.45	97.33	16.11	96.74	17.76	96.09	19.38
26...	97.85	14.51	97.31	16.17	96.72	17.81	96.07	19.43
28...	97.83	14.56	97.29	16.22	96.70	17.86	96.05	19.48
30...	97.82	14.62	97.28	16.28	96.68	17.92	96.03	19.54
32...	97.80	14.67	97.26	16.33	96.66	17.97	96.00	19.59
34...	97.78	14.73	97.24	16.39	96.64	18.03	95.98	19.64
36...	97.76	14.79	97.22	16.44	96.62	18.08	95.96	19.70
38...	97.75	14.84	97.20	16.50	96.60	18.14	95.93	19.75
40...	97.73	14.90	97.18	16.55	96.57	18.19	95.91	19.80
42...	97.71	14.95	97.16	16.61	96.55	18.24	95.89	19.86
44...	97.69	15.01	97.14	16.66	96.53	18.30	95.86	19.91
46...	97.68	15.06	97.12	16.72	96.51	18.35	95.84	19.96
48...	97.66	15.12	97.10	16.77	96.49	18.41	95.82	20.02
50...	97.64	15.17	97.08	16.83	96.47	18.46	95.79	20.07
52...	97.62	15.23	97.06	16.88	96.45	18.51	95.77	20.12
54...	97.61	15.28	97.04	16.94	96.42	18.57	95.75	20.18
56...	97.59	15.34	97.02	16.99	96.40	18.62	95.72	20.23
58...	97.57	15.40	97.00	17.05	96.38	18.68	95.70	20.28
60...	97.55	15.45	96.98	17.10	96.36	18.73	95.68	20.34
$+f=$ 0.75	0.74	0.11	0.74	0.12	0.74	0.14	0 73	0.15
$+f=$ 1.00	0.99	0.15	0.99	0.16	0.98	0.18	0.98	0.20
$+f=$ 1.25	1.23	0.18	1.23	0.21	1.23	0.23	1.22	0.25

TABLE V.—*Continued*

HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS

Minutes.	12°		13°		14°		15°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0...	95.68	20.34	94.94	21.92	94.15	23.47	93.30	25.00
2...	95.65	20.39	94.91	21.97	94.12	23.52	93.27	25.05
4...	95.63	20.44	94.89	22.02	94.09	23.58	93.24	25.10
6...	95.61	20.50	94.86	22.08	94.07	23.63	93.21	25.15
8...	95.58	20.55	94.84	22.13	94.04	23.68	93.18	25.20
10...	95.56	20.60	94.81	22.18	94.01	23.73	93.16	25.25
12...	95.53	20.66	94.79	22.23	93.98	23.78	93.13	25.30
14...	95.51	20.71	94.76	22.28	93.95	23.83	93.10	25.35
16...	95.49	20.76	94.73	22.34	93.93	23.88	93.07	25.40
18...	95.46	20.81	94.71	22.39	93.90	23.93	93.04	25.45
20...	95.44	20.87	94.68	22.44	93.87	23.99	93.01	25.50
22...	95.41	20.92	94.66	22.49	93.84	24.04	92.98	25.55
24...	95.39	20.97	94.63	22.54	93.81	24.09	92.95	25.60
26...	95.36	21.03	94.60	22.60	93.79	24.14	92.92	25.65
28...	95.34	21.08	94.58	22.65	93.76	24.19	92.89	25.70
30...	95.32	21.13	94.55	22.70	93.73	24.24	92.86	25.75
32...	95.29	21.18	94.52	22.75	93.70	24.29	92.83	25.80
34...	95.27	21.24	94.50	22.80	93.67	24.34	92.80	25.85
36...	95.24	21.29	94.47	22.85	93.65	24.39	92.77	25.90
38...	95.22	21.34	94.44	22.91	93.62	24.44	92.74	25.95
40...	95.19	21.39	94.42	22.96	93.59	24.49	92.71	26.00
42...	95.17	21.45	94.39	23.01	93.56	24.55	92.68	26.05
44...	95.14	21.50	94.36	23.06	93.53	24.60	92.65	26.10
46...	95.12	21.55	94.34	23.11	93.50	24.65	92.62	26.15
48...	95.09	21.60	94.31	23.16	93.47	24.70	92.59	26.20
50...	95.07	21.66	94.28	23.22	93.45	24.75	92.56	26.25
52...	95.04	21.71	94.26	23.27	93.42	24.80	92.53	26.30
54...	95.02	21.76	94.23	23.32	93.39	24.85	92.49	26.35
56...	94.99	21.81	94.20	23.37	93.36	24.90	92.46	26.40
58...	94.97	21.87	94.17	23.42	93.33	24.95	92.43	26.45
60...	94.94	21.92	94.15	23.47	93.30	25.00	92.40	26.50
$c+f=$ 0.75	0.73	0.16	0.73	0.17	0.73	0.19	0.72	0.20
$c+f=$ 1.00	0.98	0.22	0.97	0.23	0.97	0.25	0.96	0.27
$c+f=$ 1.25	1.22	0.27	1.21	0.29	1.21	0.31	1.20	0.34

TABLE V.—*Continued*

HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS

Minutes.	16°		17°		18°		19°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0...	92.40	26.50	91.45	27.96	90.45	29.39	89.40	30.78
2...	92.37	26.55	91.42	28.01	90.42	29.44	89.36	30.83
4...	92.34	26.59	91.39	28.06	90.38	29.48	89.33	30.87
6...	92.31	26.64	91.35	28.10	90.35	29.53	89.29	30.92
8...	92.28	26.69	91.32	28.15	90.31	29.58	89.26	30.97
10...	92.25	26.74	91.29	28.20	90.28	29.62	89.22	31.01
12...	92.22	26.79	91.26	28.25	90.24	29.67	89.18	31.06
14...	92.19	26.84	91.22	28.30	90.21	29.72	89.15	31.10
16...	92.15	26.89	91.19	28.34	90.18	29.76	89.11	31.15
18...	92.12	26.94	91.16	28.39	90.14	29.81	89.08	31.19
20...	92.09	26.99	91.12	28.44	90.11	29.86	89.04	31.24
22...	92.06	27.04	91.09	28.49	90.07	29.90	89.00	31.28
24...	92.03	27.09	91.06	28.54	90.04	29.95	88.96	31.33
26...	92.00	27.13	91.02	28.58	90.00	30.00	88.93	31.38
28...	91.97	27.18	90.99	28.63	89.97	30.04	88.89	31.42
30...	91.93	27.23	90.96	28.68	89.93	30.09	88.86	31.47
32...	91.90	27.28	90.92	28.73	89.90	30.14	88.82	31.51
34...	91.87	27.33	90.89	28.77	89.86	30.19	88.78	31.56
36...	91.84	27.38	90.86	28.82	89.83	30.23	88.75	31.60
38...	91.81	27.43	90.82	28.87	89.79	30.28	88.71	31.65
40...	91.77	27.48	90.79	28.92	89.76	30.32	88.67	31.69
42...	91.74	27.52	90.76	28.96	89.72	30.37	88.64	31.74
44...	91.71	27.57	90.72	29.01	89.69	30.41	88.60	31.78
46...	91.68	27.62	90.69	29.06	89.65	30.46	88.56	31.83
48...	91.65	27.67	90.66	29.11	89.61	30.51	88.53	31.87
50...	91.61	27.72	90.62	29.15	89.58	30.55	88.49	31.92
52...	91.58	27.77	90.59	29.20	89.54	30.60	88.45	31.96
54...	91.55	27.81	90.55	29.25	89.51	30.65	88.41	32.01
56...	91.52	27.86	90.52	29.30	89.47	30.69	88.38	32.05
58...	91.48	27.91	90.48	29.34	89.44	30.74	88.34	32.09
60...	91.45	27.96	90.45	29.39	89.40.	30.78	88.30	32.14
$c+f=$ 0.75	0.72	0.21	0.72	0.23	0.71	0.24	0.71	0.25
$c+f=$ 1.00	0.96	0.28	0.95	0.30	0.95	0.32	0.94	0.33
$c+f=$ 1.25	1.20	0.35	1.19	0.38	1.19	0.40	1.18	0.42

TABLE V.—*Continued*

HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS

Minutes.	20°		21°		22°		23°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0...	88.30	32.14	87.16	33.46	85.97	34.73	84.73	35.97
2...	88.26	32.18	87.12	33.50	85.93	34.77	84.69	36.01
4...	88.23	32.23	87.08	33.54	85.89	34.82	84.65	36.05
6...	88.19	32.27	87.04	33.59	85.85	34.86	84.61	36.09
8...	88.15	32.32	87.00	33.63	85.80	34.90	84.57	36.13
10...	88.11	32.36	86.96	33.67	85.76	34.94	84.52	36.17
12...	88.08	32.41	86.92	33.72	85.72	34.98	84.48	36.21
14...	88.04	32.45	86.88	33.76	85.68	35.02	84.44	36.25
16...	88.00	32.49	86.84	33.80	85.64	35.07	84.40	36.29
18...	87.96	32.54	86.80	33.84	85.60	35.11	84.35	36.33
20...	87.93	32.58	86.77	33.89	85.56	35.15	84.31	36.37
22...	87.89	32.63	86.73	33.93	85.52	35.19	84.27	36.41
24...	87.85	32.67	86.69	33.97	85.48	35.23	84.23	36.45
26...	87.81	32.72	86.65	34.01	85.44	35.27	84.18	36.49
28...	87.77	32.76	86.61	34.06	85.40	35.31	84.14	36.53
30...	87.74	32.80	86.57	34.10	85.36	35.36	84.10	36.57
32...	87.70	32.85	86.53	34.14	85.31	35.40	84.06	36.61
34...	87.66	32.89	86.49	34.18	85.27	35.44	84.01	36.65
36...	87.62	32.93	86.45	34.23	85.23	35.48	83.97	36.69
38...	87.58	32.98	86.41	34.27	85.19	35.52	83.93	36.73
40...	87.54	33.02	86.37	34.31	85.15	35.56	83.89	36.77
42...	87.51	33.07	86.33	34.35	85.11	35.60	83.84	36.80
44...	87.47	33.11	86.29	34.40	85.07	35.64	83.80	36.84
46...	87.43	33.15	86.25	34.44	85.02	35.68	83.76	36.88
48...	87.39	33.20	86.21	34.48	84.98	35.72	83.72	36.92
50...	87.35	33.24	86.17	34.52	84.94	35.76	83.67	36.96
52...	87.31	33.28	86.13	34.57	84.90	35.80	83.63	37.00
54...	87.27	33.33	86.09	34.61	84.86	35.85	83.59	37.04
56...	87.24	33.37	86.05	34.65	84.82	35.89	83.54	37.08
58...	87.20	33.41	86.01	34.69	84.77	35.93	83.50	37.12
60..	87.16	33.46	85.97	34.73	84.73	35.97	83.46	37.16
$c+f=$ 0.75	0.70	0.26	0.70	0.27	0.69	0.29	0.69	0.30
$c+f=$ 1.00	0.94	0.35	0.93	0.37	0.92	0.38	0.92	0.40
$c+f=$ 1.25	1.17	0.44	1.16	0.46	1.15	0.48	1.15	0.50

TABLE V.—*Continued*

HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS

Minutes.	24°		25°		26°		27°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0...	83.46	37.16	82.14	38.30	80.78	39.40	79.39	40.45
2...	83.41	37.20	82.09	38.34	80.74	39.44	79.34	40.49
4...	83.37	37.23	82.05	38.38	80.69	39.47	79.30	40.52
6...	83.33	37.27	82.01	38.41	80.65	39.51	79.25	40.55
8...	83.28	37.31	81.96	38.45	80.60	39.54	79.20	40.59
10...	83.24	37.35	81.92	38.49	80.55	39.58	79.15	40.62
12...	83.20	37.39	81.87	38.53	80.51	39.61	79.11	40.66
14...	83.15	37.43	81.83	38.56	80.46	39.65	79.06	40.69
16...	83.11	37.47	81.78	38.60	80.41	39.69	79.01	40.72
18...	83.07	37.51	81.74	38.64	80.37	39.72	78.96	40.76
20...	83.02	37.54	81.69	38.67	80.32	39.76	78.92	40.79
22...	82.98	37.58	81.65	38.71	80.28	39.79	78.87	40.82
24...	82.93	37.62	81.60	38.75	80.23	39.83	78.82	40.86
26...	82.89	37.66	81.56	38.78	80.18	39.86	78.77	40.89
28...	82.85	37.70	81.51	38.82	80.14	39.90	78.73	40.92
30...	82.80	37.74	81.47	38.86	80.09	39.93	78.68	40.96
32...	82.76	37.77	81.42	38.89	80.04	39.97	78.63	40.99
34...	82.72	37.81	81.38	38.93	80.00	40.00	78.58	41.02
36...	82.67	37.85	81.33	38.97	79.95	40.04	78.54	41.06
38...	82.63	37.89	81.28	39.00	79.90	40.07	78.49	41.09
40...	82.58	37.93	81.24	39.04	79.86	40.11	78.44	41.12
42...	82.54	37.96	81.19	39.08	79.81	40.14	78.39	41.16
44...	82.49	38.00	81.15	39.11	79.76	40.18	78.34	41.19
46...	82.45	38.04	81.10	39.15	79.72	40.21	78.30	41.22
48...	82.41	38.08	81.06	39.18	79.67	40.24	78.25	41.26
50...	82.36	38.11	81.01	39.22	79.62	40.28	78.20	41.29
52...	82.32	38.15	80.97	39.26	79.58	40.31	78.15	41.32
54...	82.27	38.19	80.92	39.29	79.53	40.35	78.10	41.35
56...	82.23	38.23	80.87	39.33	79.48	40.38	78.06	41.39
58...	82.18	38.26	80.83	39.36	79.44	40.42	78.01	41.42
60...	82.14	38.30	80.78	39.40	79.39	40.45	77.96	41.45
$+f=$ 0.75	0.68	0.31	0.68	0.32	0.67	0.33	0.66	0.35
$+f=$ 1.00	0.91	0.41	0.90	0.43	0.89	0.45	0.89	0.46
$+f=$ 1.25	1.14	0.52	1.13	0.54	1.12	0.56	1.11	0.58

TABLE V.—*Continued*

HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS

Minutes.	28°		29°		30°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	77.96	41.45	76.50	42.40	75.00	43.30
2	77.91	41.48	76.45	42.43	74.95	43.33
4	77.86	41.52	76.40	42.46	74.90	43.36
6	77.81	41.55	76.35	42.49	74.85	43.39
8	77.77	41.58	76.30	42.53	74.80	43.42
10	77.72	41.61	76.25	42.56	74.75	43.45
12	77.67	41.65	76.20	42.59	74.70	43.47
14	77.62	41.68	76.15	42.62	74.65	43.50
16	77.57	41.71	76.10	42.65	74.60	43.53
18	77.52	41.74	76.05	42.68	74.55	43.56
20	77.48	41.77	76.00	42.71	74.49	43.59
22	77.42	41.81	75.95	42.74	74.44	43.62
24	77.38	41.84	75.90	42.77	74.39	43.65
26	77.33	41.87	75.85	42.80	74.34	43.67
28	77.28	41.90	75.80	42.83	74.29	43.70
30	77.23	41.93	75.75	42.86	74.24	43.73
32	77.18	41.97	75.70	42.89	74.19	43.76
34	77.13	42.00	75.65	42.92	74.14	43.79
36	77.09	42.03	75.60	42.95	74.09	43.82
38	77.04	42.06	75.55	42.98	74.04	43.84
40	76.99	42.09	75.50	43.01	73.99	43.87
42	76.94	42.12	75.45	43.04	73.93	43.90
44	76.89	42.15	75.40	43.07	73.88	43.93
46	76.84	42.19	75.35	43.10	73.83	43.95
48	76.79	42.22	75.30	43.13	73.78	43.98
50	76.74	42.25	75.25	43.16	73.73	44.01
52	76.69	42.28	75.20	43.18	73.68	44.04
54	76.64	42.31	75.15	43.21	73.63	44.07
56	76.59	42.34	75.10	43.24	73.58	44.09
58	76.55	42.37	75.05	43.27	73.52	44.12
60	76.50	42.40	75.00	43.30	73.47	44.15
$c+f=0.75$	0.66	0.36	0.65	0.37	0.65	0.38
$c+f=1.00$	0.88	0.48	0.87	0.49	0.86	0.51
$c+f=1.25$	1.10	0.60	1.09	0.62	1.08	0.64

TABLE VI

Natural Sines and Cosines

′	0°		1°		2°		3°		4°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.00000	One.	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	One.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	One.	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.00087	One.	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.00116	One.	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.00145	One.	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.00175	One.	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.00204	One.	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.00233	One.	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.00262	One.	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.00291	One.	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.00727	.99997	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	.99997	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00844	.99996	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.01018	.99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.01193	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.01542	.99988	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.01600	.99987	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.01629	.99987	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	89°		88°		87°		86°		85°		

TABLE VI.—*Continued*

Natural Sines and Cosines

′	5°		6°		7°		8°		9°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	.15672	.98764	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	.15701	.98760	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	.15730	.98755	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	.15758	.98751	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	.15787	.98746	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	.15816	.98741	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	.15845	.98737	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	.15873	.98732	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	.15902	.98728	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	.15931	.98723	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	.15959	.98718	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	.15988	.98714	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	.16017	.98709	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	.16046	.98704	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	.16074	.98700	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	.16103	.98695	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	.16132	.98690	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	.16160	.98686	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	.16189	.98681	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	.16218	.98676	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	.16246	.98671	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	.16275	.98667	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	.16304	.98662	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	.16333	.98657	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	.16361	.98652	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	.16390	.98648	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	.16419	.98643	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	.16447	.98638	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	.16476	.98633	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	.16505	.98629	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	.16533	.98624	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	.16562	.98619	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	.16591	.98614	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	.16620	.98609	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	.16648	.98604	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	.16677	.98600	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	.16706	.98595	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	.16734	.98590	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	.16763	.98585	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	.16792	.98580	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	.16820	.98575	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	.16849	.98570	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	.16878	.98565	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	.16906	.98561	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	.16935	.98556	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	.16964	.98551	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	.16992	.98546	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	.17021	.98541	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	.17050	.98536	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	.17078	.98531	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	.17107	.98526	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	.17136	.98521	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	.17164	.98516	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	.17193	.98511	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	.17222	.98506	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	.17250	.98501	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	.17279	.98496	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	.17308	.98491	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	.17336	.98486	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	.17365	.98481	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	84°		83°		82°		81°		80°		

TABLE VI.—*Continued*

NATURAL SINES AND COSINES

′	10°		11°		12°		13°		14°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17708	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.98033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24954	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96829	32
29	.18195	.98331	.19908	.97998	.21616	.97636	.23316	.97244	.25010	.96822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25038	.96815	30
31	.18252	.98320	.19965	.97987	.21672	.97623	.23373	.97230	.25066	.96807	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96800	28
33	.18309	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25122	.96793	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96786	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96778	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96771	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96764	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96756	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.25291	.96749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	20
41	.18538	.98267	.20250	.97928	.21956	.97560	.22656	.97162	.25348	.96734	19
42	.18567	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25376	.96727	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96719	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25432	.96712	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96705	15
46	.18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96697	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96690	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96682	12
49	.18767	.98223	.20478	.97881	.22183	.97508	.23882	.97106	.25573	.96675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25601	.96667	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96660	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96653	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96645	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96638	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96630	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96623	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96615	3
58	.19024	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96608	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	79°		78°		77°		76°		75°		

TABLE VI.—*Continued*

NATURAL SINES AND COSINES

′	15°		16°		17°		18°		19°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.25882	.96593	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	60
1	.25910	.96585	.27592	.96118	.29265	.95622	.30929	.95097	.32584	.94542	59
2	.25938	.96578	.27620	.96110	.29293	.95613	.30957	.95088	.32612	.94533	58
3	.25966	.96570	.27648	.96102	.29321	.95605	.30985	.95079	.32639	.94523	57
4	.25994	.96562	.27676	.96094	.29348	.95596	.31012	.95070	.32667	.94514	56
5	.26022	.96555	.27704	.96086	.29376	.95588	.31040	.95061	.32694	.94504	55
6	.26050	.96547	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54
7	.26079	.96539	.27759	.96070	.29432	.95571	.31095	.95043	.32749	.94485	53
8	.26107	.96532	.27787	.96062	.29460	.95562	.31123	.95033	.32777	.94476	52
9	.26135	.96524	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51
10	.26163	.96517	.27843	.96046	.29515	.95545	.31178	.95015	.32832	.94457	50
11	.26191	.96509	.27871	.96037	.29543	.95536	.31206	.95006	.32859	.94447	49
12	.26219	.96502	.27899	.96029	.29571	.95528	.31233	.94997	.32887	.94438	48
13	.26247	.96494	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47
14	.26275	.96486	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46
15	.26303	.96479	.27983	.96005	.29654	.95502	.31316	.94970	.32969	.94409	45
16	.26331	.96471	.28011	.95997	.29682	.95493	.31344	.94961	.32997	.94399	44
17	.26359	.96463	.28039	.95989	.29710	.95485	.31372	.94952	.33024	.94390	43
18	.26387	.96456	.28067	.95981	.29737	.95476	.31399	.94943	.33051	.94380	42
19	.26415	.96448	.28095	.95972	.29765	.95467	.31427	.94933	.33079	.94370	41
20	.26443	.96440	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40
21	.26471	.96433	.28150	.95956	.29821	.95450	.31482	.94915	.33134	.94351	39
22	.26500	.96425	.28178	.95948	.29849	.95441	.31510	.94906	.33161	.94342	38
23	.26528	.96417	.28206	.95940	.29876	.95433	.31537	.94897	.33189	.94332	37
24	.26556	.96410	.28234	.95931	.29904	.95424	.31565	.94888	.33216	.94322	36
25	.26584	.96402	.28262	.95923	.29932	.95415	.31593	.94878	.33244	.94313	35
26	.26612	.96394	.28290	.95915	.29960	.95407	.31620	.94869	.33271	.94303	34
27	.26640	.96386	.28318	.95907	.29987	.95398	.31648	.94860	.33298	.94293	33
28	.26668	.96379	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32
29	.26696	.96371	.28374	.95890	.30043	.95380	.31703	.94842	.33353	.94274	31
30	.26724	.96363	.28402	.95882	.30071	.95372	.31730	.94832	.33381	.94264	30
31	.26752	.96355	.28429	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29
32	.26780	.96347	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28
33	.26808	.96340	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94235	27
34	.26836	.96332	.28513	.95849	.30182	.95337	.31841	.94795	.33490	.94225	26
35	.26864	.96324	.28541	.95841	.30209	.95328	.31868	.94786	.33518	.94215	25
36	.26892	.96316	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94206	24
37	.26920	.96308	.28597	.95824	.30265	.95310	.31923	.94768	.33573	.94196	23
38	.26948	.96301	.28625	.95816	.30292	.95301	.31951	.94758	.33600	.94186	22
39	.26976	.96293	.28652	.95807	.30320	.95293	.31979	.94749	.33627	.94176	21
40	.27004	.96285	.28680	.95799	.30348	.95284	.32006	.94740	.33655	.94167	20
41	.27032	.96277	.28708	.95791	.30376	.95275	.32034	.94730	.33682	.94157	19
42	.27060	.96269	.28736	.95782	.30403	.95266	.32061	.94721	.33710	.94147	18
43	.27088	.96261	.28764	.95774	.30431	.95257	.32089	.94712	.33737	.94137	17
44	.27116	.96253	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16
45	.27144	.96246	.28820	.95757	.30486	.95240	.32144	.94693	.33792	.94118	15
46	.27172	.96238	.28847	.95749	.30514	.95231	.32171	.94684	.33819	.94108	14
47	.27200	.96230	.28875	.95740	.30542	.95222	.32199	.94674	.33846	.94098	13
48	.27228	.96222	.28903	.95732	.30570	.95213	.32227	.94665	.33874	.94088	12
49	.27256	.96214	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11
50	.27284	.96206	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10
51	.27312	.96198	.28987	.95707	.30653	.95186	.32309	.94637	.33956	.94058	9
52	.27340	.96190	.29015	.95698	.30680	.95177	.32337	.94627	.33983	.94049	8
53	.27368	.96182	.29042	.95690	.30708	.95168	.32364	.94618	.34011	.94039	7
54	.27396	.96174	.29070	.95681	.30736	.95159	.32392	.94609	.34038	.94029	6
55	.27424	.96166	.29098	.95673	.30763	.95150	.32419	.94599	.34065	.94019	5
56	.27452	.96158	.29126	.95664	.30791	.95142	.32447	.94590	.34093	.94009	4
57	.27480	.96150	.29154	.95656	.30819	.95133	.32474	.93580	.34120	.93999	3
58	.27508	.96142	.29182	.95647	.30846	.95124	.32502	.94571	.34147	.93989	2
59	.27536	.96134	.29209	.95639	.30874	.95115	.32529	.94561	.34175	.93979	1
60	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	74°		73°		72°		71°		70°		

TABLE VI.—*Continued*

NATURAL SINES AND COSINES

′	20°		21°		22°		23°		24°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93959	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.34311	.93929	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93909	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.34393	.93899	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93879	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93849	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93839	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93809	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93799	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93779	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93738	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93688	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93677	.36623	.93052	.28241	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93647	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93637	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93626	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.35184	.93606	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	.93596	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.35266	.93575	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93555	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93514	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93503	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.35481	.93493	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93431	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93420	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.35701	.93410	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.35728	.93400	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93389	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93379	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93368	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.35837	.93358	.37461	.92718	.39072	.92050	.40674	.91355	.42262	.90631	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	69°		68°		67°		66°		65°		

TABLE VI.—*Continued*

Natural Sines and Cosines

′	25°		26°		27°		28°		29°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	.48506	.87448	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	.48532	.87434	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	.48557	.87420	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	.48583	.87406	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	.48608	.87391	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	.48634	.87377	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	.48659	.87363	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	.48684	.87349	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	.48710	.87335	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	.48735	.87321	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	.48761	.87306	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	.48786	.87292	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	.48811	.87278	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	.48837	.87264	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	.48862	.87250	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	.48888	.87235	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	.48913	.87221	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	.48938	.87207	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	.48964	.87193	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	.48989	.87178	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	.49014	.87164	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	.49040	.87150	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	.49065	.87136	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	.49090	.87121	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	.49116	.87107	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	.49141	.87093	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	.49166	.87079	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	.49192	.87064	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	.49217	.87050	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	.49242	.87036	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	.49268	.87021	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	.49293	.87007	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	.49318	.86993	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	.49344	.86978	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	.49369	.86964	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	.49394	.86949	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	.49419	.86935	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	.49445	.86921	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	.49470	.86906	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	.49495	.86892	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	.49521	.86878	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	.49546	.86863	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	.49571	.86849	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	.49596	.86834	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	.49622	.86820	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	.49647	.86805	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	.49672	.86791	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	.49697	.86777	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	.49723	.86762	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	.49748	.86748	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	.49773	.86733	9
52	.43628	.89981	.45192	.89206	.46742	.88404	.48277	.87575	.49798	.86719	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	.49824	.86704	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	.49849	.86690	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	.49874	.86675	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	.49899	.86661	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	.49924	.86646	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	.49950	.86632	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	.49975	.86617	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	.50000	.86603	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	64°		63°		62°		61°		60°		

TABLE VI.—*Continued*

NATURAL SINES AND COSINES

′	30°		31°		32°		33°		34°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.34759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82822	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86427	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86413	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83645	.56256	.82675	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82659	45
16	.50403	.86369	.51902	.85476	.53386	.84557	.54854	.83613	.56305	.82643	44
17	.50428	.86354	.51927	.85461	.53411	.84542	.54878	.83597	.56329	.82626	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82610	42
19	.50478	.86325	.51977	.85421	.53460	.84511	.54927	.83565	.56377	.82593	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	.50528	.86295	.52026	.85401	.53509	.84480	.54975	.83533	.56425	.82561	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.51999	.83517	.56449	.82544	38
23	.50578	.86266	.52076	.85370	.53558	.84448	.55024	.83501	.56473	.82528	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82511	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82495	35
26	.50654	.86222	.52151	.85325	.53632	.84402	.55097	.83453	.56545	.82478	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82462	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82446	32
29	.50729	.86178	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
31	.50779	.86148	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82396	29
32	.50804	.86133	.52299	.85234	.53779	.84308	.55242	.83356	.56689	.82380	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83340	.56713	.82363	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82347	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82330	25
36	.50904	.86074	.52399	.85173	.53877	.84245	.55339	.83292	.56784	.82314	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82297	23
38	.50954	.86045	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82281	22
39	.50979	.86030	.52473	.85127	.53951	.84198	.55412	.83244	.56856	.82264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55460	.83212	.56904	.82231	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55484	.83195	.56928	.82214	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	.56952	.82198	17
44	.51104	.85956	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82181	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82165	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82148	14
47	.51179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82132	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55630	.83098	.57071	.82115	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55654	.83082	.57095	.82098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55702	.83050	.57143	.82065	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55726	.83034	.57167	.82048	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83017	.57191	.82032	7
54	.51354	.85806	.52844	.84897	.54317	.83962	.55775	.83001	.57215	.82015	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82985	.57238	.81999	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82969	.57262	.81982	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82953	.57286	.81965	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	.82936	.57310	.81949	2
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	59°		58°		57°		56°		55°		

TABLE VI.—*Continued*

NATURAL SINES AND COSINES

′	35°		36°		37°		38°		39°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60
1	.57381	.81899	.58802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59
2	.57405	.81882	.58826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58
3	.57429	.81865	.58849	.80850	.60251	.79811	.61635	.78747	.63000	.77660	57
4	.57453	.81848	.58873	.80833	.60274	.79793	.61658	.78729	.63022	.77641	56
5	.57477	.81832	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55
6	.57501	.81815	.58920	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54
7	.57524	.81798	.58943	.80782	.60344	.79741	.61726	.78676	.63090	.77586	53
8	.57548	.81782	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52
9	.57572	.81765	.58990	.80748	.60390	.79706	.61772	.78640	.63135	.77550	51
10	.57596	.81748	.59014	.80730	.60414	.79688	.61795	.78622	.63158	.77531	50
11	.57619	.81731	.59037	.80713	.60437	.79671	.61818	.78604	.63180	.77513	40
12	.57643	.81714	.59061	.80696	.60460	.79653	.61841	.78586	.63203	.77494	48
13	.57667	.81698	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47
14	.57691	.81681	.59108	.80662	.60506	.79618	.61887	.78550	.63248	.77458	46
15	.57715	.81664	.59131	.80644	.60529	.79600	.61909	.78532	.63271	.77439	45
16	.57738	.81647	.59154	.80627	.60553	.79583	.61932	.78514	.63293	.77421	44
17	.57762	.81631	.59178	.80610	.60576	.79565	.61955	.78496	.63316	.77402	43
18	.57786	.81614	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42
19	.57810	.81597	.59225	.80576	.60622	.79530	.62001	.78460	.63361	.77366	41
20	.57833	.81580	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	.57857	.81563	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39
22	.57881	.81546	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.77310	38
23	.57904	.81530	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37
24	.57928	.81513	.59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36
25	.57952	.81496	.59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35
26	.57976	.81479	.59389	.80455	.60784	.79406	.62160	.78333	.63518	.77236	34
27	.57999	.81462	.59412	.80438	.60807	.79388	.62183	.78315	.63540	.77218	33
28	.58023	.81445	.59436	.80420	.60830	.79371	.62206	.78297	.63563	.77199	32
29	.58047	.81428	.59459	.80403	.60853	.79353	.62229	.78279	.63585	.77181	31
30	.58070	.81412	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	.58094	.81395	.59506	.80368	.60899	.79318	.62274	.78243	.63630	.77144	29
32	.58118	.81378	.59529	.80351	.60922	.79300	.62297	.78225	.63653	.77125	28
33	.58141	.81361	.59552	.80334	.60945	.79282	.62320	.78206	.63675	.77107	27
34	.58165	.81344	.59576	.80316	.60968	.79264	.62342	.78188	.63698	.77088	26
35	.58189	.81327	.59599	.80299	.60991	.79247	.62365	.78170	.63720	.77070	25
36	.58212	.82310	.59622	.80282	.61015	.79229	.62388	.78152	.63742	.77051	24
37	.58236	.81293	.59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23
38	.58260	.81276	.59669	.80247	.61061	.79193	.62433	.78116	.63787	.77014	22
39	.58283	.81259	.59693	.80230	.61084	.79176	.62456	.78098	.63810	.76996	21
40	.58307	.81242	.59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	.58330	.81225	.59739	.80195	.61130	.79140	.62502	.78061	.63854	.76959	19
42	.58354	.81208	.59763	.80178	.61153	.79122	.62524	.78043	.63877	.76940	18
43	.58378	.81191	.59786	.80160	.61176	.79105	.62547	.78025	.63899	.76921	17
44	.58401	.81174	.59809	.80143	.61199	.79087	.62570	.78007	.63922	.76903	16
45	.58425	.81157	.59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15
46	.58449	.81140	.59856	.80108	.61245	.79051	.62615	.77970	.63966	.76866	14
47	.58472	.81123	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13
48	.58496	.81106	.59902	.80073	.61291	.79016	.62660	.77934	.64011	.76828	12
49	.58519	.81089	.59926	.80056	.61314	.78998	.62683	.77916	.64033	.76810	11
50	.58543	.81072	.59949	.80038	.61337	.78980	.62706	.77897	.64056	.76791	10
51	.58567	.81055	.59972	.80021	.61360	.78962	.62728	.77879	.64078	.76772	9
52	.58590	.81038	.59995	.80003	.61383	.78944	.62751	.77861	.64100	.76754	8
53	.58614	.81021	.60019	.79986	.61406	.78926	.62774	.77843	.64123	.76735	7
54	.58637	.81004	.60042	.79968	.61429	.78908	.62796	.77824	.64145	.76717	6
55	.58661	.80987	.60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5
56	.58684	.80970	.60089	.79934	.61474	.78873	.62842	.77788	.64190	.76679	4
57	.58708	.80953	.60112	.79916	.61497	.78855	.62864	.77769	.64212	.76661	3
58	.58731	.80936	.60135	.79899	.61520	.78837	.62887	.77751	.64234	.76642	2
59	.58755	.80919	.60158	.79881	.61543	.78819	.62909	.77733	.64256	.76623	1
60	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	54°		53°		52°		51°		50°		

TABLE VI.—*Continued*

NATURAL SINES AND COSINES

′	40°		41°		42°		43°		44°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.64279	.76604	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.64323	.76567	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.64346	.76548	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.64368	.76530	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.64390	.76511	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.64457	.76455	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.64524	.76398	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.64546	.76380	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.64590	.76342	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.64635	.76304	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.64657	.76286	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.64679	.76267	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.64746	.76210	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.64768	.76192	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.64790	.76173	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.64812	.76154	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.64834	.76135	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.64856	.76116	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.64923	.76059	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.64967	.76022	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.64989	.76003	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.65033	.75965	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.65077	.75927	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.65100	.75908	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.65122	.75889	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.65144	.75870	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.65166	.75851	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.65210	.75813	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.65232	.75794	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.65254	.75775	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.65276	.75756	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.65298	.75738	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.65320	.75719	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.65342	.75700	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.65364	.75680	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.65430	.75623	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.65474	.75585	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.65540	.75528	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.65562	.75509	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.65584	.75490	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	49°		48°		47°		46°		45°		

TABLE VII

Natural Tangents and Cotangents

′	0°		1°		2°		3°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.00000	Infinite.	.01746	57.2900	.03492	28.6363	.05241	19.0811	60
1	.00029	3437.75	.01775	56.3506	.03521	28.3994	.05270	18.9755	59
2	.00058	1718.87	.01804	55.4415	.03550	28.1664	.05299	18.8711	58
3	.00087	1145.92	.01833	54.5613	.03579	27.9372	.05328	18.7678	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.0750	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	.05562	17.9802	49
12	.00349	286.478	.02095	47.7395	.03842	26.0307	.05591	17.8863	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	.05620	17.7934	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	.05649	17.7015	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	42
19	.00553	180.932	.02298	43.5081	.04046	24.7185	.05795	17.2558	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	.05854	17.0837	39
22	.00640	156.259	.02386	41.9158	.04133	24.1957	.05883	16.9990	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	.06029	16.5874	33
28	.00815	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15.6762	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	17
44	.01280	78.1263	.03026	33.0452	.04774	20.9460	.06525	15.3254	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15.2571	15
46	.01338	74.7292	.03084	32.4213	.04833	20.6932	.06584	15.1893	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	.06759	14.7954	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	.06817	14.6685	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14.4823	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	89°		88°		87°		86°		

TABLE VII.—*Continued*

NATURAL TANGENTS AND COTANGENTS

′	4°		5°		6°		7°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.06993	14.3007	.08749	11.4301	.10510	9.51436	.12278	8.14435	60
1	.07022	14.2411	.08778	11.3919	.10540	9.48781	.12308	8.12481	59
2	.07051	14.1821	.08807	11.3540	.10569	9.46141	.12338	8.10536	58
3	.07080	14.1235	.08837	11.3163	.10599	9.43515	.12367	8.08600	57
4	.07110	14.0655	.08866	11.2789	.10628	9.40904	.12397	8.06674	56
5	.07139	14.0079	.08895	11.2417	.10657	9.38307	.12426	8.04756	55
6	.07168	13.9507	.08925	11.2048	.10687	9.35724	.12456	8.02848	54
7	.07197	13.8940	.08954	11.1681	.10716	9.33155	.12485	8.00948	53
8	.07227	13.8378	.08983	11.1316	.10746	9.30599	.12515	7.99058	52
9	.07256	13.7821	.09013	11.0954	.10775	9.28058	.12544	7.97176	51
10	.07285	13.7267	.09042	11.0594	.10805	9.25530	.12574	7.95302	50
11	.07314	13.6719	.09071	11.0237	.10834	9.23016	.12603	7.93438	49
12	.07344	13.6174	.09101	10.9882	.10863	9.20516	.12633	7.91582	48
13	.07373	13.5634	.09130	10.9529	.10893	9.18028	.12662	7.89734	47
14	.07402	13.5098	.09159	10.9178	.10922	9.15554	.12692	7.87895	46
15	.07431	13.4566	.09189	10.8829	.10952	9.13093	.12722	7.86064	45
16	.07461	13.4039	.09218	10.8483	.10981	9.10646	.12751	7.84242	44
17	.07490	13.3515	.09247	10.8139	.11011	9.08211	.12781	7.82428	43
18	.07519	13.2996	.09277	10.7797	.11040	9.05789	.12810	7.80622	42
19	.07548	13.2480	.09306	10.7457	.11070	9.03379	.12840	7.78825	41
20	.07578	13.1969	.09335	10.7119	.11099	9.00983	.12869	7.77035	40
21	.07607	13.1461	.09365	10.6783	.11128	8.98598	.12899	7.75254	39
22	.07636	13.0958	.09394	10.6450	.11158	8.96227	.12929	7.73480	38
23	.07665	13.0458	.09423	10.6118	.11187	8.93867	.12958	7.71715	37
24	.07695	12.9962	.09453	10.5789	.11217	8.91520	.12988	7.69957	36
25	.07724	12.9469	.09482	10.5462′	.11246	8.89185	.13017	7.68208	35
26	.07753	12.8981	.09511	10.5136	.11276	8.86862	.13047	7.66466	34
27	.07782	12.8496	.09541	10.4813	.11305	8.84551	.13076	7.64732	33
28	.07812	12.8014	.09570	10.4491	.11335	8.82252	.13106	7.63005	32
29	.07841	12.7536	.09600	10.4172	.11364	8.79964	.13136	7.61287	31
30	.07870	12.7062	.09629	10.3854	.11394	8.77689	.13165	7.59575	30
31	.07899	12.6591	.09658	10.3538	.11423	8.75425	.13195	7.57872	29
32	.07929	12.6124	.09688	10.3224	.11452	8.73172	.13224	7.56176	28
33	.07958	12.5660	.09717	10.2913	.11482	8.70931	.13254	7.54487	27
34	.07987	12.5199	.09746	10.2602	.11511	8.68701	.13284	7.52806	26
35	.08017	12.4742	.09776	10.2294	.11541	8.66482	.13313	7.51132	25
36	.08046	12.4288	.09805	10.1988	.11570	8.64275	.13343	7.49465	24
37	.08075	12.3838	.09834	10.1683	.11600	8.62078	.13372	7.47806	23
38	.08104	12.3390	.09864	10.1381	.11629	8.59893	.13402	7.46154	22
39	.08134	12.2946	.09893	10.1080	.11659	8.57718	.13432	7.44509	21
40	.08163	12.2505	.09923	10.0780	.11688	8.55555	.13461	7.42871	20
41	.08192	12.2067	.09952	10.0483	.11718	8.53402	.13491	7.41240	19
42	.08221	12.1632	.09981	10.0187	.11747	8.51259	.13521	7.39616	18
43	.08251	12.1201	.10011	9.98931	.11777	8.49128	.13550	7.37999	17
44	.08280	12.0772	.10040	9.96007	.11806	8.47007	.13580	7.36389	16
45	.08309	12.0346	.10069	9.93101	.11836	8.44896	.13609	7.34786	15
46	.08339	11.9923	.10099	9.90211	.11865	8.42795	.13639	7.33190	14
47	.08368	11.9504	.10128	9.87338	.11895	8.40705	.13669	7.31600	13
48	.08397	11.9087	.10158	9.84482	.11924	8.38625	.13698	7.30018	12
49	.08427	11.8673	.10187	9.81641	.11954	8.36555	.13728	7.28442	11
50	.08456	11.8262	.10216	9.78817	.11983	8.34496	.13758	7.26873	10
51	.08485	11.7853	.10246	9.76009	.12013	8.32446	.13787	7.25310	9
52	.08514	11.7448	.10275	9.73217	.12042	8.30406	.13817	7.23754	8
53	.08544	11.7045	.10305	9.70441	.12072	8.28376	.13846	7.22204	7
54	.08573	11.6645	.10334	9.67680	.12101	8.26355	.13876	7.20661	6
55	.08602	11.6248	.10363	9.64935	.12131	8.24345	.13906	7.19125	5
56	.08632	11.5853	.10393	9.62205	.12160	8.22344	.13935	7.17594	4
57	.08661	11.5461	.10422	9.59490	.12190	8.20352	.13965	7.16071	3
58	.08690	11.5072	.10452	9.56791	.12219	8.18370	.13995	7.14553	2
59	.08720	11.4685	.10481	9.54106	.12249	8.16398	.14024	7.13042	1
60	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	85°		84°		83°		82°		

TABLE VII.—*Continued*

NATURAL TANGENTS AND COTANGENTS

′	8°		9°		10°		11°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.14054	7.11537	.15838	6.31375	.17633	5.67128	.19438	5.14455	60
1	.14084	7.10038	.15868	6.30189	.17663	5.66165	.19468	5.13658	59
2	.14113	7.08546	.15898	6.29007	.17693	5.65205	.19498	5.12862	58
3	.14143	7.07059	.15928	6.27829	.17723	5.64248	.19529	5.12069	57
4	.14173	7.05579	.15958	6.26655	.17753	5.63295	.19559	5.11279	56
5	.14202	7.04105	.15988	6.25486	.17783	5.62344	.19589	5.10490	55
6	.14232	7.02637	.16017	6.24321	.17813	5.61397	.19619	5.09704	54
7	.14262	6.91174	.16047	6.23160	.17843	5.60452	.19649	5.08921	53
8	.14291	6.99718	.16077	6.22003	.17873	5.59511	.19680	5.08139	52
9	.14321	6.98268	.16107	6.20851	.17903	5.58573	.19710	5.07360	51
10	.14351	6.96823	.16137	6.19703	.17933	5.57638	.19740	5.06584	50
11	.14381	6.95385	.16167	6.18559	.17963	5.56706	.19770	5.05809	49
12	.14410	6.93952	.16196	6.17419	.17993	5.55777	.19801	5.05037	48
13	.14440	6.92525	.16226	6.16283	.18023	5.54851	.19831	5.04267	47
14	.14470	6.91104	.16256	6.15151	.18053	5.53927	.19861	5.03499	46
15	.14499	6.89688	.16286	6.14023	.18083	5.53007	.19891	5.02734	45
16	.14529	6.88278	.16316	6.12899	.18113	5.52090	.19921	5.01971	44
17	.14559	6.86874	.16346	6.11779	.18143	5.51176	.19952	5.01210	43
18	.14588	6.85475	.16376	6.10664	.18173	5.50264	.19982	5.00451	42
19	.14618	6.84082	.16405	6.09552	.18203	5.49356	.20012	4.99695	41
20	.14648	6.82694	.16435	6.08444	.18233	5.48451	.20042	4.98940	40
21	.14678	6.81312	.16465	6.07340	.18262	5.47548	.20073	4.98188	39
22	.14707	6.79936	.16495	6.06240	.18293	5.46648	.20103	4.97438	38
23	.14737	6.78564	.16525	6.05143	.18323	5.45751	.20133	4.96690	37
24	.14767	6.77199	.16555	6.04051	.18353	5.44857	.20164	4.95945	36
25	.14796	6.75838	.16585	6.02962	.18384	5.43966	.20194	4.95201	35
26	.14826	6.74483	.16615	6.01878	.18414	5.43077	.20224	4.94460	34
27	.14856	6.73133	.16645	6.00797	.18444	5.42192	.20254	4.93721	33
28	.14886	6.71789	.16674	5.99720	.18474	5.41309	.20285	4.92984	32
29	.14915	6.70450	.16704	5.98646	.18504	5.40429	.20315	4.92249	31
30	.14945	6.69116	.16734	5.97576	.18534	5.39552	.20345	4.91516	30
31	.14975	6.67787	.16764	5.96510	.18564	5.38677	.20376	4.90785	29
32	.15005	6.66463	.16794	5.95448	.18591	5.37805	.20406	4.90056	28
33	.15034	6.65144	.16824	5.94390	.18624	5.36936	.20436	4.89330	27
34	.15064	6.63831	.16854	5.93335	.18654	5.36070	.20466	4.88605	26
35	.15094	6.62523	.16884	5.92283	.18684	5.35206	.20497	4.87882	25
36	.15124	6.61219	.16914	5.91236	.18714	5.34345	.20527	4.87162	24
37	.15153	6.59921	.16944	5.90191	.18745	5.33487	.20557	4.86444	23
38	.15183	6.58627	.16974	5.89151	.18775	5.32631	.20588	4.85727	22
39	.15213	6.57339	.17004	5.88114	.18805	5.31778	.20618	4.85013	21
40	.15243	6.56055	.17033	5.87080	.18835	5.30928	.20648	4.84300	20
41	.15272	6.54777	.17063	5.86051	.18865	5.30080	.20679	4.83590	19
42	.15302	6.53503	.17093	5.85024	.18895	5.29235	.20709	4.82882	18
43	.15332	6.52234	.17123	5.84001	.18925	5.28393	.20739	4.82175	17
44	.15362	6.50970	.17153	5.82982	.18955	5.27553	.20770	4.81471	16
45	.15391	6.49710	.17183	5.81966	.18986	5.26715	.20800	4.80769	15
46	.15421	6.48456	.17213	5.80953	.19016	5.25880	.20830	4.80068	14
47	.15451	6.47206	.17243	5.79944	.19046	5.25048	.20861	4.79370	13
48	.15481	6.45961	.17273	5.78938	.19076	5.24218	.20891	4.78673	12
49	.15511	6.44720	.17303	5.77936	.19106	5.23391	.20921	4.77978	11
50	.15540	6.43484	.17333	5.76937	.19136	5.22566	.20952	4.77286	10
51	.15570	6.42253	.17363	5.75941	.19166	5.21744	.20982	4.76595	9
52	.15600	6.41026	.17393	5.74949	.19197	5.20925	.21013	4.75906	8
53	.15630	6.39804	.17423	5.73960	.19227	5.20107	.21043	4.75219	7
54	.15660	6.38587	.17453	5.72974	.19257	5.19293	.21073	4.74534	6
55	.15689	6.37374	.17483	5.71992	.19287	5.18480	.21104	4.73851	5
56	.15719	6.36165	.17513	5.71013	.19317	5.17671	.21134	4.73170	4
57	.15749	6.34961	.17543	5.70037	.19347	5.16863	.21164	4.72490	3
58	.15779	6.33761	.17573	5.69064	.19378	5.16058	.21195	4.71813	2
59	.15809	6.32566	.17603	5.68094	.19408	5.15256	.21225	4.71137	1
60	.15838	6.31375	.17633	5.67128	.19438	5.14455	.21256	4.70463	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	81°		80°		79°		78°		

TABLE VII.—*Continued*

NATURAL TANGENTS AND COTANGENTS

′	12°		13°		14°		15°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	60
1	.21286	4.69791	.23117	4.32573	.24964	4.00582	.26826	3.72771	59
2	.21316	4.69121	.23148	4.32001	.24995	4.00086	.26857	3.72338	58
3	.21347	4.68452	.23179	4.31430	.25026	3.99592	.26888	3.71907	57
4	.21377	4.67786	.23209	4.30860	.25056	3.99099	.26920	3.71476	56
5	.21408	4.67121	.23240	4.30291	.25087	3.98607	.26951	3.71046	55
6	.21438	4.66458	.23271	4.29724	.25118	3.98117	.26982	3.70616	54
7	.21469	4.65797	.23301	4.20159	.25149	3.97627	.27013	3.70188	53
8	.21499	4.65138	.23332	4.28595	.25180	3.97139	.27044	3.69761	52
9	.21529	4.64480	.23363	4.28032	.25211	3.96651	.27076	3.69335	51
10	.21560	4.63825	.23393	4.27471	.25242	3.96165	.27107	3.68909	50
11	.21590	4.63171	.23424	4.26911	.25273	3.95680	.27138	3.68485	49
12	.21621	4.62518	.23455	4.26352	.25304	3.95196	.27169	3.68061	48
13	.21651	4.61868	.23485	4.25795	.25335	3.94713	.27201	3.67638	47
14	.21682	4.61219	.23516	4.25239	.25366	3.94232	.27232	3.67217	46
15	.21712	4.60572	.23547	4.24685	.25397	3.93751	.27263	3.66796	45
16	.21743	4.59927	.23578	4.24132	.25428	3.93271	.27294	3.66376	44
17	.21773	4.59283	.23608	4.23580	.25459	3.92793	.27326	3.65957	43
18	.21804	4.58641	.23639	4.23030	.25490	3.92316	.27357	3.65538	42
19	.21834	4.58001	.23670	4.22481	.25521	3.91839	.27388	3.65121	41
20	.21864	4.57363	.23700	4.21933	.25552	3.91364	.27419	3.64705	40
21	.21895	4.56726	.23731	4.21387	.25583	3.90890	.27451	3.64289	39
22	.21925	4.56091	.23762	4.20842	.25614	3.90417	.27482	3.63874	38
23	.21956	4.55458	.23793	4.20298	.25645	3.89945	.27513	3.63461	37
24	.21986	4.54826	.23823	4.19756	.25676	3.89474	.27545	3.63048	36
25	.22017	4.54196	.23854	4.19215	.25707	3.89004	.27576	3.62636	35
26	.22047	4.53568	.23885	4.18675	.25738	3.88536	.27607	3.62224	34
27	.22078	4.52941	.23916	4.18137	.25769	3.88068	.27638	3.61814	33
28	.22108	4.52316	.23946	4.17600	.25800	3.87601	.27670	3.61405	32
29	.22139	4.51693	.23977	4.17064	.25831	3.87136	.27701	3.60996	31
30	.22169	4.51071	.24008	4.16530	.25862	3.86671	.27732	3.60588	30
31	.22200	4.50451	.24039	4.15997	.25893	3.86208	.27764	3.60181	29
32	.22231	4.49832	.24069	4.15465	.25924	3.85745	.27795	3.59775	28
33	.22261	4.49215	.24100	4.14934	.25955	3.85284	.27826	3.59370	27
34	.22292	4.48600	.24131	4.14405	.25986	3.84824	.27858	3.58966	26
35	.22322	4.47986	.24162	4.13877	.26017	3.84364	.27889	3.58562	25
36	.22353	4.47374	.24193	4.13350	.26048	3.83906	.27921	3.58160	24
37	.22383	4.46764	.24223	4.12825	.26079	3.83449	.27952	3.57758	23
38	.22414	4.46155	.24254	4.12301	.26110	3.82992	.27983	3.57357	22
39	.22444	4.45548	.24285	4.11778	.26141	3.82537	.28015	3.56957	21
40	.22475	4.44942	.24316	4.11256	.26172	3.82083	.28046	3.56557	20
41	.22505	4.44338	.24347	4.10736	.26203	3.81630	.28077	3.56159	19
42	.22536	4.43735	.24377	4.10216	.26235	3.81177	.28109	3.55761	18
43	.22567	4.43134	.24408	4.09699	.26266	3.80726	.28140	3.55364	17
44	.22597	4.42534	.24439	4.09182	.26297	3.80276	.28172	3.54968	16
45	.22628	4.41936	.24470	4.08666	.26328	3.79827	.28203	3.54573	15
46	.22658	4.41340	.24501	4.08152	.26359	3.79378	.28234	3.54179	14
47	.22689	4.40745	.24532	4.07639	.26390	3.78931	.28266	3.53785	13
48	.22719	4.40152	.24562	4.07127	.26421	3.78485	.28297	3.53393	12
49	.22750	4.39560	.24593	4.06616	.26452	3.78040	.28329	3.53001	11
50	.22781	4.38969	.24624	4.06107	.26483	3.77595	.28360	3.52609	10
51	.22811	4.38381	.24655	4.05599	.26515	3.77152	.28391	3.52219	9
52	.22842	4.37793	.24686	4.05092	.26546	3.76709	.28423	3.51829	8
53	.22872	4.37207	.24717	4.04586	.26577	3.76268	.28454	3.51441	7
54	.22903	4.36623	.24747	4.04081	.26608	3.75828	.28486	3.51053	6
55	.22934	4.36040	.24778	4.03578	.26639	3.75388	.28517	3.50666	5
56	.22964	4.35459	.24809	4.03076	.26670	3.74950	.28549	3.50279	4
57	.22995	4.34879	.24840	4.02574	.26701	3.74512	.28580	3.49894	3
58	.23026	4.34300	.24871	4.02074	.26733	3.74075	.28612	3.49509	2
59	.23056	4.33723	.24902	4.01576	.26764	3.73640	.28643	3.49125	1
60	.23087	4.33148	.24933	4.01078	.26795	3.73205	.28675	3.48741	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	
	77°		76°		75°		74°		

TABLE VII.—*Continued*

Natural Tangents and Cotangents

′	16°		17°		18°		19°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35020	2.85555	42
19	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.29337	3.40869	.31242	3.20079	.33169	3.01489	.35118	2.84758	39
22	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	26
25	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35346	2.82914	32
29	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.30033	3.32965	.31946	3.13027	.33881	2.95155	.35838	2.79033	17
44	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.30097	3.32264	.32010	3.12400	.33945	2.94591	.35904	2.78523	15
46	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.30192	3.31216	.32106	3.11464	.34043	2.93748	.36002	2.77761	12
49	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2.76750	8
53	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.30446	3.28452	.32363	3.08991	.34303	2.91523	.36265	2.75746	4
57	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75246	1
59	.30541	3.27426	.32460	3.08073	.34400	2.90696	.36364	2.74997	0
60	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	2
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	73°		72°		71°		70°		

TABLE VII.—*Continued*

NATURAL TANGENTS AND COTANGENTS

′	20°		21°		22°		23°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43622	.43101	2.32012	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	39
22	.37123	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43239	2.31271	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38863	.43932	2.27626	17
44	.37853	2.64177	.39862	2.50864	.41899	2.38668	.43966	2.27447	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	5
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	69°		68°		67°		66°		

TABLE VII.—*Continued*

NATURAL TANGENTS AND COTANGENTS

′	24°		25°		26°		27°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.44523	2.24604	.46631	2.14451	.48773	2.05030	.50953	1.96261	60
1	.44558	2.24428	.46666	2.14288	.48809	2.04879	.50989	1.96120	59
2	.44593	2.24252	.46702	2.14125	.48845	2.04728	.51026	1.95979	58
3	.44627	2.24077	.46737	2.13963	.48881	2.04577	.51063	1.95838	57
4	.44662	2.23902	.46772	2.13801	.48917	2.04426	.51099	1.95698	56
5	.44697	2.23727	.46808	2.13639	.48953	2.04276	.51136	1.95557	55
6	.44732	2.23553	.46843	2.13477	.48989	2.04125	.51173	1.95417	54
7	.44767	2.23378	.46879	2.13316	.49026	2.03975	.51209	1.95277	53
8	.44802	2.23204	.46914	2.13154	.49062	2.03825	.51246	1.95137	52
9	.44837	2.23030	.46950	2.12993	.49098	2.03675	.51283	1.94997	51
10	.44872	2.22857	.46985	2.12832	.49134	2.03526	.51319	1.94858	50
11	.44907	2.22683	.47021	2.12671	.49170	2.03376	.51356	1.94718	49
12	.44942	2.22510	.47056	2.12511	.49206	2.03227	.51393	1.94579	48
13	.44977	2.22337	.47092	2.12350	.49242	2.03078	.51430	1.94440	47
14	.45012	2.22164	.47128	2.12190	.49278	2.02929	.51467	1.94301	46
15	.45047	2.21992	.47163	2.12030	.49315	2.02780	.51503	1.94162	45
16	.45082	2.21819	.47199	2.11871	.49351	2.02631	.51540	1.94023	44
17	.45117	2.21647	.47234	2.11711	.49387	2.02483	.51577	1.93885	43
18	.45152	2.21475	.47270	2.11552	.49423	2.02335	.51614	1.93746	42
19	.45187	2.21304	.47305	2.11392	.48459	2.02187	.51651	1.93608	41
20	.45222	2.21132	.47341	2.11233	.49495	2.02039	.51688	1.93470	40
21	.45257	2.20961	.47377	2.11075	.49532	2.01891	.51724	1.93332	39
22	.45292	2.20790	.47412	2.10916	.49568	2.01743	.51761	1.93195	38
23	.45327	2.20619	.47448	2.10758	.49604	2.01596	.51798	1.93057	37
24	.45362	2.20449	.47483	2.10600	.49640	2.01449	.51835	1.92920	36
25	.45397	2.20278	.47519	2.10442	.49677	2.01302	.51872	1.92782	35
26	.45432	2.20108	.47555	2.10284	.49713	2.01155	.51909	1.92645	34
27	.45467	2.19938	.47590	2.10126	.49749	2.01008	.51946	1.92508	33
28	.45502	2.19769	.47626	2.09969	.49786	2.00862	.51983	1.92371	32
29	.45538	2.19599	.47662	2.09811	.49822	2.00715	.52020	1.92235	31
30	.45573	2.19430	.47698	2.09654	.49858	2.00569	.52057	1.92098	30
31	.45608	2.19261	.47733	2.09498	.49894	2.00423	.52094	1.91962	29
32	.45643	2.19092	.47769	2.09341	.49931	2.00277	.52131	1.91826	28
33	.45678	2.18923	.47805	2.09184	.49967	2.00131	.52168	1.91690	27
34	.45713	2.18755	.47840	2.09028	.50004	1.99986	.52205	1.91552	26
35	.45748	2.18587	.47876	2.08872	.50040	1.99841	.52242	1.91414	25
36	.45784	2.18419	.47912	2.08716	.50076	1.99695	.52279	1.91288	24
37	.45819	2.18251	.47948	2.08560	.50113	1.99550	.52316	1.91142	23
38	.45854	2.18084	.47984	2.08405	.50149	1.99406	.52353	1.91017	22
39	.45889	2.17916	.48019	2.08250	.50185	1.99261	.52390	1.90876	21
40	.45924	2.17749	.48055	2.08094	.50222	1.99116	.52427	1.90741	20
41	.45960	2.17582	.48091	2.07939	.50258	1.98972	.52464	1.90607	19
42	.45995	2.17416	.48127	2.07785	.50295	1.98828	.52501	1.90472	18
43	.46030	2.17249	.48163	2.07630	.50331	1.98684	.52538	1.90337	17
44	.46065	2.17083	.48198	2.07476	.50368	1.98540	.52575	1.90203	16
45	.46101	2.16917	.48234	2.07321	.50404	1.98396	.52613	1.90069	15
46	.46136	2.16751	.48270	2.07167	.50441	1.98253	.52650	1.89935	14
47	.46171	2.16585	.48306	2.07014	.50477	1.98110	.52687	1.89801	13
48	.46206	2.16420	.48342	2.06860	.50514	1.97966	.52724	1.89667	12
49	.46242	2.16255	.48378	2.06706	.50550	1.97823	.52761	1.89533	11
50	.46277	2.16090	.48414	2.06553	.50587	1.97681	.52798	1.89400	10
51	.46312	2.15925	.48450	2.06400	.50623	1.97538	.52836	1.89266	9
52	.46348	2.15760	.48486	2.06247	.50660	1.97395	.52873	1.89133	8
53	.46383	2.15596	.48521	2.06094	.50696	1.97253	.52910	1.89000	7
54	.46418	2.15432	.48557	2.05942	.50733	1.97111	.52947	1.88867	6
55	.46454	2.15268	.48593	2.05790	.50769	1.96969	.52985	1.88734	5
56	.46489	2.15104	.48629	2.05637	.50806	1.96827	.53022	1.88602	4
57	.46525	2.14940	.48665	2.05485	.50843	1.96685	.53059	1.88469	3
58	.46560	2.14777	.48701	2.05333	.50879	1.96544	.53096	1.88337	2
59	.46595	2.14614	.48737	2.05182	.50916	1.96402	.53134	1.88205	1
60	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	65°		64°		63°		62°		

TABLE VII.—*Continued*

NATURAL TANGENTS AND COTANGENTS

′	28°		29°		30°		31°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.53171	1.88073	.55431	1.80405	.57735	1.73205	.60086	1.66428	60
1	.53208	1.87941	.55469	1.80281	.57774	1.73089	.60126	1.66318	59
2	.53246	1.87809	.55507	1.80158	.57813	1.72973	.60165	1.66209	58
3	.53283	1.87677	.55545	1.80034	.57851	1.72857	.60205	1.66099	57
4	.53320	1.87546	.55583	1.79911	.57890	1.72741	.60245	1.65990	56
5	.53358	1.87415	.55621	1.79788	.57929	1.72625	.60284	1.65881	55
6	.53395	1.87283	.55659	1.79665	.57968	1.72509	.60324	1.65772	54
7	.53432	1.87152	.55697	1.79542	.58007	1.72393	.60364	1.65663	53
8	.53470	1.87021	.55736	1.79419	.58046	1.72278	.60403	1.65554	52
9	.53507	1.86891	.55774	1.79296	.58085	1.72163	.60443	1.65445	51
10	.53545	1.86760	.55812	1.79174	.58124	1.72047	.60483	1.65337	50
11	.53582	1.86630	.55850	1.79051	.58162	1.71932	.60522	1.65228	49
12	.53620	1.86499	.55888	1.78929	.58201	1.71817	.60562	1.65120	48
13	.53657	1.86369	.55926	1.78807	.58240	1.71702	.60602	1.65011	47
14	.53694	1.86239	.55964	1.78685	.58279	1.71588	.60642	1.64903	46
15	.53732	1.86109	.56003	1.78563	.58318	1.71473	.60681	1.64795	45
16	.53769	1.85979	.56041	1.78441	.58357	1.71358	.60721	1.64687	44
17	.53807	1.85850	.56079	1.78319	.58396	1.71244	.60761	1.64579	43
18	.53844	1.85720	.56117	1.78198	.58435	1.71129	.60801	1.64471	42
19	.53882	1.85591	.56156	1.78077	.58474	1.71015	.60841	1.64363	41
20	.53920	1.85462	.56194	1.77955	.58513	1.70901	.60881	1.64256	40
21	.53957	1.85333	.56232	1.77834	.58552	1.70787	.60921	1.64148	39
22	.53995	1.85204	.56270	1.77713	.58591	1.70673	.60960	1.64041	38
23	.54032	1.85075	.56309	1.77592	.58631	1.70560	.61000	1.63934	37
24	.54070	1.84946	.56347	1.77471	.58670	1.70446	.61040	1.63826	36
25	.54107	1.84818	.56385	1.77351	.58709	1.70332	.61080	1.63719	35
26	.54145	1.84689	.56424	1.77230	.58748	1.70219	.61120	1.63612	34
27	.54183	1.84561	.56462	1.77110	.58787	1.70106	.61160	1.63505	33
28	.54220	1.84433	.56501	1.76990	.58826	1.69992	.61200	1.63398	32
29	.54258	1.84305	.56539	1.76869	.58865	1.69879	.61240	1.63292	31
30	.54296	1.84177	.56577	1.76749	.58905	1.69766	.61280	1.63185	30
31	.54333	1.84049	.56616	1.76629	.58944	1.69653	.61320	1.63079	29
32	.54371	1.83922	.56654	1.76510	.58983	1.69541	.61360	1.62972	28
33	.54409	1.83794	.56693	1.76390	.59022	1.69428	.61400	1.62866	27
34	.54446	1.83667	.56731	1.76271	.59061	1.69316	.61440	1.62760	26
35	.54484	1.83540	.56769	1.76151	.59101	1.69203	.61480	1.62654	25
36	.54522	1.83413	.56808	1.76032	.59140	1.69091	.61520	1.62548	24
37	.54560	1.83286	.56846	1.75913	.59179	1.68979	.61561	1.62442	23
38	.54597	1.83159	.56885	1.75794	.59218	1.68866	.61601	1.62336	22
39	.54635	1.83033	.56923	1.75675	.59258	1.68754	.61641	1.62230	21
40	.54673	1.82906	.56962	1.75556	.59297	1.68643	.61681	1.62125	20
41	.54711	1.82780	.57000	1.75437	.59336	1.68531	.61721	1.62019	19
42	.54748	1.82654	.57039	1.75319	.59376	1.68419	.61761	1.61914	18
43	.54786	1.82528	.57078	1.75200	.59415	1.68308	.61801	1.61808	17
44	.54824	1.82402	.57116	1.75082	.59454	1.68196	.61842	1.61703	16
45	.54862	1.82276	.57155	1.74964	.59494	1.68085	.61882	1.61598	15
46	.54900	1.82150	.57193	1.74846	.59533	1.67974	.61922	1.61493	14
47	.54938	1.82025	.57232	1.74728	.59573	1.67863	.61962	1.61388	13
48	.54975	1.81899	.57271	1.74610	.59612	1.67752	.62003	1.61283	12
49	.55013	1.81774	.57309	1.74492	.59651	1.67641	.62043	1.61179	11
50	.55051	1.81649	.57348	1.74375	.59691	1.67530	.62083	1.61074	10
51	.55089	1.81524	.57386	1.74257	.59730	1.67419	.62124	1.60970	9
52	.55127	1.81399	.57425	1.74140	.59770	1.67309	.62164	1.60865	8
53	.55165	1.81274	.57464	1.74022	.59809	1.67198	.62204	1.60761	7
54	.55203	1.81150	.57503	1.73905	.59849	1.67088	.62245	1.60657	6
55	.55241	1.81025	.57541	1.73788	.59888	1.66978	.62285	1.60553	5
56	.55279	1.80901	.57580	1.73671	.59928	1.66867	.62325	1.60449	4
57	.55317	1.80777	.57619	1.73555	.59967	1.66757	.62366	1.60345	3
58	.55355	1.80653	.57657	1.73438	.60007	1.66647	.62406	1.60241	2
59	55393	1.80529	.57696	1.73321	.60046	1.66538	.62446	1.60137	1
60	.55431	1.80405	.57735	1.73205	.60086	1.66428	.62487	1.60033	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	61°		60°		59°		58°		

TABLE VII.—*Continued*

NATURAL TANGENTS AND COTANGENTS

′	32°		33°		34°		35°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	60
1	.62527	1.59930	.64982	1.53888	.67493	1.48163	.70064	1.42726	59
2	.62568	1.59826	.65024	1.53791	.67536	1.48070	.70170	1.42638	58
3	.62608	1.59723	.65065	1.53693	.67578	1.47977	.70151	1.42550	57
4	.62649	1.59620	.65106	1.53595	.67620	1.47885	.70194	1.42462	56
5	.62689	1.59517	.65148	1.53497	.67663	1.47792	.70238	1.42374	55
6	.62730	1.59414	.65189	1.53400	.67705	1.47699	.70281	1.42286	54
7	.62770	1.59311	.65231	1.53302	.67748	1.47607	.70325	1.42198	53
8	.62811	1.59208	.65272	1.53205	.67790	1.47514	.70368	1.42110	52
9	.62852	1.59105	.65314	1.53107	.67832	1.47422	.70412	1.42022	51
10	.62892	1.59002	.65355	1.53010	.67875	1.47330	.70455	1.41934	50
11	.62933	1.58900	.65397	1.52913	.67917	1.47238	.70499	1.41847	49
12	.62973	1.58797	.65438	1.52816	.67960	1.47146	.70542	1.41759	48
13	.63014	1.58695	.65480	1.52719	.68002	1.47053	.70586	1.41672	47
14	.63055	1.58593	.65521	1.52622	.68045	1.46962	.70629	1.41584	46
15	.63095	1.58490	.65563	1.52525	.68088	1.46870	.70673	1.41497	45
16	.63136	1.58388	.65604	1.52429	.68130	1.46778	.70717	1.41409	44
17	.63177	1.58286	.65646	1.52332	.68173	1.46686	.70760	1.41322	43
18	.63217	1.58184	.65688	1.52235	.68215	1.46595	.70804	1.41235	42
19	.63258	1.58083	.65729	1.52139	.68258	1.46503	.70848	1.41148	41
20	.63299	1.57981	.65771	1.52043	.68301	1.46411	.70891	1.41061	40
21	.63340	1.57879	.65813	1.51946	.68343	1.46320	.70935	1.40974	39
22	.63380	1.57778	.65854	1.51850	.68386	1.46229	.70979	1.40887	38
23	.63421	1.57676	.65896	1.51754	.68429	1.46137	.71023	1.40800	37
24	.63462	1.57575	.65938	1.51658	.68471	1.46046	.71066	1.40714	36
25	.63503	1.57474	.65980	1.51562	.68514	1.45955	.71110	1.40627	35
26	.63544	1.57372	.66021	1.51466	.68557	1.45864	.71154	1.40540	34
27	.63584	1.57271	.66063	1.51370	.68600	1.45773	.71198	1.40454	33
28	.63625	1.57170	.66105	1.51275	.68642	1.45682	.71242	1.40367	32
29	.63666	1.57069	.66147	1.51179	.68685	1.45592	.71285	1.40281	31
30	.63707	1.56969	.66189	1.51084	.68728	1.45501	.71329	1.40195	30
31	.63748	1.56868	.66230	1.50988	.68771	1.45410	.71373	1.40109	29
32	.63789	1.56767	.66272	1.50893	.68814	1.45320	.71417	1.40022	28
33	.63830	1.56667	.66314	1.50797	.68857	1.45229	.71461	1.39936	27
34	.63871	1.56566	.66356	1.50702	.68900	1.45139	.71505	1.39850	26
35	.63912	1.56466	.66398	1.50607	.68942	1.45049	.71549	1.39764	25
36	.63953	1.56366	.66440	1.50512	.68985	1.44958	.71593	1.39679	24
37	.63994	1.56265	.66482	1.50417	.69028	1.44868	.71637	1.39593	23
38	.64035	1.56165	.66524	1.50322	.69071	1.44778	.71681	1.39507	22
39	.64076	1.56065	.66566	1.50228	.69114	1.44688	.71725	1.39421	21
40	.64117	1.55966	.66608	1.50133	.69157	1.44598	.71769	1.39336	20
41	.64158	1.55866	.66650	1.50038	.69200	1.44508	.71813	1.39250	19
42	.64199	1.55766	.66692	1.49944	.69243	1.44418	.71857	1.39165	18
43	.64240	1.55666	.66734	1.49849	.69286	1.44329	.71901	1.39079	17
44	.64281	1.55567	.66776	1.49755	.69329	1.44239	.71946	1.38994	16
45	.64322	1.55467	.66818	1.49661	.69372	1.44149	.71990	1.38909	15
46	.64363	1.55368	.66860	1.49566	.69416	1.44060	.72034	1.38824	14
47	.64404	1.55269	.66902	1.49472	.69459	1.43970	.72078	1.38738	13
48	.64446	1.55170	.66944	1.49378	.69502	1.43881	.72122	1.38653	12
49	.64487	1.55071	.66986	1.49284	.69545	1.43792	.72167	1.38568	11
50	.64528	1.54972	.67028	1.49190	.69588	1.43703	.72211	1.38484	10
51	.64569	1.54873	.67071	1.49097	.69631	1.43614	.72255	1.38399	9
52	.64610	1.54774	.67113	1.49003	.69675	1.43525	.72299	1.38314	8
53	.64652	1.54675	.67155	1.48909	.69718	1.43436	.72344	1.38229	7
54	.64693	1.54576	.67197	1.48816	.69761	1.43347	.72388	1.38145	6
55	.64734	1.54478	.67239	1.48722	.69804	1.43258	.72432	1.38060	5
56	.64775	1.54379	.67282	1.48629	.69847	1.43169	.72477	1.37976	4
57	.64817	1.54281	.67324	1.48536	.69891	1.43080	.72521	1.37891	3
58	.64858	1.54183	.67366	1.48442	.69934	1.42992	.72565	1.37807	2
59	.64899	1.54085	.67409	1.48349	.69977	1.42903	.72610	1.37722	1
60	.64941	1.53986	.67451	1.48256	.70021	1.42815	.72654	1.37638	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	57°		56°		55°		54°		

TABLE VII.—*Continued*

NATURAL TANGENTS AND COTANGENTS

′	36°		37°		38°		39°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.73457	1.36134	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	53°		52°		51°		50°		

TABLE VII.—*Continued*

NATURAL TANGENTS AND COTANGENTS

′	40°		41°		42°		43°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	33
28	.85308	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	24
37	.85761	1.16603	.88836	1.12657	.92008	1.08686	.95284	1.04949	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	12
49	.86368	1.15783	.89463	1.11778	.92655	1.07927	.95952	1.04218	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	1
60	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	49°		48°		47°		46°		

TABLE VII.—*Continued*

NATURAL TANGENTS AND COTANGENTS

′	44°		′	′	44°		′	′	44°		′
	Tang	Cotang			Tang	Cotang			Tang	Cotang	
0	.96569	1.03553	60	20	.97700	1.02355	40	40	.98843	1.01170	20
1	.96625	1.03493	59	21	.97756	1.02295	39	41	.98901	1.01112	19
2	.96681	1.03433	58	22	.97813	1.02236	38	42	.98958	1.01053	18
3	.96738	1.03372	57	23	.97870	1.02176	37	43	.99016	1.00994	17
4	.96794	1.03312	56	24	.97927	1.02117	36	44	.99073	1.00935	16
5	.96850	1.03252	55	25	.97984	1.02057	35	45	.99131	1.00876	15
6	.96907	1.03192	54	26	.98041	1.01998	34	46	.99189	1.00818	14
7	.96963	1.03132	53	27	.98098	1.01939	33	47	.99247	1.00759	13
8	.97020	1.03072	52	28	.98155	1.01879	32	48	.99304	1.00701	12
9	.97076	1.03012	51	29	.98213	1.01820	31	49	.99362	1.00642	11
10	.97133	1.02952	50	30	.98270	1.01761	30	50	.99420	1.00583	10
11	.97189	1.02892	49	31	.98327	1.01702	29	51	.99478	1.00525	9
12	.97246	1.02832	48	32	.98384	1.01642	28	52	.99536	1.00467	8
13	.97302	1.02772	47	33	.98441	1.01583	27	53	.99594	1.00408	7
14	.97359	1.02713	46	34	.98499	1.01524	26	54	.99652	1.00350	6
15	.97416	1.02652	45	35	.98556	1.01465	25	55	.99710	1.00291	5
16	.97472	1.02593	44	36	.98613	1.01406	24	56	.99768	1.00233	4
17	.97529	1.02533	43	37	.98671	1.01347	23	57	.99826	1.00175	3
18	.97586	1.02474	42	38	.98728	1.01288	22	58	.99884	1.00116	2
19	.97643	1.02414	41	39	.98786	1.01229	21	59	.99942	1.00058	1
20	.97700	1.02355	40	40	.98843	1.01170	20	60	1.00000	1.00000	0
′	Cotang	Tang	′	′	Cotang	Tang	′	′	Cotang	Tang	′
	45°				45°				45°		

TABLE VIII

CO-ORDINATES OF POINTS OF INTERSECTION OF PARALLELS AND MERIDIANS IN POLYCONIC PROJECTION. § 506

Latitude.	Length of 1° Latitude, in Statute Miles.	Length of Side of Tangent Cone, in Statute Miles.	Meridian Distances for 1° Longitude.				Divergence of Parallels for 1° Longitude			
			In Yards.	In Metres.	In Miles.	Factor.	In Yards.	In Metres.	In Miles.	Factor.
30°	68.875	6869	105507	96476	59.95	$n \cos (0.288n°)$	460.4	421.0	0.2617	n^2
32°	68.897	6348	103327	94481	58.71	$n \cos (0.304n°)$	477.8	436.8	0.2715	n^2
34°	68.918	5881	101022	92373	57.40	$n \cos (0.320n°)$	493.0	450.7	0.2800	n^2
36°	68.941	5461	98593	90152	56.02	$n \cos (0.337n°)$	505.7	462.4	0.2873	n^2
38°	68.964	5079	96044	87822	54.57	$n \cos (0.353n°)$	516.0	471.8	0.2932	n^2
40°	68.987	4729	93377	85383	53.06	$n \cos (0.369n°)$	523.8	479.0	0.2976	n^2
42°	69.011	4408	90596	82840	51.48	$n \cos (0.386n°)$	529.0	483.8	0.3006	n^2
44°	69.036	4110	87704	80197	49.83	$n \cos (0.402n°)$	531.7	486.2	0.3022	n^2
46°	69.060	3833	84704	77452	48.13	$n \cos (0.418n°)$	531.7	486.2	0.3022	n^2
48°	69.084	3575	81601	74615	46.37	$n \cos (0.435n°)$	529.2	484.0	0.3007	n^2
50°	69.108	3332	78398	71686	44.54	$n \cos (0.451n°)$	524.1	479.2	0.2978	n^2

n = number degrees of longitude between the given meridian and the prime meridian of the map.

TABLE IX

Giving Values of C in Kutter's Formula when $s = 0.001$

r in Feet.	Values of n.												r in Feet.
	.009	.010	.011	.012	.013	.015	.017	.020	.0225	.025	.030	.035	
.1	108.3	93.8	82.2	72.7	65.0	53.2	44.6	35.5	30.0	25.9	20.1	16.3	.1
.2	129.5	113.1	100.0	89.1	80.2	66.3	56.2	45.2	38.6	33.6	26.3	21.5	.2
.3	141.8	123.8	111.0	98.8	90.2	75.0	63.4	51.8	44.6	38.4	30.3	25.1	.3
.4	150.3	132.5	118.0	106.0	96.2	80.4	68.8	56.2	48.4	42.4	33.7	27.8	.4
.5	156.8	138.6	123.8	111.2	101.2	85.1	72.8	60.0	51.8	45.4	36.2	30.0	.5
.6	161.9	143.3	128.3	115.7	105.3	88.8	76.4	62.9	54.5	48.0	38.5	32.0	.6
.7	166.1	147.4	131.9	119.3	108.7	92.0	79.3	65.4	56.9	50.2	40.3	33.6	.7
.8	169.7	150.8	135.1	122.3	111.6	94.6	81.9	67.7	59.0	52.2	42.0	35.1	.8
.9	172.8	153.7	137.8	125.1	114.2	97.0	84.2	69.7	60.8	53.8	43.4	36.3	.9
1.0	175.4	156.2	140.5	127.4	116.5	99.1	86.0	71.5	62.5	55.4	44.9	37.7	1.0
1.2	180.0	160.4	144.6	131.5	120.4	102.7	89.4	74.5	65.3	58.1	47.1	39.7	1.2
1.4	183.6	164.0	147.9	134.7	123.7	105.7	92.2	77.0	67.7	60.2	49.2	41.5	1.4
1.6	186.7	167.0	150.8	137.4	126.2	108.2	94.5	79.3	69.9	62.3	51.0	43.2	1.6
1.8	189.2	169.5	153.2	139.7	128.7	110.3	96.6	81.1	71.6	64.0	52.5	44.6	1.8
2.0	191.4	171.6	155.4	141.8	130.5	112.3	98.4	82.9	73.4	65.5	54.0	45.9	2.0
2.2	193.3	173.5	157.3	143.7	132.3	114.0	100.0	84.3	74.7	66.9	55.2	47.0	2.2
2.4	195.0	175.2	159.0	145.3	133.9	115.4	101.4	85.6	76.0	68.1	56.3	48.0	2.4
2.6	196.7	176.8	160.5	146.8	135.3	116.8	102.8	87.0	77.1	69.2	57.4	49.0	2.6
2.8	198.0	178.2	161.8	148.1	136.7	118.0	104.0	88.2	78.2	70.3	58.4	49.9	2.8
3.0	199.3	179.4	163.2	149.3	137.9	119.2	105.1	89.3	79.2	71.3	59.2	50.6	3.0
3.4	201.7	181.7	165.3	151.4	140.0	121.3	107.1	91.1	81.0	73.0	60.8	52.0	3.4
3.8	203.7	183.6	167.2	153.3	141.8	123.0	108.8	92.7	82.5	74.5	62.3	53.5	3.8
4.2	205.4	185.3	168.8	155.0	143.4	124.6	110.3	94.3	83.9	75.8	63.5	54.7	4.2
4.6	207.0	186.8	170.3	156.4	144.8	125.9	111.6	95.4	85.2	77.0	64.7	55.7	4.6
5.0	208.3	188.1	171.6	157.7	146.0	127.2	112.9	96.6	86.3	78.1	65.7	56.7	5.0

TABLE X

Giving Diameters in Feet of Circular Brick Conduits for Various Inclinations and Rates of Discharge

Conduit full to point of maximum discharge. (By Kutter's formula.)

Cubic Feet per Second.	Fall per 100 Feet.													Cubic Feet per Second.
	.10	.15	.20	.30	.50	.75	1.0	1.5	2.0	3.0	4.0	6.0	10.0	
1	.9	.9	.8	.8	.7	.7	.6	.6	.6	.5	.5	.5	.4	1
2	1.2	1.1	1.1	1.0	.9	.8	.8	.7	.7	.7	.6	.6	.5	2
3	1.4	1.3	1.3	1.2	1.1	1.0	.9	.9	.8	.8	.7	.7	.6	3
4	1.6	1.5	1.4	1.3	1.2	1.1	1.0	1.0	.9	.8	.8	.7	.7	4
5	1.7	1.6	1.5	1.4	1.3	1.2	1.1	1.0	1.0	.9	.9	.8	.7	5
10	2.2	2.0	1.9	1.8	1.6	1.5	1.4	1.3	1.3	1.2	1.1	1.0	.9	10
15	2.6	2.4	2.3	2.1	1.9	1.8	1.7	1.6	1.5	1.4	1.3	1.2	1.1	15
20	2.9	2.6	2.5	2.3	2.1	2.0	1.9	1.7	1.6	1.5	1.4	1.3	1.2	20
25	3.1	2.9	2.7	2.5	2.3	2.1	2.0	1.9	1.8	1.6	1.6	1.5	1.3	25
30	3.3	3.1	2.9	2.7	2.5	2.3	2.2	2.0	1.9	1.8	1.7	1.5	1.4	30
35	3.5	3.3	3.1	2.9	2.6	2.4	2.3	2.1	2.0	1.9	1.8	1.6	1.5	35
40	3.7	3.4	3.2	3.0	2.7	2.5	2.4	2.2	2.1	2.0	1.9	1.7	1.6	40
45	3.9	3.6	3.4	3.1	2.9	2.6	2.5	2.3	2.2	2.0	1.9	1.8	1.6	45
50	4.0	3.7	3.5	3.3	3.0	2.7	2.6	2.4	2.3	2.1	2.0	1.9	1.7	50
60	4.3	4.0	3.7	3.5	3.2	2.9	2.8	2.6	2.5	2.3	2.1	2.0	1.8	60
70	4.5	4.2	4.0	3.7	3.4	3.1	2.9	2.7	2.6	2.4	2.3	2.1	1.9	70
80	4.8	4.4	4.2	4.0	3.5	3.3	3.1	2.9	2.7	2.5	2.4	2.2	2.0	80
90	5.0	4.6	4.4	4.1	3.7	3.4	3.2	3.0	2.9	2.7	2.5	2.3	2.1	90
100	5.2	4.8	4.6	4.2	3.8	3.6	3.4	3.1	3.0	2.7	2.6	2.4	2.2	100
125	5.7	5.2	5.0	4.6	4.2	3.9	3.6	3.4	3.2	3.0	2.8	2.6	2.4	125
150	6.1	5.6	5.3	4.9	4.5	4.1	3.9	3.6	3.4	3.2	3.0	2.8	2.6	150
175	6.4	5.9	5.6	5.2	4.6	4.4	4.2	3.8	3.6	3.4	3.2	3.0	2.7	175
200	6.8	6.3	5.9	5.5	5.0	4.6	4.4	4.0	3.8	3.6	3.4	3.1	2.8	200
250	7.3	6.8	6.4	5.9	5.4	5.0	4.7	4.4	4.2	3.9	3.7	3.4	3.1	250
300	7.9	7.3	6.9	6.4	5.8	5.4	5.1	4.7	4.5	4.1	3.9	3.6	3.3	300
350	8.3	7.7	7.3	6.8	6.1	5.7	5.4	5.0	4.7	4.4	4.2	3.9	3.5	350
400	8.8	8.1	7.7	7.1	6.5	6.0	5.7	5.2	5.0	4.6	4.4	4.0	3.7	400
500	9.5	8.8	8.4	7.8	7.0	6.5	6.2	5.7	5.4	5.0	4.7	4.4	4.0	500
600	10.2	9.5	9.0	8.3	7.5	7.0	6.6	6.1	5.8	5.4	5.1	4.7	4.3	600
700	10.8	10.0	9.5	8.8	8.0	7.4	7.0	6.5	6.1	5.7	5.4	5.0	4.5	700
800	11.4	10.6	10.0	9.3	8.4	7.8	7.4	6.8	6.5	6.0	5.7	5.3	4.7	800
900	11.9	11.1	10.5	9.7	8.8	8.1	7.7	7.1	6.7	6.2	5.9	5.5	5.0	900
1000	12.4	11.5	10.9	10.1	9.2	8.5	8.0	7.4	7.0	6.5	6.2	5.7	5.2	1000

TABLE XI

Volumes by the Prismoidal Formula.

Widths.	Heights.										Corrections for tenths in height.	
	1	2	3	4	5	6	7	8	9	10		
1	0	1	1	1	2	2	2	2	3	3	.1	0
2	1	1	2	2	3	3	4	5	6	6	.2	0
3	1	2	3	4	5	6	6	7	8	9	.3	0
4	1	2	4	5	6	7	9	10	11	12	.4	1
5	—2	—3	—5	—6	—8	—9	—11	—12	—14	—15	.5	1
6	2	4	6	7	9	11	13	15	17	19	.6	1
7	2	4	6	9	11	13	15	17	19	22	.7	1
8	2	5	7	10	12	15	17	20	22	25	.8	1
9	3	6	8	11	14	17	19	22	25	28	.9	1
10	3	6	9	12	15	19	22	25	28	31		
11	3	7	10	14	17	20	24	27	31	34	.1	0
12	4	7	11	15	19	22	26	30	33	37	.2	1
13	4	8	12	16	20	24	28	32	36	40	.3	1
14	4	9	13	17	22	26	30	35	39	43	.4	2
15	—5	—9	—14	—19	—23	—28	—32	—37	—42	—46	.5	2
16	5	10	15	20	25	30	35	40	44	49	.6	3
17	5	10	16	21	26	31	37	42	47	52	.7	3
18	6	11	17	22	28	33	39	44	50	56	.8	4
19	6	12	18	23	29	35	41	47	53	59	.9	4
20	6	12	19	25	31	37	43	49	56	62		
21	6	13	19	26	32	39	45	52	58	65	.1	1
22	7	14	20	27	34	41	48	54	61	68	.2	2
23	7	14	21	28	35	43	50	57	64	71	.3	2
24	7	15	22	30	37	44	52	59	67	74	.4	3
25	—8	—15	—23	—31	—39	—46	—54	—62	—69	—77	.5	4
26	8	16	24	32	40	48	56	64	72	80	.6	5
27	8	17	25	33	42	50	58	67	75	83	.7	5
28	9	17	26	35	43	52	60	69	78	86	.8	6
29	9	18	27	36	45	54	63	72	81	90	.9	7
30	9	19	28	37	46	56	65	74	83	93		
31	10	19	29	38	48	57	67	77	86	96	.1	1
32	10	20	30	40	49	59	69	79	89	99	.2	2
33	10	20	31	41	51	61	71	81	92	102	.3	3
34	10	21	31	42	52	63	73	84	94	105	.4	4
35	—11	—22	—32	—43	—54	—65	—76	—86	—97	—108	.5	5
36	11	22	33	44	56	67	78	89	100	111	.6	6
37	11	23	34	46	57	69	80	91	103	114	.7	8
38	12	23	35	47	59	70	82	94	106	117	.8	9
39	12	24	36	48	60	72	84	96	108	120	.9	10
40	12	25	37	49	62	74	86	99	111	123		
41	13	25	38	51	63	76	89	101	114	127	.1	1
42	13	26	39	52	65	78	91	104	117	130	.2	3
43	13	27	40	53	66	80	93	106	119	133	.3	4
44	14	27	41	54	68	81	95	109	122	136	.4	6
45	—14	—28	—42	—56	—69	—83	—97	—111	—125	—139	.5	7
46	14	28	43	57	71	85	99	114	128	142	.6	8
47	15	29	44	58	73	87	102	116	131	145	.7	10
48	15	30	44	59	74	89	104	119	133	148	.8	11
49	15	30	45	60	76	91	106	121	136	151	.9	13
50	15	31	46	62	77	93	108	123	139	154		
	1	2	3	4	5	6	7	8	9	10		
	.1	.2	.3	.4	.5	.6	.7	.8	.9		Corrections for tenths in width	
	0	0	0	1	1	1	1	1	1			

TABLE XI—*Continued*

Volumes by the Prismoidal Formula

Widths.	Heights. 1	2	3	4	5	6	7	8	9	10	Corrections for tenths in height.	
51	16	31	47	63	79	94	110	126	142	157	.1	2
52	16	32	48	64	80	96	112	128	144	160	.2	3
53	16	33	49	65	82	98	115	131	147	163	.3	5
54	17	33	50	67	83	100	117	133	150	167	.4	7
55	—17	—34	—51	—68	—85	—102	—119	—136	—153	—170	.5	8
56	17	35	52	69	86	104	121	138	156	173	.6	10
57	18	35	53	70	88	106	123	141	158	176	.7	12
58	18	36	54	72	90	107	125	143	161	179	.8	14
59	18	36	55	73	91	109	127	146	164	182	.9	15
60	19	37	56	74	93	111	130	148	167	185		
61	19	38	56	75	94	113	132	151	169	188	.1	2
62	19	38	57	77	96	115	134	153	172	191	.2	4
63	19	39	58	78	97	117	136	156	175	194	.3	6
64	20	40	59	79	99	119	138	158	178	197	.4	8
65	—20	—40	—60	—80	—100	—120	—140	—160	—181	—201	.5	10
66	20	41	61	81	102	122	143	163	183	204	.6	12
67	21	41	62	83	103	124	145	165	186	207	.7	14
68	21	42	63	84	105	126	147	168	189	210	.8	16
69	21	43	64	85	106	128	149	170	192	213	.9	18
70	22	43	65	86	108	130	151	173	194	216		
71	22	44	66	88	100	131	153	175	197	219	.1	2
72	22	44	67	89	111	133	156	178	200	222	.2	5
73	23	45	68	90	113	135	158	180	203	225	.3	7
74	23	46	69	91	114	137	160	183	206	228	.4	9
75	—23	—46	—69	—93	—116	—139	—162	—185	—208	—231	.5	12
76	23	47	70	94	117	141	164	188	211	235	.6	14
77	24	48	71	95	119	143	166	190	214	238	.7	16
78	24	48	72	96	120	144	169	193	217	241	.8	19
79	24	49	73	98	122	146	171	195	219	244	.9	21
80	25	49	74	99	123	148	173	198	222	247		
81	25	50	75	100	125	150	175	200	225	250	.1	3
82	25	51	76	101	127	152	177	202	228	253	.2	5
83	26	51	77	102	128	154	179	205	231	256	.3	8
84	26	52	78	104	130	156	181	207	233	259	.4	10
85	—26	—52	—79	—105	—131	—157	—184	—210	—236	—262	.5	13
86	27	53	80	106	133	159	186	212	239	265	.6	16
87	27	54	81	107	134	161	188	215	242	269	.7	18
88	27	54	81	109	136	163	190	217	244	272	.8	21
89	27	55	82	110	137	165	192	220	247	275	.9	24
90	28	56	83	111	139	167	194	222	250	278		
91	28	56	84	112	140	169	197	225	253	281	.1	3
92	28	57	85	114	142	170	199	227	256	284	.2	6
93	29	57	86	115	144	172	201	230	258	287	.3	9
94	29	58	87	116	145	174	203	232	261	290	.4	12
95	—29	—59	—88	—117	—147	—176	—205	—235	—264	—293	.5	15
96	30	59	89	119	148	178	207	237	267	296	.6	18
97	30	60	90	120	150	180	210	240	269	299	.7	21
90	30	60	91	121	151	181	212	242	272	302	.8	23
99	31	61	92	122	153	183	214	244	275	306	.9	26
100	31	62	93	123	154	185	216	247	278	309		
	1	2	3	4	5	6	7	8	9	10		
	.1	.2	.3	.4	.5	.6	.7	.8	.9		Corrections for tenths in width.	
	0	0	0	1	1	1	1	1	1			

TABLE XI—*Continued*

Volumes by the Prismoidal Formula

Widths.	Heights.										Corrections for tenths in height.	
	11	12	13	14	15	16	17	18	19	20		
1	3	4	4	4	5	5	5	6	6	6	.1	0
2	7	7	8	9	9	10	10	11	12	12	.2	0
3	10	11	12	13	14	15	16	17	18	19	.3	0
4	14	15	16	17	19	20	21	22	23	25	.4	1
5	—17	—19	—20	—22	— 23	—25	—26	—28	—29	—31	.5	1
6	20	22	24	26	28	30	31	33	35	37	.6	1
7	24	26	28	30	32	35	37	39	41	43	.7	1
8	27	30	32	35	37	40	42	44	47	49	.8	1
9	31	33	36	39	42	44	47	50	53	56	.9	1
10	34	37	40	43	46	49	52	56	59	62		
11	37	41	44	48	51	54	58	61	65	68	.1	0
12	41	44	48	52	56	59	63	67	70	74	.2	1
13	44	48	52	56	60	64	68	72	76	80	.3	1
14	48	52	56	60	65	69	73	78	82	86	.4	2
15	—51	—56	—60	—65	—69	—74	—79	—83	—88	—93	.5	2
16	54	59	64	69	74	79	84	89	94	99	.6	3
17	58	63	68	73	79	84	89	94	100	105	.7	3
18	61	67	72	78	83	89	94	100	106	111	.8	4
19	65	70	76	82	88	94	100	106	111	117	.9	4
20	68	74	80	86	93	99	105	111	117	123		
21	71	78	84	91	97	104	110	117	123	130	.1	1
22	75	81	88	95	102	109	115	122	129	136	.2	2
23	78	85	92	99	106	114	121	128	135	142	.3	2
24	81	89	96	104	111	119	126	133	141	148	.4	3
25	—85	—93	—100	—108	—116	—123	—131	—139	—147	—154	.5	4
26	88	96	104	112	120	128	136	144	152	160	.6	5
27	92	100	108	117	125	133	142	150	158	167	.7	5
28	95	104	112	121	130	138	147	156	164	173	.8	6
29	98	107	116	125	134	143	152	161	170	179	.9	7
30	102	111	120	130	139	148	157	167	176	185		
31	105	115	124	134	144	153	163	172	182	191	.1	1
32	109	119	128	138	148	158	168	178	188	198	.2	2
33	112	122	132	143	153	163	173	183	194	204	.3	3
34	115	126	136	147	157	168	178	189	199	210	.4	4
35	—119	—130	—140	—151	—162	—173	—184	—194	—205	—216	.5	5
36	122	133	144	156	167	178	189	200	211	222	.6	6
37	126	137	148	160	171	183	194	206	217	228	.7	8
38	129	141	152	164	176	188	199	211	223	235	.8	9
39	132	144	156	169	181	193	205	217	229	241	.9	10
40	136	148	160	173	185	198	210	222	235	247		
41	139	152	165	177	190	202	215	228	240	253	.1	1
42	143	156	169	181	194	207	220	233	246	259	.2	3
43	146	159	173	186	199	212	226	239	252	265	.3	4
44	149	163	177	190	204	217	231	244	258	272	.4	6
45	—153	—167	—181	—194	—208	—222	—236	—250	—264	—278	.5	7
46	156	170	185	199	213	227	241	256	270	284	.6	8
47	160	174	189	203	218	232	247	261	276	290	.7	10
48	163	178	193	207	222	237	252	267	281	296	.8	11
49	166	181	197	212	227	242	257	272	287	302	.9	13
50	170	185	201	216	231	247	262	278	293	309		
	11	12	13	14	15	16	17	18	19	20		
	.1	.2	.3	.4	.5	.6	.7	.8	.9		Corrections for tenths in width.	
	0	1	1	2	2	3	3	4	4			

TABLE XI—*Continued*

Volumes by the Prismoidal Formula

Widths.	Heights.										Corrections for tenths in height.	
	11	12	13	14	15	16	17	18	19	20		
51	173	189	205	220	236	252	268	283	299	315	.1	2
52	177	193	209	225	241	257	273	289	305	321	.2	3
53	180	196	213	229	245	262	278	294	311	327	.3	5
54	183	200	217	233	250	267	283	300	317	333	.4	7
55	187	204	221	238	255	272	289	306	323	340	.5	8
56	190	207	225	242	259	277	294	311	328	346	.6	10
57	194	211	229	246	264	281	299	317	334	352	.7	12
58	197	215	233	251	269	286	304	322	340	358	.8	14
59	200	219	237	255	273	291	310	328	346	364	.9	15
60	204	222	241	259	278	296	315	333	352	370		
61	207	226	245	264	282	301	320	339	358	377	.1	2
62	210	230	249	268	287	306	325	344	364	383	.2	4
63	214	233	253	272	292	311	331	350	369	389	.3	6
64	217	237	257	277	296	316	336	356	375	395	.4	8
65	221	241	261	281	301	321	341	361	381	401	.5	10
66	224	244	265	285	306	326	346	367	387	407	.6	12
67	227	248	269	290	310	331	352	372	393	414	.7	14
68	231	252	273	294	315	336	357	378	399	420	.8	16
69	234	256	277	298	319	341	362	383	405	426	.9	18
70	238	259	281	302	324	346	367	389	410	432		
71	241	263	285	307	329	351	373	394	416	438	.1	2
72	244	267	289	311	333	356	378	400	422	444	.2	5
73	248	270	293	315	338	360	383	406	428	451	.3	7
74	251	274	297	320	343	365	388	411	434	457	.4	9
75	255	278	301	324	347	370	394	417	440	463	.5	12
76	258	281	305	328	352	375	399	422	446	469	.6	14
77	261	285	309	333	356	380	404	428	452	475	.7	16
78	265	289	313	337	361	385	409	433	457	481	.8	19
79	268	293	317	341	366	390	415	439	463	488	.9	21
80	272	296	321	346	370	395	420	444	469	494		
81	275	300	325	350	375	400	425	450	475	500	.1	3
82	278	304	329	354	380	405	430	456	481	506	.2	5
83	282	307	333	359	384	410	435	461	487	512	.3	8
84	285	311	337	363	389	415	441	467	493	519	.4	10
85	289	315	341	367	394	420	446	472	498	525	.5	13
86	292	319	345	372	398	425	541	478	504	531	.6	16
87	295	322	349	376	403	430	456	483	510	537	.7	18
88	299	326	353	380	407	435	462	489	516	543	.8	21
89	303	330	357	385	412	440	467	494	522	549	.9	24
90	306	333	361	389	417	444	472	500	528	556		
91	309	337	365	393	421	449	477	506	534	562	.1	3
92	312	341	369	398	426	454	483	511	540	568	.2	6
93	316	344	373	402	431	459	488	517	545	574	.3	9
94	319	348	377	406	435	464	493	522	551	580	.4	12
95	323	352	381	410	440	469	498	528	557	586	.5	15
96	326	356	385	415	444	474	504	533	563	593	.6	18
97	329	359	389	419	449	479	509	539	569	599	.7	21
98	333	363	393	423	454	484	514	544	575	605	.8	23
99	336	367	397	428	458	489	519	550	581	611	.9	26
100	340	370	401	432	463	494	525	556	586	617		
	11	12	13	14	15	16	17	18	19	20		
	.1	.2	.3	.4	.5	.6	.7	.8	.9		Corrections for tenths in width.	
	0	1	1	2	2	3	3	4	4			

TABLE XI—*Continued*

Volumes by the Prismoidal Formula

Widths.	Heights.										Corrections for tenths in height.	
	21	22	23	24	25	26	27	28	29	30		
1	6	7	7	7	8	8	8	9	9	9	.1	0
2	13	14	14	15	15	16	17	17	18	19	.2	0
3	19	20	21	22	23	24	25	26	27	28	.3	0
4	26	27	28	30	31	32	33	35	36	37	.4	1
5	—32	—34	—35	—37	—39	—40	—42	—43	—45	—46	.5	1
6	39	41	43	44	46	48	50	52	54	56	.6	1
7	45	48	50	52	54	56	58	60	63	65	.7	1
8	52	54	57	59	62	64	67	69	72	74	.8	1
9	58	61	64	67	69	72	75	78	81	83	.9	1
10	65	68	71	74	77	80	83	86	90	93		
11	71	75	78	81	85	88	92	95	98	102	.1	0
12	78	81	85	89	93	96	100	104	107	111	.2	1
13	84	88	92	96	100	104	108	112	116	120	.3	1
14	91	95	99	104	108	112	117	121	125	130	.4	2
15	—97	—102	—106	—111	—116	—120	—125	—130	—134	—139	.5	2
16	104	109	114	119	123	128	133	138	143	148	.6	3
17	110	115	121	126	131	136	142	147	152	157	.7	3
18	117	122	128	133	139	144	150	156	161	167	.8	4
19	123	129	135	141	147	152	158	164	170	176	.9	4
20	130	136	142	148	154	160	167	173	179	185		
21	136	143	149	156	162	169	175	181	188	194	.1	1
22	143	149	156	163	170	177	183	190	197	204	.2	2
23	149	156	163	170	177	185	192	199	206	213	.3	2
24	156	163	170	178	185	193	200	207	215	222	.4	3
25	—162	—170	—177	—185	—193	—201	—208	—216	—224	—231	.5	4
26	169	177	185	193	201	209	217	225	233	241	.6	5
27	175	183	192	200	208	217	225	233	242	250	.7	5
28	181	190	199	207	216	225	233	242	251	259	.8	6
29	188	197	206	215	224	233	242	251	260	269	.9	7
30	194	204	213	222	231	241	250	259	269	278		
31	201	210	220	230	239	249	258	268	277	287	.1	1
32	207	217	227	237	247	257	267	277	286	296	.2	2
33	214	224	234	244	255	265	275	285	295	306	.3	3
34	220	231	241	252	262	273	283	294	304	315	.4	4
35	—227	—238	—248	—259	—270	—281	—292	—302	—313	—324	.5	5
36	233	244	256	267	278	289	300	311	322	333	.6	6
37	240	251	263	274	285	297	308	320	331	343	.7	8
38	246	258	270	281	293	305	317	328	340	352	.8	9
39	253	265	277	289	301	313	325	337	349	361	.9	10
40	259	272	284	296	309	321	333	346	358	370		
41	266	278	291	304	316	329	342	354	367	380	.1	1
42	272	285	298	311	324	337	350	363	376	389	.2	3
43	279	292	305	319	332	345	358	372	385	398	.3	4
44	285	299	312	326	340	353	367	380	394	407	.4	6
45	—292	—306	—319	—333	—347	—361	—375	—389	—403	—417	.5	7
46	298	312	327	341	355	369	383	398	412	426	.6	8
47	305	319	334	348	363	377	392	406	421	435	.7	10
48	311	326	341	356	370	385	400	415	430	444	.8	11
49	318	333	348	363	378	393	408	423	439	454	.9	13
50	324	340	355	370	386	401	417	432	448	463		
	21	22	23	24	25	26	27	28	29	30		
	.1	.2	.3	.4	.5	.6	.7	.8	.9		Corrections for tenths in width.	
	1	2	2	3	4	5	5	6	7			

TABLE XI—*Continued*

Volumes by the Prismoidal Formula

Widths.	Heights.										Corrections for tenths in height.	
	21	22	23	24	25	26	27	28	29	30		
51	331	346	362	378	394	409	425	441	456	472	.1	2
52	337	353	369	385	401	417	433	449	465	481	.2	3
53	344	360	376	393	409	425	442	458	474	491	.3	5
54	350	367	383	400	417	433	450	467	483	500	.4	7
55	356	373	390	407	424	441	458	475	492	509	.5	8
56	363	380	398	415	432	449	467	484	501	519	.6	10
57	369	387	405	422	440	457	475	493	510	528	.7	12
58	376	394	412	430	448	465	483	501	519	537	.8	14
59	382	401	419	437	455	473	492	510	528	546	.9	15
60	389	407	426	444	463	481	500	519	537	556		
61	395	414	433	452	471	490	508	527	546	565	.1	2
62	402	421	440	459	478	498	517	536	555	574	.2	4
63	408	428	447	467	486	506	525	544	564	583	.3	6
64	415	435	454	474	494	514	533	553	573	593	.4	8
65	421	441	461	481	502	522	542	562	582	602	.5	10
66	428	448	469	489	509	530	550	570	591	611	.6	12
67	431	455	476	496	517	538	558	579	600	620	.7	14
68	441	462	483	504	525	546	567	588	609	630	.8	16
69	447	469	490	511	532	554	575	596	618	639	.9	18
70	454	475	497	519	540	562	583	605	627	648		
71	460	482	504	526	548	570	592	614	635	657	.1	2
72	467	489	511	533	556	578	600	622	644	667	.2	5
73	473	496	518	541	563	586	608	631	653	676	.3	7
74	480	502	525	548	571	594	617	640	662	685	.4	9
75	486	509	532	556	579	601	625	648	671	694	.9	12
76	493	516	540	563	586	610	633	657	680	704	.6	14
77	499	523	547	570	594	618	642	665	689	713	.7	16
78	506	530	554	578	602	626	650	674	698	722	.8	19
79	512	536	561	585	610	634	658	683	707	731	.9	21
80	519	543	568	593	617	642	667	691	716	741		
81	525	550	575	600	625	650	675	700	725	750	.1	3
82	531	557	582	607	633	658	683	709	734	759	.2	5
83	538	564	589	615	640	666	692	717	743	769	.3	8
84	544	570	596	622	648	674	700	726	752	778	.4	10
85	551	577	603	630	656	682	708	735	761	787	.5	13
86	557	584	610	637	664	690	717	743	770	796	.6	16
87	564	591	618	644	671	698	725	752	779	806	.7	18
88	570	598	625	652	679	706	733	760	788	815	.8	21
89	577	604	632	659	687	714	742	769	797	824	.9	24
90	583	611	639	667	694	722	750	777	806	833		
91	590	618	646	674	702	730	758	786	815	843	.1	3
92	596	625	653	681	710	738	767	795	823	852	.2	6
93	603	631	660	689	718	746	775	804	832	861	.3	9
94	609	638	667	696	725	754	783	812	841	870	.4	12
95	616	645	674	704	733	762	792	821	850	880	.5	15
96	622	652	681	711	741	770	800	830	859	889	.6	18
97	629	659	689	719	748	778	808	838	868	898	.7	21
98	635	665	696	726	756	786	817	847	877	907	.8	23
99	642	672	703	733	764	794	825	856	886	917	.9	26
100	648	679	710	741	772	802	833	864	895	926		
	21	22	23	24	25	26	27	28	29	30		
	.1	.2	.3	.4	.5	.6	.7	.8	.9		Corrections for tenths in width.	
	1	2	2	3	4	5	5	6	7			

TABLE XI—*Continued*

Volumes by the Prismoidal Formula

Widths.	Heights.										Corrections for tenths in height.	
	31	32	33	34	35	36	37	38	39	40		
1	10	10	10	10	11	11	11	12	12	12	.1	0
2	19	20	20	21	22	22	23	23	24	25	.2	0
3	29	30	31	31	32	33	34	35	36	37	.3	0
4	38	40	41	42	43	44	46	47	48	49	.4	1
5	—48	—49	—51	—52	— 54	—56	—57	—59	—60	—62	.5	1
6	57	59	61	63	65	67	68	70	72	74	.6	1
7	67	69	71	73	76	78	80	82	84	86	.7	1
8	77	79	81	84	86	89	91	94	96	97	.8	1
9	86	89	92	94	97	100	103	106	108	111	.9	1
10	96	99	102	105	108	111	114	117	120	123		
11	105	109	112	115	119	122	126	129	132	136	.1	0
12	115	119	122	126	130	133	137	141	144	148	.2	1
13	124	128	132	136	140	144	148	152	156	160	.3	1
14	134	138	143	147	151	156	160	164	169	173	.4	2
15	—144	—148	—153	—157	—162	—167	—171	—176	—181	—185	.5	2
16	153	158	163	168	173	178	183	188	193	198	.6	3
17	163	168	173	178	183	189	194	199	205	210	.7	3
18	172	178	183	189	194	200	206	211	217	222	.8	4
19	182	188	194	199	205	211	217	223	229	235	.9	4
20	191	198	204	210	216	222	228	235	241	247		
21	201	207	214	220	227	233	240	246	253	259	.1	1
22	210	217	224	231	238	244	251	258	265	272	.2	2
23	220	227	234	241	248	256	263	270	277	284	.3	2
24	230	237	244	252	259	267	274	281	289	296	.4	3
25	—239	—247	—255	—262	—270	—278	—285	—293	—301	—309	.5	4
26	249	257	265	273	281	289	297	305	313	321	.6	5
27	258	267	275	283	292	300	308	317	325	333	.7	5
28	268	277	285	294	302	311	320	328	337	346	.8	6
29	277	286	295	304	313	322	331	340	349	358	.9	7
30	287	296	306	315	324	333	343	352	361	370		
31	297	306	316	325	335	344	354	364	373	383	.1	1
32	306	316	326	336	346	356	365	375	385	395	.2	2
33	316	326	336	346	356	367	377	387	397	407	.3	3
34	325	336	346	357	367	378	388	399	409	420	.4	4
35	—335	—346	—356	—367	—378	—389	—400	—410	—421	—432	.5	5
36	344	356	367	378	389	400	411	422	433	444	.6	6
37	354	365	377	388	400	411	423	434	445	457	.7	8
38	364	375	387	399	410	422	434	446	457	469	.8	9
39	373	385	397	409	421	433	445	457	469	481	.9	10
40	383	395	407	420	432	444	457	469	481	494		
41	392	405	418	430	443	456	468	481	494	506	.1	1
42	402	415	428	441	454	467	480	493	506	519	.2	3
43	411	425	438	451	465	478	491	504	518	531	.3	4
44	421	435	448	462	475	489	502	516	530	543	.4	6
45	—431	—444	—458	—472	—486	—500	—514	—528	—542	—556	.5	7
46	440	454	469	483	497	511	525	540	554	568	.6	8
47	450	464	479	493	508	522	537	551	566	580	.7	10
48	459	474	489	504	519	533	548	563	578	593	.8	11
49	469	484	499	514	529	544	560	575	590	605	.9	13
50	478	494	509	525	540	556	571	586	602	617		
	31	32	33	34	35	36	37	38	39	40		
	.1	.2	.3	.4	.5	.6	.7	.8	.9			
	1	2	3	4	5	6	8	9	10		Corrections for tenths in width.	

TABLE XI.—*Continued.*

Volumes by the Prismoidal Formula.

Widths.	Heights.										Corrections for tenths in height.	
	31	32	33	34	35	36	37	38	39	40		
51	488	504	519	535	551	567	582	598	614	630	.1	2
52	498	514	530	546	562	578	594	610	626	642	.2	3
53	507	523	540	556	573	589	605	622	638	654	.3	5
54	517	533	550	567	583	600	617	633	650	667	.4	7
55	—526	—543	—560	—577	—594	—611	—628	—645	—662	—679	.5	8
56	536	553	570	588	605	622	640	657	674	691	.6	10
57	545	563	581	598	616	633	651	669	686	704	.7	12
58	555	573	591	609	627	644	662	680	698	716	.8	14
59	565	583	601	619	637	656	674	692	710	728	.9	15
60	574	593	611	630	648	667	685	704	722	741		
61	584	602	621	640	659	678	697	715	734	753	.1	2
62	593	612	631	651	670	689	708	727	746	765	.2	4
63	603	622	642	661	681	700	719	739	758	778	.3	6
64	612	632	652	672	691	711	731	751	770	790	.4	8
65	—622	—642	—662	—682	—702	—722	—742	—762	—782	—802	.5	10
66	631	652	672	693	713	733	754	774	794	815	.6	12
67	641	662	682	703	724	744	765	786	806	827	.7	14
68	651	672	693	714	735	756	777	798	819	840	.8	16
69	660	681	703	724	745	767	788	809	831	852	.9	18
70	670	691	713	735	756	778	799	821	843	864		
71	679	701	723	745	767	789	811	833	855	877	.1	2
72	689	711	733	756	778	800	822	844	867	889	.2	5
73	698	721	744	766	789	811	834	856	879	901	.3	7
74	708	731	754	777	799	822	845	868	891	914	.4	9
75	—718	—741	—764	—787	—810	—833	—856	—880	—903	—926	.5	12
76	727	751	774	798	821	844	868	891	915	938	.6	14
77	737	760	784	808	832	856	879	903	927	951	.7	16
78	746	770	794	819	843	867	891	915	939	963	.8	19
79	756	780	805	829	853	878	902	927	951	975	.9	21
80	765	790	815	840	864	889	914	938	963	988		
81	775	800	825	850	875	900	925	950	975	1000	.1	3
82	785	810	835	860	886	911	936	962	987	1012	.2	5
83	794	820	845	871	897	922	948	973	999	1025	.3	8
84	804	830	856	881	907	933	959	985	1011	1037	.4	10
85	—813	—840	—866	—892	—918	—944	—971	—997	—1023	—1049	.5	13
86	823	849	876	902	929	956	982	1009	1035	1062	.6	16
87	832	859	886	913	940	967	994	1020	1047	1074	.7	18
88	842	869	896	923	951	978	1005	1032	1059	1086	.8	21
89	852	879	906	934	961	989	1016	1044	1071	1098	.9	24
90	861	889	917	944	972	1000	1028	1056	1083	1111		
91	871	899	927	955	983	1011	1039	1067	1095	1123	.1	3
92	880	909	937	965	994	1022	1051	1079	1107	1136	.2	6
93	890	919	947	976	1005	1033	1062	1091	1119	1148	.3	9
94	899	928	957	986	1015	1044	1073	1102	1131	1160	.4	12
95	—909	—938	—968	—997	—1026	—1056	—1085	—1114	—1144	—1173	.5	15
96	919	948	978	1007	1037	1067	1096	1126	1156	1185	.6	18
97	928	958	988	1018	1048	1078	1108	1138	1168	1198	.7	21
98	938	968	998	1028	1059	1089	1119	1491	1180	1210	.8	23
99	947	978	1008	1039	1069	1100	1131	1161	1192	1222	.9	26
100	957	988	1019	1049	1080	1111	1142	1173	1204	1235		
	31	32	33	34	35	36	37	38	39	40		
	.1	.2	.3	.4	.5	.6	.7	.8	.9		Corrections for tenths in width.	
	1	2	3	4	5	6	8	9	10			

TABLE XI.—*Continued.*

VOLUMES BY THE PRISMOIDAL FORMULA.

Widths.	Heights.										Corrections for tenths in height.	
	41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**		
1	13	13	13	14	14	14	15	15	15	15	.1	0
2	25	26	27	27	28	28	29	30	30	31	.2	0
3	38	39	40	41	42	43	44	44	45	46	.3	0
4	51	52	53	54	56	57	58	59	60	62	.4	1
5	63	65	66	68	69	71	73	74	76	77	.5	1
6	76	78	80	81	83	85	87	89	91	93	.6	1
7	89	91	93	95	97	99	102	104	106	108	.7	1
8	101	104	106	109	111	114	116	119	121	123	.8	1
9	114	117	119	122	125	128	131	136	136	139	.9	1
10	127	130	133	136	139	142	142	148	151	154		
11	139	143	146	149	153	156	160	163	166	170	.1	0
12	152	156	159	163	167	170	174	178	181	185	.2	1
13	165	169	173	177	181	185	189	193	197	201	.3	1
14	177	181	186	190	194	199	203	207	212	216	.4	2
15	190	194	199	204	208	213	218	222	227	231	.5	2
16	203	207	212	217	222	227	232	237	242	247	.6	3
17	215	220	226	231	236	241	247	252	257	262	.7	3
18	228	233	239	244	250	256	261	267	272	278	.8	4
19	240	246	252	258	264	270	276	281	287	293	.9	4
20	253	259	265	272	278	284	290	296	302	309		
21	266	272	279	285	292	298	305	311	318	324	.1	1
22	278	285	292	299	306	312	319	326	333	340	.2	2
23	291	298	305	312	319	327	334	341	348	355	.3	2
24	304	311	319	326	333	341	348	356	363	370	.4	3
25	316	324	332	340	347	355	363	370	378	386	.5	4
26	329	337	345	353	361	369	377	385	393	401	.6	5
27	342	350	358	367	375	383	392	400	408	417	.7	5
28	354	363	372	380	389	398	406	415	423	432	.8	6
29	367	376	385	394	403	412	421	430	439	448	.9	7
30	380	389	398	407	417	426	435	444	454	463		
31	392	402	411	421	431	440	450	459	469	478	.1	1
32	405	415	425	435	444	454	464	474	484	494	.2	2
33	418	428	438	448	458	469	479	489	499	509	.3	3
34	430	441	451	462	472	483	493	504	514	525	.4	4
35	443	454	465	475	486	497	508	519	529	540	.5	5
36	456	467	478	489	500	511	522	533	544	556	.6	6
37	468	480	491	502	514	525	537	548	560	571	.7	8
38	481	493	504	516	528	540	551	563	575	586	.8	9
39	494	506	518	530	542	554	566	578	590	602	.9	10
40	506	519	531	543	556	568	580	593	605	617		
41	519	531	544	557	569	582	595	607	620	633	.1	1
42	531	544	557	570	583	596	609	622	635	648	.2	3
43	544	557	571	584	597	610	624	637	650	664	.3	4
44	557	570	584	598	611	625	638	652	665	679	.4	6
45	569	583	597	611	625	639	653	667	681	694	.5	7
46	582	596	610	625	639	653	667	681	696	710	.6	8
47	595	609	624	638	653	667	682	696	711	725	.7	10
48	607	622	637	652	667	681	696	711	726	741	.8	11
49	620	635	650	665	681	696	710	726	741	756	.9	13
50	633	648	664	679	694	710	725	741	756	772		
	41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**		
	.1	.2	.3	.4	.5	.6	.7	.8	.9		Corrections for tenths in width.	
	1	3	4	6	7	8	10	11	13			

TABLE XI.—*Continued.*

Volumes by the Prismoidal Formula.

Widths.	Heights.										Corrections for tenths in height.	
	41	42	43	44	45	46	47	48	49	50		
51	645	661	677	693	708	724	740	756	771	787	.1	2
52	658	674	690	706	722	738	754	770	786	802	.2	3
53	671	687	703	720	736	752	768	785	802	818	.3	5
54	683	700	717	733	750	767	783	800	817	833	.4	7
55	—696	—713	—730	—747	—764	—781	—798	—815	—832	—849	.5	8
56	709	726	743	760	778	795	812	830	847	864	.6	10
57	721	739	756	774	792	809	827	844	862	860	.7	12
58	734	752	770	788	806	823	841	859	877	895	.8	14
59	747	765	783	801	819	833	856	874	892	910	.9	15
60	759	778	796	815	833	852	870	889	907	926		
61	772	791	810	828	847	866	885	894	923	941	.1	2
62	785	804	823	842	861	880	880	919	938	957	.2	4
63	797	817	836	856	875	894	914	933	953	972	.3	6
64	810	830	849	869	889	909	928	948	968	988	.4	8
65	—823	—843	—863	—883	—903	—923	—943	—963	—983	—1003	.5	10
66	835	856	876	896	917	937	957	978	998	1019	.6	12
67	848	869	889	910	931	951	972	993	1013	1034	.7	14
68	860	881	902	923	944	965	986	1007	1028	1049	.8	16
69	873	894	916	937	958	980	1001	1022	1044	1065	.9	18
70	886	907	929	951	972	994	1015	1037	1059	1080		
71	898	920	942	964	986	1008	1030	1052	1074	1096	.1	2
72	911	933	956	978	1000	1022	1044	1067	1089	1111	.2	3
73	924	946	969	991	1014	1036	1059	1081	1104	1127	.3	7
74	936	959	989	1005	1028	1051	1073	1096	1119	1142	.4	9
75	—949	—972	—995	—1019	—1042	—1065	—1088	—1111	—1134	—1157	.5	12
76	962	985	1009	1032	1056	1079	1102	1126	1149	1173	.6	14
77	974	998	1022	1046	1069	1093	1117	1141	1165	1188	.7	16
78	987	1011	1035	1059	1083	1107	1131	1156	1180	1204	.8	19
79	1000	1024	1048	1073	1097	1122	1146	1170	1195	1219	.9	21
80	1012	1037	1062	1086	1111	1136	1160	1185	1210	1235		
81	1025	1050	1075	1100	1125	1150	1175	1200	1225	1250	.1	3
82	1038	1063	1088	1114	1139	1164	1190	1215	1240	1265	.2	5
83	1050	1076	1102	1127	1153	1178	1204	1230	1255	1281	.3	8
84	1063	1089	1115	1141	1167	1193	1219	1244	1270	1296	.4	10
85	—1076	—1102	—1128	—1154	—1181	—1207	—1233	—1259	—1285	—1312	.5	13
86	1088	1115	1141	1168	1194	1221	1248	1274	1301	1327	.6	16
87	1101	1128	1155	1181	1208	1235	1262	1289	1316	1343	.7	18
88	1114	1141	1168	1195	1222	1249	1277	1304	1331	1358	.8	21
89	1126	1154	1181	1209	1236	1264	1291	1319	1346	1373	.9	24
90	1139	1167	1194	1222	1250	1278	1306	1333	1361	1389		
91	1152	1180	1208	1236	1264	1292	1320	1348	1376	1404	.1	3
92	1164	1193	1221	1249	1278	1306	1335	1363	1391	1420	.2	6
93	1177	1206	1234	1263	1292	1320	1349	1378	1406	1435	.3	9
94	1190	1219	1248	1277	1306	1335	1364	1393	1422	1451	.4	12
95	—1202	—1231	—1261	—1290	—1319	—1349	—1378	—1407	—1437	—1466	.5	15
95	1215	1244	1274	1304	1333	1363	1393	1422	1452	1481	.6	18
97	1227	1257	1287	1317	1347	1377	1407	1437	1467	1497	.7	21
98	1240	1270	1301	1331	1361	1391	1422	1452	1482	1512	.8	23
99	1253	1283	1314	1344	1375	1406	1436	1457	1497	1528	.9	26
100	1265	1296	1327	1358	1389	1420	1451	1481	1512	1543		
	41	42	43	44	45	46	47	48	49	50		
	.1	.2	.3	.4	.5	.6	.7	.8	.9		Corrections for tenths in width.	
	1	3	4	6	7	8	10	11	13			

TABLE XII

LENGTHS OF CIRCULAR ARCS; RADIUS = 1

Sec.	Length.	Min.	Length.	Deg.	Length.	Deg.	Length.
1	.0000048	1	.0002909	1	.0174533	61	1.0646508
2	.0000097	2	.0005818	2	.0349066	62	1.0821041
3	.0000145	3	.0008727	3	.0523599	63	1.0995574
4	.0000194	4	.0011636	4	.0698132	64	1.1170107
5	.0000242	5	.0014544	5	.0872665	65	1.1344640
6	.0000291	6	.0017453	6	.1047198	66	1.1519173
7	.0000339	7	.0020362	7	.1221730	67	1.1693706
8	.0000388	8	.0023271	8	.1396263	68	1.1868239
9	.0000436	9	.0026180	9	.1570796	69	1.2042772
10	.0000485	10	.0029089	10	.1745329	70	1.2217305
11	.0000533	11	.0031998	11	.1919862	71	1.2391838
12	.0000582	12	.0034907	12	.2094395	72	1.2566371
13	.0000630	13	.0037815	13	.2268928	73	1.2740904
14	.0000679	14	.0040724	14	.2443461	74	1.2915436
15	.0000727	15	.0043633	15	.2617994	75	1.3089969
16	.0000776	16	.0046542	16	.2792527	76	1.3264502
17	.0000824	17	.0049451	17	.2967060	77	1.3439035
18	.0000873	18	.0052360	18	.3141593	78	1.3613568
19	.0000921	19	.0055269	19	.3316126	79	1.3788101
20	.0000970	20	.0058178	20	.3490659	80	1.3962634
21	.0001018	21	.0061087	21	.3665191	81	1.4137167
22	.0001067	22	.0063995	22	.3839724	82	1.4311700
23	.0001115	23	.0066904	23	.4014257	83	1.4486233
24	.0001164	24	.0069813	24	.4188790	84	1.4660766
25	.0001212	25	.0072722	25	.4363323	85	1.4835299
26	.0001261	26	.0075631	26	.4537856	86	1.5009832
27	.0001309	27	.0078540	27	.4712389	87	1.5184364
28	.0001357	28	.0081449	28	.4886922	88	1.5358897
29	.0001406	29	.0084358	29	.5061455	89	1.5533430
30	.0001454	30	.0087266	30	.5235988	90	1.5707963
31	.0001503	31	.0090175	31	.5410521	91	1.5882496
32	.0001551	32	.0093084	32	.5585054	92	1.6057029
33	.0001600	33	.0095993	33	.5759587	93	1.6231562
34	.0001648	34	.0098902	34	.5934119	94	1.6406095
35	.0001697	35	.0101811	35	.6108652	95	1.6580628
36	.0001745	36	.0104720	36	.6283185	96	1.6755161
37	.0001794	37	.0107629	37	.6457718	97	1.6929694
38	.0001842	38	.0110538	38	.6632251	98	1.7104227
39	.0001891	39	.0113446	39	.6806784	99	1.7278760
40	.0001939	40	.0116355	40	.6981317	100	1.7453293
41	.0001988	41	.0119264	41	.7155850	101	1.7627825
42	.0002036	42	.0122173	42	.7330383	102	1.7802358
43	.0002085	43	.0125082	43	.7504916	103	1.7976891
44	.0002133	44	.0127991	44	.7679449	104	1.8151424
45	.0002182	45	.0130900	45	.7853982	105	1.8325957
46	.0002230	46	.0133809	46	.8028515	106	1.8500490
47	.0002279	47	.0136717	47	.8203047	107	1.8675023
48	.0002327	48	.0139626	48	.8377580	108	1.8849556
49	.0002376	49	.0142535	49	.8552113	109	1.9024089
50	.0002424	50	.0145444	50	.8726646	110	1.9198622
51	.0002473	51	.0148353	51	.8901179	111	1.9373155
52	.0002521	52	.0151262	52	.9075712	112	1.9547688
53	.0002570	53	.0154171	53	.9250245	113	1.9722221
54	.0002618	54	.0157080	54	.9424778	114	1.9896753
55	.0002666	55	.0159989	55	.9599311	115	2.0071286
56	.0002715	56	.0162897	56	.9773844	116	2.0245819
57	.0002763	57	.0165806	57	.9948377	117	2.0420352
58	.0002812	58	.0168715	58	1.0122910	118	2.0594885
59	.0002860	59	.0171624	59	1.0297443	119	2.0769418
60	.0002909	60	.0174533	60	1.0471976	120	2.0943951

TABLE XIII[1]

Azimuth of Polaris when at Elongation for any Year between 1910 and 1920

Lat.	1910	1911	1912	1913	1914	1915	1916	1917	1918	1919	1920
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
10	1 11.5	1 11.2	1 10.9	1 10.6	1 10.3	1 10.0	1 09.6	1 09.3	1 09.0	1 08.7	1 08.4
12	12.0	11.7	11.4	11.1	10.8	10.4	10.1	09.8	09.5	09.2	08.9
14	12.6	12.3	12.0	11.6	11.3	11.0	10.7	10.4	10.0	09.7	09.4
16	13.3	13.0	12.6	12.3	12.0	11.7	11.4	11.0	10.7	10.4	10.1
18	14.1	13.7	13.4	13.1	12.8	12.4	12.1	11.8	11.5	11.1	10.7
20	15.0	14.6	14.3	14.0	13.6	13.3	13.0	12.7	12.3	12.0	11.7
22	16.0	15.6	15.3	15.0	14.6	14.3	14.0	13.6	13.3	13.0	12.6
24	17.1	16.8	16.4	16.1	15.8	15.4	15.1	14.7	14.4	14.1	13.7
26	18.4	18.0	17.7	17.3	17.0	16.6	16.3	16.0	15.6	15.3	14.9
28	19.8	19.4	19.1	18.7	18.4	18.0	17.7	17.3	17.0	16.6	16.3
30	21.3	21.0	20.6	20.3	19.9	19.6	19.2	18.8	18.5	18.1	17.8
32	23.1	22.7	22.3	22.0	21.6	21.2	20.9	20.5	20.1	19.8	19.4
34	25.0	24.6	24.2	23.8	23.5	23.1	22.7	22.4	22.0	21.6	21.2
36	27.1	26.7	26.3	25.9	25.5	25.2	24.8	24.4	24.0	23.6	23.3
38	29.4	29.0	28.6	28.2	27.8	27.4	27.0	26.6	26.2	25.9	25.5
40	32.0	31.6	31.1	30.7	30.3	29.9	29.5	29.1	28.7	28.3	27.9
42	34.8	34.4	34.0	33.5	33.1	32.7	32.3	31.9	31.5	31.0	30.6
44	37.9	37.5	37.1	36.6	36.2	35.8	35.3	34.9	34.5	34.1	33.6
46	41.4	41.0	40.5	40.1	39.6	39.2	38.7	38.3	37.8	37.4	37.0
48	45.3	44.8	44.4	43.9	43.4	43.0	42.5	42.0	41.6	41.1	40.7
50	1 49.6	1 49.1	1 48.6	1 48.2	1 47.7	1 47.2	1 46.7	1 46.2	1 45.7	1 45.3	1 44.8

Note. The above table was computed with the mean declination of Polaris for each year. A more accurate result will be had by applying to the tabular values the following correction, which depend on the difference of the mean and the apparent place of the star. The deduced azimuth will, in general, be correct within o′.3.

For Middle of	Correction.	For Middle of	Correction.
	′		′
January	−0.5	July	+0.2
February	−0.4	August	+0.1
March	−0.3	September	−0.1
April	0.0	October	−0.4
May	+0.1	November	−0.6
June	+0.2	December	−0.8

[1] Computed by the U. S. Coast and Geodetic Survey.

LETTERING

CIVIL DIVISIONS

States, Counties, Townships, Land Grants, Reservations, Capitals, Cities, Towns, Villages, Localities.

ABCDEFGHIJKLMNOPQRSTUVWXYZ

abcdefghijklmnopqrstuvwxyz

HYDROGRAPHY

Oceans, Seas, Gulfs, Bays, Lakes, Ponds, Rivers, Creeks, Brooks, Springs, Falls, Rapids, Marshes, Glaciers.

ABCDEFGHIJKLMNOPQRSTUVWXYZ

abcdefghijklmnopqrstuvwxyz

868

Thickness of letter ⅐ of height.
Slope of letter 3 parts of base to 8 of height.

PUBLIC WORKS

Railroads, Canals, Tunnels, Mines, Bridges, Ferries, Wagon-roads, Trails, Fords, Dams. Parks, Cemeteries, Forts, Institutions.

ABCDEFGHIJKLMNOPQRSTUVWXYZ

CONTOUR NUMBERS AND ELEVATIONS

1234567890 1234567890

HYPSOGRAPHY

Peaks, Valleys, Plateaus, Plains, Cliffs, Canyons, Peninsulas, Islands

ABCDEFGHIJKLMNOPQRSTUVWXYZ

abcdefghijklmnopqrstuvwxyz

MARGINAL LETTERING

ABCDEFGHIJKLMNOPQRSTUVWXYZ

abcdefghijklmnopqrstuvwxyz

1234567890

TRNSEW1234567890

BOUNDARY LINES

State and National

County

Civil Township

Reservation

Land Grant

City, Village, and Borough

Cemetery, Small Park, etc

U. S. Township Line

U. S. Section Line

Lettering on Boundary Lines

State

NEW YORK / VERMONT — BOUNDARY LINE

County

HAMILTON CO. / UNION CO. — BOUNDARY LINE

PUBLIC WORKS

Aqueducts and Waterpipes

Aqueduct Tunnels

Canals and Ditches

Intermittent

Abandoned

Railroads
- Single Track
- Double Track
- Juxtaposition of
- Electric
- In Wagon-road or Street

Tunnels

Railroad Stations

Wagon-roads
- Metaled
- Good
- Poor or closed to the public

892

INDEX

PAGE

ACCURACY OF SURVEYING OPERATIONS
- Factors Controlling in Stream Measurements 345
- Nickel Steel Tape Measurements 591
- Precise Levelling 703
- Stadia Measurements 313
- Stadia Levels 316
- Steel Tape Measurements 591

Adverse Possession, Defined 186
- Result of Overlooking 732
- Significance of 185

Adjustment, Methods of Studying:
- General Principle of Reversion of Compass 20
- of Level, Dumpy 47
  - Precise 694–5
  - Wye 39

Adjustments:
- of Angles in Triangulation Systems 615
- of Plane Table 121
- of Precise Level Net 707
- of Sextant 114
- of Solar Transit 97
- of Transit 74
  - Triangle 614
  - Quadrilateral 615
  - Larger Systems 624
- of Traverses 233, 235, 257

Agreement, Want of, between Surveyors 461

Alignment, Correction for, in Base-line Measurements 581
- to Invisible Stations 545

Aneroid Barometer 129
- Use of 138

Angle Measurement in Triangulation:
- Programme with Direction Instrument 604
- Programme with Repeater Instrument 602

PAGE

Areas of Land:
by Triangular Subdivision 223
from Boundary Lines 229
from Rectangular Co-ordinates of the Corners 245
of Irregular Figures 256
Astronomy:
Azimuth, Time, Latitude 651
By Polaris Orientation Method 651
Example of Method 653
Azimuth by Elongation of Circum-polar Star 656
Conversion of Time 643
Finding the Stars 643
Fundamental Concepts and Definitions:
Celestial Equator 638
Celestial Meridian 639
Celestial Poles 638
Declination 639
Hour Angle 639
Hour Circle 638
Mean Solar Time 640
Right Ascension 639
Siderial Time 640
Latitude by Circum-Meridian Altitudes of the Sun 655
Latitude from Differences of Meridian Altitudes 664
Field Notes for 667
Reduction of Notes 667
Table of Differential Refraction 665
Longitude 669
Longitude and Time 642
Polaris Orientation Tables 648
Example of Use 650
References on 672
Relation of Siderial and Mean Solar Time 641
Tables of Time, Stars 645
Example of Use 647
Tables of Polaris Orientation 671
Tables of Time Stars 670
Time and Azimuth 660
Two-Star Method 660
Field Notes of 663
Reduction of 663

PAGE

Azimuth Defined:
Two-Star Method ... 660
Elongation of Polaris Method ... 656
Polaris Orientation Method ... 651
Table of Azimuths of Polaris at Elongation ... 889

BALANCING A CLOSED SURVEY ... 233
Compass Rule ... 235
Transit Rule ... 235
Comparison of Above ... 236
Barometric Formulas ... 130
Barometric Elevations:
Correction Coefficients ... 137
Table of ... 135
Base Lines:
on the U. S. Land Surveys, List of ... 797
Base Line Measurement:
Accuracy with Steel Tape and Metallic Wire Measurements ... 590
Base Apparatus ... 559
Contact Apparatus ... 561
Compensating Apparatus ... 563
Duplex Bars ... 563
Essential Features ... 559
Ice-bar Apparatus ... 563
Optical Apparatus ... 561
Computing Portion of Base not Directly Measured ... 588
Base Lines:
Connection with Triangulation ... 538
Base Net ... 539
Base Line Measurement:
Nickel-Steel Tapes, Coefficients of ... 573
Preparation of Line ... 559
Base Lines:
References on ... 637
Base Line Measurement:
Reduction of Base to Sea Level ... 585
Secondary-Base Apparatus ... 566
Steel Tape Apparatus ... 566
Accuracy of ... 567
Mounting and Stretching ... 568
M. Jäderin's Method ... 571

Base Line Measurements: PAGE
Steel Tape Measurements:
Comparison of Costs ... 574
Comparison of Errors ... 573
Coefficient of Expansion ... 575
Pull Correction ... 585
Summary of Corrections ... 586
Steel Tapes:
Alignment Correction ... 581
Modulus of Elasticity ... 576
Sag Correction ... 577, 583
Temperature Correction ... 579
Metallic Thermometers ... 579
Grade Correction ... 587
Table of Errors in Grade Corrections ... 584
To Reduce a Broken Base ... 585
True Length of the Tape ... 574
Table Showing Accuracy with Steel Tapes ... 591
Table Showing Accuracy with Invar Tapes ... 591
Base Net, Connection of ... 538
Figures of ... 539
Bed Ownership in Water Fronts ... 592
Bench-marks:
in City Surveys ... 464
in Hydrographic Surveying ... 334
in Precise Levelling ... 705
Borrow-Pits ... 523
Boundaries, Identification of:
Water Boundaries, etc ... 192
Bubble, Value of One Division of ... 36
Bubble, Level. *See* Level Vials.
Buoys and Buoy Flags ... 328

Catenary, Effect with Steel Tapes ... 577, 583
Celestial Sphere ... 92
Certificates for City Plats ... 812, 814
Check Readings in Topographic Surveys ... 294, 296
Chain:
Use of the Chain ... 10
On Level Ground ... 11
On Uneven Ground ... 11

PAGE

Chain:
Use of the Chain:
Number and Use of Pins ... 11
Chain Engineers ... 7
Chain, Gunter's ... 7
Testing the Chain ... 8
City Surveying:
City Engineer's Tape ... 449
City Engineer's Transit and Level ... 447
City Plats, Methods of Filing ... 812, 814
City Resurveys:
Causes of Disagreement between Surveyors ... 461
City Standards ... 462
In Monumented Cities ... 456
In Non-monumented Cities ... 457
Preservation of Lines ... 461
Value of an Existing Monument ... 459
What Included ... 445
Cadastral, and Topographic Surveys:
General Description ... 463
Precise Level Bench-marks ... 464
Steel Tape Traverses ... 464
The Datum Plane ... 465
Triangulation System ... 463
Topographic Methods ... 465
City Land Surveys:
Defective Plats ... 454
Manner of Setting Monuments ... 454
Methods of Filing Plats ... 813
Monuments Necessary ... 452
Requirements of ... 449
Subdivisional Work ... 451
City Plan:
Curved Streets ... 477
Location of Streets ... 468
City of Washington ... 468
City of Paris ... 470
Location of the Site ... 467
Size of City Blocks and Lots ... 471
Street Plan ... 467
Street Grades ... 474
Maximum and Minimum ... 476

City Surveying: PAGE
Street Grades :
Vertical Curves in ... 476
Width of Streets ... 474
Circular Curve, Principles of a ... 478
Laying Out Circular Curves ... 479
Deflection Angles and Chords ... 482
Form of Field Notes ... 485
Setting Lot Stakes at Fixed Arc Distances ... 483
When the Whole Curve is not Visible from One End ... 485
References:
City Layout ... 492
City Planning ... 493
City Resurveys ... 493
Street Grades and Cross-sections ... 494
Cadastral and Topographic Surveys ... 479
Maps and Profiles ... 490
Indexing City Plans and Records ... 492
Importance of ... 445
Street Cross-sections ... 486
Level Section ... 488
Side-hill Section ... 488
Surveys for Sewers ... 489
Field Notes ... 489
Circular Curves, Laying Out ... 49
Form of Field Notes ... 485
Principles of ... 478
Clinometer ... 144
Convergence of Meridians ... 210, 723
Contour Map, Use of in City Improvements ... 463
Contour Lines ... 307
Examples of ... 309, 311
Used in Computing Earth Work ... 500
Coefficient of Expansion of Steel Tapes ... 575
of Nickel Steel Tapes ... 573, 574
of Brass Wires ... 575
Co-ordinate Protractor ... 170
Corners of Sections, How Relocated ... 821
Compass:
Exercises for Compass or Compass and Chain ... 31
Use of Needle Compass ... 27
To Establish Line of a Given Bearing ... 30

PAGE

Compass:
Use of Needle Compass:
To Find the True Bearing of a Line 30
To Retrace an Old Line 30
To Set Off the Declination 28
Local Attraction of 29
Magnetism of the Earth 23
Description of Parts 17
Methods of Reading Compass Bearings 18
Adjustments of 20
Remagnetizing the Needle of 22
Corners, Restoration of 817, 823
Cross-wires, Setting of 285
Cross-section of River 334
Cross-sectioning in Earthwork 508
Cross-section Polar Protractor 116
Curved Streets 477
Curvature of the Earth 544
Current Meters. *See* also Stream Measurements.
Advantages and Disadvantages 364
Computations of Flow 368–370
Instruments Described 346
Rating the Meter 347
Use of in Stream Measurement 364
in Conduits 371

Dams as Weirs 386
Datum Plane in Cities 465
Declination of the Sun:
Example of Computation 102
How Computed 100
How Corrected for Refraction 101
Declination of Needle:
Table of Corrections 24
Declination of Magnetic Needle 23
Daily Variation 23
Lines of Equal Declination 24
Other Variations 26
Secular Variation 23
Variations in 23
Deeds, Investigation and Interpretation 189
Defective Plats 454

Direction Instruments 601
Discharge of Streams. *See* Current Meter, Rod Float 344, 389
Duplex Base Bars: PAGE
Construction of 563
Method of Use 564

Earthwork Tables 878
Earthwork. *See* Measurement of Volumes.
Ellis Current Meter 347
Errors—Appendix D. *See* also Probable Error 788
Compensating and Cumulative 3, 789
Elimination of 790
Relative Importance of 791
Sources of 788
Theory of 792
Estimates, Preliminary, in Earthwork 521
Excess, Spherical 613
Excavation under Water 524

Field Signals 89
Field Notes, Changes in 3
in Differential Levelling 57
in Land Surveying 213
in Precise Levelling 703
in Profile Levelling 60
in Stream Measurements 355
in Topographic Surveying 299
Filar Micrometer 599
Figure of the Earth 529
Minor Irregularities in the 530
References on 635
Finite Differences:
The Construction of Tables 782
Flow of Water in Open Channels. *See* Stream Measurements.
In Brick Conduits, Table X 877
over Weirs 377
References on 389
Fteley's Current Meter 347

Gauge, Water 335
Bottle Gauge 335
Chain Gauge 336
Self-registering 337

PAGE

Geodesy:
Figure of the Earth ... 529
History of ... 529
Geodetic Night Signals ... 556
Advantages of ... 557
Geodetic Positions:
Computation of ... 630
Inverse Problem ... 634
L. M. Z. Coefficient ... 631
Geodetic Levelling. *See* Trigonometric and Precise Spirit Levelling.
Geodetic Surveys, Objects of ... 528
Government Land Surveys:
Convergence of the Meridians ... 210
Table of Convergency ... 211
Fractional Sections ... 212
Guide Meridians ... 205
Initial Points ... 204
Marking Corners ... 214
Meander Corners ... 215
Meandering ... 215
Original Notes, Where Kept ... 217
Objects Noted ... 215
Other Systems Used ... 212
Principal Meridian ... 204
Relocating Lost Corners of ... 220
Relocating Obliterated Corners ... 221
Requirement of the Work ... 211
Specimen of Field Notes ... 213
Standard Parallels ... 205
Subdivision of Townships ... 207
Subdivision of Sections ... 218
To Run a Parallel of Latitude ... 198
The Secant Method ... 198
The Tangent Method ... 201
Tables for ... 203
Township Exteriors ... 205
Witnessing Corners ... 214
Witness Corners ... 215
Grade Corrections ... 584
Grading over Extended Surfaces ... 497
Gradienter Attachment ... 90
Guide Meridians ... 205

PAGE

HASKELL'S Current Meter 347
Hydrographical Surveying :
Areas of Cross-section 334
Bench-marks 334
Making the Soundings 329
The Lead 329
The Line 330
Sounding Poles 331
In Running Water 331
Plane of Reference 332
Lines of Equal Depth 333
On Fixed Cross-sections 333
Buoys, Buoy Flags and Range Poles 328
Heights of Stations in Triangulation 543
Heliotropes 553
Examples of 554
Cone of Rays 555
Hook Gauge 386
Hydraulics, General 388
Hydrographic Surveying:
Location of Soundings:
By Transit and Stadia 341
Advantages of 342
Cords or Wires 329
Intersecting Ranges 329
One Range and Time Intervals 328
One Range and One Angle 327
Two Angles Read in the Boat 324
Two Angles Read on Shore 324
Objects of 324
River Slope 341
Sounding by Sweeping 332
Water Levels 339
How Run 339
Accuracy of 340
Water Gauges 335
Bottle Gauges 335
Chain Gauges 336
Self-registering Gauge 337
INCLINATION OF HORIZONTAL AXIS, EFFECT OF 82
Inclination of Vertical Axis, Effect of 81

PAGE

Inclination of Horizontal Axis, Effect of :
Line of Sight not Perpendicular to Horizontal Axis, Effect of..... 83
Intervisibility of Stations..... 545
Case 1..... 545
Cases 2 and 3..... 546
Indexing City Plans and Records..... 492
Initial Point. Government Surveys..... 204
Instruments Used in City Surveying..... 447–449
Interpretation of Descriptions..... 189
"Invar" or Nickel Steel Tapes..... 573
Cost of Work with..... 574
Error of Work with..... 573
Description of..... 573, 574
Islands, How Surveyed..... 735, 737
Isogonic Map. Plate I..... 726
Isogonic Lines..... 26

Kutter's Formula..... 372

Lakes, Riparian Rights in..... 192, 734, 737
Latitude, Astronomic and Geodetic..... 530
L. M. Z. Coefficients, Table of..... 631
Land Surveying. *See* also Government Land Surveys.
Authority of Monuments..... 182
Examples for Practice..... 267
Adverse Possession:
Four Kinds of..... 186
Area of Land Determined:
By Compass or Transit and Chain..... 224
Field Notes of Compass Survey..... 226
By Triangular Subdivision..... 223
By Transit and Stadia Survey..... 225
Area of Irregular Tracts:
By Method of Offsets..... 256
Offsets at Irregular Intervals..... 258
Offsets at Regular Intervals..... 259
Computing the Area:
Method Stated..... 229
By Rectangular Co-ordinates..... 678
Conditions of Application..... 244
Form of Reduction..... 247

Land Surveying: PAGE
By Rectangular Co-ordinates :
Method Explained ... 245
Rule for Method ... 246
Computing Latitudes and Depar ures ... 231
Example of Such Work ... 232
Latitudes, Departures and Meridian Distances ... 229
Correction for Erroneous Chain ... 242
Example of Computation ... 240
The Form of Computation ... 238
Balancing the Survey ... 233
Error of Closure ... 237
Compass Rule ... 235
Transit Rule ... 235
Comparison of Rules ... 236
Land Corners, Restoration of ... 817, 821, 823
Land Surveying:
Laying Out Land ... 180
Lost Monuments ... 183
Monuments of ... 181
Purposes of ... 180
Plotting Field Notes ... 256
Records of Federal Land Surveys ... 829
Rerunning Old Surveys from a Deed ... 184
Land Surveying Instruments:
References on ... 270
Specification for ... 800
Resurveys of Private Lands, General Rule for ... 821
Relocation of an Old Traverse ... 251
Interpretation of Descriptions ... 189
General Rules ... 189
Particular Cases ... 191
Surplus and Deficiency ... 193
Water Boundaries and Meandered Lines ... 192
The Problem Stated ... 187
Land Surveying:
Subdivision of Land:
References on ... 269
By a Line Starting at a Given Point ... 261
By a Line Running in a Given Direction ... 263
Example of the Above ... 264
Rule for Above Case ... 267

Land Surveying: PAGE
Significance of Adverse Possession ... 185
Supplying Missing Data:
Conditions to be Filled ... 247
One Bearing and Length Unknown ... 249
Bearing of one Course and Length of Another Unknown ... 249
Two Bearings Unknown ... 250
Two Lengths Unknown ... 251
To Relocate an Old Traverse ... 251
Latitudes and Departures, How Computed ... 231
Latitude by Circum-meridian Altitudes of the Sun ... 655
by Polaris Orientation Method ... 651
Laws Governing Land Surveys ... 189
Leads used in Soundings ... 329
Least Squares ... 792
Length, Standards of ... 462
Absolute Length of Steel Tapes ... 574
Lettering on Maps ... 724
Level, Dumpy Described ... 45
Adjustments of ... 47
Bench-marks ... 55
Engineer's Wye Level ... 38
Adjustments of ... 39
Cross-section of ... 40
Relative Importance of Adjustments ... 45
Exercises with the ... 62
Hand ... 62
Levels, Length of Sight ... 54
Level, Profile Levelling ... 58
Field Record ... 59
Precautions in Level Work ... 57
Specification of ... 800
Levels, The Field Record ... 56
Turning Point ... 57
Level, Use of the Engineer's ... 52
Back and Fore Sights ... 52
Differential Levelling ... 53
Level Trier ... 37
Level Surface Defined ... 33
Level Vials:
Angular Value of Bubble Division ... 36
Axis of Bubble Defined ... 34

Level Vials: PAGE
Construction of 33
Propositions Concerning 34
Use of—in Measuring Vertical Angles 35
With Chamber 37
Levelling:
Plane of Reference 53
Levelling Rods 49
Levelling to Fix a Grade 61
Lines, Preservation of 461
Line, Sounding 330
Land Monuments. *See* Land Surveying.
Longitude and Time 669, 642
Lost Corners, Method of Locating 220
Lost Government Corner 817

Maps, Subdivional 811
Projection of 710–724
Map Projection. *See* Projection of Maps 710
Map Lettering 724
Plate II 726
Plate III 726
Plate V 890
Plate VI 891
See also Plates II, III, V, VI, VII.
Map of Topographic Surveys 307, 311, 463
Magnetic Declination:
Rate of Change 26
Mean Velocity of Water Currents 355–357
Meander Lines, Extension of, in Boundaries 192
Men's Steps, Length of 141
Mean Sea Level, Determination of, Elevation of 684, 708
Meridian Distances, Defined 229
Meridians, Convergence of 210, 723
Metallic Thermometer, Temperature Corrections 579
Metallic Tapes 13
Meridian Observation:
By Observing on the Sun 98
Method of Least Squares 792
Miner's Inch 388
Mineral Surveyors, Instructions to 739

Mining Surveying: PAGE
Application for a Survey of Claim ... 751
Conflict in Claims ... 744
Court Maps ... 438
Definitions of Mining Terms ... 441
Exercises in ... 442
Mining Claims, U. S. Instructions for the Survey of ... 740
Adverse Surveys ... 409
Amended Surveys ... 409
Conditions Controlling Patent Survey ... 401
Field Notes ... 402
Field Work and Adjustments of Claim ... 400
Instrumental Work ... 398
Location of ... 392
Location Monuments ... 743
Location Surveys ... 393
Mill Sites ... 409
Preparing the Plat ... 406
Preliminary Surveys ... 404
Preliminary Plat ... 405
Patent Surveying ... 397
Placer Claim ... 396, 408
Shape of Lode Claims ... 393
Shape of Tunnel Sites ... 397
Stations and Methods of Measuring ... 399
Title to ... 391
Underground Surveys:
Carrying Azimuth, Wire and Plumb Line ... 426
Carrying the Meridian:
By a Transit ... 420
By Plumb-lines ... 421
Single Shaft ... 423
Two Shafts ... 422
General Statement ... 410
Instruments ... 411
Mapping the Survey ... 431
Example ... 432
Property Lines in ... 430
Side and Top Telescopes ... 416
Stations, How Located ... 418
To Intersect a Known Vein ... 437
Underground Connection ... 434

Mining Surveying:
Underground Surveying: PAGE
Underground Levelling ... 430
Underground Traversing ... 427
Field Notes of ... 429
Top Telescope ... 411
Surface Surveys ... 438
Surveying Boreholes ... 439
To Determine Dip and Strike of a Vein ... 441
References to ... 444
Miner's Inch ... 388
Missing Data, Method of Supplying. *See* under Land Surveying.
Modulus of Electricity of Steel Tapes ... 576
Monuments:
Authority of ... 181, 182
at Section Corners ... 214
in Triangulation ... 557
Lost ... 183
Manner of Setting ... 454
Number Necessary ... 452
Monuments, Value of an Existing ... 459

Nickel-Steel Tapes ... 573, 574
Night Signals, in Triangulation ... 556

Obliterated Corners, Definition of ... 221, 817
Odometer ... 143
Office Records ... 492
Ownership of Surveys ... 807
Optical Square ... 451
Original City Surveys:
Requirements of ... 449
Subdivisional Work ... 541

Pantograph, Theory of ... 164
Use of ... 167
Parallax, How Removed ... 48
Parallel Rules ... 171
Parallel Ruler ... 171
Pedometer Described ... 140
Pivot Correction in Levelling ... 698
Plane of Reference, in Sounding ... 332

Plane Table: PAGE
Adjustments of ... 121
Description of ... 119
Exercises with ... 128
Location by Resection ... 125
Three-point Problem ... 125
Two-point Problem ... 126
Use with Stadia Measurements ... 128
Use of ... 122
Planimeter:
Accuracy of ... 160, 163
Polar, Length of Arm ... 152
Polar Planimeter ... 145
Rolling ... 155
Suspended ... 155
Polar, Theory of ... 147
Rolling, Theory of ... 156
Use of ... 160
Polar, Zero Circumference ... 149
Planning of Cities:
City Blocks and Lots, Size of ... 471
Examples of ... 468
Street Plan ... 467
Location of Site ... 467
References on ... 492, 493
Width of Streets ... 474
Plotting in Land Surveying ... 301–307
in Mining Surveying ... 431
in City Surveying ... 449
Polyconic Projection of Maps ... 716
Plats, to be Geometrically Consistent ... 451
Michigan Regulation Concerning Filing ... 813
Plumb Line:
Deviations of ... 530
Use in Chaining ... 11
Polaris, Table of Azimuths at Elongation ... 889
Precise Spirit Levelling:
Adjustment of a Level Net ... 707
Adjustment of Coast Survey Dumpy Level ... 695
Allowable error in ... 696
Form of Field Notes of ... 703
Adjustment of Wye Type of Instrument ... 694

Precise Spirit Levelling: PAGE
Bench-marks ... 705
Coast Survey Precise Level ... 688
Comparison with Other Types ... 688
Field Methods ... 701
Atmospheric Errors ... 701
Settlement Errors ... 701
Field Notes, form of ... 703
General Instruction U. S. Coast and Geodetic Survey ... 703
General Description ... 684
Instrumental Constants ... 696
Absolute Length of Levelling Rods ... 700
Angular Value of One Division ... 696
Inequality of Pivot Rings ... 698
Magnifying Power ... 700
Wire Interval ... 700
Importance of ... 685
Kern Level ... 685
Level Rods ... 691
Levelling Instruments ... 685
Limits of Error ... 705
Mean Sea Level ... 708
Mendenhall Level ... 687
References on Precise Levelling ... 708
Rod Supports ... 691
Stability of ... 693
Railroad Rail as ... 693
Speed and Cost ... 706
Sensibility of Level Vial ... 686
Preparation of Base Line ... 559
Probable Error, Formulas for ... 794
Use made of ... 793
Principal Meridians ... 204
Prismoidal Formula ... 505
Prismatic Compass ... 31
Price Current Meter ... 347
Projection of Maps:
Bonne's Projection ... 716
Conic Projection ... 713
Simple Conic ... 713
Convergence of Meridian ... 723
Mercator's Conic ... 715

PAGE

Projection of Maps:
Cylinder Projection ... 710
Equidistant Flat Projection ... 711
Mercator's Cylinder Projection ... 711
Explanation of Table VIII ... 720
Formulas Used in the ... 718
General Statement of ... 710
Polyconic Projection ... 716
Description of Methods ... 717
Tables for ... 717
Table VIII ...
Summary of Methods ... 722
Trapezoidal Projection ... 712
Protractors:
Full Circle ... 168
Paper ... 170
Semicircular ... 168
Three-arm ... 169
The Co-ordinate ... 170
Public Lands of the United States. *See* also Land Surveying.

Ranges and Range Poles in Sounding ... 238
Rating Curves of a River ... 374, 375
of a Current Meter ... 347
Records Office, in City Work ... 492
Refraction:
A Simple Way of Computing the Effect of ... 106
Abnormal Coefficients ... 547
Determination of ... 677
Effect of ... 544
In Trigonometric Levelling ... 675
Latitude Coefficient of ... 105
Tables of Differential ... 665
Tables of ... 104
Relocating an Old Traverse ... 251
Repeating Instruments ... 592
Records of U. S. Land Surveys, Where Kept ... 829
Reduction to Centre ... 606
Refraction. *See* Trigonometric Levelling.
Resurveys in Cities, What Included ... 445
City Standards of Length ... 462
in Monumented Cities ... 456

Resurveys in Cities, What Included: PAGE
in Non-monumented Cities ... 457
Preservation of Lines ... 461
References on ... 493
Riparian Rights in Water Fronts ... 192, 734-736
River Slope ... 341
Rod Floats in Stream Measurement:
Description of Method ... 360
Computation of Discharge ... 363
Advantages and Disadvantages ... 364
Subsurface Float ... 360
Rod Levels ... 50
Rod Floats. *See* Stream Measurements.

Scales ... 172
Triangular ... 173
Sea Level Datum:
How Determined ... 337
Sections, How Subdivided ... 218, 83c
Fractional ... 212
Sewer Surveys ... 489
Field Notes of ... 489
Sextant:
Theory of ... 113
Description of ... 111
Adjustments of ... 114
Use of ... 315
Exercises with ... 116
Shrinkage of Earthwork ... 524
Signals, Geodetic Night. *See* also Triangulation Signals ... 556
Sides, Computation of, in Triangulation ... 626
Simpson's Rule ... 259
Slide Rule, Pocket ... 173
Computations with ... 176
Thatcher's ... 174
Slope Method of Stream Measurement ... 369
Sounding ... 324
The Lead ... 329
The Line ... 330
Sounding Pole ... 331
Soundings, Location of ... 324

PAGE

Solar Azimuth:
Table of Errors in Azimuth ... 108
Solar Observation for the Meridian ... 98
Solar Attachment, Use of:
To Determine the Meridian ... 95
To Lay off the Co-latitude ... 96
How Oriented ... 96
Adjustments of Saegmuller Attachment ... 97
Described ... 91
Stadia Rods ... 286
Stadia Measurements, Accuracy of ... 313
Stadia Surveying. *See* Topographic Surveying.
Station Signals ... 548
Stretching Steel Tapes ... 568
Balance Used in ... 570
Pulley and Weight for ... 568
Straight Lines Run by Transit ... 86
Stream Measurement:
Analytical Method of Rating Meter ... 352
Field Notes ... 355
Factors Controlling Accuracy ... 345
General Statement ... 344
Graphic Method of Rating Meter ... 352
Instruments Described ... 346
Price ... 347
Ellis ... 347
Haskell ... 347
Fteley ... 347
Rating the Meter ... 347
Field Notes of ... 351
Computations ... 368
Example of ... 370
How Made ... 364
From Bridge ... 364
From Cable ... 366
Measurements on Large Rivers ... 369
Velocity in Vertical Plane:
Open Conditions ... 355
Frozen Conditions ... 357
Variation of Velocity in a Cross-section ... 354
General Conclusions ... 370

Stream Measurement: PAGE
Kutter's Formula ... 372
Miner's Inch ... 388
Methods of Measuring Velocity ... 346
Rod Floats:
Advantages of ... 364
Disadvantages of ... 364
Rod Float Method:
Computation of Discharge ... 363
Rod Floats:
Sub-surface Floats ... 360
Method Described ... 360
Rating Curve of a River ... 374
Frozen and Open Condition ... 375
References on:
General Hydraulics ... 388
Stream Gauging ... 389
Flow over Weirs ... 389
Slope Method ... 369
Vertical Velocity Curves ... 358
Weir Method ... 6
General Description ... 376
Sharp-crested Weirs ... 377
Submerged Weirs ... 380
Values of N for ... 381
Table of Coefficients:
Contracted Weirs ... 379
Suppressed Weirs ... 379
Use of Dams as Weirs ... 386
Advantages of ... 387
Weirs of Varying Section ... 381
Table of Coefficient C (Bazin) ... 383
Table of Coefficient C (Cornell) ... 385
Stations, Height in Triangulation ... 543, 458
How Constructed ... 548
Marks in Triangulation ... 557
Station Marks ... 557
Stars for Time Determination ... 670
Steps, Length of Men's ... 141
Standard Parallels ... 205
How Run ... 198-201
Street Cross-sections ... 486

PAGE
Street, Width of 474
Street Grades 474
Maximum and Minimum 476
Vertical Curves in 476
Standards of Length in City Work 462
Spirit Level 33
Specifications for Engineer's Transit and Level 800
Submerged Floats 360
Surplus and Deficiency, Treatment of 193
Subdivision of Land:
Examples for Practice 267
By a Line Running in a Given Direction 263
Example of the Above 264
Rule for Above Case 267
By a Line Starting at a Given Point 261
Rule Stated 263
Surveys, Ownership of 807
Surveyors:
Adverse Possession 185
Bed Ownership in Water Fronts 193
Cannot "Establish" Corners 730
Cannot Change Original Monuments 731
Judicial Functions of 729
Meander Lines, not Boundary Lines 192
Relocation of Lost Corners 220, 817
Riparian Rights in Water Fronts 192
Surplus and Deficiency 193
Want of Agreement 461
Stream Measurement:
Applications of 373

TABLES:
I. Trigonometric Formulæ 835
II. For Converting Feet, Metres and Chains 839
III. Logarithms of Numbers to Four Places 840
IV. Logarithmic Sines and Cosines, Four Places 842
V. Stadia Reduction Tables 846
VI. Natural Sines and Cosines, Five Places 854
VII. Natural Tangents and Cotangents, Five Places 863
VIII. Co-ordinates of Polyconic Projections 875
IX. Value of Coefficient C'' in Kutter's Formula 876
X. Diameters of Brick Conduits 877

Tables: PAGE
XI. Volumes by Prismoidal Formulæ ... 878
XII. Lengths of Circular Arcs ... 888
XIII. Azimuths of Polaris at Elongation ... 889
Tape, Steel. *See* Steel Tape.
Targets, Examples of ... 551, 552
Heliotropes ... 553
Requisites of ... 551
Used in Levelling ... 51
Used in Lining with Transit ... 89
Tapes:
Exercises with ... 15
Tape, Steel:
Common Sources of Error in Measurement ... 14
Common Mistakes in Reading Tapes ... 14
Varieties of ... 12
Use of Steel Tapes ... 13
Telescope:
Cross-section of ... 68
Cross-wires ... 68
Eye-piece ... 68
Magnifying Power ... 69
Objective ... 67
Reticule ... 68
Temperature Correction of Steel Tapes ... 579
Thermometers, on Base Line Work ... 568
Time Stars, Tables of ... 670
Time, by Polaris Orientation Method ... 651
by Two-star Method ... 660
Conversion of ... 643
Definition of ... 640
Relation between ... 641
Three-point Problem, Four Solutions ... 122
Topographic Symbols ... 725
Topographic Methods, City Surveys ... 465
General Description of ... 463
Topographical Surveying:
Adjustable Stadia Wires ... 276
Check Readings ... 294
Check on Azimuth ... 296
Contour Lines Explained ... 307
Principles of Sketching ... 308

Topographical Surveying: PAGE
Cost of Topographic Maps ... 320
Definition of ... 271
Field Work Described ... 289
Final Map ... 311
Example of ... 312
Field Notes Required ... 291
Sample of ... 299
Horizontal Control by Triangulation ... 290
Vertical Control ... 290
Interval Factor ... 276
Interval Table ... 278
Organization of Party ... 290
Plotting the Stadia Traverse ... 301
Plotting the Side Readings ... 303
Molitor's Protractor ... 304
Reducing the Field Notes ... 296
References on ... 321
Stadia Measurements:
Accuracy of ... 313
Effect of Refraction on ... 280
Formula for Inclined Sights ... 282
Design of Stadia Rod ... 286
Prevention of Systematic Errors ... 279
Table V Explained ... 283
Stadia Interval ... 274
Field Notes of ... 275
A Simple Way of Determining ... 278
Stadia Reduction Diagrams ... 297
Where Obtained ... 319
Stadia Traverse ... 291
Setting the Cross-wires ... 285
Theory of Stadia Measurements ... 273
Transit and Stadia Method ... 272
Topography Outlined ... 289
Transit and Stadia Method:
Accuracy of Levels ... 316
Relation of Vertical Angle ... 317
St. Louis Levels ... 318
Townships, Subdivision of ... 207
Township Exteriors ... 205
Townships in Land Surveying ... 196

	PAGE
Transit, Engineer's:	
Adjustment of	74, 598
Care of	90
Cross-section of	66
Description of	64
Exercises with	109
Eccentricity of Centers	80
Eccentricity of Verniers	80
Transit in City Work:	447
in Geodetic Work	592
in Mining Work	309–10
in Topographic Work	800
Transit, Engineer's:	
Relative Importance of Adjustments	79
Specifications for	800
Use of	
Setting up Transit	83–85
To Lay off an Angle by Repetition	85
To Measure a Horizontal Angle	84
To Measure a Vertical Angle	87
To Measure an Angle by Repetition	84
To Prolong a Straight Line	86
To Run a Straight Line	85
To Run a Traverse	87
Transit Verniers	71
Traversing with Transit:	
Triangulation in City Surveys	463
Triangulation:	
Adjustment of	615
Equation of Conditions	610
Local or Station	611
Triangle	614
Adjustment of a Quadrilateral	615
Angle Equation Adjustment	615
Geometric Conditions	615
Side Equation Adjustment	617
Rigorous Adjustment	621
Example of	624
Altazimuth Instrument	594
Adjustment of the Theodolite	598
Computing the Sides of Triangles	626
Example of	628

Triangulation in City Surveys:
Triangulation: PAGE
Computing the Geodetic Position 630
The Direct Problem 630
Example of 633
The Inverse Problem 634
Example of 635
Classification of 523
Primary 534
Secondary 534
Tertiary 534
Direction Instrument 594
Programme for 604
Reading the Circle 601
Direction of Invisible Stations 543
Filar Micrometer 599
Correction for Run 601
Reading the Circle 601
Triangulation Stations, How Constructed 548
Triangulation:
Instruments for 592
Method Used in Primary Triangulation 624
Reduction to Centre 606
Example of 609
Repeating Instrument 592
Programme with 602
References on 636
Reconnoissance 541
How Made 541
Instrumental Outfit 542
Station Marks 557
Spherical Excess 613
Stations, Construction of 548
Strength of Figures 535
How Determined 536, 537
Distance Angles 537
Azimuth Angles 537
Stations:
Height of 543, 458
Table of Curvature 544
Intervisibility of 545
Trans-Continental 533

PAGE

Triangulation:
Triangulation Figures ... 531
Time for Observing ... 606
Trigonometric Levelling:
Curvature and Refraction, Table of Correction for ... 682
Difference of Heights by ... 679
General Description of ... 673
Instruments and Methods ... 673
Levelling by the Sea Horizon ... 682
Refraction, How Measured ... 676
Refraction, Minimum Values of ... 679
Over Water ... 679
Refraction ... 675
Coefficient of ... 675
References on ... 683
Two-star Method ... 660
Two-point Problem ... 126
Town-site, Laying Out ... 451
Traversing with a Transit ... 87

U. S. Land Corners, General Rule for Relocation ... 821
Synopsis of Acts of Congress ... 819

Velocity of Rivers:
Method of Measuring ... 346
Velocity of Water Flow:
in Vertical Planes ... 355, 357
in Horizontal Planes ... 369, 354
Verniers:
Smallest Reading of ... 72
Construction of ... 69
Vertical Velocity Curves ... 358
Open Conditions ... 355
Frozen Conditions ... 357
Volumes, Measurement of:
Area of a Three-level Section ... 508
By Means of Contours ... 500
Example of ... 500, 503
Borrow Pits ... 523
Cross-sectioning ... 508
Centre and Side Heights ... 507
Comparison of Method by Diagonals and by Warped Surfaces ... 519

Volumes, Measurement of: PAGE
Construction of Tables for Prismoidal Computation 512
Example 512
Demonstration of Principle 495
Excavations under Water 524
Grading over Extended Surfaces 497
Prismoid Defined 504
Prismoidal Formula 505
Preliminary Estimate from the Profile 521
Shrinkage of Earthwork 524
Three-level Sections Divided into Four Planes by Diagonals 517
Three-level Sections 510

WATER GAUGES 335
Water Fronts, Riparian Rights in 192, 193
Water Boundaries 192
Water Currents, Mean Velocity of 355-357
Water Levels 339
Weir Methods 376
Weirs, Tables of Coefficients 379
Weirs Used in Stream Measurement. *See* also Stream Measurement.
Contracted Weirs 379
Submerged Weirs 380
Suppressed Weirs 379
Sharp-crested Weirs 377
References on 389
Wire Interval 274, 276

Quiz in Plane Surveying. Nov. 7, 1924.

Instructions - Read before answering questions.

No Questions will be answered.
It is not expected that anyone will be able to answer all the questions during the allotted time nor need the questions be answered in order.
After each questions appears a number which gives the weight in points. The student who gets the highest total of points during the allotted time will be graded 100%. The average number of points will be graded 75%.

1. Describe the level rod in various forms, and its accessories. 15

2. Describe the transit. How is it used? 20

3. How does refraction affect observations? 5

4. Describe the 3-arm protractor and the cross-section polar protractor. 10

5. Describe how to measure an angle with tape only. 20

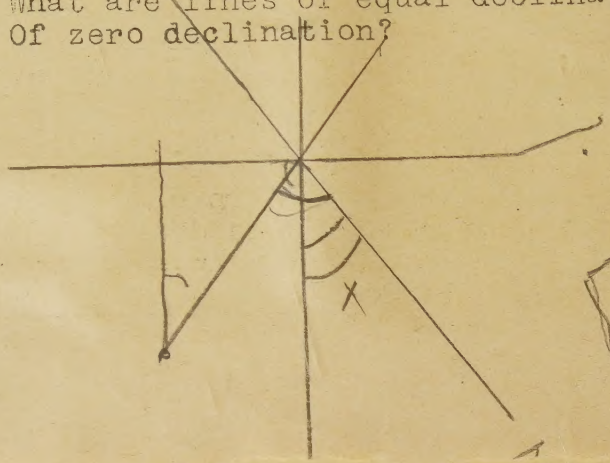
6. Given the following bearings of a closed traverse:

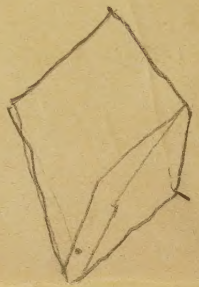
7. Name three types of planimeters and what are their essential differences? 15

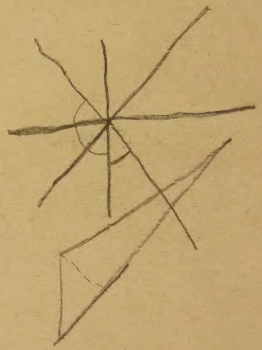
8. Define: (a) declination of the needle.
(b) variation of the needle. 10

9. Describe the Manuheim slide rule. 5

10. (a) What are lines of equal declination called?
(b) Of zero declination? 10







Y0-CIN-128